Online Appendix for "Dynamic Procurement under Uncertainty: Optimal Design and Implications for Incomplete Contracts"

By Malin Arve and David Martimort*

I. Concavity and Implementability Conditions

In this Appendix, we provide conditions that ensure that the allocation $(\mathcal{U}^{sb}, q_1^{sb}, u_1^{sb}, \varepsilon^{sb})$ characterized in the Proofs of Propositions 1, 2 and 3 through a set of necessary conditions is indeed the solution. This first requires that the principal's problem is concave and second that the rent profile $\mathcal{U}^{sb}$ is convex (which from Lemma 1 is a sufficient condition for implementability). In the rest of this Section, we provide upper bounds on the degree of risk-aversion in the $CARA$ case that ensure that those conditions hold. Beyond the $CARA$ case, we demonstrate that these conditions always hold when β is small enough.

• *Concavity of the principal's problem in the $CARA$ case.* We start with the special case of $CARA$ utility function. Using the expression of $w(z,\varepsilon)$ given in the text, we obtain:

$$\varphi(\zeta,\varepsilon) = -\frac{1}{\tau}\ln(1-\tau\zeta) + \frac{1}{\tau}\ln(\eta(\tau,\varepsilon)).$$

Inserting into (A7) yields the following expression of the Hamiltonian:

(O.1)

$$\mathcal{H}(\mathcal{U}, q_1, u_1, \varepsilon, \lambda, \theta_1) = f(\theta_1)\Bigg(S_1(q_1) - \theta_1 q_1 - (1-\beta)u_1 + \frac{\beta}{\tau}\ln\left(1-\tau\frac{\mathcal{U}-(1-\beta)u_1}{\beta}\right)$$

$$-\frac{\beta}{\tau}\ln(\eta(\tau,\varepsilon)) + \nu\beta\left(S_2(q_2^{fb}(\underline{\theta}_2)) - \underline{\theta}_2 q_2^{fb}(\underline{\theta}_2)\right) + (1-\nu)\beta\left(S_2\left(\frac{\varepsilon}{\Delta\theta_2}\right) - \overline{\theta}_2\frac{\varepsilon}{\Delta\theta_2}\right)\Bigg)$$

$$-\lambda q_1\left(1-\tau(\mathcal{U}-(1-\beta)u_1)\right).$$

For the value taken by the costate variable, namely $\lambda(\theta_1) = F(\theta_1)$ obtained from (A13) at the allocation $(\mathcal{U}^{sb}, q_1^{sb}, u_1^{sb}, \varepsilon^{sb})$ characterized by necessary conditions above, we now proceed by computing a partially maximized Hamiltonian $\hat{\mathcal{H}}(\mathcal{U}, q_1, \varepsilon, \lambda(\theta_1), \theta_1) = \max_{u_1}\mathcal{H}(\mathcal{U}, q_1, u_1, \varepsilon, \lambda(\theta_1), \theta_1)$. We shall check that

* Arve: NHH Norwegian School of Economics, malin.arve@nhh.no. Martimort: Paris School of Economics-EHESS, david.martimort@parisschoolofeconomics.eu.

this partially maximized Hamiltonian is concave in $(\mathcal{U}, q_1, \varepsilon)$ which provides sufficiency conditions for optimality by mixing Arrow-Kurz (partial maximization of the Hamiltonian with respect to one control variable, namely u_1) and Mangasarian conditions (checking that the so obtained partially maximized Hamiltonian remains concave in the remaining variables $(\mathcal{U}, q_1, \varepsilon)$). (See Seierstad and Sydsaeter (1987), Chapter 2, Theorems 4 and 5.)[1]

The first step is to observe that $\mathcal{H}(\mathcal{U}, q_1, u_1, \varepsilon, \lambda(\theta_1), \theta_1)$ is concave in u_1 so that the above optimum is obtained for $\hat{u}_1(\mathcal{U}, q_1, \theta_1)$ defined as:

$$1-\tau\frac{\mathcal{U}-(1-\beta)\hat{u}_1(\mathcal{U}, q_1, \theta_1)}{\beta}=\frac{1}{1+\tau\frac{F(\theta_1)}{f(\theta_1)}q_1}.$$

Inserting this value into the maximand gives us the following expression of the partially maximized Hamiltonian $\hat{\mathcal{H}}(\mathcal{U}, q_1, \varepsilon, \lambda(\theta_1), \theta_1)$:

(O.2)

$$\hat{\mathcal{H}}(\mathcal{U}, q_1\varepsilon, \lambda(\theta_1), \theta_1) = f(\theta_1)\left(S_1(q_1)-\theta_1 q_1-\mathcal{U}-\frac{\beta}{\tau}\ln\left(1+\tau\frac{F(\theta_1)}{f(\theta_1)}q_1\right)-\frac{\beta}{\tau}\ln(\eta(\tau,\varepsilon))\right.$$
$$\left.+\nu\beta\left(S_2(q_2^{fb}(\underline{\theta}_2))-\underline{\theta}_2 q_2^{fb}(\underline{\theta}_2)\right)+(1-\nu)\beta\left(S_2\left(\frac{\varepsilon}{\Delta\theta_2}\right)-\overline{\theta}_2\frac{\varepsilon}{\Delta\theta_2}\right)\right)-(1-\beta)F(\theta_1)q_1.$$

It is straightforward to check that $\hat{\mathcal{H}}(\mathcal{U}, q_1, \varepsilon, \lambda(\theta_1), \theta_1)$ is now concave in $(\mathcal{U}, q_1, \varepsilon)$ provided that the following condition holds:

$$S_1''(q_1)+\frac{\beta\tau\left(\frac{F(\theta_1)}{f(\theta_1)}\right)^2}{\left(1+\tau\frac{F(\theta_1)}{f(\theta_1)}q_1\right)^2}\leq 0,\quad \forall q_1\in R_+,\quad \forall\theta_1\in\Theta_1. \tag{O.3}$$

This condition is satisfied when the degree of risk aversion τ is small compared to S_1''.

• *Concavity of the principal's problem beyond the CARA case.* We proceed as above; the difficulty being now that closed-form solutions are not available. To perform a partial optimization with respect to u_1, we have to solve:

$$\max_{u_1} - f(\theta_1)\left((1-\beta)u_1+\beta\varphi\left(\frac{\mathcal{U}-(1-\beta)u_1}{\beta},\varepsilon\right)\right)$$
$$-\lambda(\theta_1)q_1\left(1-\beta+\beta w_z\left(\varphi\left(\frac{\mathcal{U}-(1-\beta)u_1}{\beta},\varepsilon\right),\varepsilon\right)\right).$$

[1] Although their proof of the sufficiency of Arrow-Kurz conditions suppose that maximized Hamiltonian is obtained by maximizing with respect to all control variables, it can be adapted *mutatis mutandis* to show that a partial maximization with respect to a subset of control variables suffices.

The necessary first-order condition for optimality gives us:

$$(1-\beta)\hat{u}_1(\mathcal{U}, q_1, \varepsilon, \theta_1) = \mathcal{U} - \beta\hat{z}(q_1, \varepsilon, \theta_1)$$

where $\hat{z}(q_1, \theta_1)$ solves

$$1 + \frac{\lambda(\theta_1)}{f(\theta_1)} q_1 w_{zz}(\hat{z}(q_1, \varepsilon, \theta_1), \varepsilon) = w_z(\hat{z}(q_1, \varepsilon, \theta_1), \varepsilon).$$

Observe that the function $w_z(z, \varepsilon) - \frac{\lambda(\theta_1)}{f(\theta_1)} q_1 w_{zz}(z, \varepsilon)$ is decreasing in z since $w_{zz} < 0$ from the concavity of v, $\lambda(\theta_1) = F(\theta_1) \geq 0$ (from (A13)) and $w_{zzz} > 0$ when $v''' > 0$ (a condition implied by Assumption 1). Hence, the necessary condition above is also sufficient.

We can thus rewrite the partially maximized Hamiltonian $\hat{\mathcal{H}}(\mathcal{U}, q_1, \varepsilon, \lambda(\theta_1), \theta_1)$ as:

$$\hat{\mathcal{H}}(\mathcal{U}, q_1, \varepsilon, \lambda(\theta_1), \theta_1) = \tag{O.4}$$

$$f(\theta_1)\Bigg(S_1(q_1) - \theta_1 q_1 - \mathcal{U} + \beta\hat{z}(q_1, \varepsilon, \theta_1) - \beta\varphi\left(\hat{z}(q_1, \varepsilon, \theta_1), \varepsilon\right) + \nu\beta\left(S_2(q_2^{fb}(\underline{\theta}_2)) - \underline{\theta}_2 q_2^{fb}(\underline{\theta}_2)\right)$$

$$+ (1-\nu)\beta\left(S_2\left(\frac{\varepsilon}{\Delta\theta_2}\right) - \overline{\theta}_2 \frac{\varepsilon}{\Delta\theta_2}\right)\Bigg) - \lambda(\theta_1) q_1 \left(1 - \beta + \beta w_z\left(\varphi\left(\hat{z}(q_1, \varepsilon, \theta_1), \varepsilon\right), \varepsilon\right)\right).$$

Now, observe that the curvature of $\hat{z}(q_1, \varepsilon, \theta_1)$ only depends on w (and thus u). Proceeding as in the $CARA$ case above, the concavity of $\hat{\mathcal{H}}(\mathcal{U}, q_1, \varepsilon, \lambda(\theta_1), \theta_1)$ in $(\mathcal{U}, q_1, \varepsilon)$ is thus ensured provided that S_1'' and S_2'' are sufficiently negative or provided that β is small enough.

• *Incentive compatibility.* We start with the $CARA$ case because of its importance for the main text. Observe that, for the allocation $(\mathcal{U}^{sb}, q_1^{sb}, u_1^{sb}, \varepsilon^{sb})$, the incentive compatibility constraint (12) can be rewritten as:

$$\dot{\mathcal{U}}^{sb}(\theta_1) = -q_1^{sb}(\theta_1)\left(1 - \beta + \frac{\beta}{1 + \tau q_1^{sb}(\theta_1)\frac{F(\theta_1)}{f(\theta_1)}}\right). \tag{O.5}$$

Differentiating (18) with respect to θ_1, we observe that:

$$\left(S_1''(q_1^{sb}(\theta_1)) + \frac{\beta\tau\left(\frac{F(\theta_1)}{f(\theta_1)}\right)^2}{\left(1+\tau\frac{F(\theta_1)}{f(\theta_1)}q_1^{sb}(\theta_1)\right)^2}\right)\dot{q}_1^{sb}(\theta_1) =$$
$$1 + \frac{d}{d\theta_1}\left(\frac{F(\theta_1)}{f(\theta_1)}\right)\left(1-\beta+\frac{\beta}{\left(1+\tau q_1^{sb}(\theta_1)\frac{F(\theta_1)}{f(\theta_1)}\right)^2}\right).$$

This condition, together with the concavity requirement (O.3) and Assumption 2 yields $\dot{q}_1^{sb}(\theta_1) < 0$.

Differentiating now the right-hand side of (O.5) with respect to θ_1, we obtain:

$$\frac{d}{d\theta_1}\left(-q_1^{sb}(\theta_1)\left(1-\beta+\frac{\beta}{1+\tau q_1^{sb}(\theta_1)\frac{F(\theta_1)}{f(\theta_1)}}\right)\right) =$$
$$-\dot{q}_1^{sb}(\theta_1)\left(1-\beta+\frac{\beta}{\left(1+\tau q_1^{sb}(\theta_1)\frac{F(\theta_1)}{f(\theta_1)}\right)^2}\right) + \frac{\tau\beta(q_1^{sb}(\theta_1))^2\frac{d}{d\theta_1}\left(\frac{F(\theta_1)}{f(\theta_1)}\right)}{\left(1+\tau q_1^{sb}(\theta_1)\frac{F(\theta_1)}{f(\theta_1)}\right)^2} > 0,$$

where the strict inequality follows from the fact that $\dot{q}_1^{sb}(\theta_1) < 0$ and Assumption 2. Hence, $\mathcal{U}^{sb}$ is always convex so that the sufficiency condition in Lemma 1 holds.

Beyond the *CARA* case, differentiating (15) with respect to θ_1 shows that the optimal output q_1^{sb} is necessarily strictly decreasing when β is small enough and Assumption 2 holds. Hence, $\mathcal{U}^{sb}$ is again convex as required by the sufficiency condition in Lemma 1.

II. First-Period Risk Aversion

Suppose that the agent also evaluates the first-period returns according to the same utility function $v(\cdot)$ as in the second period. We first analyze the case of a durable project. Then, and for the sake of completeness, we also report on the case of a non-durable, i.e., q_1 only arises in the first period. For simplicity and under both scenarios, we suppose that θ_2 remains common knowledge.

A. *The case of a durable first-period project*

The next proposition shows that the *Income Effect* disappears as suggested in the text. The principal finds no value in shifting payments towards the second period. As a result, the basic service is produced at its Baron-Myerson level.

PROPOSITION 1: *Suppose that θ_2 remains common knowledge and that the first-period project is durable. The optimal contract has the following features:*

- *Constant profit over time for the durable:*

$$y_2^{sb}(\theta_1) = 0. \tag{O.6}$$

- *The durable is produced at its Baron-Myerson level:*

$$q_1^{sb}(\theta_1) = q_1^{bm}(\theta_1). \tag{O.7}$$

To show these results, observe that the principal's expected payoff can now be written as:

(O.8)
$$E_{\theta_1}\Bigg(S_1(q_1(\theta_1)) - \theta_1 q_1(\theta_1) - (1-\beta)u_1(\theta_1) + \beta E_{\theta_2}\left(S_2(q_2(\theta_1,\theta_2)) - \theta_2 q_2(\theta_1,\theta_2)\right) - \beta\varphi\left(\frac{\mathcal{U}(\theta_1) - (1-\beta)v(u_1(\theta_1))}{\beta}, 0\right)\Bigg).$$

Omitting the sufficiency condition for incentive compatibility given by (A3) and focusing on a so called relaxed optimization problem, the principal's problem is to maximize (O.8) among all possible allocations $(\mathcal{U}(\theta_1), u_1(\theta_1), q_1(\theta_1))$ subject to the necessary condition for first-period incentive compatibility (22) and the firm's participation constraint (A6) that again turns out to be binding at the optimum.

• Optimizing w.r.t. $q_2(\theta_1,\theta_2)$ gives $q_2^{sb}(\theta_1,\theta_2) = q_2^{fb}(\theta_2)$ for all (θ_1,θ_2). Therefore, we may simplify the expression of the principal's payoff from the add-on to:

$$E_{\theta_2}(S_2(q_2^{fb}(\theta_2)) - \theta_2 q_2^{fb}(\theta_2)).$$

Equipped with this expression, and denoting by λ the costate variable for (22) we can write the Hamiltonian for the principal's problem as:

(O.9)
$$\mathcal{H}(\mathcal{U}, q_1, u_1, \lambda, \theta_1) =$$
$$f(\theta_1)\left(S_1(q_1) - \theta_1 q_1 - (1-\beta)u_1 - \beta\varphi\left(\frac{\mathcal{U} - (1-\beta)v(u_1)}{\beta}, 0\right) + E_{\theta_2}(S_2(q_2^{fb}(\theta_2)) - \theta_2 q_2^{fb}(\theta_2))\right)$$
$$-\lambda q_1\left((1-\beta)v'(u_1) + \beta v'\left(\varphi\left(\frac{\mathcal{U} - (1-\beta)v(u_1)}{\beta}, 0\right)\right)\right).$$

We shall assume that $\mathcal{H}(\mathcal{U}, q_1, u_1, \lambda, \theta_1)$ is concave in $(\mathcal{U}, q_1, u_1)$ and use the Pontryagin Principle to get optimality conditions satisfied by an extremal arc $(\mathcal{U}^{sb}(\theta_1), u_1^{sb}(\theta_1), q_1^{sb}(\theta_1))$.

• *Costate variable.* $\lambda(\theta_1)$ is continuous, piecewise continuously differentiable and such that:

$$\dot{\lambda}(\theta_1)v'(u_2^{sb}(\theta_1)) = f(\theta_1) + \lambda(\theta_1)q_1^{sb}(\theta_1)v''\left(u_2^{sb}(\theta_1)\right) \tag{O.10}$$

where the second-period profit is

$$u_2^{sb}(\theta_1) = u_1^{sb}(\theta_1) + y^{sb}(\theta_1) = \varphi\left(\frac{\mathcal{U}^{sb}(\theta_1) - (1-\beta)v(u_1^{sb}(\theta_1))}{\beta}, 0\right). \tag{O.11}$$

• *Transversality condition.* Because (A6) is binding at the optimum, this condition is:

$$\lambda(\underline{\theta}_1) = 0. \tag{O.12}$$

• *First-order optimality condition w.r.t. u_1:*

$$f(\theta_1)\frac{v'(u_1^{sb}(\theta_1))}{v'(u_2^{sb}(\theta_1))} = f(\theta_1) + \lambda(\theta_1)q_1^{sb}(\theta_1)\left(v''(u_1^{sb}(\theta_1)) - \frac{v''(u_2^{sb}(\theta_1))}{v'(u_2^{sb}(\theta_1))}v'(u_1^{sb}(\theta_1))\right). \tag{O.13}$$

• *First-order optimality condition w.r.t. q_1:*

$$S_1'(q_1^{sb}(\theta_1)) = \theta_1 + \frac{\lambda(\theta_1)}{f(\theta_1)}\left((1-\beta)v'(u_1^{sb}(\theta_1)) + \beta v'(u_2^{sb}(\theta_1))\right). \tag{O.14}$$

A solution to (O.13) is given by:

$$u_1^{sb}(\theta_1) = u_2^{sb}(\theta_1). \tag{O.15}$$

Inserting into (22) and (O.11) yields respectively:

$$\dot{\mathcal{U}}^{sb}(\theta_1) = -q_1^{sb}(\theta_1)v'(u_1^{sb}(\theta_1)) \text{ where } \mathcal{U}^{sb}(\theta_1) = v(u_1^{sb}(\theta_1)) \tag{O.16}$$

which implies

$$\dot{u}_1^{sb}(\theta_1) = -q_1^{sb}(\theta_1). \tag{O.17}$$

Inserting into (O.10) and using again (O.16) gives:

$$\frac{d}{d\theta}(\lambda(\theta_1)v'(u_1^{sb}(\theta))) = f(\theta_1).$$

Integrating and using (O.12) we obtain:

$$\lambda(\theta_1)v'(u_1^{sb}(\theta)) = F(\theta_1).$$

Inserting into (O.14) and again taking into account (O.16) gives (O.7).

B. The case of a non-durable first-period project

The next proposition shows that the principal wants to push profits for the first-period project into the second period even if the first-period project is not a durable one. This project is produced below the first-best level.

PROPOSITION 2: *Suppose that θ_2 remains common knowledge and that the first-period project's surplus and costs only arise in the first period. The optimal contract has the following features.*

- *The first-period project is rewarded in both periods but with declining profits:*

  $$u_1^{sb}(\theta_1) \geq u_2^{sb}(\theta_1) \tag{O.18}$$

  with an equality only in the case of risk neutrality.

- *The first-period production is:*

  $$S_1'(q_1^{sb}(\theta_1)) = \theta_1 + \frac{v'(u_1^{sb}(\theta_1))}{f(\theta_1)} \int_{\underline{\theta}_1}^{\theta_1} \frac{f(\tilde{\theta})}{v'(u_2^{sb}(\tilde{\theta}))} d\tilde{\theta}. \tag{O.19}$$

We first notice that, with a short-term project, the envelope condition for incentive compatibility becomes:

$$\dot{\mathcal{U}}(\theta_1) = -(1-\beta)q_1(\theta_1)v'(u_1(\theta_1)). \tag{O.20}$$

The principal's expected payoff also takes into account that surplus and cost for q_1 only arise in the first period and have to be weighted accordingly:

(O.21)

$$E_{\theta_1}\left((1-\beta)(S_1(q_1(\theta_1)) - \theta_1 q_1(\theta_1) - u_1(\theta_1)) + \beta E_{\theta_2}\left(S_2(q_2^{fb}(\theta_2)) - \theta_2 q_2^{fb}(\theta_2)\right)\right.$$
$$\left. -\beta\varphi\left(\frac{\mathcal{U}(\theta_1) - (1-\beta)v(u_1(\theta_1))}{\beta}, 0\right)\right).$$

Omitting the sufficiency condition for incentive compatibility given by (A3) and focusing on a so called relaxed optimization problem, the principal's problem is to maximize (O.21) among all possible allocations $(\mathcal{U}(\theta_1), u_1(\theta_1), q_1(\theta_1))$ subject to the necessary condition for first-period incentive compatibility (O.20) and the firm's participation constraint (A6) that again turns out to be binding at the optimum.

Denoting by λ the costate variable for (O.20) we can write the Hamiltonian for the principal's problem as:

$$\mathcal{H}(\mathcal{U}, q_1, u_1, \lambda, \theta_1) = \tag{O.22}$$

$$f(\theta_1)\left((1-\beta)(S_1(q_1)-\theta_1 q_1-u_1)-\beta\varphi\left(\frac{\mathcal{U}-(1-\beta)v(u_1)}{\beta},0\right)+E_{\theta_2}(S_2(q_2^{fb}(\theta_2))-\theta_2 q_2^{fb}(\theta_2))\right)$$

$$-\lambda(1-\beta)q_1 v'(u_1).$$

We shall assume that $\mathcal{H}(\mathcal{U}, q_1, u_1, \lambda, \theta_1)$ is concave in $(\mathcal{U}, q_1, u_1)$ and use the Pontryagin Principle to get optimality conditions satisfied by an extremal arc $(\mathcal{U}^{sb}(\theta_1), u_1^{sb}(\theta_1), q_1^{sb}(\theta_1))$.

• *Costate variable.* $\lambda(\theta_1)$ is continuous, piecewise continuously differentiable and such that:

$$\dot{\lambda}(\theta_1)v'(u_2^{sb}(\theta_1)) = f(\theta_1). \tag{O.23}$$

• *Transversality condition.* Because (A6) is binding at the optimum, this condition is:

$$\lambda(\underline{\theta}_1) = 0. \tag{O.24}$$

• *First-order optimality condition w.r.t. u_1:*

$$f(\theta_1)\frac{v'(u_1^{sb}(\theta_1))}{v'(u_2^{sb}(\theta_1))} = f(\theta_1) + \lambda(\theta_1)q_1^{sb}(\theta_1)v''(u_1^{sb}(\theta_1)). \tag{O.25}$$

• *First-order optimality condition w.r.t. q_1:*

$$S_1'(q_1^{sb}(\theta_1)) = \theta_1 + \frac{\lambda(\theta_1)}{f(\theta_1)}v'(u_1^{sb}(\theta_1)). \tag{O.26}$$

From (O.23) and (O.24), $\lambda(\theta_1)$ satisfies:

$$\lambda(\theta_1) = \int_{\underline{\theta}_1}^{\theta_1} \frac{f(\tilde{\theta})}{v'(u_2^{sb}(\tilde{\theta}))}d\tilde{\theta} \geq 0. \tag{O.27}$$

Inserting into (O.25) immediately gives (O.18). Finally, inserting (O.27) into (O.26) yields (O.19).

III. Robustness: Lumpy Add-On with Continuous Costs

In the main text, the analysis has been simplified by assuming that the cost of the uncertain add-on was drawn from a binary distribution. Although this assumption allows us to consider the consequences of an endogenous background risk on earlier incentives in a stripped down manner, a more symmetric treatment requires the cost of the add-on to take a continuum of values. We thus assume that θ_2 is distributed according to a continuous and atomless cumulative distribution $F_2(\theta_2)$ (with a positive density $f_2(\theta_2)$) on $\Theta_2 = \left[\underline{\theta}_2, \overline{\theta}_2\right]$. The technical difficulty pointed out by both Salanié (1990) and Laffont and Rochet (1998) for such models is that, even in simpler static settings, complicated areas of bunching might arise for the optimal level of add-on when the degree of risk aversion is sufficiently large.

One way to extend our analysis without falling into such technicalities is to consider a setting where the second-period project is lumpy. Possible examples would be the expansion of an existing infrastructure, or the addition of services into new geographical areas or new segments of demand. This add-on, whose fixed value is denoted by S_2, is only pursued when the principal pays a price that covers the cost θ_2. Bunching thus takes a simpler form: The project is only done for costs below a threshold. We also assume that $\underline{\theta}_2 < S_2 < \overline{\theta}_2$, meaning that implementing the add-on is not always efficient even under complete information. This assumption stands in contrast to our previous analysis where an Inada condition imposed on the second-period surplus implied that the add-on was always valuable and thus always provided even in the second-best scenario. It nevertheless still implies that the firm's second-period returns remain risky with part of the risk coming from the possibility to give up the project if it turns out to be too costly.

The firm's expected second-period payoff with an arbitrary price $p \in \Theta_2$ for the add-on can now be written as:

$$w(z,p) = \int_{\underline{\theta}_2}^{p} v(z+p-\theta_2) f_2(\theta_2) d\theta_2 + v(z)(1 - F_2(p)). \tag{O.28}$$

Observe also that an increase in p makes it more likely to implement the add-on. It thus shifts the distribution of second-period profits in the sense of first-order stochastic dominance and this reduces the firm's second-period marginal utility of income since:

$$w_{zp}(z,p) = \int_{\underline{\theta}_2}^{p} v''(z+p-\theta_2) f_2(\theta_2) d\theta_2 \leq 0. \tag{O.29}$$

Following the same procedure as previously, we may also re-define two new functions $H(z,p) = w_{zp}(z,p) - \frac{w_{zz}(z,p)}{w_z(z,p)} w_p(z,p)$ and $\varphi(\zeta,p)$ such that $\zeta = w(\varphi(\zeta,p),p)$.[2] Assumption 1 then ensures that $H(\cdot)$ remains non-negative.

LEMMA 1: *Suppose that $v(\cdot)$ is DARA (resp. CARA). Then,*

$$H(z,p) \geq 0 \ (\textit{resp.} = 0) \quad \forall (z,p) \in R \times \Theta_2. \tag{O.30}$$

[2]In particular, we have $\varphi_p(\zeta,p) = -\frac{w_p(\varphi(\zeta,p),p)}{w_z(\varphi(\zeta,p),p)} < 0$ and $\varphi_\zeta(\zeta,p) = \frac{1}{w_z(\varphi(\zeta,p),p)} > 0$.

PROOF OF LEMMA 1:

Simple properties of $w(\cdot)$ immediately follow from its definition (O.28):

$$w_p(z,p) = \int_{\underline{\theta}_2}^{p} v'(z+p-\theta_2) f_2(\theta_2) d\theta_2 > 0, w_{zp}(z,p) = \int_{\underline{\theta}_2}^{p} v''(z+p-\theta_2) f_2(\theta_2) d\theta_2 \leq 0,$$
$$w_z(z,p) = \int_{\underline{\theta}_2}^{p} v'(z+p-\theta_2) f_2(\theta_2) d\theta_2 + v'(z)(1-F_2(p)) > 0,$$
$$w_{zz}(z,p) = \int_{\underline{\theta}_2}^{p} v''(z+p-\theta_2) f_2(\theta_2) d\theta_2 + v''(z)(1-F_2(p)) \leq 0.$$

The inequality in (O.30) can be rewritten as:

$$\left(\int_{\underline{\theta}_2}^{p} v''(z+p-\theta_2) f_2(\theta_2) d\theta_2\right)\left(\int_{\underline{\theta}_2}^{p} v'(z+p-\theta_2) f_2(\theta_2) d\theta_2 + v'(z)(1-F_2(p))\right)$$
$$\geq \left(\int_{\underline{\theta}_2}^{p} v''(z+p-\theta_2) f_2(\theta_2) d\theta_2 + v''(z)(1-F_2(p))\right)\left(\int_{\underline{\theta}_2}^{p} v'(z+p-\theta_2) f_2(\theta_2) d\theta_2\right),$$
$$\forall (z,p) \in R \times \Theta_2.$$

Developing and rearranging, this amounts to demonstrating that:

$$v'(z)\left(\int_{\underline{\theta}_2}^{p} v''(z+p-\theta_2) f_2(\theta_2) d\theta_2\right) \geq v''(z)\left(\int_{\underline{\theta}_2}^{p} v'(z+p-\theta_2) f_2(\theta_2) d\theta_2\right), \quad \forall (z,p) \in R \times \Theta_2. \tag{O.31}$$

Now, observe that $v(\cdot)$ $DARA$ (resp. $CARA$) implies:

$$-\frac{v''(z+p-\theta_2)}{v'(z+p-\theta_2)} \leq -\frac{v''(z)}{v'(z)} \quad \forall \theta_2 \leq p, \quad \forall z.$$

Multiplying both terms by $v'(z+p-\theta_2)$ and $v'(z)$ and integrating over $[\underline{\theta}_2, p]$ yields (O.31) and proves the Lemma.

Thus, $\frac{dw_z}{dp}(\varphi(\zeta,p),p)$ is also non-negative. Contrary to our main scenario, this condition implies that the *Direct Effect* of increasing p is now dominated by the *Substitution Effect.* Increasing p reduces the marginal utility of income but is also requires to decrease the second-period profit made on the basic service to maintain second-period utility constant, which in turn increases the marginal utility of income more than the direct decrease.

Although details of the model differ, the analysis bears some resemblance to our previous findings. A first common feature is that the principal can reduce the cost of information rent by decreasing the firm's marginal utility of income in the second period. Indeed, at the optimal contract, we have:

(O.32)
$$w_z(u_1^{sb}(\theta_1)+y^{sb}(\theta_1), p^{sb}(\theta_1)) = 1+q_1^{sb}(\theta_1)\frac{F(\theta_1)}{f(\theta_1)}w_{zz}(u_1^{sb}(\theta_1)+y^{sb}(\theta_1), p^{sb}(\theta_1)) \leq 1.$$

As a result and by a mechanism which is now familiar, output distortions for the basic service are also less pronounced than in the Baron and Myerson (1982) outcome:

$$S_1'(q_1^{sb}(\theta_1)) = \theta_1 + \frac{F(\theta_1)}{f(\theta_1)}(1-\beta+\beta w_z(u_1^{sb}(\theta_1)+y^{sb}(\theta_1), p^{sb}(\theta_1))). \quad \text{(O.33)}$$

Because now the *Substitution Effect* dominates, relaxing the firm's first-period incentive constraint calls for decreasing the second-period price below its level in the absence of a first-period incentive problem. Indeed, the following condition holds:

(O.34)
$$(S_2 - p^{sb}(\theta_1))f_2(p^{sb}(\theta_1)) - F_2(p^{sb}(\theta_1)) \geq \varphi_p(w(u_1^{sb}(\theta_1)+y^{sb}(\theta_1), p^{sb}(\theta_1)), p^{sb}(\theta_1)).$$

Even though details differ, there is a common thread to this setting and our previous model. To isolate the first-period agency problem from the second-period one, the principal makes the second-period project less relevant either by reducing its size (in our main model) or by reducing the likelihood of its implementation in the present setting.

PROOFS :

Trade in the second period occurs only when $\theta_2 \leq p(\theta_1)$. Adapting the general expression (1) to the present context, the principal's expected payoff becomes:

$$E_{\theta_1}\left(S_1(q_1(\theta_1)) - \theta_1 q_1(\theta_1) - u_1(\theta_1) + \beta E_{\theta_2}\left((S_2 - p(\theta_1))F_2(p(\theta_1)) - y(\theta_1)\right)\right),$$

which can again be re-expressed as:

(O.35)
$$E_{\theta_1}\Bigg(S_1(q_1(\theta_1)) - \theta_1 q_1(\theta_1) - (1-\beta)u_1(\theta_1) + \beta E_{\theta_2}\left((S_2 - p(\theta_1))F_2(p(\theta_1))\right) - \beta\varphi\left(\frac{\mathcal{U}(\theta_1) - (1-\beta)u_1(\theta_1)}{\beta}, p(\theta_1)\right)\Bigg).$$

In terms of first-period incentive compatibility, (12) is readily replaced with:

$$\dot{\mathcal{U}}(\theta_1) = -q_1(\theta_1)\left(1-\beta+\beta w_z\left(\varphi\left(\frac{\mathcal{U}(\theta_1)-(1-\beta)u_1(\theta_1)}{\beta},\varepsilon(\theta_1)\right),p(\theta_1)\right)\right). \tag{O.36}$$

We now proceed as in the Proof of Propositions 1, 2 and 3 by relying on necessary conditions for optimality (details are omitted). Denoting again by λ the costate variable for (O.36), we now write the Hamiltonian for the principal's problem as:

$$\mathcal{H}(\mathcal{U},q_1,u_1,p,\lambda,\theta_1) = f(\theta_1)\Bigg(S_1(q_1)-\theta_1 q_1-(1-\beta)u_1-\beta\varphi\left(\frac{\mathcal{U}-(1-\beta)u_1}{\beta},p\right)$$
$$+\beta(S_2-p)F_2(p)\Bigg)-\lambda q_1\left(1-\beta+\beta w_z\left(\varphi\left(\frac{\mathcal{U}-(1-\beta)u_1}{\beta},p\right),p\right)\right).$$

Relying on the Pontryagin Principle to write the necessary conditions for an optimum $(\mathcal{U}^{sb}(\theta_1),u_1^{sb}(\theta_1),q_1^{sb}(\theta_1),p^{sb}(\theta_1))$, we obtain:

$$\frac{\dot{\lambda}(\theta_1)}{\varphi_\zeta\left(\frac{\mathcal{U}^{sb}(\theta_1)-(1-\beta)u_1^{sb}(\theta_1)}{\beta},p^{sb}(\theta_1)\right)} = f(\theta_1)$$
$$+\lambda(\theta_1)q_1^{sb}(\theta_1)w_{zz}\left(\varphi\left(\frac{\mathcal{U}^{sb}(\theta_1)-(1-\beta)u_1^{sb}(\theta_1)}{\beta},p^{sb}(\theta_1)\right),p^{sb}(\theta_1)\right). \tag{O.37}$$

The transversality condition is still given by (A9) and optimality w.r.t. u_1, q_1 and p yields:

$$\frac{f(\theta_1)}{\varphi_\zeta\left(\frac{\mathcal{U}^{sb}(\theta_1)-(1-\beta)u_1^{sb}(\theta_1)}{\beta},p^{sb}(\theta_1)\right)} = f(\theta_1)$$
$$+\lambda(\theta_1)q_1^{sb}(\theta_1)w_{zz}\left(\varphi\left(\frac{\mathcal{U}^{sb}(\theta_1)-(1-\beta)u_1^{sb}(\theta_1)}{\beta},p^{sb}(\theta_1)\right),p^{sb}(\theta_1)\right), \tag{O.38}$$

$$S_1'(q_1^{sb}(\theta_1)) = \theta_1+\frac{\lambda(\theta_1)}{f(\theta_1)}\left(1-\beta+\beta w_z\left(\varphi\left(\frac{\mathcal{U}^{sb}(\theta_1)-(1-\beta)u_1^{sb}(\theta_1)}{\beta},p^{sb}(\theta_1)\right),p^{sb}(\theta_1)\right)\right), \tag{O.39}$$

$$(S_2-p^{sb}(\theta_1))f_2(p^{sb}(\theta_1))-F_2(p^{sb}(\theta_1)) = \varphi_p\left(\frac{\mathcal{U}^{sb}(\theta_1)-(1-\beta)u_1^{sb}(\theta_1)}{\beta},p^{sb}(\theta_1)\right)$$
$$+q_1^{sb}(\theta_1)\frac{\lambda(\theta_1)}{f(\theta_1)}H\left(\varphi\left(\frac{\mathcal{U}^{sb}(\theta_1)-(1-\beta)u_1^{sb}(\theta_1)}{\beta},p^{sb}(\theta_1)\right),p^{sb}(\theta_1)\right). \tag{O.40}$$

Notice that (O.37), (O.38) and (A9) still imply (A13). Condition (O.32) immediately follows from inserting (A13) into (O.38). From (O.32), we know that $w_z(u_1^{sb}(\theta_1) + y^{sb}(\theta_1), p^{sb}(\theta_1)) \leq 1$. Therefore, (O.33) implies that $q_1^{sb}(\theta_1) \geq q_1^{bm}(\theta_1)$. Finally, inserting (A13) into (O.40), simplifying and taking into account the result of Lemma 1 gives us (O.34).

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