Online Appendix to the paper
Temporal Aggregation for the Synthetic Control Method
Liyang Sun, Eli Ben-Michael, and Avi Feller

Theorem 1. *In addition to assumptions stated above, suppose some oracle weights exist in the set $\mathcal{C}$. Denote $\boldsymbol{\mu}_{tk} \in \mathbb{R}^r$ as the time factors and assume that they are bounded above by M. Furthermore, denoting $\sigma_{min}(A)$ as the smallest singular value of a matrix A, assume that (i) $\sigma_{min}\left(\frac{1}{T_0 K}\sum_{tk}\mu_{tk}\mu_{tk}'\right) \geq \underline{\xi}^{dis} > 0$; and (ii) $\sigma_{min}\left(\frac{1}{T_0}\sum_t (\bar{\mu}_t)(\bar{\mu}_t)'\right) \geq \underline{\xi}^{agg} > 0$ where $\bar{\mu}_t = \frac{1}{K}\sum_{k=1}^K \mu_{tk}$. For any $\delta > 0$, let $\tilde{\sigma} = \left(2C\sqrt{\log 2N_0} + (1+C)\delta\right)(1 + 1/\sqrt{T_0 K})\sigma$, with probability at least $1 - 8\exp\left(-\frac{\delta^2}{2}\right) - 4\exp\left(-\frac{T_0 K \delta^2}{2\sigma^2(1+C^2)}\right)$, the absolute bias satisfies the bound,*

$$\left|Bias(\hat{\gamma}^{dis})\right| \leq \frac{rM^2}{\underline{\xi}^{dis}}\left(4(1+C)\sigma + 2\delta + \frac{\tilde{\sigma}}{\sqrt{T_0 K}}\right), \tag{1}$$

$$\left|Bias(\hat{\gamma}^{agg})\right| \leq \frac{rM^2}{\underline{\xi}^{agg}}\left(\frac{4(1+C)\sigma}{\sqrt{K}} + 2\delta + \frac{\tilde{\sigma}}{\sqrt{T_0 K}}\right). \tag{2}$$

PROOF:

Results follow directly from Theorem 1 of Sun, Ben-Michael and Feller (2023) where $\hat{\gamma}^{\text{dis}}$ correspond to the concatenated weights, and $\hat{\gamma}^{\text{agg}}$ correspond to the average weights in their notation.

Lemma 1. *Suppose there exists $\nu^* \in [0,1]$ such that for $\hat{\gamma}^{com} \in \arg\min_{\gamma\in\mathcal{C}} \nu^* q^{dis}(\gamma) + (1-\nu^*)q^{agg}(\gamma)$, we have $q^{dis}(\gamma^{com}) \leq q^{dis}(\gamma^*)$ and $q^{agg}(\gamma^{com}) \leq q^{agg}(\gamma^*)$ almost surely. For any $\delta > 0$, with probability at least $1 - 8\exp\left(-\frac{\delta^2}{2}\right) - 4\exp\left(-\frac{T_0 K\delta^2}{2\sigma^2(1+C^2)}\right)$, the absolute bias satisfies the bound,*

$$\left|Bias(\hat{\gamma}^{com})\right| \leq \min\left\{\frac{rM^2}{\underline{\xi}^{dis}}\left(4(1+C)\sigma + 2\delta + \frac{\tilde{\sigma}}{\sqrt{T_0 K}}\right), \frac{rM^2}{\underline{\xi}^{agg}}\left(\frac{4(1+C)\sigma}{\sqrt{K}} + 2\delta + \frac{\tilde{\sigma}}{\sqrt{T_0 K}}\right)\right\}.$$

PROOF:

Since $q^{\text{dis}}(\gamma^{\text{com}}) \leq q^{\text{dis}}(\gamma^*)$ and $q^{\text{agg}}(\gamma^{\text{com}}) \leq q^{\text{agg}}(\gamma^*)$ almost surely, either of the two bias bounds stated in Theorem 1 is a valid upper bound for the estimate based on the combined weights $\hat{\gamma}^{\text{com}}$. We may therefore take the minimum of the two bounds to bound $|Bias(\hat{\gamma}^{\text{com}})|$.

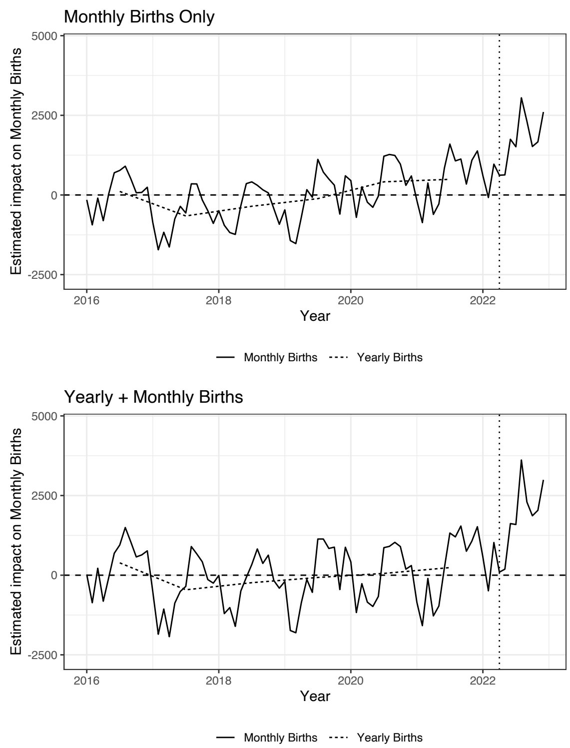

FIGURE 1. BALANCE ONLY IN MONTHLY VS IN BOTH MONTHLY AND YEARLY

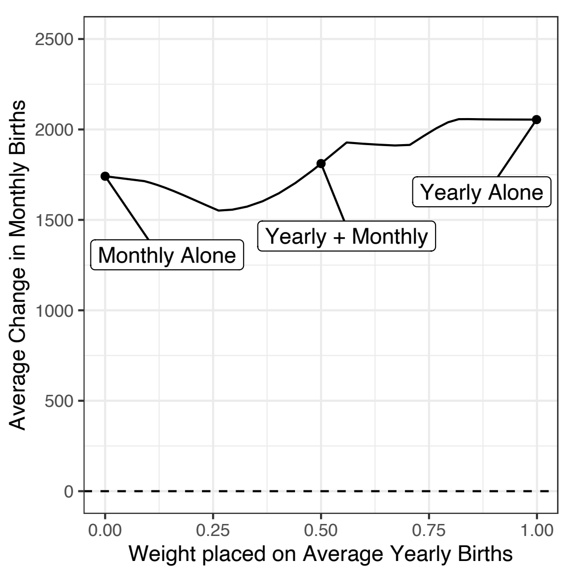

FIGURE 2. CHANGE IN AVERAGE ESTIMATED IMPACT ON MONTHLY BIRTHS AS THE RELATIVE WEIGHT ON MONTHLY VS YEARLY BIRTHS VARIES FROM 0 TO 1.