

For Online Publication

Appendix to:

The Power of Forward Guidance Revisited

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September 25, 2015

A Computational Methods

Here we describe the procedure used to find an equilibrium path of the heterogeneous agent model along a perfect foresight transition for the zero-lower-bound episode considered in Section 4.3. The algorithm used to compute the results for a one-time change in the real interest rate is closely related to what we present here.

Writing the firm's first order condition recursively. For the numerical analysis it is convenient to rewrite equation (7) in the main text recursively. Define

$$P_t^A \equiv \sum_{s=t}^{\infty} \beta^{s-t} (1-\theta)^{s-t} \left(\frac{P_t}{P_s} \right)^{\mu/(1-\mu)} Y_s \mu W_s \quad (1)$$

$$P_t^B \equiv \sum_{s=t}^{\infty} \beta^{s-t} (1-\theta)^{s-t} \left(\frac{P_t}{P_s} \right)^{1/(1-\mu)} Y_s. \quad (2)$$

then equation (7) in the main text becomes

$$\frac{p_t^*}{P_t} = \frac{P_t^A}{P_t^B}. \quad (3)$$

Equations (17) and (18) in the main text can be written recursively

$$P_t^A = \mu W_t Y_t + (1-\theta) \beta \mathbb{E}_t (1 + \pi_{t+1})^{-\mu/(1-\mu)} P_{t+1}^A \quad (4)$$

$$P_t^B = Y_t + \beta (1-\theta) \mathbb{E}_t (1 + \pi_{t+1})^{-1/(1-\mu)} P_{t+1}^B. \quad (5)$$

Initial guess. We assume that the economy has returned to steady state after $T = 250$ periods and look for equilibrium values for endogenous variables between dates $t = 0$ to T . In this explanation of our methods we use variables without subscripts to represent sequences from 0 to T . Let X denote a path for all endogenous aggregate variables from date 0 to date T . These variables include aggregate quantities and prices

$$X \equiv \{C_t, L_t, N_t, Y_t, D_t, i_t, W_t, \pi_t, r_t, p_t^*/P_t, S_t, \tau_t, P_t^A, P_t^B\}_{t=0}^T.$$

The dimension of X is given by 14 variables for each date and 251 dates. We require an initial guess X^0 . In most cases we found it sufficient to guess that the economy remains in steady state.

Solving the household’s problem. The household’s decision problem depends on X through r , W , τ , and D . For a given X^i we solve the household’s problem using the endogenous grid point method (Carroll, 2006). We approximate the household consumption function $c(b, z)$ with a shape-preserving cubic spline with 200 unequally-spaced knot points for each value of z with more knots placed at low asset levels where the consumption function exhibits more curvature. Given the consumption function we calculate labor supply from the household’s intratemporal optimality condition and savings from the budget constraint.

Simulating the population of households. We simulate the population of households in order to compute aggregate consumption and aggregate labor supply. We use a non-stochastic simulation method. We approximate the distribution of wealth with a histogram with 1000 unequally-spaced bins for each value of z again placing more bins at low asset levels. We then update the distribution of wealth according to the household savings policies and the exogenous transitions across z . When households choose levels of savings between the center of two bins, we allocate these households to the adjacent bins in a way that preserves total savings. See Young (2010) for a description of non-stochastic simulation in this manner.

Checking the equilibrium conditions. An equilibrium value of X must satisfy equations (4), (5), (9), (10), (11), (13), (15), (16) in the main text as well as equations (3), (4), and (5) and the monetary policy rule $i_t = \max[0, \bar{r} + \phi * \pi_t + \epsilon_t]$, where ϵ_t is the exogenous deviation from the Taylor rule that takes a negative value under our “extended” policy. Call these 12 equations the “analytical” equilibrium conditions. The remaining two equilibrium conditions that pin down X

are that C and L are consistent with household optimization and the dynamics of the distribution of wealth given the prices. Call these the “computational” equilibrium conditions.

To check whether a given X represents an equilibrium of the model is straightforward. We can easily verify whether the analytical equilibrium conditions hold at X . In addition, we can solve the household problem and simulate the population of households to verify that aggregated choices for consumption and labor supply of the heterogeneous households match with the values of C and L that appear in X .

Updating X^i The difficult part of the solution method arises when X^i is not an equilibrium. In this case we need to find a new guess X^{i+1} that moves us towards an equilibrium. To do this, we construct an auxiliary model by replacing the computational equilibrium conditions with additional analytical equilibrium conditions that approximate the behavior of the population of heterogeneous households but are easier to analyze. Specifically we use the equations

$$C_t^{-\gamma} = \eta_t^1 \beta (1 + r_t) C_{t+1}^{-\gamma} \quad (6)$$

$$C_t^{-\gamma} W_t = \eta_t^2 L_t^\psi. \quad (7)$$

where η^1 and η^2 are treated as parameters of the auxiliary model. For a given X^i , we have computed C and L from the computational equilibrium conditions. We then calibrate η^1 and η^2 from (6) and (7). We then solve for a new value of X from the 12 analytical equilibrium conditions and (6) and (7). This is a problem of solving for 14 unknowns at each date from 14 non-linear equations at each date for a total of 3514 unknowns and 3514 non-linear equations. We solve this system using the method described by Juillard (1996) for computing perfect foresight transition paths for non-linear models. This method is a variant of Newton’s method that exploits the sparsity of the Jacobian matrix. Call this solution $X^{i'}$. We then form X^{i+1} by updating partially from X^i towards $X^{i'}$. We iterate until X^i satisfies the equilibrium conditions within a tolerance of 5×10^{-6} .

References

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