Export-Platform FDI: Cannibalization or Complementarity?

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Online Appendix

In this Appendix, we present details that were omitted from the main text. We provide proofs for the three Propositions in the paper, and for other results claimed (without proof) in the main text.

A1. Formal Definition of Price Indexes

Denoting by $p_i(\varphi, k)$ the price charged for variety k, the overall price index $\boldsymbol{p}_i(\varphi)$ for varieties sold by firm φ is given by

$$\boldsymbol{p}_i(\varphi) = \left(\sum_{k\in\mathcal{K}} p_i(\varphi,k)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{A1}$$

The economy-wide ideal price index is in turn given by

$$P_i = \left(\int_{\varphi\in\Omega_i} \boldsymbol{p}_i(\varphi)^{1-\sigma}\mathrm{d}\varphi\right)^{\frac{1}{1-\sigma}}. \tag{A2}$$

A2. Optimal Prices

In this Appendix, we show that firms have an incentive to charge a constant markup over marginal cost for its goods, with the markup being governed by the cross-firm demand elasticity σ.

To simplify matters, we assume, without loss of generality, that $P_i^{\sigma-1}E_i = 1$. Because we focus throughout on a firm-level problem, we often omit φ subscripts in variables that are firm-specific, to make the notation a bit less cumbersome.

A firm solves the following problem in each market i:

$$\begin{aligned} \max_{q_i(k)} \quad & \sum_{k\in\mathcal{K}} (p_i(k) - c_i(k)) \cdot q_i(k) \\ \text{s.t.} \quad & q_i(k) = p_i(k)^{-\varepsilon}\boldsymbol{p}_i^{\varepsilon-\sigma} \end{aligned} \tag{A3}$$

where $\mathcal{K}$ is the set of active assembly plants, $c_i(k)$ is the marginal cost of production from plant k when selling to market i, and

$$\boldsymbol{p}_i = \left(\sum_{k\in\mathcal{K}} p_i(k)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}},$$

as indicated in equation (A1). The constraint in (A3) can easily be derived from equation (3) after setting $P_i^{\sigma-1}E_i = 1$.

It is straightforward to verify that:

$$\sum_{k\in\mathcal{K}} p_i(k)\cdot q_i(k) = \boldsymbol{p}_i \cdot \boldsymbol{q}_i \text{ where } \boldsymbol{q}_i \equiv \left(\sum_{k\in\mathcal{K}} q_i(k)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = \boldsymbol{p}_i^{-\sigma}.$$

Therefore, problem (A3) can be written as a one-dimensional profit maximization

$$\max_{\boldsymbol{q}_i} \quad \boldsymbol{q}_i^{1-\frac{1}{\sigma}} - \boldsymbol{c}_i \cdot \boldsymbol{q}_i, \tag{A4}$$

where the marginal cost $\boldsymbol{c}_i$ for producing a bundle $\boldsymbol{q}_i$ is obtained from cost minimization:

$$\begin{aligned} \boldsymbol{c}_i &= \min_{q_i(k)} \sum_{k\in\mathcal{K}} c_i(k)\cdot q_i(k) \\ \text{s.t.} & \left(\sum_{k\in\mathcal{K}} q_i(k)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = 1. \end{aligned} \tag{A5}$$

Solving (A4) and (A5), and substituting optimal $\boldsymbol{q}_i$ and $\{q_i(k)\}_{k\in\mathcal{K}}$ into the demand equations in (A3) gives the following optimal prices

$$p_i(k) = \frac{\sigma}{\sigma-1} c_i(k) \text{ and } \boldsymbol{p}_i = \frac{\sigma}{\sigma-1}\cdot\left(\sum_{k\in\mathcal{K}} c_i(k)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}, \tag{A6}$$

which are a constant markup $\sigma/(\sigma-1)$ over marginal cost.

A3. Expressions in the Main Text

In this Appendix, we explicitly derive the key expressions in the main text. We begin by using the optimal prices in (A6) to derive sales from plant k to market i (we again omit φ subscripts, for simplicity).

Starting with equation (3) in the main text, we obtain:

$$S_{ki} = p_i(k)^{1-\varepsilon}\boldsymbol{p}_i^{\varepsilon-\sigma}P_i^{\sigma-1}E_i = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\mathcal{I}_k^a\cdot c_i(k)^{1-\varepsilon}\cdot\left(\sum_{k\in J}\mathcal{I}_k^a\cdot c_i(k)^{1-\varepsilon}\right)^{\frac{\sigma-\varepsilon}{\varepsilon-1}}\cdot P_i^{\sigma-1}E_i,$$

where $\mathcal{I}_k^a = 1$ if a firm paid fixed costs of assembly in location $k\in J$, and $\mathcal{I}_k^a = 0$ otherwise. For a firm with productivity φ, the marginal costs are

$$c_i(k) = \frac{1}{\varphi}\cdot\frac{w_k}{Z_k^a}\cdot\tau_{ki}^a,$$

thereby delivering the expression in equation (4) in the main text.

The overall profit for firm φ, equation (6), is

$$\pi(\varphi) = \frac{1}{\sigma}\sum_{i\in J}\sum_{k\in J} S_{ki} = \kappa_\pi\varphi^{\sigma-1}\sum_{i\in J} P_i^{\sigma-1}E_i\cdot(\Psi_i(\varphi))^{\frac{\sigma-1}{\varepsilon-1}} - \sum_{k\in J}\mathcal{I}_k^a\cdot w_k f_k^a,$$

where $\kappa_\pi = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}$, $\mathcal{I}_k^a = 1$ if $k\in\mathcal{K}(\varphi)$, and

$$\Psi_i(\varphi) = \sum_{k\in J}\mathcal{I}_k^a\cdot\xi_k^a\left(\tau_{ki}^a\right)^{1-\varepsilon}.$$

With firm-level fixed costs of exporting, the profit function, equation (7), is

$$\pi(\varphi) = \kappa_\pi \varphi^{\sigma-1} \sum_{i\in J} \mathcal{I}_i^x \cdot P_i^{\sigma-1} E_i \cdot (\Psi_i(\varphi))^{\frac{\sigma-1}{\varepsilon-1}} - \sum_{i\in J} \mathcal{I}_i^x \cdot w_i f_i^x - \sum_{k\in J} \mathcal{I}_k^a \cdot w_k f_k^a .$$

In section IV of the main text, we introduce tradable intermediate inputs. Formally, we assume that firm φ has the following production

$$F_\varphi(\ell, Q_s) = \frac{\varphi}{(1-\alpha)^{1-\alpha}\alpha^\alpha} \ell^{1-\alpha} Q_s^{1-\alpha},$$

where ℓ is labor, and Q_s is a bundle of inputs

$$Q_s = \left(\sum_{j\in J} \mathcal{I}_j^s \cdot \left(q_j^s\right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \quad \text{where } \mathcal{I}_j^s = 1 \text{ if } j \in \mathcal{J}(\varphi) \text{ and } \rho > 1.$$

This production function has the following marginal costs

$$c_i(\varphi, k) = \frac{1}{\varphi} \cdot \left(\frac{w_k}{Z_k^a}\right)^{1-\alpha} \cdot \left(\sum_{j\in J} \mathcal{I}_j^s \cdot \left(\frac{w_j \tau_{jk}^s}{Z_j^s}\right)^{1-\rho} \right)^{\frac{\alpha}{1-\rho}} .$$

Substituting these marginal costs into the optimal prices in (A6) we get the sales from plant k to market i, written in equation (8) in the main text.

Finally, the profit function with intermediate inputs can be written as

$$\pi(\varphi) = \kappa_\pi \varphi^{\sigma-1} \sum_{i\in J} P_i^{\sigma-1} E_i \cdot \Lambda_i(\varphi) - \sum_{j\in J} \mathcal{I}_j^s \cdot w_j f_j^s - \sum_{k\in J} \mathcal{I}_k^a \cdot w_k f_k^a,$$

where

$$\Lambda_i(\varphi) = \left[\sum_{k\in J} \mathcal{I}_k^a \cdot \left(\xi_k^a\right)^{1-\alpha} \left(\tau_{ki}^a\right)^{1-\varepsilon} \cdot \left(\sum_{j\in J} \mathcal{I}_j^s \cdot \xi_j^s \left(\tau_{jk}^s\right)^{1-\rho} \right)^{\frac{\alpha(\varepsilon-1)}{\rho-1}} \right]^{\frac{\sigma-1}{\varepsilon-1}}$$

and

$$\xi_k^a = \left(\frac{w_k}{Z_k^a}\right)^{1-\varepsilon} \text{ and } \xi_j^s = \left(\frac{w_j}{Z_j^s}\right)^{1-\rho} .$$

A4. Relaxing the Armington Assumption

In section II of the main text, we argue that our main results are not dependent on the Armington assumption implicit in equation (2). We prove this claim in this Appendix.

Labor Substitutability in the Armington Model

We first demonstrate that, in our baseline model, ε corresponds to the within-firm elasticity of labor substitution across an MNE's plants. In that model, when figuring out the optimal way to allocate labor across plants to sell goods in market i, for a given assembly strategy $\mathcal{K}$, the firm solves the following problem

$$c_i = \min_{\{\ell_{k,i}(\nu)\}} \sum_{k\in\mathcal{K}} w_k \ell_{k,i}$$

$$\text{s.t.} \left(\sum_{k\in\mathcal{K}(\varphi)} q_i\left(k\right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} = 1 \qquad \text{(bundle of products)}$$

$$\text{s.t. } q_i(k) = \frac{Z_k^a}{\tau_{ki}^a} \cdot \ell_{k,i} \qquad \text{(production technology).}$$

The solution to this problem delivers the following cost function

$$c_i = \left(\sum_{k\in\mathcal{K}(\varphi)} \left(\tau_{ki}^a \frac{w_k}{Z_k^a} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{A7}$$

Define the conditional elasticity of labor demand in location k to changes in location l as

$$\mathcal{E}_{k,l}^i = \frac{\partial \ell_{k,i}}{\partial w_l} \frac{w_l}{\ell_{k,i}},$$

and define the share of variable labor costs associated with selling goods to i paid to labor in location l as:

$$S_l^i = \frac{w_l \ell_{l,i}}{c_i} = \frac{w_l \ell_{l,i}}{\sum\limits_{k\in\mathcal{K}} w_k \ell_{k,i}}.$$

The Allen partial elasticity of substitution is defined as

$$\varepsilon_{k,l}^i \equiv \frac{\mathcal{E}_{k,l}^i}{S_l^i}.$$

For our CES-Armington cost function in (A7), we can invoke Shephard's lemma to find:

$$\ell_{k,i} = \frac{\partial c_i}{\partial w_k} = \left(c_i\right)^{\varepsilon} \left(\frac{\tau_{ki}^a}{Z_k^a} \right)^{1-\varepsilon} \left(w_k\right)^{-\varepsilon}. \tag{A8}$$

The conditional elasticity of labor demand in location k to changes in location l is thus

$$\mathcal{E}_{k,l}^i = \frac{\partial \ell_{k,i}}{\partial w_l} \frac{w_l}{\ell_{k,i}} = \left(\frac{\tau_{ki}^a}{Z_k^a} \right)^{1-\varepsilon} \varepsilon \left(c_i\right)^{\varepsilon-1} \frac{\partial c_i}{\partial w_l} \left(w_k\right)^{-\varepsilon} \frac{w_l}{\ell_{k,i}}$$

Invoking Shephard's lemma and plugging in (A8) delivers

$$\mathcal{E}_{k,l}^i = \varepsilon \frac{w_l \ell_{l,i}}{c_i},$$

so the Allen partial elasticity of labor substitution across plants is

$$\varepsilon_{k,l}^i \equiv \frac{\mathcal{E}_{k,l}^i}{S_l^i} = \varepsilon.$$

It is also simple to see from equation (A8) that, for two locations k and l,

$$\frac{\ell_{k,i}}{\ell_{l,i}} = (c_i)^{\varepsilon} \left(\frac{\tau_{ki}^{a}/Z_k^a}{\tau_{li}^{a}/Z_l^a} \right)^{1-\varepsilon} \left(\frac{w_k}{w_l} \right)^{-\varepsilon}$$

and thus ε also corresponds to the more traditional Hicks elasticity of substitution, defined as

$$\tilde{\mathcal{E}}_{k,l}^{i} = \frac{\partial \ln \left(\ell_{k,i}/\ell_{l,i} \right)}{\partial \ln \left(w_l/w_k \right)}.$$

It is important to stress that ε measures the *intensive-margin* elasticity of labor substitution, taking as fixed the location of the various plants and without consideration to the labor investments that might have been incurred when setting up those plants.

Labor Substitutability with Productivity differences à la Eaton-Kortum

We next explore the robustness of our results to a version of our model in which goods are *not* differentiated based on where they are produced. This version constitutes a simple extension of the model in Tintelnot (2017).

There is an endogenous measure Ω_i of manufacturing firms selling goods in country i. As in Tintelnot (2017), each of these firms produces and sells a continuum of measure one of varieties of manufactured goods. We continue to index firms by φ and varieties within firms by ω. We assume a nested-CES structure in which the degree of substitutability σ across varieties produced by different firms, and the degree of substitutability σ_w across varieties produced by the same firm may differ from each other:

$$U_{Mi} = \left(\int\limits_{\varphi \in \Omega_i} \left(\int_0^1 q_i \left(\varphi, \omega \right)^{(\sigma_w - 1)/\sigma_w} \mathrm{d}\omega \right)^{\frac{\sigma_w}{\sigma_w - 1} \frac{(\sigma - 1)}{\sigma}} \mathrm{d}\varphi \right)^{\sigma/(\sigma-1)}, \quad \sigma_w, \sigma > 1.$$

These preferences imply that consumers in country i spend a share

$$s_i(\varphi) = \left(\frac{p_i(\varphi)}{P_i} \right)^{1-\sigma} E_i \tag{A9}$$

of their income on firm φ. In this expression, E_i is total spending on manufactured goods in country $i \in J$,

$$\boldsymbol{p}_i(\varphi) = \left(\int\limits_0^1 p_i(\varphi, \omega)^{1-\sigma_w} \mathrm{d}v \right)^{\frac{1}{1-\sigma_w}} \tag{A10}$$

is the overall price index for varieties sold by firm φ, and P_i is the economy-wide ideal price index in country i (given again by equation (A2)). Note that, as in our baseline model, σ continues to govern the cross-firm elasticity of demand faced by firm φ.

On the production side, we let firms produce their continuum of products in multiple countries. Given fixed costs of assembly (identical to those in our baseline model), firms will typically produce only in a subset of all countries in the world, and we denote this set $\mathcal{K} \subseteq J$ as the firm's *global assembly strategy.* Shipping final goods from country k to country i entails variable (iceberg) trade costs τ_{ki}^{a}. In line with our baseline model and with Tintelnot (2017), we abstract from fixed costs of exporting.

The marginal cost for firm φ to produce units of final-good variety ω in country k is given by

$$c(\varphi, k, \omega) = \frac{1}{\varphi} \frac{1}{z_k(\varphi, \omega)} w_k, \tag{A11}$$

where $z_k(\varphi, \omega)$ is a firm- and location-specific labor productivity term. Following Tintelnot (2017), we assume that these firm- and location-specific assembly productivity shifters are drawn from the following Fréchet distribution:

$$\Pr(1/z_k(\varphi, \omega) \geq a) = e^{-(Z_k^a a)^\theta}, \quad \text{with } Z_k^a > 0. \tag{A12}$$

Z_k^a governs the average productivity of plant k, while θ determines the dispersion of productivity draws across final-good varieties, with a lower θ indicating a higher variance, and thus greater benefits from producing final-good varieties in various locations. To ensure a well-defined solution, we follow Tintelnot (2017) in imposing a lower bound on the dispersion in the final-good productivity draws $z_k(\varphi, \omega)$:

Technical Assumption: $\sigma_\omega - 1 < \theta$.

Following the derivations in Tintelnot (2017), it is possible to show that this Eaton-Kortum formulation results in a marginal cost for firm φ of selling its bundle of goods to market i, which is given by

$$c_i(\varphi) = \kappa \cdot \left(\sum_{k \in \mathcal{K}(\varphi)} \left(\tau_{ki}^a \frac{w_k}{Z_k^a} \right)^{-\theta} \right)^{-1/\theta}, \tag{A13}$$

where κ is a constant. As claimed in the main text, this marginal cost is identical (up to a constant) to that in equation (A7), with θ replacing $\varepsilon - 1$. Because firms charge a constant markup $\sigma/(\sigma-1)$ over this marginal cost, the rest of the equilibrium conditions of this version of our model, i.e., the analogues of equations (4)–(6), are identical to those in the main text with θ replacing $\varepsilon - 1$. The isomorphism between (A7) and (A13) also makes it clear that the Allen partial elasticity of labor substitution across plants is now given by $\theta + 1$, and whether assembly locations are complements or substitutes depends on the relative size of the (cross-firm) demand elasticity σ and this labor substitution elasticity $\theta + 1$.

It is also worth pointing out that Tintelnot (2017) focused on symmetric CES preferences with a common degree of substitutability across varieties produced by different firms and across varieties produced by the same firm, or $\sigma = \sigma_\omega$. The technical assumption $\sigma_\omega - 1 < \theta$ then led him to assume $\sigma - 1 < \theta$, which implies that assembly locations were necessarily substitutes in his framework. But if $\sigma_\omega < \sigma$, under our more general nested CES structure, it is perfectly possible for assembly locations to be complements ($\sigma - 1 > \theta$) while ensuring a well-defined firm-level problem ($\sigma_\omega - 1 < \theta$).

A More General Production Structure

We finally consider a more general production structure that encompasses to two models developed above and more general settings. We focus on the problem of a firm that produces a set of varieties $\mathcal{V}$ (for simplicity we drop firm-specific subscripts). For each destination $i \in J$, varieties are bundled according to

$$Q_i = F_i\left(\{q_i(\nu)\}_{\nu \in \mathcal{V}}\right),$$

and consumers have CES preferences over Q_i across firms, with elasticity of substitution σ. Each variety is produced using labor from different locations in the firm's global assembly strategy according to

$$q_i(\nu) = F_i^{\nu}\left(\{\ell_{k,i}(\nu)\}_{k\in\mathcal{K}}\right).$$

The operating profit function (excluding fixed costs) can be written as

$$\pi^o = \kappa \cdot \sum_{i\in J} c_i^{1-\sigma} \cdot P_i^{\sigma-1} E_i$$

where c_i is the marginal cost of producing a bundle of goods to be sold in destination i. These marginal costs come from a cost-minimization problem:

$$\begin{aligned} c_i = \min_{\{\ell_{k,i}(\nu)\}} & \sum_{\nu\in\mathcal{V}}\sum_{k\in\mathcal{K}} w_k \ell_{k,i}(\nu) & \\ \text{s.t. } & F_i\left(\{q_i(\nu)\}_{\nu\in\mathcal{V}}\right) = 1 & \text{(bundle of products)} \\ \text{s.t. } & q_i(\nu) = F_i^{\nu}\left((\ell_{k,i}(\nu))_{k\in\mathcal{K}}\right) & \text{(production technology)} \end{aligned}$$

We shall say that assembly locations are (local) substitutes if $\frac{\partial^2\pi^o}{\partial w_k \partial w_l} < 0$ and (local) complements if $\frac{\partial^2\pi^o}{\partial w_k \partial w_l} > 0$ for $k \neq l$.[2] To compute these expressions, we calculate

$$\begin{aligned} \frac{\partial^2 c_i^{1-\sigma}}{\partial w_k \partial w_l} &= \frac{\partial}{\partial w_l}\left[(1-\sigma)c_i^{-\sigma}\cdot\frac{\partial c_i}{\partial w_k}\right] = \frac{\partial}{\partial w_l}\left[(1-\sigma)c_i^{-\sigma}\cdot \ell_{k,i}\right] = \\ &= (1-\sigma)\cdot\left[c_i^{-\sigma}\frac{\partial \ell_{k,i}}{\partial w_l} - \sigma\cdot c_i^{-\sigma-1}\ell_{l,i}\cdot\ell_{k,i}\right] \end{aligned}$$

where we use Shephard's lemma to derive the total demand for labor from location k, $\frac{\partial c_i}{\partial w_k} = \sum_{\nu\in\mathcal{V}} \ell_{k,i}(\nu) \equiv \ell_{k,i}$, and location l, $\frac{\partial c_i}{\partial w_l} = \sum_{\nu\in\mathcal{V}} \ell_{l,i}(\nu) \equiv \ell_{l,i}$.

It thus follows that assembly locations are (local) substitutes or complements, respectively, if

$$\text{(A14)} \qquad \min_{i,l,k}\left\{\frac{\mathcal{E}_{k,l}^i}{S_l^i}\right\} > \sigma \text{ or } \max_{i,l,k}\left\{\frac{\mathcal{E}_{k,l}^i}{S_l^i}\right\} < \sigma,$$

where $\mathcal{E}_{k,l}^i$ is the elasticity of substitution of conditional demand for labor in location k with respect to the price of labor in location l, and S_l^i is share of spending on labor from l in total spending on labor from different countries to serve market i:

$$\mathcal{E}_{k,l}^i = \frac{\partial \ell_{k,i}}{\partial w_l}\frac{w_l}{\ell_{k,i}} \text{ and } S_l^i = \frac{w_l \ell_{l,i}}{c_i} = \frac{w_l \ell_{l,i}}{\sum_{k\in\mathcal{K}} w_k \ell_{k,i}}.$$

In sum, we have that assembly locations are (local) substitutes or complements, respectively, if

$$\min_{i,l,k}\left\{\varepsilon_{k,l}^i\right\} > \sigma \text{ or } \max_{i,l,k}\left\{\varepsilon_{k,l}^i\right\} < \sigma,$$

where $\varepsilon_{k,l}^i$ is the (Allen) partial elasticity of substitution of labor across locations k and l, when producing goods for sale in market i.

Special Cases. The Armington setting in the main text corresponds to the following

[2]We assume that wages are firm-specific, so the aggregate demand $P_i^{\sigma-1}E_i$ is constant.

assumptions

$$\mathcal{V} = \mathcal{K}$$
$$F_i\left(\{q_i(\nu)\}\right) = \left(\sum_{\nu \in \mathcal{K}} q_i(\nu)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
$$q_i(\nu) = Z^a_{k,i} \cdot \ell_{k,i}(\nu) \text{ for } \nu = k \text{ and } Z^a_{k,i} = \frac{Z^a_k}{\tau^a_{k,i}} > 0$$
$$q_i(\nu) = 0 \text{ for } \nu \neq k,$$

while the setting in Tintelnot (2017) (extended to nested CES preferences) corresponds to[3]

$$\mathcal{V} = [0,1]$$
$$F_i\left(\{q_i(\nu)\}\right) = \left(\int_0^1 q_i(\nu)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
$$q_i(\nu) = \sum_{k \in \mathcal{K}} Z^a_{k,i}(\nu) \cdot \ell_{k,i}(\nu).$$

A5. Proofs of Propositions 1-3

Notation

Consider the general problem with firm- and plant-level fixed costs. Denote by $\mathcal{I}^x_i = 1$ if a firm paid firm-level fixed costs of marketing to destination i, $w_i f^x_i$, and $\mathcal{I}^x_i = 0$ otherwise; by $\mathcal{I}^s_j = 1$ if a firm paid firm-level fixed costs of importing from sourcing location j, $w_j f^s_j$, and $\mathcal{I}^s_j = 0$ otherwise; by $\mathcal{I}^a_k = 1$ if a firm paid firm-level fixed costs of assembly in location k, $w_k f^a_k$, and $\mathcal{I}^a_k = 0$ otherwise; by $\mathcal{I}^x_{ki} = 1$ if a firm paid plant-destination specific fixed costs of exporting from plant k to destination i, $w_i f^x_{ki}$, and $\mathcal{I}^x_{ki} = 0$ otherwise; by $\mathcal{I}^s_{jk} = 1$ if a firm paid sourcing-assembly specific fixed costs of importing from sourcing location j to assembly plant k, $w_j f^s_{jk}$, and $\mathcal{I}^s_{jk} = 0$ otherwise.

We denote by $\mathcal{I}^a = (\mathcal{I}^a_1, \ldots, \mathcal{I}^a_J)$ the vector of optimal decisions for assembly locations under ξ^a, and by $\hat{\mathcal{I}}^a = (\hat{\mathcal{I}}^a_1, \ldots, \hat{\mathcal{I}}^a_J)$ the optimal solution under $\hat{\xi}^a$. In a similar way, we denote by $\mathcal{I}^x$, $\mathcal{I}^s$, $\hat{\mathcal{I}}^x$, $\hat{\mathcal{I}}^s$ the vectors of optimal decisions for exporting and sourcing. We also denote by $\mathcal{I}^a_{-k}$ and $\hat{\mathcal{I}}^a_{-k}$ the vectors $\mathcal{I}^a$ and $\hat{\mathcal{I}}^a$ without elements $\mathcal{I}^a_k$ and $\hat{\mathcal{I}}^a_k$, respectively. For vectors X and Y, we say that $X \geq Y$ if $X_i \geq Y_i$ for all i, and $X > Y$ if $X \geq Y$ and $X_j > Y_j$ for some j.

In all propositions, we assume that $\xi^a_k > 0$ and $\xi^s_j > 0$ for all $k \in J$ and $j \in J$.

[3]We replaced the sum with an integral.

General Profit Function

Consider the general profit function with firm- and plant-level fixed costs:

(A15)

$$\pi = \kappa_\pi \varphi^{\sigma-1} \cdot \overbrace{\sum_{i\in J} \mathcal{I}_i^x \cdot E_i P_i^{\sigma-1}}^{\text{Destinations}} \left[\sum_{k\in J} \overbrace{\mathcal{I}_{k,i}^x \mathcal{I}_k^a \cdot \xi_k^a \left(\tau_{ki}^a\right)^{1-\varepsilon}}^{\text{Assembly}} \left(\overbrace{\sum_{j\in J} \mathcal{I}_{j,k}^s \mathcal{I}_j^s \xi_j^s \left(\tau_{jk}^s\right)^{1-\rho}}^{\text{Sourcing}} \right)^{\mu} \right]^{\theta} -$$

$$- \underbrace{\sum_{i\in J}\sum_{k\in J} \mathcal{I}_{k,i}^x \cdot w_i f_{k,i}^x - \sum_{k\in J}\sum_{j\in J} \mathcal{I}_{j,k}^s \cdot w_j f_{j,k}^x}_{\text{Plant-Level FC}} - \underbrace{\sum_{i\in J} \mathcal{I}_i^x \cdot w_i f_i^x - \sum_{k\in J} \mathcal{I}_k^a \cdot w_k f_k^a - \sum_{j\in J} \mathcal{I}_j^s \cdot w_j f_j^s}_{\text{Firm-Level FC}},$$

where

$$\theta = \frac{\sigma-1}{\varepsilon-1} \text{ and } \mu = \frac{\alpha(\varepsilon-1)}{\rho-1}.$$

If $\sigma \geq \varepsilon$ and $\alpha(\varepsilon-1) \geq \rho-1$, then the profit function in (A15) is supermodular in $(\mathcal{I}', \mathcal{I}'')$ and has increasing differences in $(\mathcal{I}, \xi_k^a)$, where $\mathcal{I}'$ and $\mathcal{I}''$ are two any indicator variables in (A15). Therefore, by Topkis' Theorem

$$\text{If } \hat{\xi}_k^a \geq \xi_k^a, \text{ then } \hat{\mathcal{I}} \geq \mathcal{I}.$$

As shown below, this result will suffice to prove all Propositions for the case of $\sigma \geq \varepsilon$ and $\alpha(\varepsilon-1) \geq \rho-1$.

Proposition 1

In our baseline model without fixed costs of exporting or intermediate inputs, a firm solves the following problem:

$$\text{(A16)} \quad \max_{\mathcal{I}^a} \ \pi\left(\mathcal{I}^a; \xi^a\right) = \kappa_\pi \varphi^{\sigma-1} \cdot \sum_{i\in J} E_i P_i^{\sigma-1} \left[\sum_{k\in J} \mathcal{I}_k^a \cdot \xi_k^a \left(\tau_{ki}^a\right)^{1-\varepsilon} \right]^{\frac{\sigma-1}{\varepsilon-1}} - \sum_{k\in J} \mathcal{I}_k^a \cdot w_k f_k^a,$$

which is a special case of (A15) under $\mu = 0$ and all fixed costs equal to zero except for the assembly ones, $f_k^a > 0$. We prove the following proposition:

PROPOSITION 1: *Consider the problem in* (A16) *and an increase in the assembly potential of plant k, $\hat{\xi}_k^a > \xi_k^a$, holding other parameters and $P_i^{\sigma-1} E_i$ fixed. If $\varepsilon \leq \sigma$, then $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$. If $\varepsilon > \sigma$ and $\mathcal{I}^a$ is a unique solution, then $\hat{\mathcal{I}}_k^a \geq \mathcal{I}_k^a$, and it is **not** possible that $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$.*

PROOF:

For the case $\varepsilon \leq \sigma$, we can apply Topkis' theorem.

Consider the case $\varepsilon > \sigma$. If $\mathcal{I}^a$ is an optimal solution under $\xi^a = (\xi_1^a, \ldots, \xi_k^a, \ldots, \xi_J^a)$, then

$$\pi(\mathcal{I}^a; \xi^a) \geq \pi(\tilde{\mathcal{I}}^a; \xi^a) \text{ for all } \tilde{\mathcal{I}}^a \in 2^J.$$

To prove that $\hat{\mathcal{I}}_k^a \geq \mathcal{I}_k^a$, assume, by contradiction, that $\hat{\mathcal{I}}_k^a = 0 < \mathcal{I}_k^a = 1$. Notice that $\pi\left(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \xi^a\right)$ is increasing in ξ_k^a while $\pi\left(\mathcal{I}_k^a = 0, \tilde{\mathcal{I}}_{-k}^a; \xi^a\right)$ is independent of ξ_k^a for all $\tilde{\mathcal{I}}_{-k}^a$ and ξ_{-k}^a, where ξ_{-k}^a is vector ξ^a without an element ξ_k^a. Therefore,

$$\pi(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \hat{\xi}^a) > \pi(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \xi^a) \geq \pi(\hat{\mathcal{I}}_k^a = 0, \hat{\mathcal{I}}_{-k}^a; \xi^a) = \pi(\hat{\mathcal{I}}_k^a = 0, \hat{\mathcal{I}}_{-k}^a; \hat{\xi}^a),$$

which is a contradiction. Therefore, $\hat{\mathcal{I}}_k^a \geq \mathcal{I}_k^a$.

For the second part, suppose, by contradiction, that $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$. Consider three cases. First, suppose that $\mathcal{I}_k^a = 1$. Then, $\hat{\mathcal{I}}_k^a = 1$, and

(A17)
$$\pi\left(\hat{\mathcal{I}}_k^a = 1, \hat{\mathcal{I}}_{-k}^a; \hat{\xi}^a\right) - \pi\left(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \hat{\xi}^a\right) < \pi\left(\hat{\mathcal{I}}_k^a = 1, \hat{\mathcal{I}}_{-k}^a; \xi^a\right) - \pi\left(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \xi^a\right) \leq 0,$$

where the the first inequality comes from $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$ and $\varepsilon > \sigma$, and the second inequality comes from the optimality of $\mathcal{I}^a$ under ξ^a. This inequality contradicts the optimality of $\hat{\mathcal{I}}^a$ under $\hat{\xi}^a$.

Second, suppose that $\hat{I}_k^a = 0$. Then, $\mathcal{I}_k^a = 0$, and $\hat{\mathcal{I}}^a$ should be the optimal solution under both ξ^a and $\hat{\xi}^a$. This result contradicts the uniqueness of the solution.

Finally, suppose that $\mathcal{I}_k^a = 0$ and $\hat{\mathcal{I}}_k^a = 1$. The optimality of $\mathcal{I}^a$ under ξ^a implies

$$\pi\left(\hat{\mathcal{I}}_k^a = 1, \hat{\mathcal{I}}_{-k}^a; \xi^a\right) - \pi\left(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \xi^a\right) \leq \pi\left(\hat{\mathcal{I}}_k^a = 1, \hat{\mathcal{I}}_{-k}^a; \xi^a\right) - \pi\left(\mathcal{I}_k^a = 0, \mathcal{I}_{-k}^a; \xi^a\right) \leq 0.$$

Combining this inequality with (A17), we get a contradiction for the optimality of $\hat{\mathcal{I}}^a$: $\pi\left(\hat{\mathcal{I}}_k^a = 1, \hat{\mathcal{I}}_{-k}^a; \hat{\xi}^a\right) < \pi\left(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \hat{\xi}^a\right)$. □

Note: If parameters in (A15) are randomly drawn from continuous distributions, the solution is generically unique. To see the problem with multiple solutions, consider the following example. There are two plant decisions and one market with $\kappa_\pi \varphi^{\sigma-1} E_i P_i^{\sigma-1} = 1$. Suppose that $w_1 f_1^a = 100$, $w_2 f_2^a = 1$, $\xi_2^a = 1$, $\tau_{1i}^a = \tau_{2i}^a = 1$, and we consider a change from $\xi_1^a = 1$ to $\hat{\xi}_2^a = 2$. The firm chooses $\mathcal{I}_1^a = \hat{\mathcal{I}}_1^a = 0$, it is indifferent between $\mathcal{I}_2^a = 1$ and $\mathcal{I}_2^a = 0$ under ξ_1^a, and between $\hat{\mathcal{I}}_2^a = 1$ and $\hat{\mathcal{I}}_2^a = 0$ under $\hat{\xi}_1^a$. Therefore, we might have $\mathcal{I}_2^a = 0$ and $\hat{\mathcal{I}}_2^a = 1$ due to multiplicity, leading to $\hat{\mathcal{I}}_{-1}^a > \mathcal{I}_{-1}^a$ for $\hat{\xi}_1^a > \xi_1^a$. If we specify a solution selection, the proposition can be refined for the case with multiple solutions, for instance, by always choosing the solution with the largest number of active plants.

Proposition 2

We add firm-level exporting fixed costs. A firm solves the following problem:

(A18)
$$\max_{\mathcal{I}^a, \mathcal{I}^x} \quad \kappa_\pi \varphi^{\sigma-1} \cdot \sum_{i \in J} \mathcal{I}_i^x \cdot E_i P_i^{\sigma-1} \left[\sum_{k \in J} \mathcal{I}_k^a \cdot \xi_k^a \left(\tau_{ki}^a\right)^{1-\varepsilon}\right]^{\frac{\sigma-1}{\varepsilon-1}} - \sum_{i \in J} \mathcal{I}_i^x \cdot w_i f_i^x - \sum_{k \in J} \mathcal{I}_k^a \cdot w_k f_k^a,$$

which is a special case of (A15) under $\mu = 0$ and all fixed costs equal to zero except assembly and firm-level exporting ones, $f_k^a > 0$ and $f_i^x > 0$. We prove the following proposition:

PROPOSITION 2: *Consider the problem with firm-level fixed costs of exporting* (A18) *and an increase in the assembly potential of plant k, $\hat{\xi}_k^a > \xi_k^a$, holding other parameters and $P_i^{\sigma-1} E_i$ fixed. If $\varepsilon \leq \sigma$, then $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$. If $\varepsilon > \sigma$, then it is possible that $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$.*

PROOF:

For the case $\varepsilon \leq \sigma$, we can apply Topkis' theorem.

Consider the case $\varepsilon > \sigma$, it is sufficient to construct an example in which a rise in ξ_k^a leads to an opening of assembly plants in $l \neq k$. For simplicity, we assume that there is only one feasible destination market i, with $\tau_{ki'}^a = \infty$ for $i' \neq i$. Suppose that all assembly fixed costs are very small and equal to $\delta > 0$. The firm-level fixed cost of exporting to i is such that

$$\kappa_\pi \varphi^{\sigma-1} \cdot E_i P_i^{\sigma-1} \cdot \left(\xi_k^a \left(\tau_{ki}\right)^{1-\varepsilon} + \sum_{l \neq k} \xi_l^a \left(\tau_{li}^a\right)^{1-\varepsilon} \right)^{\frac{\sigma-1}{\varepsilon-1}} < w_i f_i^x$$

$$\kappa_\pi \varphi^{\sigma-1} \cdot E_i P_i^{\sigma-1} \cdot \left(\hat{\xi}_k^a \left(\tau_{ki}\right)^{1-\varepsilon} + \sum_{l \neq k} \xi_l^a \left(\tau_{li}^a\right)^{1-\varepsilon} \right)^{\frac{\sigma-1}{\varepsilon-1}} > w_i f_i^x.$$

For sufficiently small δ, an increase in ξ_k^a leads from an optimum with no assembly plants to the optimum in which all plants are activated. □

Proposition 3

We add firm-level importing fixed costs. A firm solves the following problem:

(A19)

$$\max_{\mathcal{I}^a, \mathcal{I}^s} \quad \kappa_\pi \varphi^{\sigma-1} \cdot \sum_{i \in J} E_i P_i^{\sigma-1} \left[\sum_{k \in J} \mathcal{I}_k^a \cdot \xi_k^a \left(\tau_{ki}^a\right)^{1-\varepsilon} \cdot \left(\sum_{j \in J} \mathcal{I}_j^s \cdot \xi_j^s \left(\tau_{jk}^s\right)^{1-\rho} \right)^{\frac{\alpha(\varepsilon-1)}{\rho-1}} \right]^{\frac{\sigma-1}{\varepsilon-1}} - \\ - \sum_{j \in J} \mathcal{I}_j^x \cdot w_j f_j^s - \sum_{k \in J} \mathcal{I}_k^a \cdot w_k f_k^a,$$

which is a special case of (A15) under $\mu > 0$ and all fixed costs equal to zero except assembly and firm-level importing ones, $f_k^a > 0$ and $f_j^s > 0$. We prove the following proposition:

PROPOSITION 3: *Consider the problem with firm-level fixed costs of importing* (A19) *and an increase in the assembly potential of plant k, $\hat{\xi}_k^a > \xi_k^a$, holding other parameters and $P_i^{\sigma-1} E_i$ fixed. If $\varepsilon \leq \sigma$, then $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$. Assume that $\alpha(\varepsilon - 1) \geq \rho - 1$. If $\varepsilon > \sigma$, then it is possible that $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$.*

PROOF:

Consider the following example. Assume that there is only one feasible destination market i, with $\tau_{ki'}^a = \infty$ for $i' \neq i$, and one sourcing location j, with $\tau_{j'k}^s = \infty$ for $j' \neq j$. Assume also that $\tau_{jk}^s = 1$ for all k. Suppose that all assembly fixed costs are very small and equal to $\delta > 0$. The firm-level fixed cost of sourcing from j is such that

$$\kappa_\pi \varphi^{\sigma-1} E_i P_i^{\sigma-1} \cdot \left(\xi_j^s\right)^{\frac{\alpha(\sigma-1)}{\rho-1}} \cdot \left(\xi_k^a \left(\tau_{ki}\right)^{1-\varepsilon} + \sum_{l \neq k} \xi_l^a \left(\tau_{li}^a\right)^{1-\varepsilon} \right)^{\frac{\sigma-1}{\varepsilon-1}} < w_j f_j^s$$

$$\kappa_\pi \varphi^{\sigma-1} E_i P_i^{\sigma-1} \cdot \left(\xi_j^s\right)^{\frac{\alpha(\sigma-1)}{\rho-1}} \cdot \left(\hat{\xi}_k^a \left(\tau_{ki}\right)^{1-\varepsilon} + \sum_{l \neq k} \xi_l^a \left(\tau_{li}^a\right)^{1-\varepsilon} \right)^{\frac{\sigma-1}{\varepsilon-1}} > w_j f_j^s.$$

For sufficiently small δ, an increase in ξ_k^a leads from an optimum with no assembly plants to the optimum in which all plants are activated. □

Plant-Level Fixed Costs

Consider the problem with plant-level fixed costs of exporting. A firm solves the following problem:

$$\text{(A20)} \quad \max_{\mathcal{I}^x,\mathcal{I}^a} \quad \kappa_\pi \varphi^{\sigma-1} \cdot \sum_{i\in J} E_i P_i^{\sigma-1} \left[\sum_{k\in J} \mathcal{I}^x_{k,i} \mathcal{I}^a_k \cdot \xi^a_k \left(\tau^a_{ki}\right)^{1-\varepsilon} \right]^{\frac{\sigma-1}{\varepsilon-1}} - \\ - \underbrace{\sum_{i\in J}\sum_{k\in J} \mathcal{I}^x_{k,i} \cdot w_i f^x_{k,i}}_{\text{Plant-Level FC}} - \underbrace{\sum_{k\in J} \mathcal{I}^a_k \cdot w_k f^a_k}_{\text{Firm-Level FC}} .$$

We can then prove that:

PROPOSITION 4: *Consider the problem in* (A20) *and an increase in the assembly potential of plant k, $\hat{\xi}^a_k > \xi^a_k$, holding other parameters and $P_i^{\sigma-1}E_i$ fixed. If $\varepsilon \leq \sigma$, then $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$ and $\hat{\mathcal{I}}^x \geq \mathcal{I}^x$. If $\varepsilon > \sigma$ and the solution is unique, then $\hat{\mathcal{I}}^a_k \geq \mathcal{I}^a_k$, and it is **not** possible that $\hat{\mathcal{I}}^a_{-k} > \mathcal{I}^a_{-k}$ and $\hat{\mathcal{I}}^x > \mathcal{I}^x$.*

PROOF:

For the case $\varepsilon \leq \sigma$, we can apply Topkis' theorem. Consider the case $\varepsilon > \sigma$. The proof follows the same steps as the proof of Proposition 1. Under $\varepsilon > \sigma$, the assumption $\hat{\mathcal{I}}^a_{-k} > \mathcal{I}^a_{-k}$ and $\hat{\mathcal{I}}^x > \mathcal{I}^x$ contradicts the optimality (or uniqueness) of the solution. □

Now consider the problem with plant-level fixed costs of importing. A firm solves the following problem:

(A21)

$$\max_{\mathcal{I}^s,\mathcal{I}^a} \quad \kappa_\pi \varphi^{\sigma-1} \cdot \sum_{i\in J} E_i P_i^{\sigma-1} \left[\sum_{k\in J} \mathcal{I}^a_k \cdot \xi^a_k \left(\tau^a_{ki}\right)^{1-\varepsilon} \cdot \left(\sum_{j\in J} \mathcal{I}^s_{j,k} \xi^s_j \left(\tau^s_{jk}\right)^{1-\rho} \right)^{\frac{\alpha(\varepsilon-1)}{\rho-1}} \right]^{\frac{\sigma-1}{\varepsilon-1}} - \\ - \underbrace{\sum_{k\in J}\sum_{j\in J} \mathcal{I}^s_{j,k} \cdot w_j f^x_{j,k}}_{\text{Plant-Level FC}} - \underbrace{\sum_{k\in J} \mathcal{I}^a_k \cdot w_k f^a_k}_{\text{Firm-Level FC}} .$$

We can then prove that:

PROPOSITION 5: *Consider the problem in* (A21) *and an increase in the assembly potential of plant k, $\hat{\xi}^a_k > \xi^a_k$, holding other parameters and $P_i^{\sigma-1}E_i$ fixed. Assume that $\alpha(\varepsilon-1) \geq \rho - 1$. If $\varepsilon \leq \sigma$, then $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$ and $\hat{\mathcal{I}}^s \geq \mathcal{I}^s$. If $\varepsilon > \sigma$ and the solution is unique, then $\hat{\mathcal{I}}^a_k \geq \mathcal{I}^a_k$, and it is **not** possible that $\hat{\mathcal{I}}^a_{-k} > \mathcal{I}^a_{-k}$ and $\hat{\mathcal{I}}^s > \mathcal{I}^s$.*

PROOF:

For the case $\varepsilon \leq \sigma$, we can apply Topkis' theorem. Consider the case $\varepsilon > \sigma$. The proof follows the same steps as the proof of Proposition 1. Under $\varepsilon > \sigma$, the assumption $\hat{\mathcal{I}}^a_{-k} > \mathcal{I}^a_{-k}$ and $\hat{\mathcal{I}}^s > \mathcal{I}^s$ contradicts the optimality (or uniqueness) of the solution. □