Online Appendix to "(Empirical) Bayes Approaches to Parallel Trends"

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1 Proof of Proposition 1

Note that since $\tau_{post} = \beta_{post} - \delta_{post}$, the linearity of the expectation operator implies that $E[\tau_{post} \mid \hat{\beta}] = E[\beta_{post} \mid \hat{\beta}] - \mathrm{E}[\delta_{post} \mid \hat{\beta}]$. To derive the alternative form for $E[\delta_{post} \mid \hat{\beta}]$ in the proposition, we claim that under the uninformative prior, $\pi_{\delta_{post}|\beta}(\delta_{post} \mid \beta) = \pi_{\delta_{post}|\delta_{pre}}(\delta_{post} \mid \beta_{pre})$. This is because

$$\begin{aligned}
\pi_{\delta|\beta}(\delta \mid \beta) &= \frac{\pi_{\beta,\delta}(\beta,\delta)}{\pi_{\beta}(\beta)} \\
&\propto \pi_{\delta,\tau_{post}}(\delta, \beta_{post} - \delta_{post}) \cdot 1[\delta_{pre} = \beta_{pre}] \\
&= \pi_{\delta}(\delta) \cdot \pi_{\tau_{post}|\delta}(\beta_{post} - \delta_{post} \mid \delta) \cdot 1[\delta_{pre} = \beta_{pre}] \\
&\propto \pi_{\delta_{post}|\delta_{pre}}(\delta_{post} \mid \beta_{pre}),
\end{aligned}$$

where we obtain the last line from the fact that $\pi_{\tau|\delta} \propto 1$ under the uninformative prior and the definition of the conditional density. It follows that

$$E_{\delta_{post}|\hat{\beta}}[\delta_{post} \mid \hat{\beta}] = E_{\beta|\hat{\beta}}[E_{\delta_{post}|\beta}[\delta_{post} \mid \beta] \mid \hat{\beta}] = E_{\beta_{pre}|\hat{\beta}}[E_{\delta_{post}|\delta_{pre}}[\delta_{post} \mid \beta_{pre}] \mid \hat{\beta}]$$

where the first equality uses iterated expectations and the fact that $\hat{\beta} \perp\!\!\!\perp \delta \mid \beta$, and the second uses the fact that $\pi_{\delta_{post}|\beta}(\delta_{post} \mid \beta) = \pi_{\delta_{post}|\delta_{pre}}(\delta_{post} \mid \beta_{pre})$ as derived above.

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2 Calculations for example with Gaussian prior

We now provide detailed calculations for the posterior for β when $\delta \sim \mathcal{N}(\mu_\delta, V_\delta)$ and we use the uninformative prior for τ_{post}. Note that $\pi_{\tau_{post}|\delta} \propto 1$ implies that

$$\begin{aligned}
\pi_\beta(\beta) &= \pi_{\beta_{pre}}(\beta_{pre})\pi_{\beta_{post}|\beta_{pre}}(\beta_{post}) \\
&= \pi_{\delta_{pre}}(\beta_{pre}) \int \pi_{\delta_{post}|\delta_{pre}}(\beta_{post} - \tau_{post})\pi_{\tau_{post}}(\tau_{post}) d\tau_{post} \\
&\propto \pi_{\delta_{pre}}(\beta_{pre})
\end{aligned}$$

where in the second line we use the fact that $\tau_{post} \perp\!\!\!\perp \delta$ (since $\pi_{\tau_{post}|\delta} \propto 1$) and in the last line we use the fact that $\pi_{\tau_{post}}(\tau_{post}) \propto 1$ and $\int \pi_{\delta_{post}|\delta_{pre}}(\beta_{post} - \tau_{post}) = 1$ since densities integrate to 1. Thus, we have that

$$\begin{aligned}
p(\beta \mid \hat{\beta}) &= \ell(\hat{\beta} \mid \beta)\pi_{\delta_{pre}}(\beta_{pre}) \\
&\propto \exp(-\frac{1}{2}(\beta - \hat{\beta})'\Sigma_{\hat{\beta}}^{-1}(\beta - \hat{\beta})) \cdot exp(-\frac{1}{2}(\beta_{pre} - \mu_{\delta_{pre}})'V_{\delta_{pre}}^{-1}(\beta_{pre} - \mu_{\delta_{pre}})) \\
&\propto \exp(-\frac{1}{2}(\beta'(\Sigma_{\hat{\beta}}^{-1} + \tilde{V}_{\delta_{pre}}^{-1})\beta - 2\beta'(\Sigma_{\hat{\beta}}^{-1}\hat{\beta} + \tilde{V}_{\delta_{pre}}^{-1}\tilde{\mu}_{\delta_{pre}}))
\end{aligned}$$

where $\tilde{V}_{\delta_{pre}}^{-1} = \begin{pmatrix} V_{\delta_{pre}}^{-1} & 0 \\ 0 & 0 \end{pmatrix}$ and $\tilde{\mu}_{\delta_{pre}} = \begin{pmatrix} \mu_{\delta_{pre}} \\ 0 \end{pmatrix}$. We thus see that the posterior for β is normal with mean

$$E[\beta \mid \hat{\beta}] = (\Sigma_{\hat{\beta}}^{-1} + \tilde{V}_{\delta_{pre}}^{-1})^{-1}(\Sigma_{\hat{\beta}}^{-1}\hat{\beta} + \tilde{V}_{\delta_{pre}}^{-1}\tilde{\mu}_{\delta_{pre}})$$

and variance

$$Var[\beta \mid \hat{\beta}] = (\Sigma_{\hat{\beta}}^{-1} + \tilde{V}_{\delta_{pre}}^{-1})^{-1}$$

Corollary 4.1 in Lu and Shiou (2002) shows that for the symmetric block matrix $M = \begin{pmatrix} A & B \\ B' & D \end{pmatrix}$,

$$M^{-1} = \begin{pmatrix} (A - BD^{-1}B')^{-1} & -(A - BD^{-1}B')^{-1}BD^{-1} \\ -D^{-1}B'(A - BD^{-1}B')^{-1} & (D - B'A^{-1}B)^{-1} \end{pmatrix} \tag{1}$$

and that

$$-D^{-1}B'(A - BD^{-1}B')^{-1} = -(D - B'A^{-1}B)^{-1}B'A^{-1} \tag{2}$$

It follows that

$$\Sigma_{\hat{\beta}}^{-1} + \tilde{V}_{\delta_{pre}}^{-1} =$$
$$\begin{pmatrix} (\Sigma_{\hat{\beta}_{pre}} - \Sigma_{\hat{\beta}_{pre},\hat{\beta}_{post}}\Sigma_{\hat{\beta}_{post}}^{-1}\Sigma'_{\hat{\beta}_{pre},\hat{\beta}_{post}})^{-1} + V_{\delta_{pre}}^{-1} & \cdot \\ -(\Sigma_{\hat{\beta}_{post}} - \Sigma'_{\hat{\beta}_{pre},\hat{\beta}_{post}}\Sigma_{\hat{\beta}_{pre}}^{-1}\Sigma_{\hat{\beta}_{pre},\hat{\beta}_{post}})^{-1}\Sigma'_{\hat{\beta}_{pre},\hat{\beta}_{post}}\Sigma_{\hat{\beta}_{pre}}^{-1} & (\Sigma_{\hat{\beta}_{post}} - \Sigma'_{\hat{\beta}_{pre},\hat{\beta}_{post}}\Sigma_{\hat{\beta}_{pre}}^{-1}\Sigma_{\hat{\beta}_{pre},\hat{\beta}_{post}})^{-1} \end{pmatrix}$$

where the upper-right block is the transpose of the lower-left. Applying (1) again to the previous display, but using the alternative formula $D^{-1} + D^{-1}B'(A - BD^{-1}B')^{-1}BD^{-1}$ for the lower-right block given in Lu and Shiou (2002), we obtain that

$$\left(\Sigma_{\hat{\beta}}^{-1}+\tilde{V}_{\delta_{pre}}^{-1}\right)^{-1}=\begin{pmatrix} (\Sigma_{\hat{\beta}_{pre}}^{-1}+V_{\delta_{pre}}^{-1})^{-1} & \cdot \\ \Sigma'_{\hat{\beta}_{pre},\hat{\beta}_{post}}\Sigma_{\hat{\beta}_{pre}}^{-1}(\Sigma_{\hat{\beta}_{pre}}^{-1}+V_{\delta_{pre}}^{-1})^{-1} & \Sigma_{\hat{\beta}_{post}|\hat{\beta}_{pre}}+\Gamma'_{\Sigma}(\Sigma_{\hat{\beta}_{pre}}^{-1}+V_{\delta_{pre}}^{-1})^{-1}\Gamma_{\Sigma} \end{pmatrix}$$

where $\Sigma_{\hat{\beta}_{post}|\hat{\beta}_{pre}}=\Sigma_{\hat{\beta}_{post}}-\Sigma'_{\hat{\beta}_{pre},\hat{\beta}_{post}}\Sigma_{\hat{\beta}_{pre}}^{-1}\Sigma_{\hat{\beta}_{pre},\hat{\beta}_{post}}$ is the conditional variance of $\hat{\beta}_{post}\mid\hat{\beta}_{pre}$, and $\Gamma_{\Sigma}=\Sigma_{\hat{\beta}_{pre}}^{-1}\Sigma_{\hat{\beta}_{pre},\hat{\beta}_{post}}$ are the coefficients in the best linear predictor of $\hat{\beta}_{post}$ given $\hat{\beta}_{pre}$.

It follows that

$$\left(\Sigma_{\hat{\beta}}^{-1}+\tilde{V}_{\delta_{pre}}^{-1}\right)^{-1}\tilde{V}_{\delta_{pre}}^{-1}=\begin{pmatrix} (\Sigma_{\hat{\beta}_{pre}}^{-1}+V_{\delta_{pre}}^{-1})^{-1}V_{\delta_{pre}}^{-1} & 0 \\ \Sigma'_{\hat{\beta}_{pre},\hat{\beta}_{post}}\Sigma_{\hat{\beta}_{pre}}^{-1}(\Sigma_{\hat{\beta}_{pre}}^{-1}+V_{\delta_{pre}}^{-1})^{-1}V_{\delta_{pre}}^{-1} & 0 \end{pmatrix}$$

and thus

$$\left(\Sigma_{\hat{\beta}}^{-1}+\tilde{V}_{\delta_{pre}}^{-1}\right)^{-1}\Sigma_{\hat{\beta}}^{-1}=I-\left(\Sigma_{\hat{\beta}}^{-1}+\tilde{V}_{\delta_{pre}}^{-1}\right)^{-1}\tilde{V}_{\delta_{pre}}=\begin{pmatrix} I-(\Sigma_{\hat{\beta}_{pre}}^{-1}+V_{\delta_{pre}}^{-1})^{-1}V_{\delta_{pre}}^{-1} & 0 \\ -\Sigma'_{\hat{\beta}_{pre},\hat{\beta}_{post}}\Sigma_{\hat{\beta}_{pre}}^{-1}(\Sigma_{\hat{\beta}_{pre}}^{-1}+V_{\delta_{pre}}^{-1})^{-1}V_{\delta_{pre}}^{-1} & I \end{pmatrix}$$

It follows that

$$\begin{aligned} E[\beta_{pre}\mid\hat{\beta}] &= (\Sigma_{\hat{\beta}_{pre}}^{-1}+V_{\delta_{pre}}^{-1})^{-1}\Sigma_{\hat{\beta}_{pre}}^{-1}\hat{\beta}_{pre}+(\Sigma_{\hat{\beta}_{pre}}^{-1}+V_{\delta_{pre}}^{-1})^{-1}V_{\delta_{pre}}^{-1}\mu_{\delta_{pre}} \\ E[\beta_{post}\mid\hat{\beta}] &= \hat{\beta}_{post}-\Sigma'_{\hat{\beta}_{pre},\hat{\beta}_{post}}\Sigma_{\hat{\beta}_{pre}}^{-1}(\Sigma_{\hat{\beta}_{pre}}^{-1}+V_{\delta_{pre}}^{-1})^{-1}V_{\delta_{pre}}^{-1}(\hat{\beta}_{pre}-\mu_{\delta_{pre}}) \\ &= \hat{\beta}_{post}-\Gamma'_{\Sigma}(\hat{\beta}_{pre}-E[\beta_{pre}\mid\hat{\beta}]) \end{aligned}$$

We showed in the main text that

$$E[\tau_{post}\mid\hat{\beta}]=\beta^*_{post}-\mu_{\delta_{post}}-V_{\delta_{post},\delta_{pre}}V_{\delta_{pre}}^{-1}(\beta^*_{pre}-\mu_{\delta_{pre}}),$$

where $\beta^*=E[\beta\mid\hat{\beta}]$ is the posterior for $\hat{\beta}$, which we derived above.

To get $Var(\tau_{post}\mid\hat{\beta})$, recall that $\tau_{post}=\beta_{post}-\delta_{post}$. Using the law of total variance, we have that

$$Var(\tau_{post}\mid\hat{\beta})=E_{\beta|\hat{\beta}}[Var(\beta_{post}-\delta_{post}\mid\beta)]+Var_{\beta|\hat{\beta}}(E[\beta_{post}-\delta_{post}\mid\beta])$$

Note, however, that[1]

$$Var(\beta_{post}-\delta_{post}\mid\beta)=Var(\delta_{post}\mid\beta)=Var(\delta_{post}\mid\delta_{pre}=\beta_{pre})=V_{\delta_{post}}-V'_{\delta_{pre},\delta_{post}}V_{\delta_{pre}}^{-1}V_{\delta_{pre},\delta_{post}}$$

and

$$\begin{aligned} Var_{\beta|\hat{\beta}}(E[\delta_{post}\mid\beta]) &= Var_{\beta|\hat{\beta}}(\beta_{post}-(\mu_{\delta_{post}}+\Gamma'_V(\beta_{pre}-\mu_{\delta_{pre}})) \\ &= M(\Sigma_{\hat{\beta}}^{-1}+\tilde{V}_{\delta_{pre}}^{-1})^{-1}M' \end{aligned}$$

where $\Gamma_V=V_{\delta_{pre}}^{-1}V_{\delta_{pre},\delta_{post}}$ and $M=(-\Gamma'_V,I)$ is the matrix such that $M\beta=\beta_{post}-\Gamma'_V\beta_{pre}$. Hence,

[1] In what follows, we use the fact that $\pi_{\delta_{post}|\beta}(\delta_{post}\mid\beta)\propto\pi_{\delta_{post}|\delta_{pre}}(\delta_{post}\mid\beta_{pre})$, which we derived in the proof to Proposition 1.

$$Var(\tau_{post} \mid \hat{\beta}) = V_{\delta_{post}} - V'_{\delta_{pre},\delta_{post}} V^{-1}_{\delta_{pre}} V_{\delta_{pre},\delta_{post}} + M(\Sigma^{-1}_{\hat{\beta}} + \tilde{V}^{-1}_{\delta_{pre}})^{-1} M'.$$

Further calculation shows that this can be simplified to

$$\begin{aligned} Var(\tau_{post} \mid \hat{\beta}) &= V_{\delta_{post}} + \Sigma_{\hat{\beta}_{post}} - (\Sigma_{\hat{\beta}_{pre},\hat{\beta}_{post}} + V_{\delta_{pre},\delta_{post}})'(\Sigma_{\hat{\beta}_{pre}} + V_{\delta_{pre}})^{-1}(\Sigma_{\hat{\beta}_{pre},\hat{\beta}_{post}} + V_{\delta_{pre},\delta_{post}}) \\ &= \tilde{\Sigma}_{post} - \tilde{\Sigma}'_{pre,post}\tilde{\Sigma}^{-1}_{pre}\tilde{\Sigma}_{pre,post}, \end{aligned}$$

where $\tilde{\Sigma} := \Sigma_{\hat{\beta}} + V_{\delta}$ with block matrix form

$$\tilde{\Sigma} = \begin{pmatrix} \tilde{\Sigma}_{pre} & \tilde{\Sigma}_{pre,post} \\ \tilde{\Sigma}'_{pre,post} & \tilde{\Sigma}_{post} \end{pmatrix}.$$

3 Informative Gaussian prior for τ_{post}

We now consider a modification of the Gaussian example in the main text in which there is a joint Gaussian prior over (δ, τ_{post}). We begin with the following lemma.

Lemma 3.1. *Suppose that*

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\alpha \\ \mu_\beta \end{pmatrix}, \begin{pmatrix} V_\alpha & V_{\alpha\beta} \\ V_{\beta\alpha} & V_\beta \end{pmatrix} \right)$$

and we observe $\hat{\beta} \mid \alpha, \beta \sim \mathcal{N}\left(\beta, \Sigma_{\hat{\beta}}\right)$. *Then the posterior for* (α, β) *is jointly normal with means*

$$E[\beta \mid \hat{\beta}] = (V^{-1}_\beta + \Sigma^{-1}_{\hat{\beta}})^{-1}(\Sigma^{-1}_{\hat{\beta}}\hat{\beta} + V^{-1}_\beta \mu_\beta) =: \mu^*_\beta$$

and

$$E[\alpha \mid \hat{\beta}] = \mu_\alpha + V_{\alpha\beta}V^{-1}_\beta(\mu^*_\beta - \mu_\beta) =: \mu^*_\alpha$$

and variances

$$Var(\beta \mid \hat{\beta}) = (V^{-1}_\beta + \Sigma^{-1}_{\hat{\beta}})^{-1} =: \Sigma^*_\beta$$

and

$$Var(\alpha \mid \hat{\beta}) = \underbrace{(V_\alpha - V_{\alpha\beta}V^{-1}_\beta V_{\beta\alpha})}_{=Var(\alpha \mid \beta)} + V_{\alpha\beta}V^{-1}_\beta \Sigma^*_\beta V^{-1}_\beta V_{\beta\alpha} := \Sigma^*_\alpha$$

Proof. Consider the reparametrized parameter $\tilde{\alpha} = \alpha - V_{\alpha\beta}V^{-1}_\beta \beta$. The prior for $(\tilde{\alpha}, \beta)$ is

$$\begin{pmatrix} \tilde{\alpha} \\ \beta \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\alpha - V_{\alpha\beta}V^{-1}_\beta \mu_\beta \\ \mu_\beta \end{pmatrix}, \begin{pmatrix} V_\alpha - V_{\alpha\beta}V^{-1}_\beta V_{\beta\alpha} & 0 \\ 0 & V_\beta \end{pmatrix} \right),$$

so the priors for $\tilde{\alpha}$ and β are independent. By Bayes' rule,

$$\begin{aligned} p(\tilde{\alpha}, \beta \mid \hat{\beta}) &\propto p(\hat{\beta} \mid \tilde{\alpha}, \beta) p(\tilde{\alpha}, \beta) \\ &= p(\hat{\beta} \mid \beta) p(\tilde{\alpha}, \beta) \\ &= p(\hat{\beta} \mid \beta) p(\beta) p(\tilde{\alpha}) \end{aligned}$$

where the first equality uses the fact that the likelihood doesn't depend on α, and the second uses prior independence. We thus see that the posteriors for β and $\tilde{\alpha}$ are independent, and the posterior for $\tilde{\alpha}$ is equal to the prior. Standard results for the normal-normal model give that the posterior for β is normal with mean $\mu_\beta^* = (V_\beta^{-1} + \Sigma_{\hat{\beta}}^{-1})^{-1}(\Sigma_{\hat{\beta}}^{-1}\hat{\beta} + V_\beta^{-1}\mu_\beta)$ and variance $\Sigma_\beta^* = (V_\beta^{-1} + \Sigma_{\hat{\beta}}^{-1})^{-1}$. Since the linear combination of independent normals is normal, we then see that the posterior for $\alpha = \tilde{\alpha} + V_{\alpha\beta}V_\beta^{-1}$ is normal with mean $(\mu_\alpha - V_{\alpha\beta}V_\beta^{-1}\mu_\beta) + V_{\alpha\beta}V_\beta^{-1}\mu_\beta^*$ and variance $(V_\alpha - V_{\alpha\beta}V_\beta^{-1}V_{\beta\alpha}) + V_{\alpha\beta}V_\beta^{-1}\Sigma_\beta^* V_\beta^{-1}V_{\beta\alpha}$, which completes the proof. □

Now suppose that the prior over (δ, τ_{post}) is joint Gaussian with independence between δ and τ_{post},

$$\begin{pmatrix} \delta_{pre} \\ \delta_{post} \\ \tau_{post} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_{\delta_{pre}} \\ \mu_{\delta_{post}} \\ \mu_{\tau_{post}} \end{pmatrix}, \begin{pmatrix} V_{\delta_{pre}} & V_{\delta_{pre},\delta_{post}} & 0 \\ V_{\delta_{post},\delta_{pre}} & V_{\delta_{post}} & 0 \\ 0 & 0 & V_{\tau_{post}} \end{pmatrix} \right).$$

This implies that

$$\begin{pmatrix} \beta_{pre} \\ \beta_{post} \\ \tau_{post} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_{\delta_{pre}} \\ \mu_{\delta_{post}} + \mu_{\tau_{post}} \\ \mu_{\tau_{post}} \end{pmatrix}, \begin{pmatrix} V_{\delta_{pre}} & V_{\delta_{pre},\delta_{post}} & 0 \\ V_{\delta_{post},\delta_{pre}} & V_{\delta_{post}} + V_{\tau_{post}} & V_{\tau_{post}} \\ 0 & V_{\tau_{post}} & V_{\tau_{post}} \end{pmatrix} \right)$$

Given $\hat{\beta} \mid \beta, \tau \sim \mathcal{N}\left(\beta, \Sigma_{\hat{\beta}}\right)$, the posterior for $\tau_{post} \mid \hat{\beta}$ then follows directly from the formulas given in Lemma 3.1, setting $\alpha = \tau_{post}$.

4 Calibration of prior in BZ application

We now describe the calibration of the prior in our application to Benzarti and Carloni (2019). Suppose, as in Benzarti and Carloni (2019), that units with $D_i = 1$ come from a single industry (restaurants) while units with $D_i = 0$ come from many other industries. Suppose that

$$E[Y_{it}(0) \mid D_i = 0] = \mu_t$$

and that

$$E[Y_{it}(0) \mid D_i = 1] = \mu_t + \alpha_t,$$

where μ_t represents aggregate shocks to the outcome common to all units and α_t is the idiosyncratic shock to the treated industry. Suppose that the industry-specific shock follows an $AR(1)$,

$$\alpha_t = \rho\alpha_{t-1} + u_t$$

where the u_t are iid with mean 0 and variance σ^2.

McGahan and Porter (1999) estimate an $AR(1)$ for the industry-component of firm profits (measured as a fraction of firm assets[2]) and obtain an estimate of ρ of 0.766. They estimate that $SD(\alpha_t)$ is 0.063 (6.3

[2]Note this differs slightly from Benzarti and Carloni (2019), who use log profits as an outcome. Let E_t and A_t respectively correspond to net earnings and assets in period t. Note that if $E_t/A_t \approx 1$ and $A_t \approx A_{t-1}$, so that assets are stable over time, then

$$\log(E_t) - \log(E_{t-1}) \approx \log(E_t/A_t) - \log(E_{t-1}/A_{t-1}) \approx \frac{E_t}{A_t} - \frac{E_{t-1}}{A_{t-1}},$$

where we use the fact that $log(x) \approx x - 1$ for $x \approx 1$, so that innovations in log profits are similar to innovations in percentage

percentage points), which using the formula $Var(\alpha_t) = \sigma^2/(1-\rho^2)$ implies a value of σ of $\sqrt{(1-\rho^2)}0.063 = \sqrt{1-0.766^2}0.063 = 0.04$.

Note that the violation of parallel trends between period 0 and period t is given by $\delta_t = \alpha_t - \alpha_0$. Under the $AR(1)$ process described above, δ_t is mean-zero. To derive its variance-covariance matrix, recall that for an $AR(1)$ process, the covariance is $Cov(\alpha_t, \alpha_{t-k}) = \frac{\rho^{|k|}}{1-\rho^2}\sigma^2$. Hence, we have that

$$\begin{aligned} Cov(\delta_t, \delta_{t'}) &= Cov(\alpha_t - \alpha_0, \alpha_{t'} - \alpha_0) \\ &= \frac{\rho^{|t-t'|} - \rho^{|t|} - \rho^{|t'|} + 1}{1-\rho^2}\sigma^2 \end{aligned}$$

We calibrate the prior covariance on δ, V_δ, using the expression in the previous display and the calibrated values of ρ and σ^2.

References

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earnings.