Online Appendix: Large shocks travel fast

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A The model economy

Household problem. Households consume a composite good C made of varieties c_i with a constant elasticity of substitution $\eta > 1$

$$C_t = \left(\int_0^1 \left(A_{it} c_{it}\right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \tag{17}$$

where A_{it} denote shocks to preferences. The households maximize utility

$$\max_{C(t), c_i(t), H(t), M(t)} \quad \int_0^\infty e^{-\rho t} \left(\frac{C_t^{1-\epsilon}}{1-\epsilon} - \alpha H_t + \log\left(\frac{M_t}{P_t}\right)\right) dt$$

where $\alpha > 0$ is a labor disutility parameter and $\rho > 0$ is the discount factor, subject to the inter-temporal budget constraint

$$M_0 + \int_0^\infty Q_t \left(H_t W_t \left(1 + \tau_\ell\right) + \Pi_t + \tau_t - R_t M_t - \int_0^1 p_{it} c_{it} di \right) dt = 0 \tag{18}$$

where R_t denotes the nominal interest rate, $Q_t = \exp(-\int_0^t R_s ds)$ is the discount factor, W_t is the nominal wage rate, τ_ℓ a labor income tax, Π_t the firms' profits, τ_t a lump-sum transfer, p_{it} the nominal price of variety i and P is the price index. The household first order conditions yield

$$C_t: \quad e^{-\rho t} C_t^{-\epsilon} - \lambda Q_t P_t = 0 \tag{19}$$

$$c_{it}: \quad e^{-\rho t} C_t^{-\epsilon} C_t^{\frac{1}{\eta}} A_{it}^{\frac{\eta-1}{\eta}} c_{it}^{-\frac{1}{\eta}} - \lambda Q_t p_{it} = 0 \tag{20}$$

$$H_t: \quad -e^{-\rho t}\alpha + Q_t W_t \left(1+\tau_\ell\right)\lambda = 0 \tag{21}$$

$$M_t: \quad e^{-\rho t}\frac{1}{M_t} - \lambda Q_t R_t = 0 \tag{22}$$

where λ is the lagrange multiplier of the intertemporal budget constraint.

Using equation (19) and equation (20)

$$c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\eta} A_{it}^{\eta-1} C_t \tag{23}$$

Rearranging and differentiating equation (22) with respect to time we obtain

$$-\rho \frac{e^{-\rho t}}{M_t} - \frac{e^{-\rho t}}{M_t}\frac{\dot{M}_t}{M_t} = -\lambda Q_t R_t^2 + \lambda Q_t \dot{R}_t$$

where $\dot{Q}_t = -Q_t R_t$. Assume a monetary policy $M_t = M_0 \exp(\mu t)$. Then simplifying the above expression

$$\rho + \mu = R_t - \frac{\dot{R}_t}{R_t}$$

which is solved by $R_t = \rho + \mu$, all t. This implies, using equation (22)

$$\lambda = \frac{\exp(-\rho t)}{M_t Q_t R_t} = \frac{1}{M_0(\rho + \mu)}.$$

From equation (21), the nominal wage rate is

$$W_t = \exp(\mu t) \frac{\alpha}{1 + \tau_\ell} M_0(\rho + \mu),$$

with growth rate equal to μ. Using equation (20) and equation (21)

$$\begin{aligned}
A_{it} c_{it} &= \left(\frac{p_{it}}{W_t A_{it}} \right)^{-\eta} \left(\frac{\alpha}{1 + \tau_\ell} \right)^{-\eta} C_t^{1-\epsilon\eta}, \\
A_{it} c_{it} &= \left(\frac{\eta}{\eta - 1} K \left(\frac{E_t}{W_t} \right)^\zeta \frac{p_{it}}{p^*_{it}} \right)^{-\eta} \left(\frac{\alpha}{1 + \tau_\ell} \right)^{-\eta} C_t^{1-\epsilon\eta}, \\
A_{it} c_{it} &= e^{-\eta x_{it}} \left(K \left(\frac{E_t}{W_t} \right)^\zeta \frac{\eta}{\eta - 1} \frac{\alpha}{1 + \tau_\ell} \right)^{-\eta} C_t^{1-\epsilon\eta}.
\end{aligned}$$

where the second line uses the definition of profit-maximizing price and the fact $A_{it} = Z_{it}^{1-\zeta}$. Integrating over varieties,

$$\begin{aligned}
C_t^{\frac{\eta-1}{\eta}} &= \left(K \left(\frac{E_t}{W_t} \right)^\zeta \frac{\eta}{\eta - 1} \frac{\alpha}{1 + \tau_\ell} \right)^{1-\eta} C_t^{\frac{\eta-1}{\eta} - \epsilon(\eta-1)} \int_{\mathbb{R}} e^{(1-\eta)x} \hat{m}(x,t) dx, \\
C_t^{\epsilon(\eta-1)} &= \left(K \left(\frac{E_t}{W_t} \right)^\zeta \frac{\eta}{\eta - 1} \frac{\alpha}{1 + \tau_\ell} \right)^{1-\eta} \int_{\mathbb{R}} e^{(1-\eta)x} \hat{m}(x,t) dx, \\
C_t &= \left(K \left(\frac{E_t}{W_t} \right)^\zeta \alpha \right)^{-\frac{1}{\epsilon}} \left(\int_{\mathbb{R}} e^{(1-\eta)x} \hat{m}(x,t) dx \right)^{-\frac{1}{\epsilon(1-\eta)}}, \qquad (24)
\end{aligned}$$

which gives aggregate consumption as a function of the distribution of price gaps $\hat{m}(x,t)$ and prices. The labor subsidy is assumed to offset markups. Further, using the aggregate price index implied by equation (23) we obtain

$$P_t^{1-\eta} = \int_0^1 \left(\frac{p_{it}}{A_{it}}\right)^{1-\eta} di,$$

$$P_t^{1-\eta} = \left(\frac{\eta}{\eta-1}\text{mc}_t\right)^{1-\eta} \int_{\mathbb{R}} e^{(1-\eta)x}\hat{m}(x,t)dx, \tag{25}$$

where mc_t is short for $KE^{\zeta}W^{1-\zeta}$. Equation (25) gives an expression for real marginal costs as a function of the distribution of price gaps $\hat{m}(x,t)$.

The firm's profit function. The technology for firm i is Cobb-Douglas $y_i = (h_i/Z_i)^{1-\zeta}m_i^{\zeta}$ where h_i are units of labor and m_i are units of energy input. The marginal cost (and average cost) of producing y_i is $\text{mc}_i = K \cdot E^{\zeta}\left(W \cdot Z_i\right)^{1-\zeta}$ where $K = \zeta^{-\zeta} \cdot (1-\zeta)^{-(1-\zeta)}$. The profit function is

$$\Pi_i = (p_i - \text{mc}_i)\left(\frac{p_i}{P}\right)^{-\eta} A_i^{\eta-1}C,$$

$$\Pi_i = \left(\frac{p_i}{\text{mc}_i} - 1\right)\left(\frac{p_i}{PA_i}\right)^{-\eta} A_i^{-1}K \cdot E^{\zeta}\left(W \cdot Z_i\right)^{1-\zeta} C,$$

$$\Pi_i = \left(\frac{p_i}{\text{mc}_i} - 1\right)\left(\frac{p_i}{PZ_i^{1-\zeta}}\right)^{-\eta} K \cdot E^{\zeta}W^{1-\zeta}C,$$

$$\Pi_i = \left(\frac{p_i}{\text{mc}_i} - 1\right)\left(\frac{p_i}{\text{mc}_i}\right)^{-\eta} P^{\eta}\left[K \cdot E^{\zeta}W^{(1-\zeta)}\right]^{1-\eta} C,$$

the first line uses equation (23), the second factorizes marginal cost out, the third uses the assumption that $A_i = Z_i^{1-\zeta}$ and the fourth rearranges. The flexible-price optimum is $p_i^* = \frac{\eta}{\eta-1}\text{mc}_i$. Let $x \equiv \log p_i/p_i^*$ denote the "price gap", namely the distance between the current price and the static profit maximizing price. Expressing profits as a function of the price gap gives

$$\Pi(x,t) = \left[e^x - \frac{\eta-1}{\eta}\right] e^{-\eta x} \cdot P_t^{\eta}\left[\frac{\eta}{\eta-1}K \cdot E_t^{\zeta}W_t^{(1-\zeta)}\right]^{1-\eta} C_t,$$

$$\frac{\Pi(x,t)}{P_t} = \left[e^x - \frac{\eta-1}{\eta}\right] e^{-\eta x}\left[\frac{\eta}{\eta-1}\frac{\text{mc}_t}{P_t}\right]^{1-\eta} C_t, \tag{26}$$

where mc_t, again, is short for $KE_t^{\zeta}W_t^{1-\zeta}$. It is worth noting that all time dependent terms of real profits can be computed from the distribution of price gaps using equation (24) and equation (25). Notice how the assumption $A_i = Z_i^{1-\zeta}$ makes the profit function independent of the productivity shock, a feature that allows us to reduce the state space of the problem to a single scalar variable x. We define the flow cost function that represents forgone profits due to price gap x along a transition and in steady-state as

$$F(x,t) \equiv 1 - \frac{\Pi(x,t)}{\Pi_{ss}(0)}, \quad F(x) \equiv 1 - \eta \left[e^{x} - \frac{\eta - 1}{\eta} \right] e^{-\eta x},$$

where $\Pi_{ss}(0)$ are profits at $x = 0$ given steady state real marginal costs and consumption.

A second order expansion of the flow cost function around $x = 0$, yields the following quadratic approximation

$$F(x) \approx \frac{\eta(\eta-1)}{2} x^2,$$

where the profits are expressed as a fraction of the maximized profits $\Pi_{ss}(0)$. Thus, profit maximization can be approximated by the minimization of the quadratic return function $\frac{\eta(\eta-1)}{2}x^2$.

The profit function $\Pi(x)$ also reveals that the aggregate consumption level C only has a second order effect on the firm's choice of the optimal price gap, since the cross partial derivative of $\frac{\partial^2 \Pi}{\partial x \partial C}$ is zero when evaluated around the optimal value $x = 0$. This fact, which is due to the constant elasticity assumption, implies that the steady state decision rules of the firm are not altered, up to a first order, by a small perturbation of the aggregate variable C. We note that the absence of strategic complementarities makes the analysis with steady state rule Λ very close to the one of a general equilibrium model with feedback to aggregates (see Appendix D).

The firm's price-setting problem. The firm's sequential problem consists in minimizing the flow costs from forgone profits and effort costs by choosing hazard rates $\ell(t)$ and the optimal reset point x^* according to

$$v(x) = \min_{\ell(t), x^*} \mathbb{E} \left\{ \int_0^\infty e^{-\rho t} \left[F(x(t)) + (\kappa\, \ell(t))^\gamma \right] dt \,\middle|\, x(0) = x \right\}$$
$$\text{s.t. } x(t) = x(0) - \mu t + \sigma z(t) + \sum_{\tau_i < t} \Delta x(\tau_i)$$

where τ_i denotes the stopping times when a resetting opportunity arrives, $\Delta x(\tau_i) = x^* - x(\tau_i)$ is the price change, and $(\kappa \ell)^\gamma$ is the effort cost of choosing hazard rate ℓ with $\kappa > 0$ and $\gamma > 1$. This sequential formulation implies the HJB equation in equation (6).

B Proof of Proposition 1

Proof. Note that upon a stopping time τ (when adjustment occurs) we have $x(\tau) = x^* + \sigma z(\tau)$ where $z(t)$ is a standard Brownian motion since $\mu = 0$. By Ito's lemma the stochastic process $u(t) \equiv (x(t) - x^*)^2$ follows the diffusion

$$du = \sigma^2 dt + 2\sigma\sqrt{u}\; dz$$

Notice that $u(t) - \sigma^2 t = 2\sigma\sqrt{u}\; z(t)$ is a martingale. Letting τ be a stopping time, we have

$$\mathbb{E}\left(u(\tau) - \sigma^2\tau\right) = 0 \;\text{ so that }\; \mathbb{E}\left(u(\tau)\right) - \sigma^2\mathbb{E}\left(\tau\right) = 0$$

Note that $\mathbb{E}(\Delta x) = \mathbb{E}(x^* - x(\tau)) = -\sigma\mathbb{E}z(\tau) = 0$, so that $\mathbb{E}\left(u(\tau)\right) \equiv Var(\Delta x)$ is the variance of the size of price changes. Then, using that the mean duration of price changes satisfies $\mathbb{E}\left(\tau\right) = 1/N$, we have

$$N \cdot \text{Var}(\Delta x) = \sigma^2.$$

From the equation above and the stated relation between variance and frequency among the two economies, we immediately obtain $\tilde{\sigma}^2 = s\,\sigma^2$. Next we prove the scaling properties stated in the proposition. The steps are: we guess and verify a solution for the value function $\tilde{v}$ using the HJB, then we obtain the hazard function $\tilde{\Lambda}$ and we guess and verify a solution to $\tilde{m}$ using the hazard and the KFE. The HJB equation for the tilde economy satisfies

$$\tilde{\rho}\,\tilde{v}(x) = \tilde{B}x^2 + \frac{\tilde{\sigma}^2}{2}\tilde{v}''(x) + \tilde{C}(x) \;\text{ where }\; \tilde{C}(x) \equiv \left(\frac{\tilde{v}(x) - \tilde{v}(x^*)}{\tilde{\kappa}\;\gamma}\right)^{\frac{\gamma}{\gamma-1}} (1-\gamma) < 0$$

Define $\beta \equiv \tilde{B}/B$ and consider $\tilde{\rho} = \frac{s}{a}\rho$, $\tilde{\sigma}^2 = s\sigma^2$ and $\tilde{\kappa} = \frac{\kappa}{s}\,a^{\frac{\gamma+1}{\gamma}}\beta^{\frac{1}{\gamma}}$. Guess that $\tilde{v}(x) = v\left(\frac{x}{\sqrt{a}}\right)\frac{a^2}{s}\beta$ and substitute in the equation above to obtain (after some algebra)

$$a\beta\;\rho\, v\left(\frac{x}{\sqrt{a}}\right) = a\beta\left(B\left(\frac{x}{\sqrt{a}}\right)^2 + \frac{\sigma^2}{2}v''\left(\frac{x}{\sqrt{a}}\right) + C\left(\frac{x}{\sqrt{a}}\right)\right)$$

This verifies our guess for $\tilde{v}$ since the HJB for v holds. Note that given the value function and parameters we have

$$\tilde{\Lambda}(x) = \frac{1}{\tilde{\kappa}}\left(\frac{\tilde{v}(x) - \tilde{v}(x^*)}{\tilde{\kappa}\;\gamma}\right)^{\frac{1}{\gamma-1}} = \frac{s}{a}\Lambda\left(\frac{x}{\sqrt{a}}\right). \tag{27}$$

Now guess a density function $\tilde{m}(x) = m\left(\frac{x}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}$. Note that the density m solves $\int_{-\infty}^{\infty} m(x)dx = 1$ hence it follows that $\int_{-\infty}^{\infty}\tilde{m}(x)dx = \int_{-\infty}^{\infty} m\left(\frac{x}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}dx = 1$ which verifies the integration to one condition. Consider the Kolmogorov forward equation for the tilde economy

$$\tilde{\Lambda}(x)\tilde{m}(x) = \frac{\tilde{\sigma}^2}{2}\tilde{m}''(x), \qquad x \neq 0$$

and rewrite it using the guessed density, $\tilde{\sigma}^2 = s\sigma^2$, and equation (27). We obtain

$$\Lambda\left(\frac{x}{\sqrt{a}}\right)m\left(\frac{x}{\sqrt{a}}\right) = \frac{\sigma^2}{2}m''\left(\frac{x}{\sqrt{a}}\right), \qquad x \neq 0$$

which verifies our guess for $\tilde{m}$ since the KFE for m holds. Our final result follows from

$$\tilde{q}(x) = \frac{\tilde{\Lambda}(x)\tilde{m}(x)}{\tilde{N}} = \frac{\Lambda\left(\frac{x}{\sqrt{a}}\right) m\left(\frac{x}{\sqrt{a}}\right)}{N} \frac{1}{\sqrt{a}} = q\left(\frac{x}{\sqrt{a}}\right) \frac{1}{\sqrt{a}}.$$

■

C Detailed Data Description

In this Appendix we describe the data-cleaning process and provide information on the data coverage. We use granular price data collected from the websites of large multi-channel retailers that sell products both online and in brick-and-mortar stores. The data were collected on a daily basis by PriceStats, a private firm related to the Billion Prices Project at Harvard and MIT. Previous research has shown that price indices constructed with this data can closely track official CPI statistics in many countries (Cavallo and Rigobon (2016)).[20] We focus on the "Food and Beverages" sector and use data for all products sold by some of the largest food retailers in six European countries (France, Germany, Italy, Netherlands, Spain, and the United Kingdom) and the United States. The sample goes from January 1st, 2019, until July 22nd, 2023, and contains daily information on price spell duration and the size of price changes.

Data-cleaning process. To minimize the impact of scraping errors and compositional changes, we keep only the products that remain in the sample for at least 365 days. For each product, we fill all the missing prices with the last available price from the previous spell. We also drop all price changes larger than 1.5 log points and equal to 1 cent in local currency. We filter temporary price discounts (sales) by using a V-shaped algorithm that detects a price drop followed by an equal price increase, back to the original price, within 90 days. When we identify a sale, we replace the discounted price with the last observed pre-sale price to obtain a regular price spell for that product.

Data coverage. The final dataset contains information on 583,788 products from 58 retailers the "Food and Beverages" sector. For each country, we have products from eleven 3-digit COICOP sectors within the 1-digit "Food and Beverages" sector. As Table 2 shows, each product remains over 1000 days in the sample with an average of 8 distinct prices.

D General equilibrium

Equilibrium. The equilibrium is characterized by $\{\hat{v}, \hat{m}, C\}$ a value function for the firms' price-setting problem, a distribution of price gaps, and a path of aggregate consumption such that for each t

[20] Also see Cavallo, A. (2013). Online and official price indexes: Measuring Argentina's inflation. *Journal of Monetary Economics 60*(2), 152-165

Table 2: Data Coverage

	Retailers	Unique Products	Average Prices per Product	Average Product Life
France	12	147,115	9.41	998
Germany	7	65,670	7.26	963
Italy	5	36,490	7.04	988
Netherlands	8	80,614	9.98	1,001
Spain	9	91,184	8.65	1,051
United Kingdom	10	86,295	4.98	1,096
United States	7	76,420	9.05	1,034
All	58	583,788	8.05	1,019

$$\rho\hat{v}(x,t) - \hat{v}_t(x,t) = F(x,t;C(t)) - \mu\,\hat{v}_x(x,t) + \frac{\sigma^2}{2}\,\hat{v}_{xx}(x,t) + \min_{\ell,x^*}\{\ell\,(\hat{v}(x^*,t) - \hat{v}(x,t)) + (\kappa\,\ell)^\gamma\}, \tag{28}$$

$$\hat{m}_t(x,t) = -\,\hat{\lambda}(x,t)\hat{m}(x,t) + \mu\,\hat{m}_x(x,t) + \frac{\sigma^2}{2}\,\hat{m}_{xx}(x,t), \quad x \neq \hat{x}^*(t), \tag{29}$$

$$C(t) = \left(K\left(\frac{E(t)}{W(t)}\right)^{\zeta}\alpha\right)^{-\frac{1}{\epsilon}}\left(\int_{\mathbb{R}} e^{(1-\eta)x}\hat{m}(x,t)dx\right)^{-\frac{1}{\epsilon(1-\eta)}}, \tag{30}$$

where F emphasizes how the firm problem depends on aggregate consumption $C(t)$. Equation (28) takes a path of aggregate demand C and solves for $\hat{v}$ given terminal condition $\lim_{t\to\infty}\hat{v}(x,t) = v(x)$, the steady-state value function. Policies $\hat{\lambda}, \hat{x}^*$ are obtained from equation (28) using the optimal return condition $\hat{v}_x(\hat{x}^*(t),t) = 0$ and the optimal hazard condition in equation (7). Equation (29) takes policies and solves for $\hat{m}$ given initial condition $\hat{m}(x,0) = m(x+\delta)$, the displaced steady-state distribution.

Equilibrium is a fixed point problem. Policies $\hat{\lambda}, \hat{x}^*$ depend on the path C and the distribution depends on a path of policies. Equation (30) is the equilibrium condition coupling the HJB and KFE, requiring that the path C is consistent with both. This is the structure of a mean-field game where optimal decisions and aggregation are coupled only through a finite set of distributional moments, see Alvarez, Lippi, and Souganidis (2023). In this case, only through the aggregate consumption path.

Fixed point problem. Following Golosov and Lucas (2007), we construct an operator Γ over paths C such that a fixed point solves the coupled system of equations described by equations (28)-(30). Consider a path C, then the correspondence Γ maps C into a path ΓC

implied by the HJB and KFE. Specifically, for each t, $(\Gamma C)(t)$ is defined as

$$(\Gamma C)(t) = \left(K \left(\frac{E(t)}{W(t)} \right)^{\zeta} \alpha \right)^{-\frac{1}{\epsilon}} \left(\int_{\mathbb{R}} e^{(1-\eta)x} \hat{m}(x,t;C) dx \right)^{-\frac{1}{\epsilon(1-\eta)}}.$$

where $\hat{m}$ emphasizes the dependence on the input path C. Aggregate consumption path $C = \Gamma C$ and associated $\hat{v}$, $\hat{m}$ solve equations (28) to (30).

Computation. To compute the equilibrium we use standard finite difference methods (Achdou et al., 2021). Iteration on the operator Γ converges in few steps. We set the two additional parameters of labor disutility and relative risk aversion to standard parameters $\alpha = 6$ and $\epsilon = 2$, following Golosov and Lucas (2007).

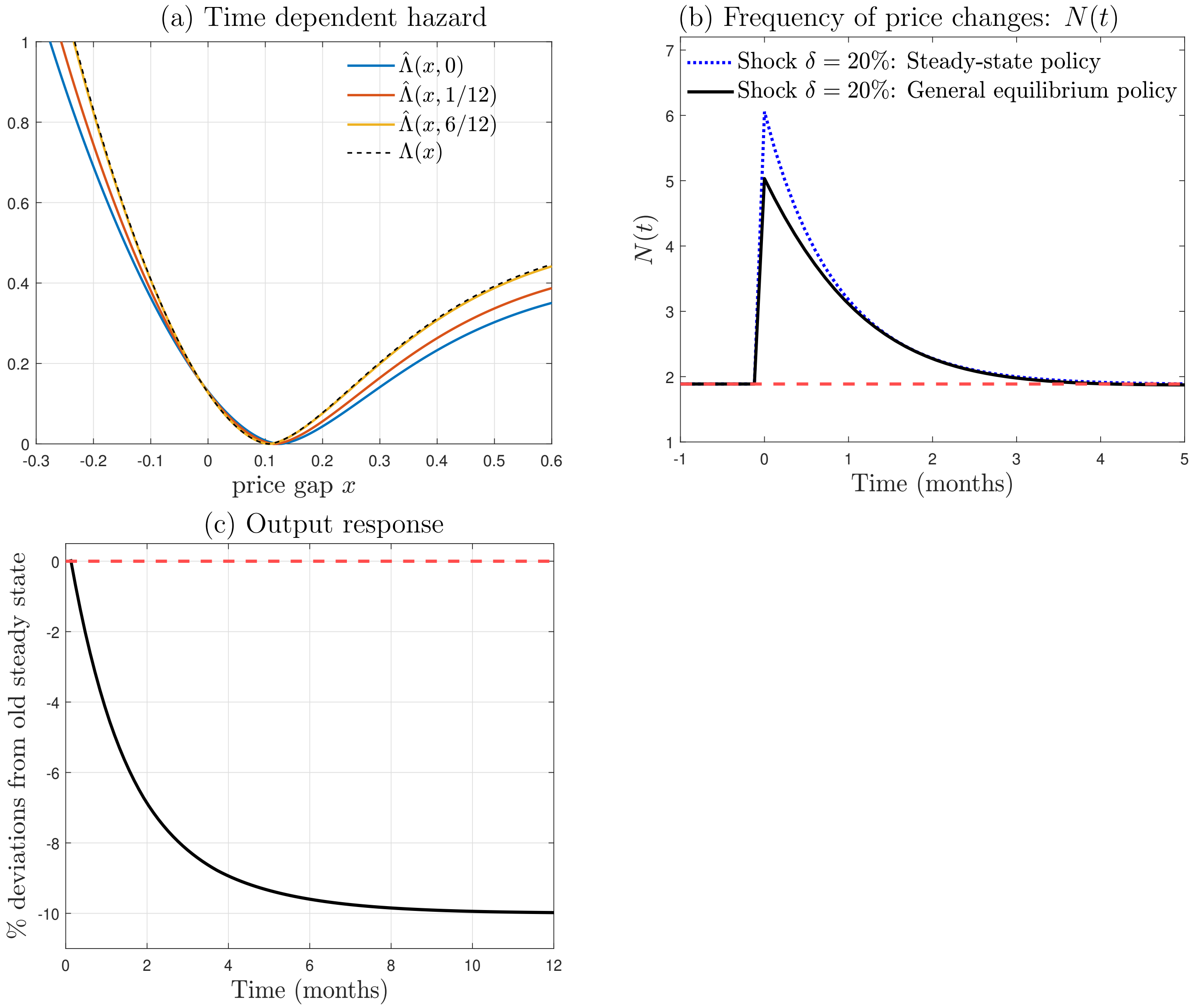
A shock to energy prices increases marginal cost by $\delta = 20\%$. We assume that energy prices and money grow at rate μ. Together these imply that (i) the shock to nominal marginal cost is permanent, drifting at rate μ for $t > 0$ and (ii) the relative price of energy to wages $E(t)/W(t)$ increases permanently.

Results. Panel (a) and (b) of Figure 5 indicate that the general equilibrium response of frequency is very close to one using steady-state policy rules. Panel (a) depicts the hazards, as a monthly probability, at several points in time, in yearly units, and shows that they are close to the stationary hazard (dashed). Panel (b) shows that the response of frequency is slightly dampened due to a lower hazard at impact but reaches a comparable peak response of 5 price changes per year.

Panel (c) shows the response of output. At impact, the increase in relative costs $E(t)/W(t)$ exactly offsets the increase in demand due to low price gaps. Afterwards, elevated relative costs and rising prices $P(t)$ generate a permanent decrease in output. This is a canonical permanent supply shock.

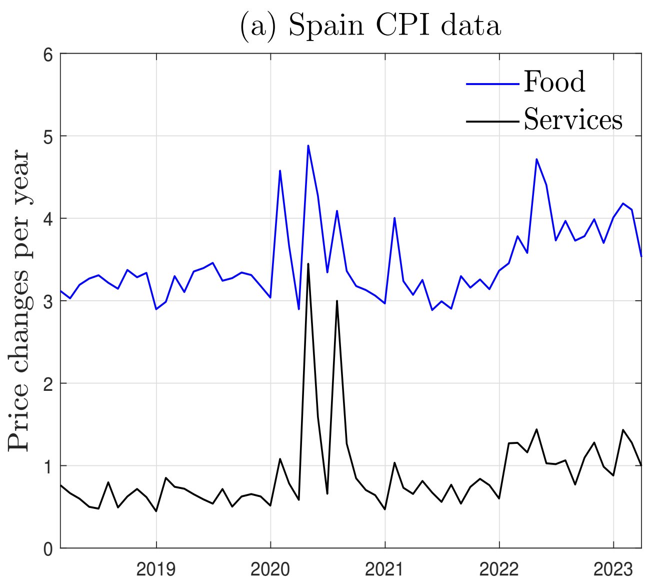
The slightly dampened response of frequency is explained by the general equilibrium effects on the flow cost function (i.e. on the profit function). Equation (26) indicates that flow costs increase with output (are more negative) and decrease with real marginal costs, with an elasticity of 1 and $\eta - 1 = 5$, respectively. Real marginal costs (in equation (25)) increase by δ at impact and gradually revert to steady state, whereas output (in equation (24)) is unresponsive at impact and gradually declines by δ/ϵ relative to the pre-shock level. Quantitatively, the real marginal cost drives the dynamics of the flow cost, reducing the marginal value of repricing efforts, and thus the hazard (see panel (a)). This in turn, marginally dampens the response of frequency.

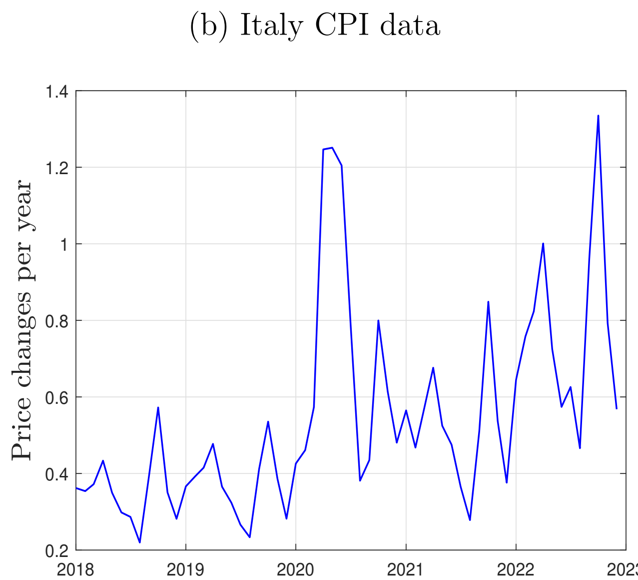
Figure 5: Propagation of a Large Shock in General Equilibrium



E More evidence on increased frequency of price changes

Figure 6: Increase in Frequency: Aggregate data





Sources: Left panel, Instituto Nacional de Estadistica and Bank of Spain staff calculations. Right panel, Istat and Bank of Italy staff calculations.