Is There a Stable Relationship between Unemployment and Future Inflation?*

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Appendix
For Online Publication

A An Old Keynesian Model

In this appendix, we present a slightly modified version of the model used by Taylor (1999) and discussed in Cochrane (2011). We show that model delivers a reduce form like the one analyzed in Section ??. The model specifies a NAIRU-type Phillips curve, where the growth rate of inflation holds a negative linear relationship with the difference between the current unemployment rate and a constant level (known as the natural rate of unemployment). Thus, we write

$$\pi_t - \pi_{t-1} = -\gamma\left(u_{t-1} - u\right) - \varepsilon(u_t - u) + e_t^{\pi},$$

where π_t is the inflation rate; u_t is the unemployment rate; γ, u, and ε are positive parameters, and e_t^{π} is a shock. This is the same equation used by Taylor (1999), except that he assumes $\varepsilon = 0$.[1] This assumption implies that the policy rate has no immediate effect on the inflation rate. By letting $\varepsilon > 0$, albeit it is small, we allow for that immediate effect.

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[1]Taylor's model is expressed in terms of output deviations instead of unemployment deviations. Our specification implies a negative linear relationship between output deviations and unemployment deviations.

The second equation establishes a negative linear relationship between unemployment and the difference between the policy interest rate and the inflation rate, so we write

$$u_t = \sigma(i_t - \pi_t - r) + e_t^u,$$

where σ, r are positive parameters and e_t^u is a shock.[2]

In what follows, we interpret the unemployment rate as deviations from its steady state level u, or, equivalently, we set $u = 0$.

Using the second in the first, we have

$$\pi_t = \pi_{t-1} - \gamma u_{t-1} - \varepsilon \left(\sigma(i_t - \pi_t - r) + e_t^u\right) + e_t^\pi$$

or

$$\pi_t = \frac{\pi_{t-1} - \gamma u_{t-1} + \sigma\varepsilon r - \sigma\varepsilon i_t - \varepsilon e_t^u + e_t^\pi}{(1 - \varepsilon\sigma)},$$

whereas using the first in the second, we have

$$u_t(1 - \varepsilon\sigma) = \sigma(i_t - \pi_{t-1} + \gamma u_{t-1} - e_t^\pi - r) + e_t^u$$

$$u_t = -\frac{\sigma}{(1 - \sigma\varepsilon)}\pi_{t-1} + \frac{\sigma\gamma}{(1 - \sigma\varepsilon)}u_{t-1} + \frac{\sigma i_t - \sigma r - \sigma e_t^\pi + e_t^u}{(1 - \sigma\varepsilon)}.$$

Thus, we can write the system as

$$\begin{aligned} \begin{bmatrix} \pi_t \\ u_t \end{bmatrix} &= \begin{bmatrix} \frac{1}{(1-\varepsilon\sigma)} & -\frac{\gamma}{(1-\varepsilon\sigma)} \\ -\frac{\sigma}{(1-\sigma\varepsilon)} & \frac{\sigma\gamma}{(1-\sigma\varepsilon)} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} -\frac{\sigma\varepsilon}{(1-\varepsilon\sigma)} \\ \frac{\sigma}{(1-\varepsilon\sigma)} \end{bmatrix} (i_t - r) + \\ &\quad \begin{bmatrix} -\frac{\varepsilon}{(1-\varepsilon\sigma)} & \frac{1}{(1-\varepsilon\sigma)} \\ \frac{1}{(1-\sigma\varepsilon)} & -\frac{\sigma}{(1-\sigma\varepsilon)} \end{bmatrix} \begin{bmatrix} e_t^u \\ e_t^\pi \end{bmatrix}. \end{aligned}$$

Recall that we had assumed that $\varepsilon > 0$, albeit it is small. Thus, the coefficient of unemployment in the inflation equation is close to $-\gamma$, which is the slope of the NAIRU Phillips curve.

[2]To the extent that the term in parentheses on the right-hand side of this equation aims at capturing movements in the real interest rate as deviations from r (presumably its steady state value), the fact that π_t rather than $E_t\pi_{t+1}$ is in this equation may appear surprising. However, as we show below, this equation—with a reinterpretation of the parameters—will arise exactly as the solution in any case, as long as ε is zero. Given the lack of microfoundations, this reinterpretation seems innocuous to us.

A.1 The Interest Rate Rule

If we assume, as Taylor (1999) and Cochrane (2011) do, that

$$i_t = r + \phi_\pi \pi_t + \phi_y y_t,$$

then the solution is

$$\begin{bmatrix} \pi_t \\ u_t \end{bmatrix} = \begin{bmatrix} \frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon} & -\frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\gamma \\ \frac{\sigma(\phi_\pi-1)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon} & -\frac{\sigma(\phi_\pi-1)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\gamma \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ u_{t-1} \end{bmatrix} + \frac{1}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon} \begin{bmatrix} -\varepsilon & 1 \\ 1 & \sigma(\phi_\pi-1) \end{bmatrix} \begin{bmatrix} e_t^u \\ e_t^\pi \end{bmatrix}.$$

The two roots are given by

$$\begin{aligned} \lambda_1\lambda_2 &= 0 \\ \lambda_1 + \lambda_2 &= \frac{(1+\sigma\phi_u)-(\phi_\pi-1)\sigma\gamma}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}, \end{aligned}$$

so one root is zero, and the other is given by

$$\frac{(1+\sigma\phi_u)-(\phi_\pi-1)\sigma\gamma}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon},$$

which is less than one as long as $\phi_\pi > 1$, as described in Taylor (1999). Therefore, the system has a unique bounded solution.

A.2 Characterizing the Optimal Policy Rule

Recall that the solution is given by

$$\begin{bmatrix} \pi_t \\ u_t \end{bmatrix} = \begin{bmatrix} \frac{1}{(1-\varepsilon\sigma)} & -\frac{\gamma}{(1-\varepsilon\sigma)} \\ -\frac{\sigma}{(1-\sigma\varepsilon)} & \frac{\sigma\gamma}{(1-\sigma\varepsilon)} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} -\frac{\sigma\varepsilon}{(1-\varepsilon\sigma)} \\ \frac{\sigma}{(1-\varepsilon\sigma)} \end{bmatrix} (i_t - r) + \begin{bmatrix} -\frac{\varepsilon}{(1-\varepsilon\sigma)} & \frac{1}{(1-\varepsilon\sigma)} \\ \frac{1}{(1-\sigma\varepsilon)} & -\frac{\sigma}{(1-\sigma\varepsilon)} \end{bmatrix} \begin{bmatrix} e_t^u \\ e_t^\pi \end{bmatrix},$$

so, in the notation of the paper,

$$\pi_{t+1} = a + b\pi_t + cu_t + di_t + \xi_t^\pi,$$

so

$$b = \frac{1}{(1-\varepsilon\sigma)}, c = -\frac{\gamma}{(1-\varepsilon\sigma)}, d = -\frac{\sigma\varepsilon}{(1-\varepsilon\sigma)},$$

and the optimal policy is

$$i_t^{Opt} = \frac{1}{d}\left[\pi_{t+1}^* - \left(a + b\pi_t + cu_t + E_t\xi_{t+1}^{\pi}\right)\right],$$

so

$$i_t^{Opt} = \frac{1}{-\frac{\sigma\varepsilon}{(1-\varepsilon\sigma)}}\left[\pi_{t+1}^* - \left(a + \frac{1}{(1-\varepsilon\sigma)}\pi_t + -\frac{\gamma}{(1-\varepsilon\sigma)}u_t + E_t\xi_{t+1}^{\pi}\right)\right]$$

or

$$i_t^{Opt} = \left[-\frac{(1-\varepsilon\sigma)}{\sigma\varepsilon}\pi_{t+1}^* + \frac{(1-\varepsilon\sigma)}{\sigma\varepsilon}a + \frac{1}{\sigma\varepsilon}\pi_t - \frac{\gamma}{\sigma\varepsilon}u_t - E_t\xi_{t+1}^{\pi}\right].$$

Thus, as long as $\sigma\varepsilon < 1$, which will hold for small values of ε, the conditions for a unique stable solution are satisfied.

A.3 The Reduced Form Parameter versus the Structural Form Parameter

The solution of the model is given by

$$\begin{bmatrix} \pi_t \\ u_t \end{bmatrix} = \begin{bmatrix} \frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon} & -\frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\gamma \\ \frac{\sigma(\phi_\pi-1)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon} & -\frac{\sigma(\phi_\pi-1)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\gamma \end{bmatrix}\begin{bmatrix} \pi_{t-1} \\ u_{t-1} \end{bmatrix} + \frac{1}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\begin{bmatrix} -\varepsilon & 1 \\ 1 & \sigma(\phi_\pi-1) \end{bmatrix}\begin{bmatrix} e_t^u \\ e_t^{\pi} \end{bmatrix},$$

so we can write the solution for inflation as

$$\pi_t = \frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\pi_{t-1} - \frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\gamma u_{t-1} + \frac{e_t^{\pi} - \varepsilon e_t^u}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}.$$

Thus, the reduced form parameter $\widehat{\gamma}$ is equal to

$$\widehat{\gamma} = \frac{(1+\sigma\phi_u)}{(1+\sigma\phi_u)+\sigma(\phi_\pi-1)\varepsilon}\gamma = \frac{1}{1+\varepsilon\frac{\sigma(\phi_\pi-1)}{(1+\sigma\phi_u)}}\gamma,$$

so it is lower than the structural parameter γ but arbitrarily close when ε is close to zero.

B Regression Specifications

In this section, we present more details on the empirical specifications presented in Section ??. In Section B.1, we provide the list of variables that are used as controls in Table ?? and their sources. In Section B.2, we present the regressions models we adopt in the main text. Section B.3 shows additional results in the reduced form analysis.

B.1 Control Variables

- ***rgdp***: state-level real GDP growth relative to national average in the same period, trend term of HP filtered series with smoothing parameter 400. Source: Bureau of Economic Analysis (BEA) `https://apps.bea.gov/regional/downloadzip.cfm`, all MSAs since 1960.

- ***temp***: MSA-level temperature relative to regional average in 1960 and 2018. Source: National Centers for Environmental Information (NCEI), `https://www.ncdc.noaa.gov/cag/`, all MSAs since 1960, except for Kansas City since 1972 and Honolulu since 1965.

- ***prec***: MSA-level precipitation relative to regional average in 1960 and 2018. Source of variable ***prec*** is the same as ***temp***.

- ***infExp***: division-level inflation expectation relative to national average in the same period. Source: Survey of Consumers from the University of Michigan, `https://data.sca.isr.umich.edu/sda-public/cgi-bin/hsda?harcsda+sca`, all divisions since 1978.

- ***bartik***: interaction of regional exposure variable (combining regional industrial employment composition and government expenditure shipment by industry) with a measure of the growth rate of real aggregate federal government consumption. Source: constructed following McLeay and Tenreyro (2020).

The variable x in region i in period t is denoted by x_t^i, we further define its cross-sectional deviation from US average Δx_t^i and its deviation from 1960–2018 regional average $\Delta^R x_t^i$ as

$$\Delta x_t^i = x_t^i - X_t^{US}$$
$$\Delta^R x_t^i = x_t^i - \frac{1}{N}\sum_{t=1960}^{2018} x_t^i$$

B.2 Regression Specifications

- We specify the OLS regression models without controls in the following form:

$$\pi_{t+1}^i = b\Delta u_t^i + c\Delta\pi_t^i + \sum_s \mathbb{I}\{t=s\}\alpha_s.$$

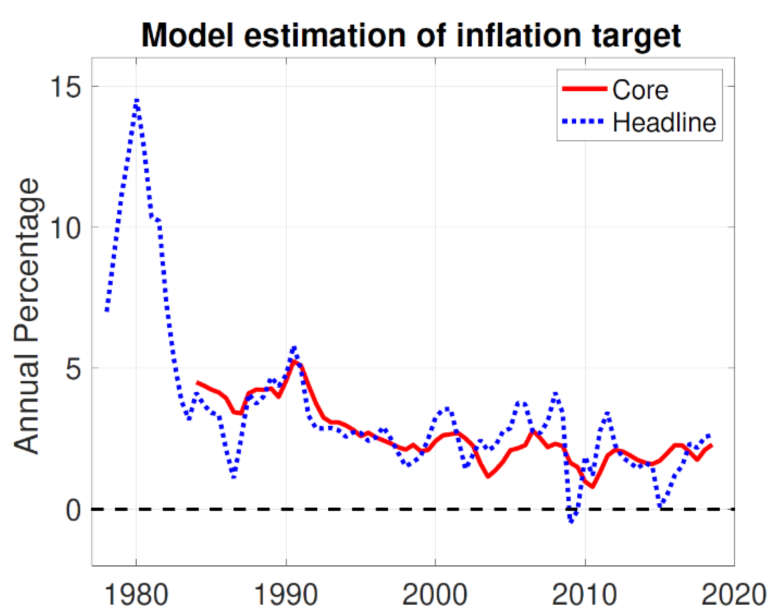
- For the 2SLS without control regression models, we use Δu_{t-1}^i as instruments for the first stage.

- For the regression models with controls (both OLS and 2SLS), we extend the models without controls to include $\Delta^R temp_t^i$, $\Delta^R prec_t^i$, $\Delta infExp_t^i$, Δu_{t-2}^i, $\Delta\pi_{t-1}^i$, $\Delta\pi_{t-2}^i$, and bartik_t as explanatory variables. However, we can use the Bartik starting in 1990 only, owing to the availability of data. We show that when adding this control to the sub-samples from 1990 onwards, the results do not change for headline or core inflation.

B.3 Full Reduced Form Results

In this subsection, we present a complete set of results corresponding to the regression analysis of Section ??. First, we show the estimated value for the time dummy, which corresponds to the estimate of the inflation target. We report the results using both core and headline inflation in Figure B.1.

Figure B.1: Model Estimation of Inflation Target



In Tables 7 to 9, we report complete results for the regressions using headline inflation, including OLS and 2SLS, with and without controls. We also show, in Table 10, results including the Bartik variable as a control, which we have only since the late 80s. As we show, the results barely change when including that additional control for the period in which we have data. We report only the case of 2SLS with controls, but the results are also robust for the other specifications and also when we use core inflation rather than headline. We then present the results for OLS and 2SLS with and without controls when using core in tables 11 to 13.

Table B.1: Headline without Controls

Model	Coefficient	1977-1984	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018	1977-2018
OLS	c	-0.31**	-0.41**	-0.31**	-0.24**	-0.21**	-0.28**	-0.28**
		(0.11)	(0.11)	(0.05)	(0.06)	(0.07)	(0.03)	(0.04)
	b	-0.17**	0.00	0.21**	0.05	0.04	0.22**	0.10**
		(0.06)	(0.06)	(0.06)	(0.06)	(0.07)	(0.04)	(0.05)
	Overall R^2	0.83	0.69	0.45	0.70	0.51	0.72	0.88
2SLS	c	-0.39**	-0.29*	-0.46**	-0.21**	-0.24**	-0.24**	-0.27**
		(0.12)	(0.15)	(0.13)	(0.08)	(0.08)	(0.03)	(0.04)
	b	-0.18**	0.05	0.14	0.06	0.03	0.24**	0.10**
		(0.06)	(0.09)	(0.09)	(0.06)	(0.07)	(0.05)	(0.05)
	Overall R^2	0.79	0.71	0.39	0.70	0.51	0.74	0.88
Observations		377	288	492	536	362	1678	2055

* significant at 5% level, ** significant at 1% level

C Data

C.1 Description of Data for Reduced Form Exercises

This appendix describes our data sources and calculations for the reduced form exercises. We analyze semiannual CPI inflation and unemployment data for the United States and for 27 metropolitan statistical areas (MSAs). All semiannual data for unemployment and CPI price indices are computed as the arithmetic average of monthly data for the first and second half of each year. Inflation and price data for MSAs are available only as non seasonally adjusted, so all the data are not seasonally adjusted.

Table B.2: Headline OLS with Controls

Coefficient	1977-1984	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018	1977-2018
c	-0.19	-0.43**	-0.14*	-0.26**	-0.17	-0.28**	-0.27**
	(0.18)	(0.09)	(0.07)	(0.07)	(0.10)	(0.04)	(0.07)
b	-0.30**	-0.01	0.19**	0.06	0.02	0.23**	0.08
	(0.10)	(0.08)	(0.06)	(0.06)	(0.07)	(0.04)	(0.04)
$e(infExp)$	-0.30**	-0.01	0.19**	0.06	0.02	0.23**	0.08
	(0.10)	(0.08)	(0.06)	(0.06)	(0.07)	(0.04)	(0.04)
$e(prec)$	-0.20	0.00	0.03	-0.01	0.07*	0.05	0.00
	(0.13)	(0.06)	(0.04)	(0.05)	(0.03)	(0.03)	(0.03)
$e(temp)$	0.19**	0.01	-0.02	-0.09*	0.07	0.00	0.00
	(0.07)	(0.06)	(0.03)	(0.04)	(0.04)	(0.01)	(0.02)
$e(u(-1))$	-0.21	-0.02	-0.27*	-0.02	-0.10	-0.07	-0.06
	(0.15)	(0.12)	(0.12)	(0.09)	(0.11)	(0.05)	(0.06)
$e(u(-2))$	-0.04	0.02	0.09	0.03	0.12	0.12*	0.08
	(0.15)	(0.07)	(0.11)	(0.12)	(0.08)	(0.05)	(0.05)
$e(\pi(-1))$	-0.22*	-0.23**	0.06	-0.09	-0.03	0.03	0.05
	(0.11)	(0.08)	(0.05)	(0.06)	(0.06)	(0.03)	(0.04)
$e(\pi(-2))$	-0.23**	-0.11	-0.09	-0.02	0.09	-0.01	0.00
	(0.06)	(0.08)	(0.05)	(0.06)	(0.06)	(0.03)	(0.02)
Overall R^2	0.80	0.64	0.46	0.70	0.54	0.73	0.88
Observations	327	288	484	532	362	1666	1993

* significant at 5% level,   ** significant at 1% level

Table B.3: Headline 2SLS with Controls

Coefficient	1977-1984	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018	1977-2018
c	-0.50**	-0.45**	-0.45**	-0.28**	-0.28*	-0.35**	-0.33**
	(0.19)	(0.14)	(0.10)	(0.10)	(0.13)	(0.05)	(0.05)
b	-0.29**	-0.02	0.14	0.06	0.01	0.22**	0.08
	(0.09)	(0.09)	(0.08)	(0.06)	(0.08)	(0.04)	(0.04)
$e(infExp)$	-0.04	-0.07	0.17	0.11	0.08	0.16	-0.13
	(0.33)	(0.18)	(0.17)	(0.25)	(0.23)	(0.11)	(0.15)
$e(prec)$	-0.20	0.00	0.04	-0.01	0.08**	0.05	0.00
	(0.13)	(0.06)	(0.04)	(0.05)	(0.03)	(0.03)	(0.03)
$e(temp)$	0.20**	0.01	-0.02	-0.09*	0.08*	0.00	0.00
	(0.08)	(0.06)	(0.03)	(0.04)	(0.03)	(0.01)	(0.02)
$e(u(-2))$	-0.06	0.02	0.00	0.02	0.09	0.10**	0.07
	(0.14)	(0.06)	(0.10)	(0.11)	(0.07)	(0.04)	(0.04)
$e(\pi(-1))$	-0.20*	-0.22**	0.07	-0.09	-0.03	0.03	0.05
	(0.10)	(0.08)	(0.06)	(0.06)	(0.06)	(0.03)	(0.04)
$e(\pi(-2))$	-0.21**	-0.10	-0.09	-0.02	0.09	-0.01	0.00
	(0.07)	(0.08)	(0.05)	(0.05)	(0.06)	(0.03)	(0.02)
Overall R^2	0.76	0.65	0.40	0.70	0.54	0.74	0.88
Observations	327	288	484	532	362	1666	1993

* significant at 5% level, ** significant at 1% level

Table B.4: Headline 2SLS with Controls Including Bartik Variable

Coefficient	1991-2000	2001-2010	2011-2018	1985-2018
c	-0.47**	-0.29**	-0.28*	-0.36**
	(0.11)	(0.10)	(0.13)	(0.06)
b	0.17*	0.06	0.00	0.20**
	(0.08)	(0.06)	(0.08)	(0.04)
$e(infExp)$	0.13	0.11	0.09	0.14
	(0.16)	(0.25)	(0.24)	(0.11)
$e(prec)$	0.06	-0.01	0.07**	0.04
	(0.03)	(0.05)	(0.03)	(0.03)
$e(temp)$	-0.02	-0.09*	0.07*	0.00
	(0.03)	(0.05)	(0.03)	(0.01)
$e(bartik)$	-1.09	3.59	2.41	-0.37
	(5.60)	(5.52)	(2.98)	(2.87)
$e(u(-2))$	0.04	0.02	0.09	0.11*
	(0.09)	(0.11)	(0.07)	(0.05)
$e(\pi(-1))$	0.07	-0.10	-0.03	0.02
	(0.05)	(0.06)	(0.06)	(0.04)
$e(\pi(-2))$	-0.06	-0.02	0.09	0.01
	(0.05)	(0.05)	(0.06)	(0.02)
Overall R^2	0.55	0.69	0.54	0.70
Observations	532	532	362	1426

* significant at 5% level, ** significant at 1% level

Table B.5: Core – without Controls

Model	Coefficient	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018
OLS	c	-0.47**	-0.33**	-0.34**	-0.29**	-0.32**
		(0.10)	(0.05)	(0.07)	(0.09)	(0.04)
	b	0.09	0.23**	0.10	0.06	0.26**
		(0.06)	(0.08)	(0.06)	(0.08)	(0.04)
	Overall R^2	0.41	0.36	0.34	0.10	0.61
2SLS	c	-0.34**	-0.41**	-0.25**	-0.27**	-0.24**
		(0.15)	(0.13)	(0.08)	(0.10)	(0.04)
	b	0.13	0.20	0.12*	0.07	0.30**
		(0.11)	(0.10)	(0.06)	(0.07)	(0.04)
	Overall R^2	0.42	0.34	0.36	0.13	0.63
Observations		288	492	536	362	1678

* significant at 5% level, ** significant at 1% level

Table B.6: Core OLS with Controls

Coefficient	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018
c	-0.44**	-0.21*	-0.37**	-0.30**	-0.36**
	(0.09)	(0.08)	(0.07)	(0.11)	(0.05)
b	0.10	0.21**	0.12*	0.03	0.26**
	(0.09)	(0.08)	(0.06)	(0.07)	(0.03)
$e(infExp)$	0.08	0.12	0.11	0.26	0.17
	(0.22)	(0.17)	(0.30)	(0.29)	(0.12)
$e(prec)$	0.00	0.02	0.03	0.07	0.05*
	(0.07)	(0.04)	(0.04)	(0.05)	(0.02)
$e(temp)$	0.03	-0.06	-0.08	0.00	-0.03*
	(0.06)	(0.04)	(0.05)	(0.05)	(0.01)
$e(u(-1))$	-0.10	-0.22	0.00	0.01	-0.06
	(0.10)	(0.15)	(0.11)	(0.12)	(0.07)
$e(u(-2))$	0.13	0.14	0.04	0.04	0.16**
	(0.10)	(0.13)	(0.14)	(0.11)	(0.05)
$e(\pi(-1))$	-0.17	-0.09*	-0.07	0.07	0.06*
	(0.09)	(0.04)	(0.06)	(0.08)	(0.03)
$e(\pi(-2))$	-0.08	-0.07	-0.10	0.01	-0.03
	(0.08)	(0.05)	(0.06)	(0.07)	(0.02)
Overall R^2	0.37	0.41	0.34	0.13	0.63
Observations	260	484	532	362	1638

* significant at 5% level, ** significant at 1% level

Table B.7: Core 2SLS with Controls

Coefficient	1985-1990	1991-2000	2001-2010	2011-2018	1985-2018
c	-0.57**	-0.46**	-0.37**	-0.29	-0.41**
	(0.15)	(0.12)	(0.13)	(0.16)	(0.06)
b	0.08	0.18*	0.12*	0.04	0.25**
	(0.09)	(0.08)	(0.06)	(0.07)	(0.03)
$e(infExp)$	0.03	0.13	0.11	0.27	0.16
	(0.19)	(0.17)	(0.30)	(0.29)	(0.13)
$e(prec)$	0.00	0.03	0.03	0.07	0.06*
	(0.07)	(0.04)	(0.04)	(0.05)	(0.02)
$e(temp)$	0.04	-0.06	-0.08	0.00	-0.03*
	(0.06)	(0.04)	(0.05)	(0.05)	(0.01)
$e(u(-2))$	0.09	0.07	0.04	0.04	0.14**
	(0.11)	(0.11)	(0.13)	(0.09)	(0.04)
$e(\pi(-1))$	-0.17	0.10*	-0.07	0.07	0.06*
	(0.09)	(0.05)	(0.06)	(0.08)	(0.03)
$e(\pi(-2))$	-0.08	-0.07	-0.10	0.01	-0.03
	(0.08)	(0.05)	(0.06)	(0.07)	(0.02)
Overall R^2	0.37	0.38	0.35	0.15	0.64
Observations	260	484	532	362	1638

* significant at 5% level, ** significant at 1% level

C.1.1 Inflation Data

The Bureau of Labor Statistics (BLS) publishes CPI data for 27 MSAs. The BLS publishes semiannual data for 13 MSAs and higher frequency data (monthly or bimonthly) for the other 14 MSAs. We use semiannual data to obtain the largest possible sample. Headline CPI is available back to 1941 for 23 MSAs, with data for the remaining MSAs starting in 1977, 1987, 1997, and 2002. Core CPI is available back to 1982 for 24 MSAs, with data for the remaining MSAs starting in 1987, 1997, and 2002. When semiannual data are not available as a published series, we compute the semiannual average following BLS methodology: first, interpolate the missing monthly indices using a geometric mean of values in adjacent months; second, calculate the arithmetic average of the monthly data in the first and second half of each year.

C.1.2 Unemployment Data

The BLS publishes a monthly unemployment rate, not seasonally adjusted, for each of the 27 MSAs with corresponding CPI price indices. Published BLS data are available back to 1990. The BLS has unpublished unemployment data back to 1976, but these data are not consistent with the published data because of changes in the MSA geographic definitions and other factors. However, the BLS also has unemployment and labor force data by county, going back to 1976. We used the county-level data to construct a geographically consistent definition of MSAs, going back to 1976. The constructed unemployment and labor force series overlap very closely with the published data in the post-1990 period. We combine our pre-1990 constructed unemployment rates with the published data to obtain unemployment rate series back to 1976. The lack of readily accessible unemployment data before 1976 is a limiting factor for our analysis.

C.2 Description of Data for Structural Model Estimation

C.2.1 State Level

We use the MSA-level inflation data, described above, and map the 27 MSA regions into 20 states with the mapping in Table C.8. For states which contain multiple MSA regions (for example, Cincinnati and Cleveland are both in Ohio), we select only the data of one of the MSA regions. In our final inflation dataset used in estimation, we drop inflation data for the Phoenix MSA, Boston MSA, and Baltimore MSA regions, as there is no inflation data for these MSAs prior to 1998. Finally, we also note that inflation data is not available in

1977:H1 to 1978:H1 for Miami.

Table C.8: MSA to State Mapping

State	MSA
AK	Anchorage
AZ	Phoenix
CA	Los Angeles
CO	Denver
FL	Miami
GA	Atlanta
HI	Honolulu
IL	Chicago
KS	Kansas City
MA	Boston
MD	Baltimore
MI	Detroit
MO	St. Louis
NY	New York
OH	Cincinnati
OR	Portland
PA	Philadelphia
TX	Dallas
WA	Seattle
WI	Minneapolis

For the other state-level data series, we use state-level data on employment, output, and compensation. The observed state data are annual. To construct the data, we first take each state's series relative to its initial value, compute the deviation of each state's observation from the state mean, regress that series on time dummies, weighted by the state's relative population, and work with the residuals. We then take out a linear trend from the resulting series, for each subsample studied.

Main estimations Here, we provide more details on each series.

- Output: We use state-level data on Gross Domestic Product in current dollars. (BEA SAGDP2S). The data are available for download at the BEA website.
- Employment: We use state-level data on total employment from the BEA annual table SA4. In our empirical analysis, we scale this measure of employment by each state's population.

- Labor Compensation: We use state-level data on compensation of employees from the BEA annual table SA6N.

- Wages: To construct our wages series, we divide total labor compensation by the number of employed individuals, using the two series described above.

- Population: We use state-level data on population from the BEA annual table SA1-3.

Robustness Exercises In robustness exercises, we use the following data series.

- Income: We use state-level data on personal income from the BEA annual table SA4.

- Household Debt: We use data from the FRBNY Consumer Credit Panel Q4 State statistics by year. Our measures of debt include auto loans, credit card debt, mortgage debt and student loans. This database also provides information on the number of individuals with credit scores in each state, which we use to express the debt data in per capita terms. We then construct a debt-to-income series by dividing this measure of per capita debt by per capita income, using the data described above on income and population from the BEA.

- House Prices: We used data on the not seasonally adjusted house price index available on the FHFA website.

- Consumption: For the robustness exercise with consumption, we use state-level data on total personal consumption expenditures by state from the BEA, net of housing. The data are available for download at the BEA website.

C.2.2 Aggregate Level

At the aggregate level, we use the GDP deflator for inflation, employment, output, wages, the Fed Funds rates, and ZLB durations from NY Federal Reserve Survey Data. The codes for each raw data series are as follows:

- Gross Domestic Product: Implicit Price Deflator (GDPDEF).

- Gross Domestic Product: (GDP).

- Cumulated nonfarm business section compensation (PRS85006062) minus employment growth (PRS85006012) and deflated by the GDP deflator.

- Total employment net of construction, over the civilian noninstitutional population.

In robustness exercises, we use:

- Household Debt from FRED (code CMDEBT) deflated by PCE deflator, and expressed relative to income (from the BEA Table 2.1).
- House Prices from Case-Logic.
- Personal Consumption Expenditures (BEA Table 2.4.5U). Current, \$. We subtract housing from consumption.

Fed Funds rate: the interest rate is the Federal Funds Rate, taken from the Federal Reserve Economic Database.

ZLB Durations: we follow the approach of Kulish, Morley and Robinson (2017) and use the ZLB durations extracted from the New York Federal Reserve Survey of Primary Dealers, conducted eight times a year from 2011Q1 onwards.[3] We take the mode of the distribution implied by these surveys. Before 2011, we use responses from the Blue Chip Financial Forecasts survey.

D Structural Model

The model description follows Jones, Midrigan and Philippon (2022). We describe the model with the full operative credit channel. But we note that absent this credit channel and the tradeable production structure, the model would reduce to the familiar 3-equation New Keynesian model.

D.1 Full Model with Credit Channel

Household problem The economy consists of a continuum of ex ante identical islands of measure 1 that belong to a trading bloc in a monetary union. Consumers on each island derive utility from the consumption of a final good, leisure, and housing. Let s index an individual island and $p_t(s)$ denote the price of the final consumption good. Individual households on each island belong to labor unions that sell differentiated varieties of labor. We assume perfect risk-sharing across households belonging to different labor unions on a given island. Labor is immobile across islands and the housing stock on each island is in

[3] See the website here. For example, in the survey conducted on January 18 2011, one of the questions asked was: "Of the possible outcomes below, please indicate the percent chance you attach to the timing of the first federal funds target rate increase" (Question 2b). Responses were given in terms of a probability distribution across future quarters.

fixed supply. The problem of a household that belongs to labor union ι is to

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{j=0}^{t-1} \beta_j(s) \right) \left[\int_0^1 v_{it}(s) \log\left(c_{it}(s)\right) \mathrm{d}i + \eta_t^h(s) \log\left(h_t(s)\right) - \frac{\eta_t^n(s)}{1+\nu} n_t(\iota, s)^{1+\nu} \right], \quad (1)$$

where $h_t(s)$ is the amount of housing the household owns, $n_t(\iota, s)$ is the amount of labor it supplies, and $c_{it}(s)$ is the consumption of an individual member i. The term $v_{it}(s) \geq 1$ represents a taste shifter, an i.i.d random variable drawn from a Pareto distribution:

$$\Pr(v_{it}(s) \leq v) = F(v) = 1 - v^{-\alpha}. \quad (2)$$

Here, $\alpha > 1$ determines the amount of uncertainty about v. A lower α implies more uncertainty. The terms $\eta_t^h(s)$ and $\eta_t^n(s)$ affect the preference for housing and the disutility from work, while $\beta_t(s)$ is the household's one-period-ahead discount factor. We assume that each of these preference shifters have an island-specific component and an aggregate component, all of which follow AR(1) processes with independent Gaussian innovations. The household's budget constraint is:

$$p_t(s)x_t(s) + e_t(s)(h_{t+1}(s) - h_t(s)) = w_t(\iota, s)n_t(\iota, s) + q_t l_t(s) - b_t(s) + (1 + \gamma q_t)a_t(s) + T_t(\iota, s), \quad (3)$$

where $x_t(s)$ are transfers made to individual members in the goods market, $e_t(s)$ is the price of housing, $w_t(\iota, s)$ is the wage rate, and $T_t(\iota, s)$ collects the profits households earn from their ownership of intermediate goods firms, transfers from the government aimed at correcting the steady state markup distortion, and the transfers stemming from the risk-sharing arrangement.[4] We let $a_t(s)$ denote the amount of coupon payments the household is entitled to receive in period t, $b_t(s)$ the amount it must repay, and q_t the economy-wide price of the securities described below. Thus, $q_t a_t(s)$ represents the household's total asset holdings (savings), while $q_t b_t(s)$ represents its outstanding debt. We describe a household's holdings of the security by recording the amount of coupon payments b_t that the household has to make period t. Letting $l_t(s)$ denote the amount of securities the household sells in period t, the date $t+1$ coupon payments are

$$b_{t+1}(s) = \sum_{i=0}^{\infty} \gamma^i l_{t-i}(s) = l_t(s) + \gamma b_t(s). \quad (4)$$

The household also faces a liquidity constraint limiting the consumption of an individual

[4]We assume that households on island s exclusively own firms on that particular island.

member to be below the amount of real balances the member holds:

$$p_t(s)c_{it}(s) \leq p_t(s)x_t(s). \tag{5}$$

The household also faces a borrowing constraint

$$q_t l_t(s) \leq m_t(s)e_t(s)h_{t+1}(s). \tag{6}$$

The law of motion for a household's assets is given by

$$q_t a_{t+1}(s) = p_t(s)\left(x_t(s) - \int_0^1 c_{it}(s)\mathrm{d}i\right). \tag{7}$$

There are no barriers to capital flows, so all islands trade securities at a common price q_t. The credit limit $m_t(s)$ evolves as the product of an island-specific and aggregate component, both of which are AR(1) processes with Gaussian disturbances.

At this point, we note that as $\alpha \to \infty$, $v_{it}(s) \to 1$. In this case, there is no idiosyncratic uncertainty. There is no meaningful role for the liquidity constraints and, since housing is separable in the utility function and exogenously fixed, there is no role for credit, and the economy collapses to the standard 3-equation New Keynesian model (see Jones, Midrigan and Philippon, 2022, for details and a discussion of this point).

Final goods producers Final goods producers on island s produce $y_t(s)$ units of the final good using $y_t^N(s)$ units of non-tradable goods produced locally and $y_t^M(s,j)$ units of tradable goods produced on island j and imported to island s:

$$y_t(s) = \left(\omega^{\frac{1}{\sigma}} y_t^N(s)^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}}\left(\int_0^1 y_t^M(s,j)^{\frac{\kappa-1}{\kappa}}\mathrm{d}j\right)^{\frac{\kappa}{\kappa-1}\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

where ω determines the share of non-traded goods, σ is the elasticity of substitution between traded and non-traded goods and κ is the elasticity of substitution between varieties of the traded goods produced on different islands. Letting $p_t^N(s)$ and $p_t^M(s)$ denote the prices of these goods on island s, the final goods price on an island is

$$p_t(s) = \left(\omega p_t^N(s)^{1-\sigma} + (1-\omega)\left(\int_0^1 p_t^M(j)^{1-\kappa}\mathrm{d}j\right)^{\frac{1-\sigma}{1-\kappa}}\right)^{\frac{1}{1-\sigma}}. \tag{9}$$

The demand for non-tradable intermediate goods produced on an island is

$$y_t^N(s) = \omega \left(\frac{p_t^N(s)}{p_t(s)} \right)^{-\sigma} y_t(s), \tag{10}$$

while demand for an island's tradable exports $y_t^X(s)$ is an aggregate of what all other islands purchase:

$$y_t^X(s) = (1-\omega) p_t^M(s)^{-\kappa} \left(\int_0^1 p_t^M(j)^{1-\kappa} \mathrm{d}j \right)^{\frac{\kappa-\sigma}{1-\kappa}} \left(\int_0^1 p_t(j)^{\sigma} y_t(j) \mathrm{d}j \right). \tag{11}$$

Intermediate goods producers Traded and non-traded goods on each island are themselves CES composites of varieties of differentiated intermediate inputs with an elasticity of substitution ϑ. The demand for an individual variety k for non-tradeable goods (for example) are

$$y_t^N(s,k) = \left(p_t^N(s,k)/p_t^N(s) \right)^{-\vartheta} y_t^N(s).$$

Individual producers of intermediate goods are subject to Calvo price adjustment frictions. Let λ_p denote the probability that a firm does not reset its price in a given period. A firm that resets its price maximizes the present discounted flow of profits weighted by the probability that the price it chooses at t will still be in effect at any particular date. As was the case earlier, the production function is linear in labor, but it is now subject to sector-specific productivity disturbances $z_t^N(s)$ and $z_t^X(s)$, so that

$$y_t^j(s,k) = z_t^j(s) n_t^j(s,k), \text{ for } j \in \{N, X\}$$

so that the unit cost of production is simply $w_t(s)/z_t^j(s)$ in both sectors.

For example, a traded intermediate goods firm that resets its price solves

$$\max_{p_t^{X*}(s)} \sum_{j=0}^{\infty} \left(\lambda_p^j \prod_{i=0}^{j-1} \beta_{t+i}(s) \right) \mu_{t+j}(s) \left(p_t^{X*}(s) - \tau_p \frac{w_{t+j}(s)}{z_{t+j}^X(s)} \right) \left(\frac{p_t^{X*}(s)}{p_{t+j}^X(s)} \right)^{-\vartheta} y_{t+j}^X(s), \tag{12}$$

where $\mu_{t+j}(s)$ is the shadow value of wealth of the representative household on island s – that is, the multiplier on the flow budget constraint (3) – and $\tau_p = \frac{\vartheta-1}{\vartheta}$ is a tax the government levies to eliminate the steady state markup distortion. This tax is rebated lump sum to households on island s. The composite price of traded exports or non-traded goods is then a weighted average of the prices of individual differentiated intermediates. For example, the price of export goods is

$$p_t^X(s) = \left[(1-\lambda_p)p_t^{X*}(s)^{1-\vartheta} + \lambda_p p_{t-1}^X(s)^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}. \tag{13}$$

Wage setting We assume that individual households are organized in unions that supply differentiated varieties of labor. The total amount of labor services available in production is

$$n_t(s) = \left(\int_0^1 n_t(\iota, s)^{\frac{\psi-1}{\psi}} \mathrm{d}\iota\right)^{\frac{\psi}{\psi-1}}, \tag{14}$$

where ψ is the elasticity of substitution between labor varieties. Demand for an individual union's labor given its wage $w_t(\iota, s)$ is therefore $n_t(\iota, s) = (w_t(\iota, s)/w_t(s))^{-\psi} n_t(s)$. The problem of a union that resets its wage is to choose a new wage $w_t^*(s)$ to

$$\max_{w_t^*(s)} \sum_{j=0}^{\infty} \left(\lambda_w^j \prod_{i=0}^{j-1} \beta_{t+i}(s)\right) \left[\tau_w \mu_{t+j}(s) w_t^*(s) \left(\frac{w_t^*(s)}{w_{t+j}(s)}\right)^{-\psi} n_{t+j}(s) - \frac{\eta_{t+j}^n(s)}{1+\nu} \left(\left(\frac{w_t^*(s)}{w_{t+j}(s)}\right)^{-\psi} n_{t+j}(s)\right)^{1+\nu}\right], \tag{15}$$

where λ_w is the probability that a given union leaves its wage unchanged and $\tau_w = \frac{\psi-1}{\psi}$ is a labor income subsidy aimed at correcting the steady state markup distortion. The composite wage at which labor services are sold to producers is

$$w_t(s) = \left[(1-\lambda_w)w_t^*(s)^{1-\psi} + \lambda_w w_{t-1}(s)^{1-\psi}\right]^{\frac{1}{1-\psi}}. \tag{16}$$

D.2 Monetary Policy

Let $y_t = \int_0^1 p_t(s)y_t(s)/p_t \, \mathrm{d}s$ be total real output in this economy, where $p_t = \int_0^1 p_t(s)\mathrm{d}s$ is the aggregate price index. Let $\pi_t = p_t/p_{t-1}$ denote the rate of inflation and

$$1 + i_t = \mathbb{E}_t R_{t+1} \tag{17}$$

be the expected one-period nominal return on the long-term security, which we refer to as the nominal interest rate. Aggregation over the pricing choices of individual producers implies, up to a first-order approximation,

$$\log(\pi_t/\bar{\pi}) = \bar{\beta}\mathbb{E}_t \log(\pi_{t+1}/\bar{\pi}) + \frac{(1-\lambda_p)(1-\lambda_p\bar{\beta})}{\lambda_p}(\log(w_t) - \log(z_t)) + \theta_t,$$

where we add an AR(1) disturbance θ_t to individual firms' desired markups, $\bar{\beta}$ is the steady state discount factor, and $\bar{\pi}$ is the steady-state level of inflation.

We assume that monetary policy is characterized by a Taylor rule when the ZLB does not bind:

$$1 + i_t = (1 + i_{t-1})^{\alpha_r} \left[(1 + \bar{\imath})\, \pi_t^{\alpha_\pi} \left(\frac{y_t}{y_t^*} \right)^{\alpha_y} \right]^{1-\alpha_r} \left(\frac{y_t / y_t^*}{y_{t-1} / y_{t-1}^*} \right)^{\alpha_x} \exp(\varepsilon_t^r),$$

where ε_t^r is a monetary policy shock; α_r determines the persistence; and α_π; α_y; and α_x determine the extent to which monetary policy responds to inflation, deviations of output from its flexible price level y_t^*, and the growth rate of the output gap, respectively. We assume that $\bar{\imath}$ is set to a level that ensures a steady state level of inflation of $\bar{\pi}$. When the ZLB binds, then

$$i_t = 0.$$

The interest rate may be at zero either because aggregate shocks cause the ZLB to bind, or because the Fed commits to keeping i_t at 0 for a longer time period than implied by the constraint. We thus implicitly assume that the Fed can manipulate expectations of how the path of interest rates evolves, as in Eggertsson and Woodford (2003) and Werning (2012). In our estimation we use survey data from the New York Federal Reserve to discipline the expected duration of the zero interest rate regime during the 2009 to 2015 period.

Since we assume that an individual island is of measure zero, monetary policy does not react to island-specific disturbances. The monetary union is closed so aggregate savings must equal aggregate debt:

$$\int_0^1 a_{t+1}(s)\mathrm{d}s = \int_0^1 b_{t+1}(s)\mathrm{d}s. \tag{18}$$

D.3 Estimation Approach

Practically, the use of equations (**??**) and (**??**) to estimate the model involves first expressing each state's observable variable as a deviation from its aggregate counterpart by subtracting time effects for each year and each variable. It also involves subtracting a state-specific fixed effect and time trend for each observable, since in the model, all islands are ex ante identical.

We estimate the model using state-level data, following the strategy described in the paper. With the purpose of comparing results, we also estimate the model using aggregate data. In doing so, we jointly estimate the structural parameters and the policy rule.

In all cases, we use Bayesian methods to estimate the model's structural parameters.[5] To construct the posterior distribution, as the island-level shocks in (**??**) are independent and

[5] We estimate λ_p, λ_w, α_r, α_p, α_x, α_y, and the persistence and standard deviations of the autoregressive exogenous processes. See Appendix E for the full estimation results.

do not affect aggregate outcomes, we can write the likelihood of the model as the product of each individual state's likelihood, computed from (??). When we estimate the model using aggregate data, we use equation (??) to compute the aggregate likelihood. For the prior distributions for the model's structural parameters, we follow standard practice and use the same priors Smets and Wouters (2007) use for the Calvo parameters λ_p and λ_w. We use this procedure for both the state-level data and the aggregate data estimations. As it turns out, assumptions regarding prior distributions of the Calvo parameters can be quite important in standard aggregate-level estimation. On the other hand, estimates using state-level data are found to be robust to the assumed priors.[6]

As we want to illustrate the role that changing policy regimes may have on the estimated values of the Calvo parameters using aggregate data, we do not wish to take a strong stand on the priors for the Taylor rule parameters. For this reason, in the estimates we report, we use uniform priors for α_r, α_p, α_x, and α_y. In Appendix Section E.3, we show that results are similar if we instead used the priors of Smets and Wouters (2007) for the Taylor rule parameters.

D.4 Mixed Frequency/Observation

As mentioned earlier, our data is such that inflation data do not exist for around half of the 51 states in our panel, and the inflation series is biannual, while other state-level observables are annual. An innovation of our analysis is to extend the estimation of the structural model to this unbalanced panel. To do this, let N be the size of the model's state-space, and define by $\mathbf{z}_t^s$ the $(\hat{N}_t^s \times 1)$ vector of state s's observable variables at time t. Note that the dimension of state s's observable vector is changing over time with the availability of data. We map each state's $\mathbf{z}_t^s$ to the $(N \times 1)$ vector of model variables $\hat{\mathbf{x}}_t^s$ by the $(\hat{N}_t^s \times N)$ matrix $\mathbf{H}_t^s$:

$$\mathbf{z}_t^s = \mathbf{H}_t^s \hat{\mathbf{x}}_t^s.$$

Thus, to allow for estimation using different frequencies and observables, the differences across states and time are encoded in the matrix $\mathbf{H}_t^s$, so that forecast errors are computed only for the data series available at each point in time.[7]

To illustrate the procedure with an example, consider an estimation using an unbalanced panel dataset consisting of two regions labeled A and B and two observables, inflation and the output gap (which, for simplicity, also define the state space; that is, $N = 2$ in the

[6] See Jones, Kulish and Nicolini (2021), who discuss in detail the role of priors in the estimation of New Keynesian models with aggregate and state-level data.

[7] We describe the full Kalman filter in Appendix D.

dimension of $\hat{\mathbf{x}}_t^s$). With two observables, $\hat{N}_t^s$ can be 0, 1, or 2, depending on data availability.

Assume the following structure for the panel: from period t, the output gap is observed every two periods for both regions, while inflation is observed every period, but only for region A. Defining $\mathbf{z}_t = \begin{bmatrix}(\mathbf{z}_t^A)' & (\mathbf{z}_t^B)'\end{bmatrix}'$ as the vector of observable variables, the panel's structure implies that $\mathbf{z}_t$ is of dimension $\hat{N}_t^A + \hat{N}_t^B = 2 + 1$ in period t and has dimension $\hat{N}_{t+1}^A + \hat{N}_{t+1}^B = 1 + 0$ in period $t+1$. To map these to the state vector, the coefficient matrices for region A are

$$\mathbf{H}_t^A = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \quad \mathbf{H}_{t+1}^A = \begin{bmatrix}1 & 0\end{bmatrix},$$

and the coefficient matrices for region B are

$$\mathbf{H}_t^B = \begin{bmatrix}0 & 1\end{bmatrix},$$

and $\mathbf{H}_{t+1}^B$ is of zero dimension. Notice that in period $t+1$, region B exits the set of observable variables that are used to compute forecast errors and the model's likelihood with the Kalman filter.

To the best of our knowledge, by using this procedure, ours is the first paper to show how to bring an unbalanced panel dataset to the estimation of a structural macro model, which could prove useful in other contexts and applications. More generally, this flexible approach opens up more possibilities of how to bring regional-level data to identify key parameters of macro models, building on the work of Nakamura and Steinsson (2014); Beraja, Hurst and Ospina (2019); and Jones, Midrigan and Philippon (2022).

D.5 Likelihood Function

We use Bayesian likelihood methods to estimate the parameters of the island economy and the shocks. We use a panel dataset across states for the state-level estimation, and aggregate data and the ZLB for the aggregate-level estimation. We formulate the state-space of the model so as to separate our estimation into a collection of regional components to make it computationally feasible.

We discuss the likelihood function of the state/regional component and then the likelihood function of the aggregate component.

D.5.1 Likelihood of the State Component

We use Bayesian methods. We first log-linearize the model. The log-linearized model has the state space representation

$$x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}\varepsilon_t \tag{19}$$

$$z_t = \mathbf{H}_t x_t. \tag{20}$$

The state vector is x_t. The error is distributed $\varepsilon_t \sim N(0, \Omega)$, where Ω is the covariance matrix of ε_t. We assume no observation error of the data z_t.

Denote by ϑ the vector of parameters to be estimated. Denote by $\mathcal{Z} = \{z_\tau\}_{\tau=1}^{T}$ the sequence of $N_z \times 1$ vectors of observable variables, combined over states. By Bayes law, the posterior $\mathcal{P}(\vartheta \mid \mathcal{Z})$ satisfies

$$\mathcal{P}(\vartheta \mid \mathcal{Z}) \propto L(\mathcal{Z} \mid \vartheta) \times \mathcal{P}(\vartheta).$$

With Gaussian errors ε_t, the likelihood function $L(\mathcal{Z} \mid \vartheta)$ is computed using the sequence of structural matrices and the Kalman filter, described below:

$$\log L(\mathcal{Z} \mid \vartheta) = -\left(\frac{N_z T}{2}\right)\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log\det \mathbf{S}_t - \frac{1}{2}\sum_{t=1}^{T}\widetilde{y}_t^{\top}\left(\mathbf{S}_t\right)^{-1}\widetilde{y}_t,$$

where $\widetilde{y}_t$ is the vector of forecast errors and $\mathbf{S}_t$ is its associated covariance matrix.

D.5.2 Kalman Filter

The Kalman filter recursion is given by the following equations. The state of the system is $(\hat{x}_t, \mathbf{P}_{t-1})$. In the predict step, the structural matrices $\mathbf{J}$, $\mathbf{Q}$ and $\mathbf{G}$ are used to compute a forecast of the state $\hat{x}_{t|t-1}$ and the forecast covariance matrix $\mathbf{P}_{t|t-1}$ as

$$\begin{aligned} \hat{x}_{t|t-1} &= \mathbf{J} + \mathbf{Q}\hat{x}_t \\ \mathbf{P}_{t|t-1} &= \mathbf{Q}\mathbf{P}_{t-1}\mathbf{Q}^{\top} + \mathbf{G}\Omega\mathbf{G}^{\top}. \end{aligned} \tag{21}$$

We update these forecasts with imperfect observations of the state vector. This update step involves computing forecast errors $\widetilde{y}_t$ and their associated covariance matrix $\mathbf{S}_t$ as

$$\begin{aligned} \widetilde{y}_t &= z_t - \mathbf{H}_t\hat{x}_{t|t-1} \\ \mathbf{S}_t &= \mathbf{H}_t\mathbf{P}_{t|t-1}\mathbf{H}_t^{\top}. \end{aligned}$$

The Kalman gain matrix is given by

$$\mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{H}_t^\top \mathbf{S}_t^{-1}.$$

With $\widetilde{y}_t$, $\mathbf{S}_t$ and $\mathbf{K}_t$ in hand, the optimal filtered update of the state x_t is

$$\hat{x}_t = \hat{x}_{t|t-1} + \mathbf{K}_t \widetilde{y}_t,$$

and for its associated covariance matrix,

$$\mathbf{P}_t = (I - \mathbf{K}_t \mathbf{H}_t)\, \mathbf{P}_{t|t-1}.$$

The Kalman filter is initialized with x_0 and $\mathbf{P}_0$ determined from their unconditional moments and is computed until the final time period T of data. We can show that the stationary $\mathbf{P}_0$ has the expression

$$\mathrm{vec}(\mathbf{P}_0) = (\mathbf{I} - \mathbf{Q} \otimes \mathbf{Q})^{-1}\, \mathrm{vec}(\mathbf{G}\Omega\mathbf{G}^\top) \tag{22}$$

D.5.3 Kalman Smoother

With the estimates of the parameters on a sample up to time period T, the Kalman smoother gives an estimate of $x_{t|T}$, or an estimate of the state vector at each point in time given all available information. With $\hat{x}_{t|t-1}$, $\mathbf{P}_{t|t-1}$, $\mathbf{K}_t$, and $\mathbf{S}_t$ in hand from the Kalman filter, the vector $x_{t|T}$ is computed by

$$x_{t|T} = \hat{x}_{t|t-1} + \mathbf{P}_{t|t-1} r_{t|T},$$

where the vector $r_{T+1|T} = 0$ and is updated with the recursion

$$r_{t|T} = \mathbf{H}_t^\top \mathbf{S}_t^{-1} \left(z_t - \mathbf{H}_t \hat{x}_{t|t-1} \right) + (I - \mathbf{K}_t \mathbf{H}_t)^\top\, \mathbf{P}_{t|t-1}^\top r_{t+1|T}.$$

Finally, to get an estimate of the shocks to each state variable under this model's shock structure, denoted by e_t, we can compute

$$e_t = \mathbf{G}\varepsilon_t = \mathbf{G} r_{t|T}.$$

D.5.4 Block Structure

The regional component of the model has a block structure separated by state. For example, consider two states so that the log-linearized state-space representation is

$$\begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix} = \begin{bmatrix} \mathbf{J}^1 \\ \mathbf{J}^2 \end{bmatrix} + \begin{bmatrix} \mathbf{Q}^1 & 0 \\ 0 & \mathbf{Q}^2 \end{bmatrix} \begin{bmatrix} x_{t-1}^1 \\ x_{t-1}^2 \end{bmatrix} + \begin{bmatrix} \mathbf{G}^1 & 0 \\ 0 & \mathbf{G}^2 \end{bmatrix} \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{bmatrix}$$

Under this block structure, the forecast covariance matrix $P_{t|t-1}$ also has a block structure. This is clear from the expressions (21) and (22).

The block structure is also helpful for computational reasons. The log-likelihood becomes a weighted sum of state-by-state log-likelihood functions. To show this: because $\mathbf{P}_{t|t-1}$ has a block structure, so does $\mathbf{S}_t$. And because $\mathbf{S}_t$ has a block structure

$$\log \det \mathbf{S}_t = \log \prod_j \det \mathbf{S}_t^j = \sum_j \log \det \mathbf{S}_t^j.$$

Also, because $\mathbf{S}_t$ has a block structure, so does its inverse, so that the last term in the log-likehood can also be separated by state. The log-likelihood is then

$$\log L(\mathcal{Z} \mid \vartheta) = \sum_s \log L^s(\mathcal{Z}^s \mid \vartheta).$$

D.6 Likelihood of the Aggregate Component

D.6.1 Solution with Zero Lower Bound

We write the model that approximates the ZLB in the following way. Under the ZLB, the economy has time variation in the evolution of the model's structural parameters $\mathbf{A}_t$, $\mathbf{B}_t$, $\mathbf{C}_t$, $\mathbf{D}_t$, and $\mathbf{F}_t$, where

$$\mathbf{A}_t x_t = \mathbf{C}_t + \mathbf{B}_t x_{t-1} + \mathbf{D}_t \mathbb{E}_t x_{t+1} + \mathbf{F}_t \epsilon_t.$$

For example, if the ZLB binds, the equation describing the Taylor rule becomes $i_t = 0$, changing the structural matrices $\mathbf{A}_t$, and so on. With time-varying structural matrices, the solution we seek is the time-varying VAR representation:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \epsilon_t, \tag{23}$$

where $\mathbf{J}_t$, $\mathbf{Q}_t$ and $\mathbf{G}_t$ are conformable matrices that are functions of the evolution of beliefs about the time-varying structural matrices $\mathbf{A}_t$, $\mathbf{B}_t$, $\mathbf{C}_t$, $\mathbf{D}_t$, and $\mathbf{F}_t$ (Kulish and Pagan, 2017). These matrices satisfy the recursion

$$\begin{aligned}
\mathbf{Q}_t &= \left[\mathbf{A}_t - \mathbf{D}_t\mathbf{Q}_{t+1}\right]^{-1}\mathbf{B}_t \\
\mathbf{J}_t &= \left[\mathbf{A}_t - \mathbf{D}_t\mathbf{Q}_{t+1}\right]^{-1}\left(\mathbf{C}_t + \mathbf{D}_t\mathbf{J}_{t+1}\right) \\
\mathbf{G}_t &= \left[\mathbf{A}_t - \mathbf{D}_t\mathbf{Q}_{t+1}\right]^{-1}\mathbf{E}_t,
\end{aligned}$$

where the final structures $\mathbf{Q}_T$ and $\mathbf{J}_T$ are known and computed from the time invariant structure above under the terminal period's structural parameters–that is, the no-ZLB case.

Given a sequence of ZLB durations, the state-space of the model is

$$\begin{aligned}
x_t &= \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t\varepsilon_t \\
z_t &= \mathbf{H}_t x_t.
\end{aligned}$$

The observation equation is time-varying because the nominal interest rate becomes unobserved when it is at its bound.

Denote by ϑ the vector of parameters to be estimated and by $\mathbf{T}$ the vector of ZLB durations that are observed each period. Denote by $\mathcal{Z} = \{z_\tau\}_{\tau=1}^{T}$ the sequence of vectors of observable variables. With Gaussian errors, the likelihood function $L(\mathcal{Z}, \mathbf{T} \mid \vartheta)$ for the aggregate component is computed using the sequence of structural matrices associated with the sequence of ZLB durations, and the Kalman filter:

$$\log L(\mathcal{Z}, \mathbf{T} \mid \vartheta) = -\left(\frac{N_z T}{2}\right)\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log\det \mathbf{H}_t\mathbf{S}_t\mathbf{H}_t^\top - \frac{1}{2}\sum_{t=1}^{T}\widetilde{y}_t^\top\left(\mathbf{H}_t\mathbf{S}_t\mathbf{H}_t^\top\right)^{-1}\widetilde{y}_t.$$

D.6.2 Kalman filter

The state of the system is $(\hat{x}_t, \mathbf{P}_{t-1})$. In the predict step, the structural matrices $\mathbf{J}_t$, $\mathbf{Q}_t$, and $\mathbf{G}_t$ are used to compute a forecast of the state $\hat{x}_{t|t-1}$ and the forecast covariance matrix $\mathbf{P}_{t|t-1}$ as

$$\begin{aligned}
\hat{x}_{t|t-1} &= \mathbf{J}_t + \mathbf{Q}_t\hat{x}_t \\
\mathbf{P}_{t|t-1} &= \mathbf{Q}_t\mathbf{P}_{t-1}\mathbf{Q}_{t|t-1}^\top + \mathbf{G}_t\Omega\mathbf{G}_t^\top.
\end{aligned}$$

This formulation differs from the time-invariant Kalman filter used at the state level, because in the forecast stage, the matrices $\mathbf{J}_t$, $\mathbf{Q}_t$ and $\mathbf{G}_t$ can vary over time. We update these forecasts with imperfect observations of the state vector. This update step involves computing forecast errors $\widetilde{y}_t$ and its associated covariance matrix $\mathbf{S}_t$ as

$$\widetilde{y}_t = z_t - \mathbf{H}_t \hat{x}_{t|t-1}$$
$$\mathbf{S}_t = \mathbf{H}_t \mathbf{P}_{t|t-1} \mathbf{H}_t^\top.$$

The Kalman gain matrix is given by

$$\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{H}_t^\top \mathbf{S}_t^{-1}.$$

With $\widetilde{y}_t$, $\mathbf{S}_t$, and $\mathbf{K}_t$ in hand, the optimal filtered update of the state x_t is

$$\hat{x}_t = \hat{x}_{t|t-1} + \mathbf{K}_t \widetilde{y}_t,$$

and for its associated covariance matrix:

$$\mathbf{P}_t = (I - \mathbf{K}_t \mathbf{H}_t) \, \mathbf{P}_{t|t-1}.$$

The Kalman filter is initialized with x_0 and $\mathbf{P}_0$ determined from their unconditional moments and is computed until the final time period T of data.

D.6.3 Kalman Smoother

With the estimates of the parameters and durations in hand at time period T, the Kalman smoother gives an estimate of $x_{t|T}$, or an estimate of the state vector at each point in time given all available information (Hamilton, 1994). With $\hat{x}_{t|t-1}$, $\mathbf{P}_{t|t-1}$, $\mathbf{K}_t$ and $\mathbf{S}_t$ in hand from the Kalman filter, the vector $x_{t|T}$ is computed by

$$x_{t|T} = \hat{x}_{t|t-1} + \mathbf{P}_{t|t-1} r_{t|T},$$

where the vector $r_{T+1|T} = 0$ and is updated with the recursion:

$$r_{t|T} = \mathbf{H}_t^\top \mathbf{S}_t^{-1} \left(z_t - \mathbf{H}_t \hat{x}_{t|t-1} \right) + (I - \mathbf{K}_t \mathbf{H}_t)^\top \mathbf{P}_{t|t-1}^\top r_{t+1|T}.$$

Finally, to get an estimate of the shocks to each state variable under this model's shock structure, denoted by e_t, we compute:

$$e_t = \mathbf{G}_t \varepsilon_t = \mathbf{G}_t r_{t|T}.$$

Table E.9: Calibrated Parameters

Parameter	Value	Description	Source/Target
ν	2	Inverse labor supply elasticity	
β	0.995	Quarterly discount factor	2% annual real rate
ω	0.7	Weight on non-traded goods	
σ	0.5	Elasticity traded/non-traded	
κ	4	Elasticity traded goods	Simonovska and Waugh (2014)
ψ	21	Elasticity labor aggregator	Christiano, Eichenbaum and Evans (2005)

D.7 Posterior Sampler

This section describes the sampler used to obtain the posterior distribution of interest. We compute the likelihood function at the state level and the aggregate level, together with the prior. The posterior of our full model $\mathcal{P}(\vartheta \mid \mathbf{T}, \mathcal{Z})$ satisfies

$$\mathcal{P}(\vartheta \mid \mathbf{T}, \mathcal{Z}) \propto L(\mathcal{Z}, \mathbf{T} \mid \vartheta) \times \mathcal{P}(\vartheta).$$

We use a Markov Chain Monte Carlo procedure to sample from the posterior. It has a single block, corresponding to the parameters ϑ.[8] The sampler at step j is initialized with the last accepted draw of the structural parameters ϑ_{j-1}.

First, start by selecting which parameters to propose new values. For those parameters, draw a new proposal ϑ_j from a proposal density centered at ϑ_{j-1} and with thick tails to ensure sufficient coverage of the parameter space and an acceptance rate of roughly 20% to 25%. The proposal ϑ_j is accepted with probability $\frac{\mathcal{P}(\vartheta_j \mid \mathbf{T}, \mathcal{Z})}{\mathcal{P}(\vartheta_{j-1} \mid \mathbf{T}, \mathcal{Z})}$. If ϑ_j is accepted, then set $\vartheta_{j-1} = \vartheta_j$.

E Additional Structural Model Estimation Results

E.1 Calibrated Parameters

Table E.9 details the small set of parameters that are calibrated prior to estimation.

[8] It is worth noting that as in Kulish, Morley and Robinson (2017), in addition to the structural parameters, one can estimate the expected zero lower bound durations, in which case an additional block is needed in the posterior sampler.

Table E.10: Structural Estimation, State Data Only

	Prior			Posterior		
Parameter	Dist	Mean	SD	Mode	5%	95%
λ_p	B	0.5	0.1	0.59	0.57	0.61
λ_w	B	0.5	0.1	0.41	0.39	0.43
ρ_z	B	0.5	0.2	0.99	0.98	0.99
ρ_n	B	0.5	0.1	0.96	0.95	0.97
ρ_b	B	0.5	0.2	0.96	0.96	0.97
ρ_z^N	B	0.5	0.2	0.96	0.95	0.97
σ_z	IG	2.0	1.4	1.49	1.45	1.54
σ_n	IG	2.0	1.4	0.03	0.03	0.04
σ_b	IG	2.0	1.4	0.63	0.56	0.70
σ_z^N	IG	2.0	1.4	1.37	1.31	1.42

E.2 Full Structural Model Estimation Results

Tables E.10 and E.11 give the full prior and posterior distributions of the estimated structural parameters using state and aggregate data, respectively.

The parameters are the Calvo parameter on prices λ_p, the Calvo parameter on wages λ_w, the persistence of TFP shocks ρ_z, the persistence of labor disutility shocks ρ_n, the persistence of preference shocks ρ_b, the persistence of non-tradeable TFP shocks ρ_z^N, and the respective standard deviations of those four shocks. At the aggregate level, we also have the persistence of markup shocks ρ_p, the standard deviation of markup shocks σ_p, and the standard deviation of policy interest rate shocks σ_r. The Taylor rule parameters are given by α_r, α_p, α_x, and α_y.

We choose the same prior as Smets and Wouters (2007) for the Calvo parameters. Our remaining priors are chosen to be wide/diffuse. We choose a somewhat tighter prior on the persistence of labor disutility shocks at the state-level as preliminary estimations took ρ_n to a value of 1. We use uniform priors over a wide range for the parameters of the Taylor rule.

E.3 Robustness

E.3.1 Estimation with Credit Channel

Results with an active credit channel and the use of household debt and house prices as observables are shown in Tables E.12 to E.16. The structure of the results is similar to that of the main tables reported in the text.

Table E.11: Structural Estimation, Aggregate Data Only

	Prior			Posterior		
Parameter	Dist	Mean	SD	Mode	5%	95%
λ_p	B	0.5	0.1	0.92	0.90	0.94
λ_w	B	0.5	0.1	0.84	0.80	0.88
ρ_z	B	0.5	0.2	0.96	0.94	0.97
ρ_n	B	0.5	0.2	0.08	0.03	0.18
ρ_b	B	0.5	0.2	0.86	0.84	0.88
ρ_p	B	0.5	0.2	0.91	0.86	0.95
σ_z	IG	2.0	1.4	0.58	0.53	0.64
σ_n	IG	2.0	1.4	0.11	0.07	0.21
σ_b	IG	2.0	1.4	2.73	2.35	3.25
σ_p	IG	2.0	1.4	0.37	0.28	0.52
σ_r	IG	2.0	1.4	1.53	1.28	1.95
α_r	U	0.5	0.3	0.80	0.73	0.84
α_p	U	4.5	2.6	2.38	1.98	2.86
α_x	U	1.0	0.6	0.46	0.36	0.62
α_y	U	1.0	0.6	0.28	0.21	0.37

Table E.12: Posterior Distributions, Relative State Data Only, with Credit

	1977 to 2017			1977 to 1998			1999 to 2017		
Parameter	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
λ_p	0.59	0.58	0.61	0.58	0.55	0.60	0.62	0.61	0.64
λ_w	0.38	0.35	0.39	0.50	0.46	0.54	0.40	0.39	0.43

Table E.13: Posterior Distributions, Aggregate Data Only, with Credit

	1977 to 2015			1977 to 1998			1999 to 2015		
Parameter	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
				Calvo Parameters					
λ_p	0.92	0.89	0.94	0.84	0.79	0.89	0.93	0.90	0.94
λ_w	0.83	0.81	0.94	0.90	0.87	0.93	0.84	0.79	0.88
				Taylor Rule Parameters					
α_r	0.80	0.72	0.85	0.68	0.40	0.78	0.77	0.70	0.84
α_p	2.38	1.93	2.82	2.00	1.52	3.13	1.06	1.03	1.85
α_x	0.44	0.37	0.65	1.56	0.89	1.94	0.18	0.13	0.24
α_y	0.27	0.20	0.36	0.07	0.01	0.24	0.23	0.20	0.30

Notes: $Beta(0.5, 0.1)$ prior on Calvos. Uniform priors on Taylor Rule parameters

Table E.14: Implied Slopes of Phillips Curve at Baseline Estimates, with Credit

	1977 to 2015	1977 to 1998	1999 to 2015
	A. State-Level Estimates		
Prices*	0.279	0.306	0.228
Wages†	1.044	0.517	0.882
	B. Aggregate-Level Estimates		
	1977 to 2017	1977 to 1998	1999 to 2017
Prices*	0.008	0.032	0.007
Wages†	0.034	0.012	0.031

*: Price Phillips curve slope is $(1-\beta\lambda_p)(1-\lambda_p)/\lambda_p$
†: Wage Phillips curve slope is $(1-\beta\lambda_w)(1-\lambda_w)/\lambda_w$

Table E.15: Posterior Distributions, Interaction with Policy Rules, with Credit

A. Aggregate Data Only, Fixed Taylor Rule Parameters

	1977 to 2015*			1977 to 2015†			1977 to 2015‡		
Parameter	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
λ_p	0.92	0.89	0.94	0.87	0.80	0.89	0.92	0.89	0.94
λ_w	0.83	0.81	0.94	0.71	0.67	0.73	0.95	0.80	0.95

B. Aggregate Data Only, Policy Regime Periods

	1965 to 2015			1965 to 1985§			1986 to 2015§		
Parameter	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
λ_p	0.86	0.83	0.90	0.72	0.67	0.77	0.93	0.90	0.95
λ_w	0.90	0.87	0.93	0.91	0.88	0.94	0.87	0.83	0.90
α_r	0.93	0.90	0.95	0.95	0.86	0.96	0.86	0.81	0.91
α_p	4.02	3.30	7.29	4.48	2.81	9.43	2.42	1.85	3.62
α_x	0.46	0.40	0.59	0.55	0.44	0.79	0.21	0.15	0.27
α_y	0.77	0.46	1.13	0.82	0.34	1.72	0.27	0.20	0.38

$\star$: Estimated Taylor Rule with uniform priors
†: Taylor Rule parameters fixed at 1977 to 1998 estimates (see Table E.13)
‡: Taylor Rule parameters fixed at 1999 to 2015 estimates (see Table E.13)
§: No credit or house price series and no credit or housing preference shocks

Table E.16: Implied Slopes of Phillips Curve at Aggregate Estimates

	1977 to 2015	1977 to 1998	1999 to 2015
A. Aggregate-Level Estimates, Fixed Taylor Rule			
Prices*	0.008	0.020	0.008
Wages†	0.034	0.117	0.003

B. Aggregate-Level Estimates, Policy Regime Periods

	1965 to 2005	1965 to 1985	1986 to 2005
Prices*	0.022	0.107	0.006
Wages†	0.012	0.009	0.020

*: Price Phillips curve slope is $(1-\beta\lambda_p)(1-\lambda_p)/\lambda_p$
†: Wage Phillips curve slope is $(1-\beta\lambda_w)(1-\lambda_w)/\lambda_w$

Table E.17: Aggregate-Level, Smets and Wouters (2007) Priors on Calvos and Taylor Rule

	1977 to 2015			1977 to 1998			1999 to 2015		
Parameter	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
λ_p	0.92	0.89	0.94	0.87	0.82	0.91	0.92	0.90	0.94
λ_w	0.84	0.79	0.88	0.90	0.87	0.93	0.84	0.80	0.89
α_r	0.79	0.75	0.82	0.80	0.74	0.84	0.78	0.72	0.82
α_p	1.70	1.53	1.87	1.61	1.44	1.81	1.41	1.24	1.62
α_x	0.30	0.26	0.35	0.33	0.28	0.39	0.15	0.11	0.20
α_y	0.20	0.16	0.25	0.18	0.13	0.24	0.23	0.19	0.26

Table E.18: Posterior of Calvo Prices λ_p and Calvo Wages λ_w

	(1)			(2)		
	Mode	10%	90%	Mode	10%	90%
λ_p	0.57	0.55	0.58	0.58	0.57	0.60
λ_w	0.33	0.31	0.35	0.34	0.33	0.36

(1): 1999 to 2015, consumption spending and no credit shocks
(2): 1999 to 2015, consumption spending

E.3.2 Smets and Wouters (2007) Priors on Calvo and Taylor Rule Parameters

Table E.17 shows the estimated structural parameters when the same priors as Smets and Wouters (2007) are used on the Calvo parameters and on the Taylor rule parameters. In these estimations, there is a role for the credit channel.

E.3.3 State-Level Estimation with Consumption

The results from an estimation using state-level consumption spending are given in Table E.18. The estimated nominal frictions are lower–in the model, nominal output equals nominal consumption, and since consumption is less volatile than output, the model estimation explains relatively more volatile prices and wages with more flexible prices. The addition of credit shocks does not change the estimated λ_p and λ_w, as for the estimation using nominal output.

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