Online Appendix: Unplanned Purchases and Retail Competition

By Justin P. Johnson[*]

In this online appendix I provide additional results.
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Here I provide more general circumstances under which, if the large firm prices any products below cost, it prices a product below cost that the small firm includes in its product portfolio. The assumptions in the body of the paper correspond to their being a single "product class" such that $x_n(p) = x_k(p)$ for each available product n and k and with equal equal marginal costs ($c_n = c_k$). Here, I instead suppose that there are J distinct classes of products, where each class $j \in \{1, 2, \ldots, J\}$ is associated with a demand function $x^j(p) = \gamma^j x(p)$, where $\gamma^1 > \cdots > \gamma^J > 0$. Additionally, the marginal cost of any good in class j is $c^j > 0$, where $0 < c^1 \leq \cdots \leq c^J$. Under this formulation, each good n of the N available products is described by (i) its class j, (ii) the objective probability $\theta_n > 0$ that a consumer will have positive demand for it, and (iii) the subjective probability $0 < \hat{\theta}_n \leq \theta_n$ that a consumer will have demand for it. If $j \leq l$, I will say that j is a "better class than l." Note that, for a given price $p \geq c_k$, if product k is in a better product class than product n, then k generates higher per-customer profits if $\theta_k \geq \theta_n$, and similarly (for any given p) k generates higher forecast utility if $\hat{\theta}_k \geq \hat{\theta}_n$.

If S sold but a single product, and if θ and $\hat{\theta}$ were identical across products, S would prefer a product in class 1 (having the highest value of γ^j and lowest value of c^j). However, when choosing a portfolio this formulation presents tradeoffs. The reason is that some products might be priced below cost, implying that a good with a higher value of γ could cause larger per-customer losses for the firm. Additionally, regardless of how many products its portfolio contains, S must also weigh how many consumers will actually demand that product and whether consumers believe they will demand that product: θ and $\hat{\theta}$ matter. For instance, S may decide to carry a product in a worse class because the values of θ and $\hat{\theta}$ associated with that product are attractive compared to alternate products in better classes.

The following lemma details how S favors products n with high values of $\hat{\theta}_n$ and θ_n. As staples by definition have the highest possible values of both θ_n and $\hat{\theta}_n$, this lemma will generalize the results from the body of the article about S's desire to carry staples.

[*] Samuel Curtis Johnson Graduate School of Management, Cornell University, Ithaca, NY, jpj25@cornell.edu.

LEMMA 1: *Suppose that the small firm carries product n in class j. If there is any product k in class l such that (i) $l \leq j$, (ii) $\theta_k \geq \theta_n$, and (iii) $\hat{\theta}_k \geq \hat{\theta}_n$, where at least one of these inequalities is strict, and $\alpha_k \geq \alpha_n$, then the small firm also carries product k.*

PROOF:

Suppose for the sake of contradiction that n is carried but k is not, and let $\pi_S(\hat{U}_S, n)$ denote S's per-customer profit function when it is carrying n and all other goods part of the supposedly optimal portfolio, and let $\pi_S(\hat{U}_S, k)$ denote the resulting profit function when n is replaced with k but all other goods in the portfolio are unchanged.

I will show that $\pi_S(\hat{U}_S, k) > \pi_S(\hat{U}_S, n)$ in the relevant range of $\hat{U}_S$. To discern the relevant range, let $\hat{U}_S^{\min}(n)$ denote the forecast utility generated by the portfolio containing n in which each good is priced to maximize that good's per-customer profits, and let $\hat{U}_S^{\min}(k)$ denote the corresponding quantity for the portfolio containing k. Because k is in a weakly better class than n and because $\hat{\theta}_k \geq \hat{\theta}_n$, it must be that $\hat{U}_S^{\min}(k) \geq \hat{U}_S^{\min}(n)$. If this equality is exactly satisfied, then n and k are from the same product class and $\hat{\theta}_n = \hat{\theta}_k$, so that by hypothesis it must be that $\theta_k > \theta_n$, which in turn means that $\pi_S(\hat{U}_S^{\min}(k), k) > \pi_S(\hat{U}_S^{\min}(n), n)$. The other possibility is that $\hat{U}_S^{\min}(k) > \hat{U}_S^{\min}(n)$. Because k is in a better product class with $\theta_k \geq \theta_n$, it must be that $\pi_S(\hat{U}_S^{\min}(k), k) \geq \pi_S(\hat{U}_S^{\min}(n), n)$. And, because $\pi_S(\hat{U}_S, n)$ is strictly decreasing for $\hat{U}_S \geq \hat{U}_S^{\min}(n)$, no value of $\hat{U}_S \in [\hat{U}_S^{\min}(n), \hat{U}_S^{\min}(k))$ can be optimal; it would be better to choose $\hat{U}_S^{\min}(k)$ and use the portfolio with good k, thereby earning higher per-customer profits and also attracting more customers.

In either of the two cases above, attention can be restricted to $\hat{U}_S \geq \hat{U}_S^{\min}(k)$, where carrying k strictly dominates carrying n at this lower forecast utility level. I will now show that per-customer profits from carrying k continue to exceed those of carrying n at higher values of $\hat{U}_S$. In particular, following earlier notation define $\hat{U}_S^{\max}(n)$ and $\hat{U}_S^{\max}(k)$ to be the highest forecast utility values consistent with zero per-customer profits for the portfolios containing n or instead k, respectively. As part of this proof I will show that $\hat{U}_S^{\max}(n) \leq \hat{U}_S^{\max}(k)$, but for the moment let the smaller of these two values be denoted by $\min\{\hat{U}_S^{\max}(n), \hat{U}_S^{\max}(k)\}$.

Now, suppose that for any $\hat{U}_S \in [\hat{U}_S^{\min}(k), \min\{\hat{U}_S^{\max}(n), \hat{U}_S^{\max}(k)\}]$ it were the case that

$$(1) \qquad \frac{d\pi_S(\hat{U}_S, k)}{d\hat{U}_S} \geq \frac{d\pi_S(\hat{U}_S, n)}{d\hat{U}_S}.$$

Then, because carrying k strictly dominates carrying n at $\hat{U}_S^{\min}(k)$, it would follow that (i) $\hat{U}_S^{\max}(n) \leq \hat{U}_S^{\max}(k)$, and (ii) carrying k strictly dominates carrying n at any relevant value of $\hat{U}_S$, which would complete the proof.

I will show that if the inequality in (1) is violated at some particular $\hat{U}_S$, then all

of the prices in the portfolio containing n are higher than those in the portfolio containing k. This will establish a contradiction, because it means that both portfolios cannot in fact be generating the target $\hat{U}_S$, given that k is in a better product class with $\hat{\theta}_k \geq \hat{\theta}_n$.

Thus, suppose for the sake of contradiction that $d\pi_S(\hat{U}_S, k)/d\hat{U}_S < d\pi_S(\hat{U}_S, n)/d\hat{U}_S$ at some particular value $\hat{U}_S$. Lemma **??** reports an expression for these derivatives. Using this expression for any product s excluding n and k, and using the fact that L_s is decreasing, it follows that the price of s must be strictly higher in the portfolio containing n than in the portfolio containing k. The same observations imply that for n and k it is the case that

$$(2) \qquad \frac{(1 + L_n(p_n^*))}{\alpha_n} < \frac{(1 + L_k(p_k^*))}{\alpha_k},$$

where p_n^* and p_k^* are the prices that are optimal given the target value $\hat{U}_S$. Recall that under the formulation in this section $L_n(p) = (p - c_n)x'(p)/x(p)$ and $L_k(p) = (p - c_k)x'(p)/x(p)$. By assumption, k is in a weakly better class than n, so that $c_k \leq c_n$. Additionally, by assumption $\alpha_k \geq \alpha_n$. Thus, for any p, $(1 + L_k(p))/\alpha_k \leq (1 + L_n(p))/\alpha_n$ (this uses the fact that by construction all prices under consideration are lower than the monopoly prices, ensuring $1 + L_n(p_n^*) > 0$). Hence, to satisfy (2), it must be that $p_n^* > p_k^*$.

I have shown that all prices are strictly higher in the portfolio containing n, at this particular $\hat{U}_S$. But this contradicts the fact that both portfolios are generating the same value of $\hat{U}_S$, given that k is in a weakly better class with $\hat{\theta}_k \geq \hat{\theta}_n$. I conclude that $d\pi_S(\hat{U}_S, k)/d\hat{U}_S \geq d\pi_S(\hat{U}_S, n)/d\hat{U}_S$ at all relevant values of $\hat{U}_S$, which completes the proof.

Recall that a staple good is one for which consumers have unbiased beliefs and which they always purchase. That is, good n is a staple good if $\theta_n = \hat{\theta}_n = 1$: consumers know that they definitely need the good. In light of Lemma 1, the following is immediate.

PROPOSITION 1: *The small firm prioritizes carrying staples (that is, products n with $\theta_n = \hat{\theta}_n = 1$). That is, if the small firm carries any product in class l that is not a staple, then it also carries all available staples in better product classes (that is, product classes $j \in \{1, 2, ..., l\}$).*

It is important to be very clear about what this proposition says and what it does not say. The result does not ensure that some arbitrary staple good will always be chosen over a non-staple good. For example, S may prefer a non-staple good to a staple good if the non-staple is in a strictly better product class (thereby having an underlying demand function that is an outward shift of that of the staple good). What this result does ensure is that, if some non-staple good is chosen by S, then any staple good must also be chosen so long as the staple good is not in an inferior product class. Thus, a sufficient condition for staples to

be preferred is that their underlying demand is not worse than some other good. Intuitively, this ensures that S will carry some staple goods as long as they are not too "sparse" amongst the better product classes.

Generalized conditions under which the loss leaders of L overlap with products chosen by S are straightforward. Recall that $\mathscr{P}_S$ is the product portfolio of S, and let $\mathscr{L}_m \subset \mathscr{P}_m$ denote the (possibly empty) set of products that firm $m \in \{S, L\}$ prices beneath cost.

PROPOSITION 2: *Suppose that the small firm carries some product from class l and that there is at least one staple good in some better product class j (that is, $j \in \{1, 2, ..., l\}$). If the large firm practices loss leading, then at least one of its loss leaders is carried by the small firm (that is, if $\mathscr{L}_L$ is nonempty, then $\mathscr{L}_L \cap \mathscr{P}_S$ is also nonempty).*

The logic is the same as in the less general model: if the large firm prices anything below cost, all staples are priced below cost, and some staple is also carried by the small firm under the assumed conditions. Proposition 2 ensures that the large firm prices below cost on some product that the small firm carries, so long as staples are not too sparse amongst the better product classes. The required condition is easiest to satisfy when l is taken to be the worst product class from which the small firm selects a product. That is, so long as there is some staple good amongst all products in classes l or better, then the small firm carries some product that the large firm prices below cost.

It is straightforward to provide conditions that ensure the hypotheses of Proposition 2 are satisfied. One immediate corollary follows if the best class (class 1) contains at least one staple good.

COROLLARY 1: *Suppose that there is at least one staple good that is in the best product class. If the large firm practices loss leading, then at least one of its loss leaders is carried by the small firm.*