Online Appendix to Accompany "Worker Selection, Hiring, and Vacancies"

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A   A Simple Two-Period Model with Wage Bargaining and Time Inconsistency

Suppose that there are two periods. Firms post vacancies to hire workers in each period and exogenously exits at the end of the second period. Assume that the cost function is strictly convex in the number of applicants, but decreases with firm size as in the text. For simplicity, assume that there are no idiosyncratic productivity and exogenous separation shocks. Further, all the workers are productive with probability 1 so that there is no worker selection either. The linear cost of posting vacancy is also set equal to zero. Deviating from the text, let also n_1 and n_2 denote the number of workers hired in the first and the second period, respectively. Then, the sequential problem of the firm is:

$$J(\varepsilon) = \max_{n_1 > 0, n_2 > 0} - \frac{c_s}{z}(n_1)^z + \varepsilon A n_1{}^{\alpha} - w^1(n_1, n_2) n_1$$
$$+ \beta \left(-\frac{c_s}{z} n_2^z (1 + n_1)^{1-z} + \varepsilon A (n_1 + n_2)^{\alpha} - w^2(n_1 + n_2)(n_1 + n_2) \right),$$

Wages in the first period and the second period are given by $w^1(n_1, n_2)$ and $w^2(n_1 + n_2)$ and they are determined before production each period by the bargaining rules in Stole and Zwiebel (1996). Since there is no continuation value, the wages in the second period splits the marginal surplus and the workers' outside option according to bargaining rules. Hence, it depends on the sum of n_1 and n_2 is same with the production wages in the text:

$$w^2(n_1 + n_2) = \frac{\alpha\phi}{1 - \phi + \alpha\phi} A \varepsilon n_2{}^{\alpha-1} + (1 - \phi)(b + \Omega) \quad .$$

Now consider the surplus of the workers hired in the first period. At the time of the bargaining in the first period, the selection costs from the first period are sunk. However, the selection costs from the first period creates a surplus for the match in addition to production surplus. Therefore, the marginal surplus of a worker hired in the first period is:

$$D_1 = \alpha A \varepsilon n_1{}^{\alpha-1} + \\ \beta \left((z-1)\frac{c_s}{z}\frac{n_2}{1+n_1} + \frac{1-\phi}{1-\phi+\alpha\phi}\alpha A \varepsilon(n_1+n_2) - (1-\phi)(b+\Omega) \right)$$

The sharing rule implies that the wage for the first period depends on n_1 and n_2 as conjectured. Then, in the sequential problem, the first order conditions for n_2 is:

$$w_{n_2}^1(n_1,n_2) = \beta \left(-c_s \left(\frac{n_2}{1+n_1} \right)^{z-1} + \frac{1-\phi}{1-\phi+\alpha\phi}\alpha A \varepsilon(n_1+n_2)^{\alpha-1} - (1-\phi)(b+\Omega) \right).$$

This equation states that the firm equates the increase in the first period's wages to the gain in the second period associated with the marginal worker hired in the second period. But, this creates a time inconsistency problem as the firm would choose n_2 such that RHS in the equation above is equal to zero. Therefore, the solution to the sequential problem would not coincide with its recursive formulation. Splitting the wages for the workers hired in the first period into two parts as in the text solves the time inconsistency problem since the recruitment wage function affects the firm's decision in the second period of the sequential problem above.

B   Recursive Stationary Equilibrium

The recursive stationary equilibrium consists of value function for firms, $J(n,\varepsilon)$, $J^h(n,\varepsilon)$, and $J^f(n,\varepsilon)$; a set of decision rules for vacancies, hiring standard, firings and employment, $g_v(n,\varepsilon)$, $g_p(n,\varepsilon)$, $g_d(n,\varepsilon)$ and $g_{n'}(n,\varepsilon)$; value functions for employed workers, $V^n(n,\varepsilon)$ and $V^p(n,\varepsilon)$; wage functions, $w^n(n',\varepsilon)$, $w^p(n',\varepsilon,p)$, and $w^r(p,n';n,\varepsilon)$; market tightness and aggregate matching probability, θ and q; value of unemployment at the beginning of the period and at the bargaining stage, $\tilde{V}^u$ and V^u; and a stationary distribution firms across productivity and employment, $\Gamma(n,\varepsilon)$, such that:

1. θ and q are related according to (2).

2. Firm's Optimization: Given q, $w^n(n',\varepsilon)$, $w^p(n',\varepsilon,p)$, and $w^r(p,n';n,\varepsilon)$, the set of decision rules, $g_v(n,\varepsilon)$, $g_p(n,\varepsilon)$, $g_d(n,\varepsilon)$ and $g_{n'}(n,\varepsilon)$, solve firms'

problem described by equations (8)-(11).

3. Worker Value Functions: Given $\theta q, \Gamma(n,\varepsilon), w^n(n',\varepsilon), w^p(n',\varepsilon,p), w^r(p,n';n,\varepsilon)$, and firms' decision rules, $g_v(n,\varepsilon), g_p(n,\varepsilon)$, and $g_{n'}(n,\varepsilon)$, value functions for workers, $V^n(n,\varepsilon), V^p(n,\varepsilon), \tilde{V}^u$ and V^u, satisfy equations (12)-(15), where retention probabilities are calculated from firm's policy functions.

4. Wage Bargaining: The wage equations, $w^n(n',\varepsilon), w^p(n',\varepsilon,p)$, and $w^r(p,n';n,\varepsilon)$, satisfy equations (16)-(18).

5. Free-entry condition in (23) holds.

6. Consistency: The stationary distribution $\Gamma(n,\varepsilon)$ is consistent with the firm's decision rules and satisfies (22).

C   The Effect of z on the Cross Sectional Patterns

Since the choice of the hiring standard threshold, p, in the worker selection model is sensitive the curvature of the worker selection cost function, I change the value of z to see the changes in the cross sectional patterns of the hires-to-vacancy ratio. I performed the calculations under quadratic, $z = 2$, and quartic, $z = 4$, specifications of the selection cost function. Whenever I change the value of z, I re-calibrate some of the parameters of the model to hit the same targets described in Section III.B. Table 1 shows the new parameter estimates under each specification.

Table 1: Calibrated Parameters of the Worker Selection Model with Quadratic and Quartic Selection Cost Functions

Parameter		$z = 2$	$z = 4$
c_s:	Selection cost, scale	0.733	6.663
σ:	Dispersion of shocks	0.153	0.159
γ:	Success probability, $x^{\gamma-1}$	2.775	2.268
ϕ:	Workers' bargaining power	0.338	0.430
c_v:	Flow cost of vacancy	4.382×10^{-4}	2.688×10^{-4}
A:	Aggregate productivity	3.305	3.197
c_e:	Fixed entry cost		

Evident from Figure 1, an increase in z makes the relationship between the hires-to-vacancy ratio and employment growth rate stronger. A cubic specification

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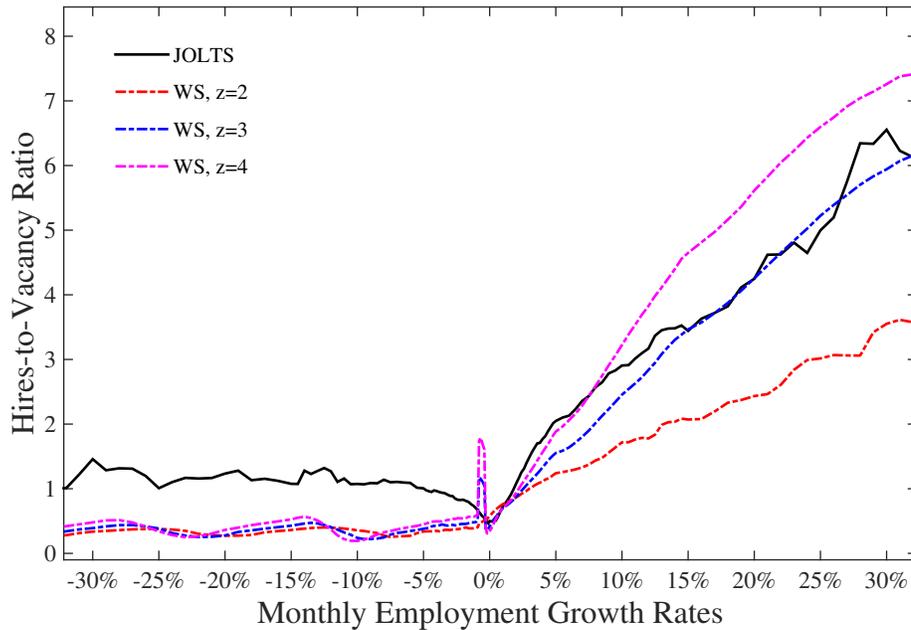generates a pattern that is very close pattern in JOLTS described in Davis et al. (2013).

Figure 1: Monthly Employment Growth Rates and Hires-to-Vacancy Ratio

Note: The data from the worker selection model, denoted by WS, are generated from the stationary distribution of the model for different values of *z*.
Source: JOLTS data is taken from DFH.

Figure 2 and Table 2 shows the effect of z on the relationship to firm size and worker turnover. Similarly, increasing *z* makes both of these relationships stronger.

D   Hires and Separation in the Cross Section

Despite the similarities in the cross sectional patterns of the hires-to-vacancy ratio, the worker selection and the directed search models differ in terms of their hiring rates. The hires rate across employment growth rates from worker selection and directed search models are plotted in Figure 3. The difference between the hires rate and the 45 degree line corresponds to the separation rates at each bin. For example, the separation rate is increasing in JOLTS reaching from 1% to 5% as we move from 0% to 30% employment growth rate.
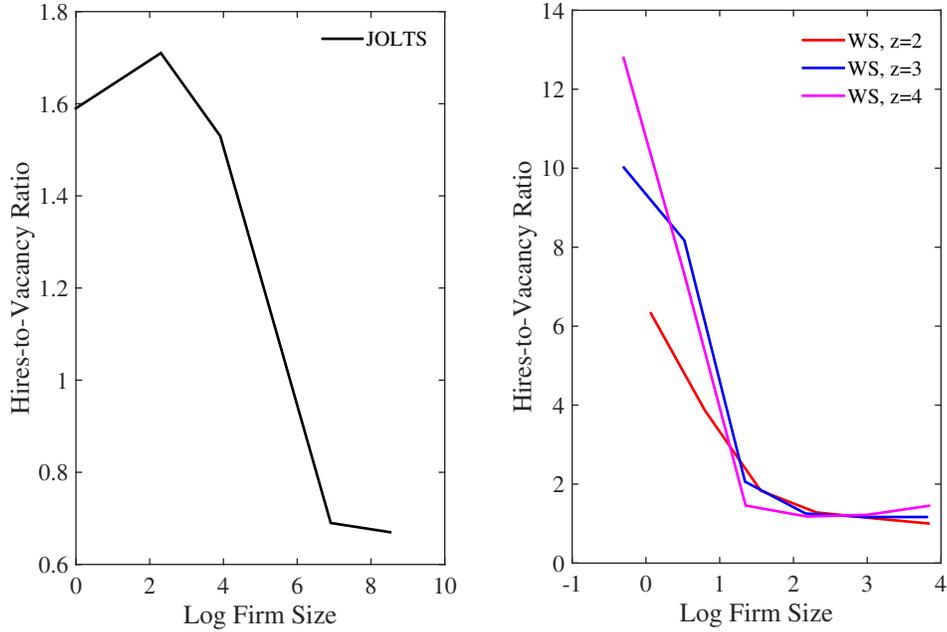
Figure 2: Log Firm Size and Hires-to-Vacancy Ratio

Note: The data from the worker selection model, denoted by WS, are generated from the stationary distribution of the model for different values of *z*.
Source: JOLTS data is taken from DFH.

Table 2: Monthly Worker Turnover and Hires-to-Vacancy Ratio

	Hires-to-Vacancy Ratio			
	JOLTS	$z = 2$	$z = 3$	$z = 4$
Low or No Turnover	0.000	0.569	0.416	0.339
First Quintile	0.290	0.623	0.465	0.405
Second Quintile	0.490	0.649	0.530	0.509
Third Quintile	0.787	0.687	0.605	0.608
Fourth Quintile	1.433	0.872	0.990	1.167
Fifth Quintile	3.077	1.945	2.681	3.255

Note: The data from the model is generated from the stationary distribution of the worker selection model for different values of *z*. The first worker turnover bin includes firms with very low worker turnover rates from the model and no worker turnover firms from JOLTS. See text for details.
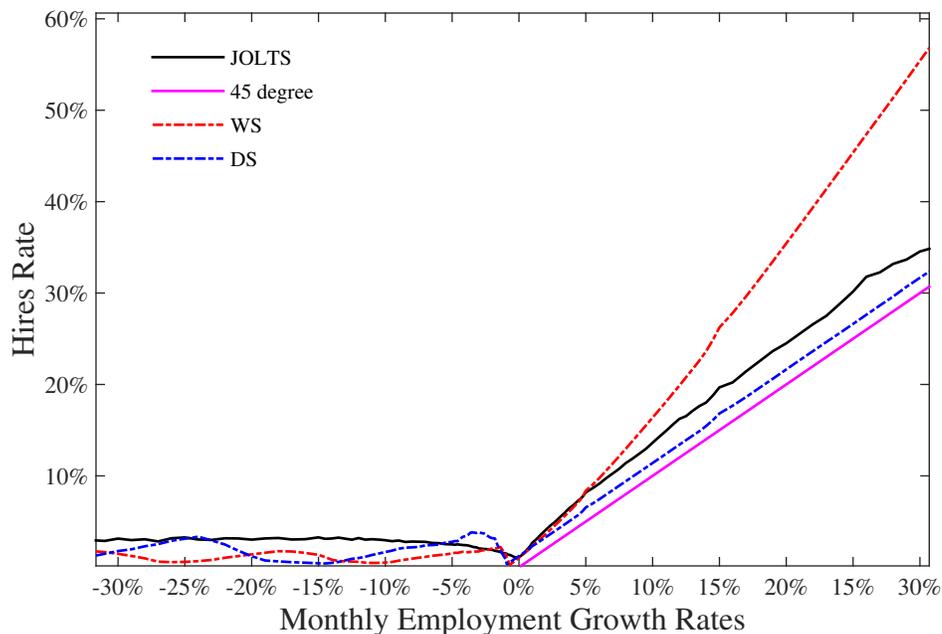*Source:* JOLTS data is taken from DFH.

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Figure 3: Comparison Across Models: Hires-to-Vacancy Ratio and Employment Growth

Note: The data from the models are generated from the stationary distribution of the corresponding model with $z = 3$. WS, DS, and DMP stand for worker selection, directed search, and standard DMP models, respectively.

Source: JOLTS data is taken from DFH.

In directed search model, the separation rate is constant because separation mostly happens due to exogenous shocks at these firms. In contrast, the separation rate is increasing in the worker selection model, but it increases rather quickly, reaching to 15% at 30% employment growth rate compared to 5% in JOLTS. This difference between the worker selection model and the data is a side effect of learning the productivity of the worker after one period.

References

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L. Stole and J. Zwiebel. Intra-firm bargaining under non-binding contracts. *Review of Economic Studies*, 63(3):375 – 410, 1996.