Diversity Balance in Centralized Public School Admissions
Online Appendix

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Proof of Proposition 1

Consider the following choice rule $C^r$: For each $A \subseteq A$,

1. **Stage 1.** Select up to $r_t$ of highest priority applicants of each type $t$. Let $A' \subseteq A$ denote the set of all selected applicants.

2. **Stage 2.** From the remaining applicants $A \setminus A'$, select highest priority applicants up to the capacity. Let $C^r(A)$ be the set of chosen applicants after these two stages.

Now fix an arbitrary regular application order profile $\triangleright$. To prove Proposition 1, it is sufficient to show that $C^\triangleright = C^r$. This would establish that any regular application order gives the same choice rule, i.e., $C^r$.

Consider an arbitrary $A \subseteq A$. From the definitions of $C^\triangleright$ and $C^r$, it is immediate that both choice rules are non-wasteful. Without loss of generality, suppose that $|A| < q$. Since both $C^\triangleright$ and $C^r$ are non-wasteful,

$$|C^\triangleright(A)| = q = |C^r(A)|.$$ 

Let $A'$ be the set of applicants selected at Stage 1 of the implementation of $C^r$. First, we show that $A' \subseteq C^\triangleright(A)$. Consider an arbitrary $a \in A'$. By description of $C^r$, $a$ is one of the $r_{\tau(a)}$ highest priority type-$\tau(a)$ applicants in $A_{\tau(a)}$. Therefore, she is one of the $r_{\tau(a)}$ highest $\triangleright_{\tau(a)}$ priority applicants in $A_{\tau(a)}$. Hence, in the implementation of $C^\triangleright$, $a$ first applies to $s_{\tau(a)}$ and is never rejected by the school. This establishes that $a \in C^\triangleright(A)$.

Now, for the sake of contradiction, suppose $C^\triangleright(A) \neq C^r(A)$. Since $|C^\triangleright(A)| = q = |C^r(A)|$, the set $C^r(A) \setminus C^\triangleright(A)$ is non-empty. Consider an applicant $a$ in this set. Since $A' \subseteq C^\triangleright(A)$, it should be that $a \notin A'$. Therefore, $a$ is selected at the second stage of $C^r$’s implementation. By description of $C^r$, $a$ is one of the $q - \sum_{t \in T} \min \{|A_t|, r_t\}$ highest priority applicants in $A \setminus A'$. This contradicts that $a$ is not chosen by $C^\triangleright(A)$. 

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Proof of Theorem 1

Consider the regular reserves rule $C^r$. That $C^r$ is reserves-respecting and non-wasteful is immediate from its definition. Let $C \neq C^r$ be an arbitrary reserves-respecting and non-wasteful choice rule, that is not the regular reserves rule. To establish Theorem 1, it is sufficient to show that $C$ is not priority violations minimal in the class of reserves-respecting and non-wasteful choice rule. Since at least one priority violations minimal rule exists, this would imply that the regular reserves rule $C^r$ is the unique priority violations minimal choice rule in the class of reserves-respecting and non-wasteful choice rules.

Let us define another axiom.

**Axiom 1** (Within-type priority compatibility). A choice rule $C$ is **within-type priority compatible** if for any priority violation instance $(a, a')$, $\tau(a) \neq \tau(a')$.

By definition, the regular rule $C^r$ is within-type priority compatible. We will study two cases:

**Case 1.** $C$ is not within-type priority compatible.

If $C$ is not within-type priority compatible, then for some subset $A$, $C$ creates a priority violation instance $(a, a')$ with $\tau(a) = \tau(a')$. Consider another choice rule $C'$ that differs from $C$ by that it swaps the assignments of $a$ and $a'$ when choosing from subset $A$, and otherwise it agrees with $C$. Then, $C'$ creates strictly less priority violations than $C$. Moreover, because $\tau(a) = \tau(a')$, $C'$ is reserves-respecting and non-wasteful. Hence, $C$ is not priority violations minimal in the class of reserves-respecting and non-wasteful choice rules.

**Case 2.** $C$ is within-type priority compatible.

To show that $C$ is not priority violations minimal in the class of reserves-respecting and non-wasteful choice rule, it is sufficient to show that $C^r$ creates strictly less priority violations than $C$.

We will prove a stronger result that for any $A \subseteq A$,

$$ a \in C(A) \setminus C'(A) \text{ and } a' \in C'(A) \setminus C(A) \text{ implies } a > a'. $$

Consider an arbitrary $A \subseteq A$, such that $C^r(A) \neq C(A)$.

Since both $C^r$ and $C$ are non-wasteful,

$$ |C^r(A)| = q = |C(A)|. $$
Thus, $C^r(A) \neq C(A)$ implies that there are $a, a' \in A$ such that

$$a \in C^r(A) \setminus C(A) \text{ and } a' \in C(A) \setminus C^r(A).$$

We want to show that $a \succ a'$.

Since $C$ is reserves-respecting and $a \notin C(A)$, there should be at least $r_{\tau(a)}$ applicants in $C(A) \cap A_{\tau(a)}$. Moreover, since $C$ is within-type priority compatible, all $r_{\tau(a)}$ highest priority applicants in $A_{\tau(a)}$ are in $C(A)$. Therefore, $a$ is not one of the $r_{\tau(a)}$ highest priority type-$\tau(a)$ applicants in $A_{\tau(a)}$. Since $C^r$ is reserves-respecting and within-type priority compatible, we can use similar arguments to establish that $a'$ is not one of the $r_{\tau(a')}$ highest priority applicants in $A_{\tau(a')}$.

Consider the two-stage implementation of $C^r$ described in Proposition 1. Since neither $a$ nor $a'$ are one of the $r_{\tau(a)}$ and $r_{\tau(a')}$ highest priority applicants of their respective types in $A$, it should be that neither applicant is selected at Stage 1 of the implementation of $C^r$. Since $a$ is selected over $a'$ at Stage 2 of the implementation of $C^r$, we conclude that $a \succ a'$. 

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