

# ONLINE APPENDIX TO “INEQUALITY, TAXATION, AND SOVEREIGN DEFAULT RISK”

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## A Data and Additional Empirical Results

### A.1 Data Sources

*State government CDS spreads*: Bloomberg.

*Municipal bond yields*: Global Public Finance database.

*State Gini index*: U.S. Census Bureau and American Community Survey.

*State party control*: National Conference of State Legislatures.

*State total output*: U.S. Bureau of Economic Analysis.

*State government debt*: U.S. Census State Finances Dataset.

*State-to-state migration*: Internal Revenue Service Migration Dataset.

*State government tax revenue*: U.S. Census Bureau, "State and Local Government Finance".

*State government expenditures*: U.S. Census Bureau, "State and Local Government Finance".

*Maximum state income tax rate*: NBER's calculations using TAXSIM model.

*State unemployment rate*: Local Area Unemployment Statistics.

*State real personal income*: Bureau of Economic Analysis.

*State price parities*: Bureau of Economic Analysis.

*Country government bond spreads*: OECD Database.

*Country Gini index*: World Income Inequality Database (WIID4).

*Country income shares by quintile groups*: World Income Inequality Database (WIID4).

*Country debt-to-GDP ratio*: central government debt as the percentage of GDP, IMF.

### A.2 State Gini Index

Table A.1 reports the average Gini index for each state from year 2000 to 2019.

### A.3 Construction of State Government Bond Spreads

The data on municipal bond issuance comes from the Global Public Finance database of the Securities Data Company (SDC). The dataset contains rich information on various characteristics of newly issued bonds at the state and local levels, including issuer information, amount issued, years to maturity, coupon, prices and yields, and credit ratings, among others.

Table A.1: State Gini Index

State	Gini	State	Gini	State	Gini
Alabama	0.472	Louisiana	0.482	Ohio	0.452
Alaska	0.412	Maine	0.44	Oklahoma	0.458
Arizona	0.456	Maryland	0.444	Oregon	0.45
Arkansas	0.46	Massachusetts	0.472	Pennsylvania	0.46
California	0.475	Michigan	0.452	Rhode Island	0.459
Colorado	0.45	Minnesota	0.436	South Carolina	0.463
Connecticut	0.485	Mississippi	0.472	South Dakota	0.439
Delaware	0.439	Missouri	0.451	Tennessee	0.468
Florida	0.474	Montana	0.443	Texas	0.473
Georgia	0.468	Nebraska	0.435	Utah	0.414
Hawaii	0.433	Nevada	0.446	Vermont	0.434
Idaho	0.431	New Hampshire	0.425	Virginia	0.458
Illinois	0.469	New Jersey	0.466	Washington	0.446
Indiana	0.437	New Mexico	0.467	West Virginia	0.457
Iowa	0.429	New York	0.501	Wisconsin	0.43
Kansas	0.446	North Carolina	0.465	Wyoming	0.425
Kentucky	0.465	North Dakota	0.444		

Note: This table reports the average Gini index for each state from year 2000 to 2019. Data source: U.S. Census Bureau and American Community Survey.

As most municipal bonds are exempt from federal and state taxes, state bond yields are adjusted by a tax-adjustment factor  $\tau_{s,t}$  specified as  $1 - \tau_{s,t} = (1 - \tau_{s,t}^{fed})(1 - \tau_{s,t}^{state})$ , where  $\tau_{s,t}^{fed}$  and  $\tau_{s,t}^{state}$  denote the top federal and maximum state income tax rates, following [Schwert \(2017\)](#).

State bond spreads are calculated as the difference in yields between a municipal bond and a synthetic treasury bond with equivalent coupon and maturity date. First, for each municipal bond, solve for the theoretical price on a synthetic treasury bond with the same maturity date and coupon rate by calculating the present value of its coupon payments and face value using the U.S. Treasury yield curve.

$$P_N^T = \sum_{n=1}^N \frac{C/2}{(1 + r_n^T/2)^n} + \frac{100}{(1 + r_N^T/2)^N}$$

where  $r_n^T$  is the set of treasury spot rates estimated in [Gürkaynak, Sack and Wright \(2007\)](#). Second, calculate the yield-to-maturity of the synthetic Treasury bond using this price, the coupon payments, and the face value. Last, take the difference between the municipal bond yield and the synthetic Treasury bond yield to generate a bond spread. This procedure

is similar to the yield spread calculation in Longstaff, Mithal and Neis (2005) and Ang, Bhansali and Xing (2014), among others.

## A.4 Institutional Details for State Government Finances

**Balanced budget requirements.** Balanced budget requirements typically only apply to state operating budgets. Bond finance for capital projects generally does not fall within any constraints of a balanced budget requirement. Less attention (if any) is given to the question of whether a state's entire budget is in balance.<sup>1</sup> The details of balanced budget requirements vary across states, and political cultures reinforce the requirements.

**State debt limits.** States structure their debt limits very differently. For authorized debt, some states have quite a strict limit, for example, Georgia restricts debt to less than 3.5% of personal income and less than \$1200 in debt per capita as specified in their Debt Management Plan.<sup>2</sup> Some states have less restrictive debt limits. For example, the policy to limit authorized debt for Illinois is that a three-fifths vote of the legislature is required to increase the state debt limit. Out of 50 states, seven states do not have any debt limits (including authorized debt and debt service): Arkansas, California, Montana, New Hampshire, New Mexico, Oklahoma, and Oregon.

**State tax and expenditure limits.** Tax and expenditure limits (TEs) restrict the growth of government revenues or spending by either capping them at fixed-dollar amounts or limiting their growth rate to match increases in population, inflation, personal income, or some combination of those factors. Most states do not have a revenue limit.<sup>3</sup> About half of the states do not have a spending limit.<sup>4</sup>

**State government expenditures over time.** State governments spent about \$2.15 trillion on general government expenditures in fiscal year 2019. State government general expen-

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<sup>1</sup>National Conference of State Legislatures Fiscal Brief, <https://docs.house.gov/meetings/JU/JU00/20170727/106327/HHRG-115-JU00-20170727-SD002.pdf>

<sup>2</sup>The Debt Management Plan is adopted by the Georgia State Financing and Investment Commission annually and sets target planning ratios for current and future debt for a five-year projection cycle.

<sup>3</sup>Only four states (Colorado, Florida, Michigan, Missouri) have a revenue limit. For Florida, for instance, its revenue is limited to the average growth rate in state personal income for the previous five years. Source: National Association of State Budget Officers, "Budget Processes in the States," Spring 2015.

<sup>4</sup>States with no limits on spending: Alabama, Arkansas, Florida, Georgia, Illinois, Kansas, Kentucky, Maryland, Massachusetts, Minnesota, Missouri, Montana, Nebraska, New Hampshire, New Mexico, New York, North Dakota, Pennsylvania, South Dakota, Vermont, Virginia, West Virginia, Wisconsin, Wyoming. Source: National Association of State Budget Officers, "Budget Processes in the States," Spring 2015.

ditures fall into one of these categories: education, public welfare, health and hospitals, highways, police protection, fire protection, corrections, natural resources, parks and recreation, housing and community development, sewerage and solid waste, and interest on general debt. Figure A.1 plots the state general expenditures by functional category from 1977 to 2019. Public welfare constitutes a large and growing portion of state spending.

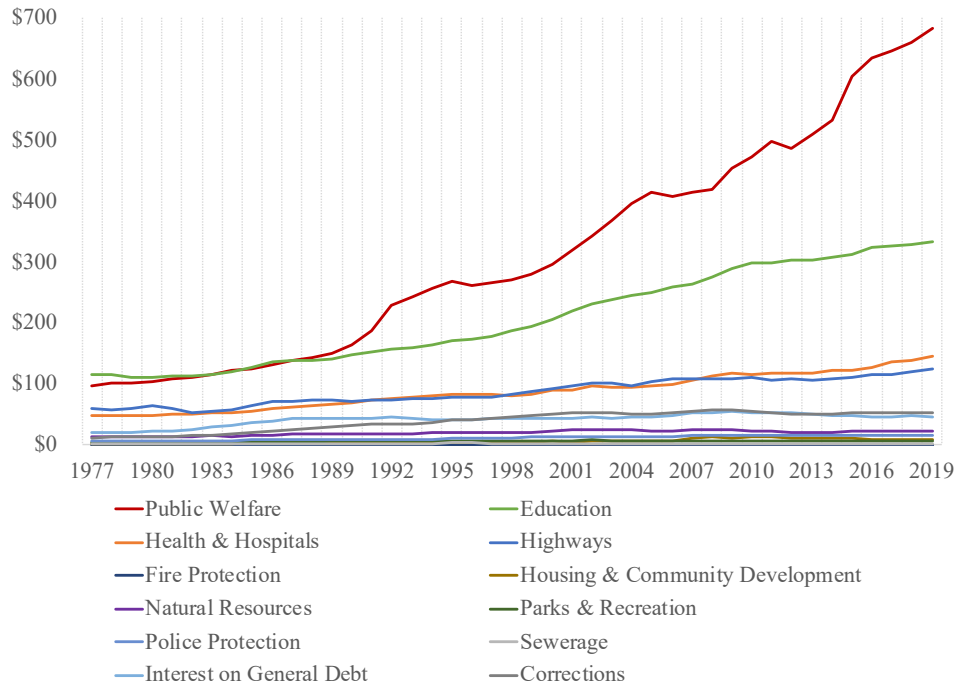


Figure A.1: State Government General Expenditures, by Function

Note: This figure plots state government general expenditures (billions of dollars, real, 2019 dollars) decomposed by function from 1977 to 2019. Data source: US Census Bureau’s Census of Governments and its associated annual survey. Compiled by the Urban-Brookings Tax Policy Center. Washington, DC. Website: <https://state-local-finance-data.taxpolicycenter.org>

## A.5 Government Spreads and Migration: Additional Figures

Figure A.2 and A.3 plot state-level net migration rates and government spreads winsorized at 1% and 5% level, respectively. The results remain robust: net migration rate is negatively correlated with government spreads.

## A.6 Additional State-level Results

A novel mechanism in this paper that generates the positive correlation between spreads and income inequality is endogenous tax progressivity. Here I use state-level data to test

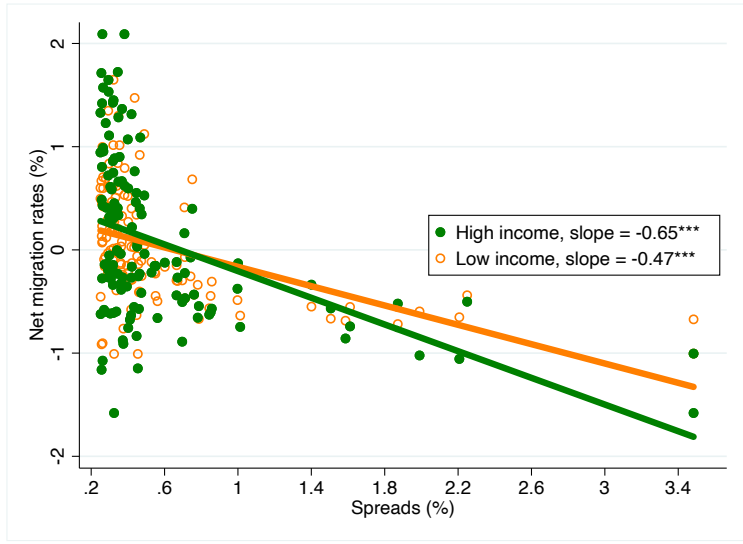


Figure A.2: Government spreads and migration: winsorize at 1%

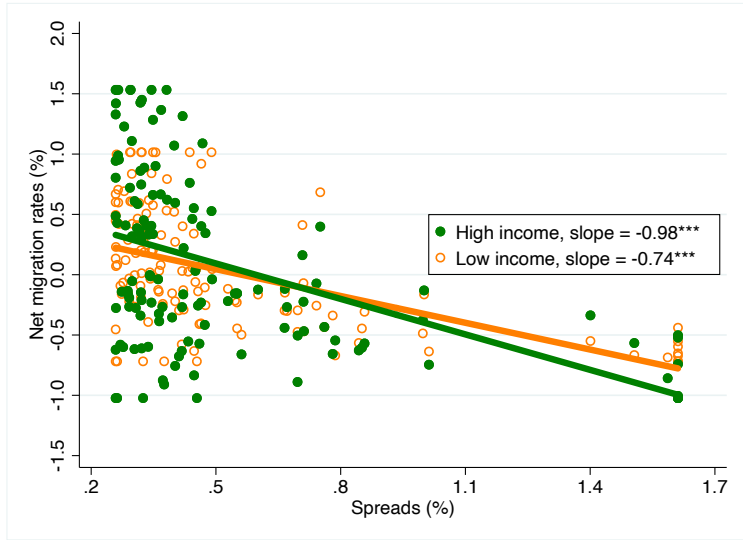


Figure A.3: Government spreads and migration: winsorize at 5%

the following two model predictions. First, with higher inequality, a government tends to impose a more progressive income tax system; second, more progressive taxation is associated with higher government spreads.

The empirical specification that explores the relationship between tax progressivity and income inequality is as follows:

$$prog_{jt} = \beta_0 + \beta_1 ineq_{j,t-1} + \Gamma' Z_{j,t-1} + \alpha_t + \epsilon_{jt}, \quad (A.1)$$

where  $prog_{jt}$  is income tax progressivity in state  $j$  in year  $t$ , which is proxied for by the maximum state income tax rate;  $ineq_{j,t-1}$  is pre-tax income inequality proxied for by the Gini index for state  $j$  in year  $t - 1$ ; and  $Z_{j,t-1}$  is a vector of control variables, including state total output, the debt-to-output ratio, and political party control of state legislatures.  $\alpha_t$  is a time fixed effect. Data covers 49 states from 2006 to 2017.<sup>5</sup> Coefficient  $\beta_1$  captures the correlation between income inequality and tax progressivity.

Table A.2 reports the result for regression (A.1), showing that a more unequal state tends to impose a more progressive income tax system. Also, the states with Democratic-controlled or split legislatures are more likely to impose a more progressive tax than those with Republican-controlled legislatures.

Table A.2: Regression of tax progressivity on inequality

	(1)	(2)
Gini	26.78 (7.64)	16.38 (8.33)
Political (=Split)		1.55 (0.47)
Political (=Democratic)		3.10 (0.36)
Year FE	Yes	Yes
Controls	Yes	Yes
$N$	408	392
$R^2$	0.05	0.20

Note: Standard errors in parentheses

To explore the correlation between government bond spreads and tax progressivity, I use the following empirical specification:

$$spread_{jt} = \beta_0 + \beta_1 prog_{j,t-1} + \Gamma' Z_{j,t-1} + \alpha_t + \epsilon_{jt}, \quad (A.2)$$

<sup>5</sup>Nebraska does not have partisan composition (political party control of state legislatures) data since it is a non-partisan unicameral legislature. Thus, after merging the variables, the panel covers 49 states.

where  $spread_{jt}$  is the average CDS spread for state  $j$  in year  $t$ . Table A.3 shows the regression results. A more progressive tax is associated with higher government bond spreads. Since CDS spreads data is available for 19 states, the number of observations is smaller than for regression (A.1).

Table A.3: Regression of spreads on tax progressivity

	(1)	(2)
Progressivity	0.03 (0.01)	0.02 (0.01)
Political (=Split)		0.33 (0.16)
Political (=Democratic)		0.29 (0.11)
Year FE	Yes	Yes
Controls	Yes	Yes
$N$	109	109
$R^2$	0.55	0.58

Note: Standard errors in parentheses

## A.7 Cross-country Empirical Evidence

To explore the correlation between government spreads and income inequality across countries, I use the following empirical specification:

$$spread_{jt} = \beta_0 + \beta_1 ineq_{j,t-1} + \Gamma' Z_{j,t-1} + \alpha_t + \epsilon_{jt}, \quad (A.3)$$

where  $spread_{jt}$  is the government bond spread of country  $j$  in period  $t$ . Spread here is defined as the 10-year government bond interest rate of country  $j$  in period  $t$  minus that of the U.S. for the same period;  $ineq_{j,t-1}$  is income inequality for country  $j$  in period  $t - 1$ . Here I use two measures for income inequality: the pre-tax Gini index and the gap between the income shares of the top 20% and the bottom 20%.  $Z_{j,t-1}$  includes real per-capita GDP and debt-to-GDP ratio as controls.  $\alpha_t$  is the time fixed effect. The panel covers 1960-2017 and contains 35 countries.<sup>6</sup>

Table A.4 shows the results of specification (A.3). Columns (1) and (2) use the Gini index as the measure of income inequality, and columns (3) and (4) use the gap between

<sup>6</sup>Countries in the sample: Australia, Austria, Belgium, Canada, Switzerland, Chile, Colombia, Costa Rica, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Hungary, India, Ireland, Iceland, Israel, Italy, Japan, Korea, Lithuania, Luxembourg, Latvia, Mexico, Netherlands, Norway, New Zealand, Poland, Portugal, Slovenia, Sweden, and South Africa.

the income shares of the top 20% and the bottom 20% to measure inequality. The results show that high inequality is associated with high government default risk. Increasing the Gini index by 0.1 (e.g., Sweden to Portugal) is associated with government bond spread increases of about 0.5%.

Table A.4: Regression of government spreads on inequality  
(cross-country)

	(1)	(2)	(3)	(4)
Gini	12.29 (1.32)	4.96 (1.59)		
top-bottom-gap			11.96 (1.34)	4.84 (1.53)
Year FE	Yes	Yes	Yes	Yes
Controls	No	Yes	No	Yes
$N$	688	540	604	486
$R^2$	0.30	0.48	0.31	0.47

Note: This table reports regression results for the cross-country sample. Columns (1) and (2) report the results when using the Gini index as measure for inequality; columns (3) and (4) instead use the gap between the income shares of the top 20% and the bottom 20%. Standard errors in parentheses.

## B Theoretical Appendix

### B.1 Model Proofs

Here I prove some results in Section II.D including 1) the monotonicity of each term in (21) with respect to tax progressivity  $\tau$ ; and 2) that the default set is larger with higher inequality. I also show the equivalence of the transformed problem (in Section II.E) and the original problem.

**Monotonicity of each term in (21).** Taking derivatives for each term in the government repayment value (21) with respect to  $\tau$  generates:

(i)

$$\frac{\partial \log(Y - B_0)}{\partial \tau} = - \frac{A\bar{z} \frac{1}{1+\gamma} (1 - \tau)^{\frac{1}{1+\gamma}-1}}{A\bar{z}(1 - \tau)^{\frac{1}{1+\gamma}} - B_0} < 0$$



(ii)

$$\frac{\partial \frac{1-\tau}{1+\gamma}}{\partial \tau} = -\frac{1}{1+\gamma} < 0$$

(iii)

$$\frac{\partial \frac{1}{2} \log[\alpha(1-\alpha)]}{\partial \tau} = \frac{1}{2} \frac{(z_H^{1-\tau} - z_L^{1-\tau})(\ln z_H - \ln z_L)}{z_L^{1-\tau} + z_H^{1-\tau}} > 0$$

Thus, in the repayment value function, total consumption is decreasing in  $\tau$ , disutility from working is decreasing in  $\tau$ , and redistribution is increasing in  $\tau$ .

**Default set is larger under higher inequality.** The government's productivity threshold  $\bar{A}$  that satisfies  $V^d(A) = V^c(B_0, A)$  is given by:

$$\bar{A} = \frac{B_0}{\bar{z}(\ell - \Theta \ell^d)}$$

where

$$\Theta = \exp\left(-\frac{1}{2} \log \frac{\alpha(1-\alpha)}{\alpha^d(1-\alpha^d)} - \frac{\tau - \tau^d}{1+\gamma}\right),$$

and

$$\alpha \equiv \frac{(\bar{z} - \sigma_z)^{1-\tau}}{(\bar{z} - \sigma_z)^{1-\tau} + (\bar{z} + \sigma_z)^{1-\tau}},$$

$$\alpha^d \equiv \frac{(\bar{z} - \sigma_z)^{1-\tau^d}}{(\bar{z} - \sigma_z)^{1-\tau^d} + (\bar{z} + \sigma_z)^{1-\tau^d}}.$$

**Lemma 1.**  $\Theta$  is increasing in  $\sigma_z$ .

Since  $\Theta$  is increasing in  $\sigma_z$ , we have  $\frac{\partial \bar{A}}{\partial \sigma_z} > 0$ . That is, higher inequality (a higher value for  $\sigma_z$ ) would lead to a higher productivity threshold  $\bar{A}$ , and thus a larger default set. Alternatively, one can write down the borrowing threshold and show that a higher  $\sigma_z$  leads to a lower borrowing threshold  $\bar{B}_0$ .

Proof of Lemma 1:

Take the derivative of  $\Theta$  with respect to  $\sigma_z$ :

$$\frac{\partial \Theta}{\partial \sigma_z} = \Theta \frac{\partial \left[-\frac{1}{2} \log \frac{\alpha(1-\alpha)}{\alpha^d(1-\alpha^d)}\right]}{\partial \sigma_z},$$

where

$$\frac{\alpha(1-\alpha)}{\alpha^d(1-\alpha^d)} = [(\bar{z} - \sigma_z)(\bar{z} + \sigma_z)]^{\tau^d - \tau} \left[ \frac{(\bar{z} - \sigma_z)^{1-\tau^d} + (\bar{z} + \sigma_z)^{1-\tau^d}}{(\bar{z} - \sigma_z)^{1-\tau} + (\bar{z} + \sigma_z)^{1-\tau}} \right]^2,$$

then

$$\frac{\partial \Theta}{\partial \sigma_z} = \frac{\Theta \bar{z}}{\ln(10)(\bar{z} - \sigma_z)(\bar{z} + \sigma_z)} \left[ (1 - \tau) \frac{(\bar{z} + \sigma_z)^{1-\tau} - (\bar{z} - \sigma_z)^{1-\tau}}{(\bar{z} + \sigma_z)^{1-\tau} + (\bar{z} - \sigma_z)^{1-\tau}} - (1 - \tau^d) \frac{(\bar{z} + \sigma_z)^{1-\tau^d} - (\bar{z} - \sigma_z)^{1-\tau^d}}{(\bar{z} + \sigma_z)^{1-\tau^d} + (\bar{z} - \sigma_z)^{1-\tau^d}} \right]$$

Since  $f(\tau) = (1 - \tau) \frac{(\bar{z} + \sigma_z)^{1-\tau} - (\bar{z} - \sigma_z)^{1-\tau}}{(\bar{z} + \sigma_z)^{1-\tau} + (\bar{z} - \sigma_z)^{1-\tau}}$  is decreasing in  $\tau$  and  $\tau^d > \tau$ , we have:

$$\frac{\partial \Theta}{\partial \sigma_z} = \frac{\Theta \bar{z}}{\ln(10)(\bar{z} - \sigma_z)(\bar{z} + \sigma_z)} \left[ (1 - \tau) \frac{(\bar{z} + \sigma_z)^{1-\tau} - (\bar{z} - \sigma_z)^{1-\tau}}{(\bar{z} + \sigma_z)^{1-\tau} + (\bar{z} - \sigma_z)^{1-\tau}} - (1 - \tau^d) \frac{(\bar{z} + \sigma_z)^{1-\tau^d} - (\bar{z} - \sigma_z)^{1-\tau^d}}{(\bar{z} + \sigma_z)^{1-\tau^d} + (\bar{z} - \sigma_z)^{1-\tau^d}} \right] > 0.$$

**Equivalence of the transformed problem and the original problem.** The following relations hold:

$$W^s(S, z) = W^s(s, z),$$

$$W(S, z, \delta) = W(s, z, \delta),$$

$$g_i(S) = g_i(s) = N'_i / N_i = (1 + m_i) e^{-\zeta(z_i)(W^m - W^s(s, z_i))} \quad (i = L, H),$$

$$\frac{N'}{N} = \frac{N'_L + N'_H}{N_L + N_H} = g_L(s) f + g_H(s) (1 - f),$$

$$f' = \frac{N'_L}{N'} = \frac{N'_L}{N_L} \frac{N_L}{N} \frac{N}{N'} = \frac{g_L(s) f}{g_L(s) f + g_H(s) (1 - f)},$$

$$\frac{B'}{N} = \frac{B'}{N'} \frac{N'}{N} = b' \frac{N'}{N} = b' [g_L(s) f + g_H(s) (1 - f)],$$

$$\frac{V(B, A, \Phi')}{N} = v(b, A, f'),$$

$$\frac{V^c(B, A, \Phi')}{N} = v^c(b, A, f'),$$

$$\frac{V^d(A, \Phi')}{N} = v^d(A, f').$$

In the original problem, the government chooses whether to repay or default:

$$V(B, A, \Phi') = \max\{V^c(B, A, \Phi'), V^d(A, \Phi')\}$$

Divide both sides of the default decision by  $N$ :

$$\frac{V(B, A, \Phi')}{N} = \max\left\{\frac{V^c(B, A, \Phi')}{N}, \frac{V^d(A, \Phi')}{N}\right\},$$

which implies

$$v(b, A, f') = \max\{v^c(b, A, f'), v^d(A, f')\}.$$

Thus the default decisions satisfy

$$D(B, A, \Phi') = d(b, A, f').$$

Let the default decision be  $d(b, A, f') = 1$  if  $v^c(b, A, f') < v^d(A, f')$ . Thus, for the bond price, we have:

$$\begin{aligned} q(B', A, \Phi') &= \frac{1 - \Pr[D(B', A', \Phi'')]}{1 + r} \\ &= \frac{1 - \Pr[d(b', A', f'')]}{1 + r} \\ &= q(b', A, f'). \end{aligned}$$

Now I derive the repayment value in the transformed problem. The repayment value function in the original problem is:

$$V^c(B, A, \Phi') = \max_{B', \tau, \lambda} \{u(c_L, \ell_L) N'_L \omega_L + u(c_H, \ell_H) N'_H \omega_H + \beta \mathbb{E}V(B', A', \Phi'')\}.$$

Divide both sides by  $N$ :

$$\begin{aligned} \frac{V^c(B, A, \Phi')}{N} &= \max_{B', \tau, \lambda} \left\{ u(c_L, \ell_L) \frac{N'_L}{N_L} \frac{N_L}{N} \omega_L + u(c_H, \ell_H) \frac{N'_H}{N_H} \frac{N_H}{N} \omega_H + \beta \frac{N'}{N} \frac{1}{N'} \mathbb{E}V(B', A', \Phi'') \right\} \\ &= \max_{B', \tau, \lambda} \left\{ u(c_L, \ell_L) g_L f \omega_L + u(c_H, \ell_H) g_H (1 - f) \omega_H \right. \\ &\quad \left. + \beta (f g_L + (1 - f) g_H) \frac{1}{N'} \mathbb{E}V(B', A', \Phi'') \right\}, \end{aligned}$$

which gives

$$\begin{aligned} v^c(b, A, f') &= \max_{b', \tau, \lambda} \left\{ g_L f u(c_L, \ell_L) \omega_L + g_H (1 - f) u(c_H, \ell_H) \omega_H \right. \\ &\quad \left. + \beta [g_L f + g_H (1 - f)] \mathbb{E}v(b', A', f'') \right\}, \end{aligned}$$

The budget constraint in the original problem is:

$$B \leq T + qB'.$$

Divide both sides by  $N$ :

$$\frac{B}{N} \leq \frac{N'_L}{N_L} \frac{N_L}{N} (y_L - c_L) + \frac{N'_H}{N_H} \frac{N_H}{N} (y_H - c_H) + q \frac{B'}{N'} \frac{N'}{N},$$

which gives

$$b \leq g_L f (y_L - c_L) + g_H (1 - f) (y_H - c_H) + [g_L f + g_H (1 - f)] q (b', A, f') b'.$$

The derivation of the defaulting value function in the transformed problem follows similar steps. The defaulting value function in the original problem is:

$$V^d(A, \Phi') = \max_{\tau, \lambda} \{ u(c_L^d, \ell_L^d) N'_L \omega_L + u(c_H^d, \ell_H^d) N'_H \omega_H + \beta [\theta \mathbb{E}V(0, A', \Phi''_{aut=0}) + (1 - \theta) \mathbb{E}V^d(A', \Phi''_{aut=1})] \}$$

Divide both sides by  $N$ :

$$\begin{aligned} \frac{V^d(A, \Phi')}{N} &= \max_{\tau, \lambda} \left\{ u(c_L^d, \ell_L^d) \frac{N'_L}{N_L} \frac{N_L}{N} \omega_L + u(c_H^d, \ell_H^d) \frac{N'_H}{N_H} \frac{N_H}{N} \omega_H \right. \\ &\quad \left. + \beta \left[ \theta \frac{N'}{N} \frac{1}{N'} \mathbb{E}V(0, A', \Phi''_{aut=0}) + (1 - \theta) \frac{N'}{N} \frac{1}{N'} \mathbb{E}V^d(A', \Phi''_{aut=1}) \right] \right\} \\ &= \max_{\tau, \lambda} \left\{ u(c_L^d, \ell_L^d) g_L f \omega_L + u(c_H^d, \ell_H^d) g_H (1 - f) \omega_H \right. \\ &\quad \left. + \beta [\theta \mathbb{E}v(0, A', f''_{aut=0}) + (1 - \theta) \mathbb{E}v^d(A', f''_{aut=1})] [f g_L + (1 - f) g_H] \right\} \end{aligned}$$

which gives

$$\begin{aligned} v^d(A, f') &= \max_{\tau, \lambda} \left\{ g_L f u(c_L^d, \ell_L^d) \omega_L + g_H (1 - f) u(c_H^d, \ell_H^d) \omega_H \right. \\ &\quad \left. + \beta [g_L f + g_H (1 - f)] [\theta \mathbb{E}v(0, A', f''_{aut=0}) + (1 - \theta) \mathbb{E}v^d(A', f''_{aut=1})] \right\}. \end{aligned}$$

The budget constraint under default in the original problem is:

$$0 \leq T.$$

Divide both sides by  $N$ :

$$0 \leq \frac{N'_L}{N_L} \frac{N_L}{N} (y_L - c_L) + \frac{N'_H}{N_H} \frac{N_H}{N} (y_H - c_H),$$

which gives

$$0 \leq g_L f (y_L - c_L) + g_H (1 - f) (y_H - c_H).$$

## B.2 Model mechanism: effect of migration

The simplified one-period model in Section II.D offers clear analytical solutions that help to demonstrate the central model mechanism through explicit representations of the repayment value  $V^c$  and default value  $V^d$ . However, it cannot be used to analyze the impact of migration. Here, we turn our attention to the infinite horizon model to investigate the effects of migration.

Recall the government chooses  $\{B', \tau, \lambda\}$  to maximize its value:

$$V^c(B, A, \Phi') = \max_{B', \tau, \lambda} \left\{ \int_{\Phi'} u(c_i, \ell_i) \omega_i di + \beta \mathbb{E} V(B', A', \Phi'') \right\},$$

subject to the government budget constraint and worker distribution implied by the worker optimal decision rules:

$$\begin{aligned} B &= \int_{\Phi'} T(y_i) di + q(B', A, \Phi') B', \\ c_i &= \frac{\lambda}{(1 + \tau_c)} y_i^{1-\tau}, \\ \Phi'' &= H_{\Phi'}. \end{aligned}$$

The worker distribution  $\Phi'$  enters into the government's problem in three ways. First, it affects the government's value function, as shown in the first term in the value function. Second, it affects the tax base, shown as the first term in the right-hand side of the government budget constraint. Third, it affects the government bond price  $q(B', A, \Phi')$  by affecting future default risk. The emigration of workers, especially high-income workers, lowers the government's future repayment capacity and suppresses the bond price. The government also internalizes the impact of its choices on  $\Phi''$ , which is the next-period worker distribution.

To illustrate the intertemporal trade-off faced by the government, here I assume differentiability of the bond price and the value function with respect to  $B'$ . Note that I do not rely on the optimality conditions to solve the equilibrium numerically. The next equation represents the intertemporal Euler equation for the government:

$$\left[ q(B', A, \Phi') + \frac{\partial q(B', A, \Phi')}{\partial B'} B' \right] \int_{\Phi'} u'(c_i, \ell_i) \omega_i di = \beta \mathbb{E} \int_{\Phi''} \left[ u(c'_i, \ell'_i) \omega_i \frac{\partial \Phi''}{\partial B'} + u'(c'_i, \ell'_i) \omega_i \right] di \quad (\text{B.4})$$

The left-hand side of equation (B.4) represents the current marginal benefit from issuing bonds. The government collects  $\left[ q(B', A, \Phi') + \frac{\partial q(B', A, \Phi')}{\partial B'} B' \right]$  additional units of the consumption good when it issues an extra bond, and the second term shows that it is costly to

lower the current bond price. A lower bond price reduces the proceeds the government obtains from issuing bonds. To measure the welfare impact of issuing additional bonds, the marginal change in current consumption is weighted by the current consumption valuation  $\int_{\Phi} u'(c_i, \ell_i) \omega_i di$ . The right-hand side of equation (B.4) represents the cost of transferring more debt to the future.

### B.3 Decision rules

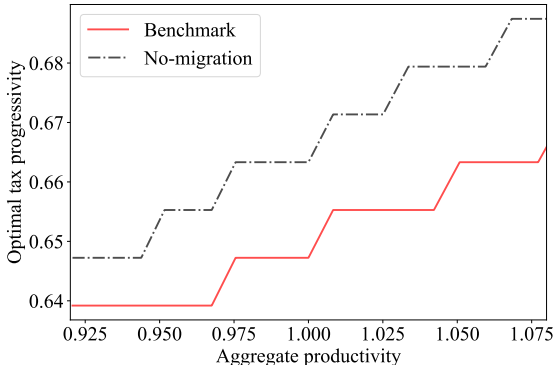
Here I plot the optimal decision rules for the government to visualize how optimal tax progressivity and borrowing depend on key variables. Figure B.4 plots the decision rules when government chooses to repay the debt. Panel (a) and (b) plot the optimal tax progressivity  $\tau$  as a function of aggregate productivity  $A$  and debt level  $B$ . Panel (c) and (d) plot the optimal next period debt  $B'$  as a function of aggregate productivity  $A$  and debt level  $B$ . The red solid lines plot correspondingly for the benchmark model and the black dash-dotted lines for the no-migration model. In the no-migration model, the worker distribution  $\Phi$  is time-invariant. The parameter values for each model follow the parameterization in Section III.

Optimal tax progressivity is increasing in aggregate productivity (Panel (a)). Intuitively, in good times, the government chooses to impose a more progressive tax to redistribute. When government has a large debt to repay, it adopts a less progressive tax (Panel (b)). As illustrated in Section II.D, with high outstanding debt, the marginal cost of increasing tax progressivity is high, leading to a less progressive tax in equilibrium. As is commonly found in sovereign default literature, with higher productivity (Panel (c)) or higher current debt (Panel (d)), next period debt is higher.

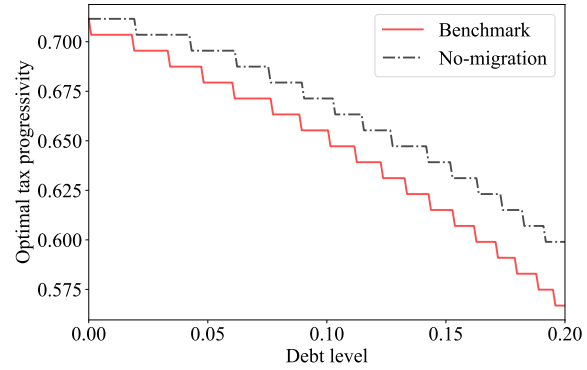
To isolate the impact of worker migration on the optimal government policies, we can compare the decision rules in the benchmark model and those in the no-migration reference model. With everything else equal, the optimal tax progressivity and next period debt level are higher in the no-migration reference model than in the benchmark model. This is because government internalizes the impact of its policies on worker migration. If workers are not allowed to emigrate (as for the black dash-dotted line), the government would impose a more progressive tax (Panel (a) and (b)) and borrow more (Panel (c) and (d)) to redistribute income.

### B.4 CRRA utility

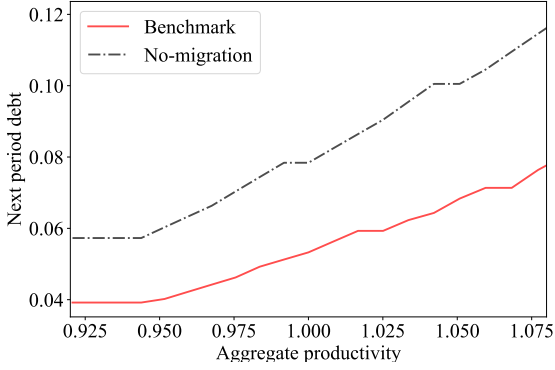
I derive the optimal labor supply choices using a constant relative risk aversion (CRRA) utility function and show that the main results stay unchanged. Assume the utility of



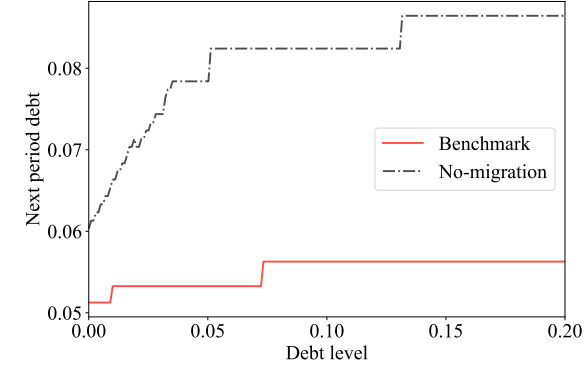
(a)



(b)



(c)



(d)

Figure B.4: Decision rules

Notes: Decision rules when government chooses to repay the debt. Panel (a) and (b) plot the optimal tax progressivity  $\tau$  as a function of aggregate productivity and debt level. Panel (c) and (d) plot for the optimal next period debt  $B'$  as a function of aggregate productivity and debt level. The red solid lines plot correspondingly for the benchmark model and the black dash-dotted lines for the no-migration model. In the no-migration model, the worker distribution  $\Phi$  is time-invariant. The parameter values for each model follow the parameterization in Section III.

worker  $i$  is given by:

$$u(c_i, \ell_i) = \frac{c_i^{1-\sigma}}{1-\sigma} - \frac{\ell_i^{1+\gamma}}{1+\gamma},$$

where  $\sigma$  is the parameter for risk aversion ( $\sigma = 1$  gives logarithmic utility). The optimal choice of labor supply for worker  $i$  satisfies:

$$\ell_i^{\sigma-\tau\sigma+\tau+\gamma} = (1-\tau)\lambda^{1-\sigma}(wz_i)^{1-\sigma+\tau\sigma-\tau}.$$

To illustrate, I calculate the optimal labor supply and  $\lambda$  under the following set of parameters:  $A = 1$ ,  $z_L = 0.3$ ,  $z_H = 0.7$ , and  $\sigma = 2$ . Then I calculate and plot the social welfare functions under different values of  $\tau$ . The optimal solutions that maximize the value function are characterized by three unknowns  $\ell_L$ ,  $\ell_H$ , and  $\lambda$  and three nonlinear equations:

$$\begin{aligned} \ell_L^{\sigma-\tau\sigma+\tau+\gamma} - (1-\tau)\lambda^{1-\sigma}(wz_L)^{1-\sigma+\tau\sigma-\tau} &= 0, \\ \ell_H^{\sigma-\tau\sigma+\tau+\gamma} - (1-\tau)\lambda^{1-\sigma}(wz_H)^{1-\sigma+\tau\sigma-\tau} &= 0, \\ \lambda - \frac{wz_L\ell_L + wz_H\ell_H - B_0}{(wz_L\ell_L)^{1-\tau} + (wz_H\ell_H)^{1-\tau}} &= 0. \end{aligned}$$

With  $\{\ell_L^*, \ell_H^*, \lambda^*\}$ , it is easy to solve for output, tax revenue, and consumption. Given consumption and labor choices, I calculate and plot social welfare under different scenarios.

Figure B.5 plots social welfare as a function of tax progressivity  $\tau$ . The blue dashed line plots for the scenario with  $z_L = 0.5$  and  $z_H = 0.5$  (no inequality). The comparison between the solid line with inequality and the dashed line without inequality shows that inequality increases the degree of optimal tax progressivity. When the government chooses to default, it can achieve a larger  $\tau^*$ , as shown in Figure B.6. These results are consistent with the predictions for logarithmic utility.

Recall that with logarithmic utility, tax progressivity  $\tau$  discourages labor. Figure B.7 shows this is still the case with CRRA utility. The yellow dashed line plots total effective labor. Total effective labor is decreasing in tax progressivity  $\tau$ , and thus the total output is decreasing in tax progressivity  $\tau$ .

Figure B.8 plots tax revenues collected from different workers and relative consumption as a function of  $\tau$ . With a more progressive tax, low-income workers pay less tax, high-income workers pay more tax, and the relative consumption of low-income workers to that of high-income workers increases.



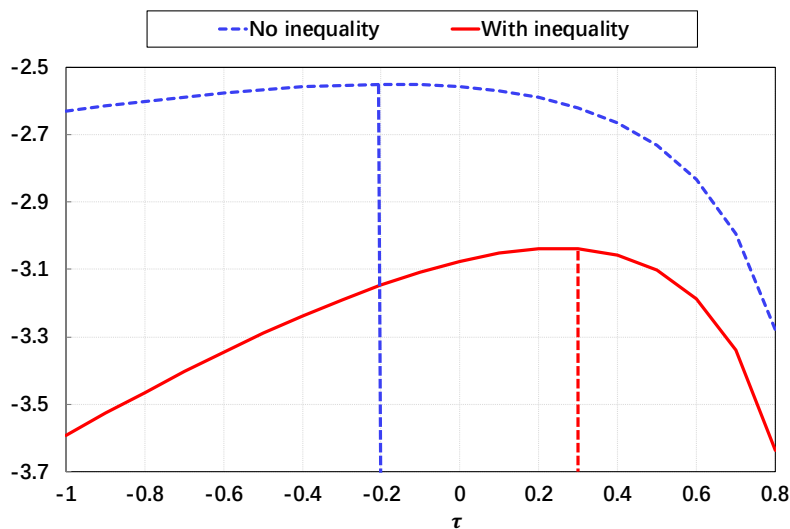


Figure B.5: CRRRA utility: inequality and optimal tax progressivity

Notes: This figure plots social welfare as a function of tax progressivity under the parameterization  $A = 1, \sigma = 2$  and  $B_0 = 0.2$ . The blue dashed line (no inequality) plots for the scenario with  $z_L = 0.5$  and  $z_H = 0.5$ . The red solid line plots for the case with inequality where  $z_L = 0.3$  and  $z_H = 0.7$ . The comparison of the two lines shows that inequality increases the degree of optimal tax progressivity.

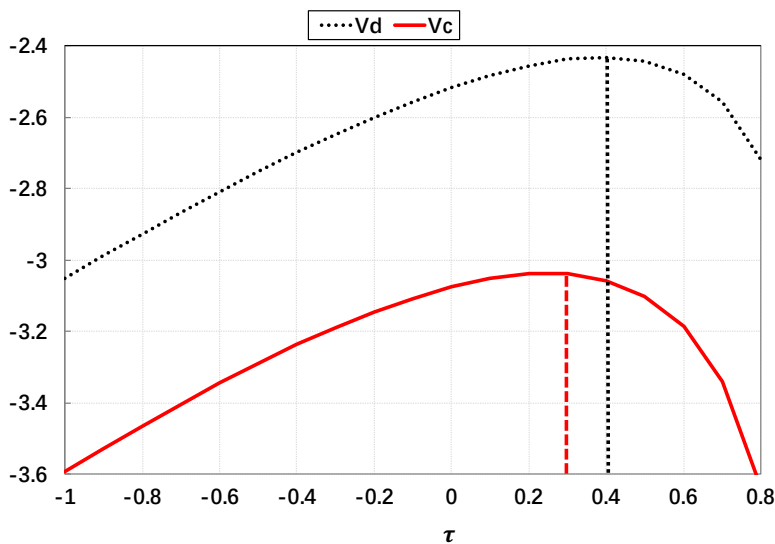


Figure B.6: CRRRA utility: default and optimal tax progressivity

Notes: This figure plots social welfare as a function of tax progressivity under the parameterization  $A = 1, \sigma = 2, B_0 = 0.2, z_L = 0.3$  and  $z_H = 0.7$ . The red solid line plots the repayment value and the black dotted plots the defaulting value. The comparison of the two lines shows that when government chooses to default, it can achieve a larger degree of optimal tax progressivity.

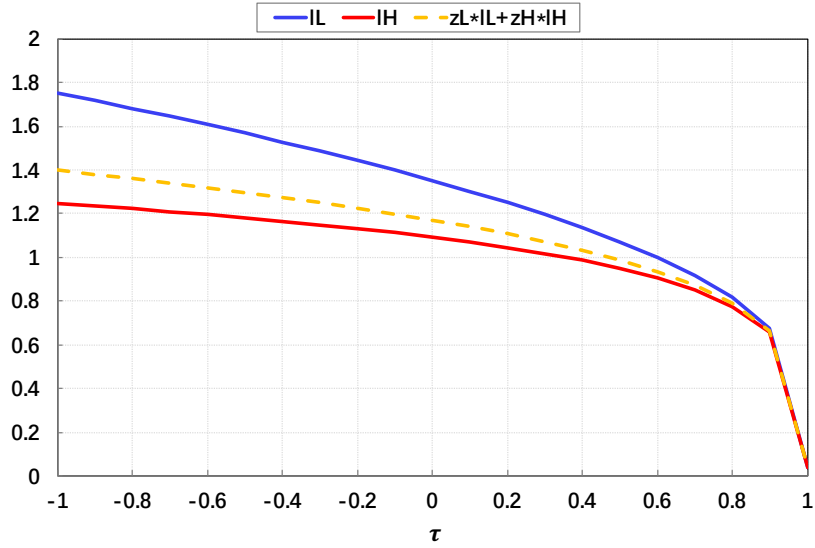


Figure B.7: CRRA utility: labor supply and tax progressivity

Notes: This figure plots labor supply as a function of tax progressivity under the parameterization  $A = 1$ ,  $\sigma = 2$ ,  $B_0 = 0.2$ ,  $z_L = 0.3$  and  $z_H = 0.7$ . The yellow dashed line plots total effective labor. Labor supply is decreasing in tax progressivity  $\tau$ . It shows that tax progressivity  $\tau$  discourages labor with CRRA utility, similar to the case with logarithmic utility.

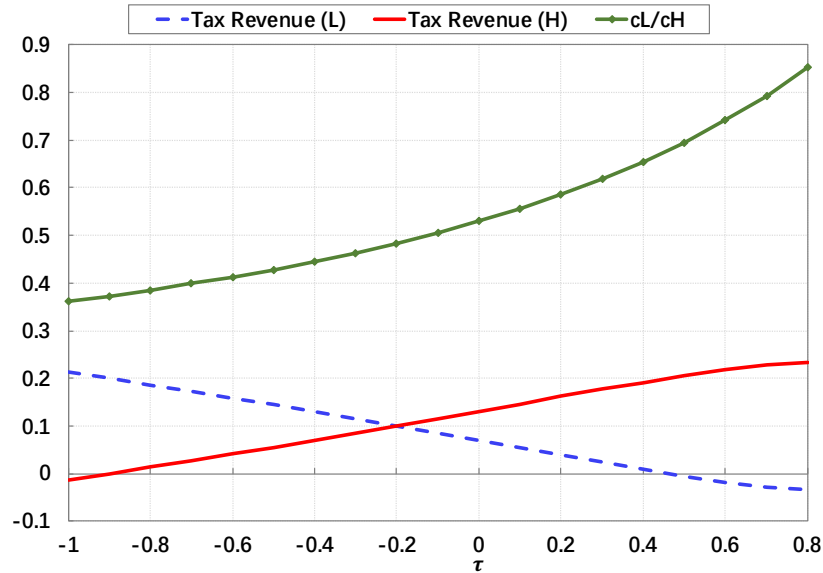


Figure B.8: CRRA utility: tax revenue, relative consumption, and tax progressivity

Notes: This figure plots tax revenues collected from different workers and relative consumption as a function of tax progressivity under the parameterization  $A = 1$ ,  $\sigma = 2$ ,  $B_0 = 0.2$ ,  $z_L = 0.3$  and  $z_H = 0.7$ . With a more progressive tax, low-income workers pay less tax, high-income workers pay more tax, and the relative consumption of low-income workers to that of high-income workers increases.

## B.5 Exogenous tax progressivity

This section solves for several economies with exogenous tax rules capturing different tax progressivities and reports the key moments in Table B.5. The moments in the table are the averages from 3,000 model simulations. The parameterization follows Benchmark model parameter values. Table B.5 shows that a more progressive tax (higher  $\tau$ ) distorts labor supply, increases emigration of high-income workers, and reduces emigration of the low-income workers. With a more progressive tax, the government has lower spreads, which is consistent with the quantitative results in Ferriere (2015) where tax progressivity is exogenous. Without endogenous tax progressivity, the government does not internalize the impact of progressivity on labor supply, migration, default risk and the cost of borrowing.

Table B.5: Exogenous tax progressivity  $\tau$

Exogenous $\tau$	labor supply	emigration rate (high-income)	emigration rate (low-income)	spread	debt-to-GDP
$\tau = 0.1$	0.965	1.419%	5.458%	1.239%	0.131
$\tau = 0.3$	0.888	1.578%	4.698%	1.226%	0.145
$\tau = 0.5$	0.794	1.883%	3.987%	0.737%	0.155

Note: This table reports the results with exogenous tax progressivity. Higher  $\tau$  reflects a more progressive tax. The numbers in the table are the averages from 3,000 model simulations. The parameterization follows Benchmark model parameter values.

## B.6 Parameters for the reference models

In section III.B, I compare the benchmark model with two reference models: no-inequality model and no-inequality-no-migration model. Both reference models share the same parameter values as the benchmark, except the following parameters shown in Table B.6.

## B.7 Solution method

I solve the government and worker problems using value function iteration. The AR(1) process for the aggregate productivity shock  $A$  is discretized using 21 equally spaced grid points with Tauchen's method. The government makes a borrowing decision  $b'$  and tax progressivity choice  $\tau$  if not in default, but makes only a tax progressivity choice  $\tau$  if in default ( $\lambda$  will be determined by the government budget constraint). For government debt, I use a grid with 200 equally spaced points on  $b \in [0, 0.2]$ . For tax progressivity, I use a grid

Table B.6: Parameters changed from benchmark

	Benchmark	No-inequality	No-inequality-no-migration
Labor heterogeneity $\sigma_z$	0.457	0	0
Migration cost distribution, low-income $\zeta_L$	0.0021	0.0021	-
Migration cost distribution, low-income $\zeta_H$	0.0028	0.0028	-
Exogenous inflow, low-income $\bar{m}_L$	0.033	0.033	0
Exogenous inflow, high-income $\bar{m}_H$	0.0246	0.0246	0

with 200 equally spaced points on  $\tau \in [-0.8, 0.8]$ . For the fraction of low-income workers  $f$ , I use a grid with 11 equally spaced points on  $f \in [0, 1]$ . Given optimal government policies, workers determine whether to migrate or not. The staying workers choose labor supply and consumption to maximize lifetime utility. Given the workers' choices, the government updates the repayment value and default value and decides whether to default. For each iteration, I update the value of the government and the value of each type of worker. The code stops running when the value function of the government and the value function for each type of worker converge. The tolerance level for the government is  $1e-4$ . The tolerance level for the each type of worker is  $1e-3$ .

Here is a more detailed description of the algorithm:

1. Create grids and discretize Markov process for the productivity shock  $A$ . Create grids for government bonds  $b$ , tax progressivity  $\tau$ , and fraction of low-income workers  $f$ .
2. Guess an initial value function of government  $v_0(b, A, f)$  and a bond price function  $q_0(b, A, f)$ ; guess the initial value functions for workers  $W_0(b, A, f, aut, z)$ .
3. Update the repayment value  $v^c(b, A, f)$  and the default value  $v^d(A, f)$ .
4. Compare  $v^c(b, A, f)$  and  $v^d(A, f)$ , and update the defaulting rule, price function, and the value function of the government  $v(b, A, f)$ .
5. Compute the optimal policy of the government with and without access to credit. With access to the financial market, the optimal policies consists of borrowing  $b'(b, A, f)$  and taxation  $\tau(b, A, f), \lambda(b, A, f)$ ; without access to the financial market, the optimal policy consists of taxation  $\{\tau(A, f), \lambda(A, f)\}$ .
6. Given government policies, update the staying value for workers  $W^s(b, A, f, aut, z)$ .

7. Update workers' value  $W(b, A, f, aut, z)$ .
8. Check the distance  $dist_g$  between the updated value function of the government and the one from the last iteration, and the distance  $dist_i$  between the updated value function of worker  $i$  and the one from last iteration. If any of these distances are larger than the given tolerance levels, then go back to 3. Otherwise, stop.

## References

- Ang, Andrew, Vineer Bhansali, and Yuhang Xing.** 2014. "The muni bond spread: Credit, liquidity, and tax." *Columbia Business School Research Paper*, , (14-37).
- Ferriere, Axelle.** 2015. "Sovereign default, inequality, and progressive taxation." Working Paper, European University Institute. 5.
- Gürkaynak, Refet S, Brian Sack, and Jonathan H Wright.** 2007. "The US Treasury yield curve: 1961 to the present." *Journal of Monetary Economics*, 54(8): 2291–2304.
- Longstaff, Francis A, Sanjay Mithal, and Eric Neis.** 2005. "Corporate yield spreads: Default risk or liquidity? New evidence from the credit default swap market." *The journal of finance*, 60(5): 2213–2253.
- Schwert, Michael.** 2017. "Municipal bond liquidity and default risk." *The Journal of Finance*, 72(4): 1683–1722.