Online Appendix A: Analytical Appendix

When Tariffs Disturb Global Supply Chains (Not for Publication)

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August 2023

Table of Contents

Section 1 Introduction

This appendix provides proofs and derivations for the Propositions and analytical expressions in the main text. Section numbers in the appendix correspond to those in the main text. We also derive the analytical expressions used in the calibration exercise.

Section 2 Foreign Sourcing with Search and Bargaining

We start from the bargaining game, which determines the payment to a supplier with inverse match productivity a for one unit of the intermediate input. The Nash bargaining solution solves

$$\rho(a) = \arg\max_q (qm - wam)^{1-\beta} \left[\mu_\rho(\bar{a})\, m + \frac{f}{G(\bar{a})} - qm\right]^\beta .$$

The first-order condition for the maximization on the right-hand side yields

$$\frac{1-\beta}{\rho(a) - wa} = \frac{\beta}{\mu_\rho(\bar{a}) + \frac{f}{mG(\bar{a})} - \rho(a)}$$

and therefore

$$\rho(a) = \beta wa + (1-\beta)\,\mu_\rho(\bar{a}) + (1-\beta)\frac{f}{mG(\bar{a})}.$$

Taking the conditional mean of both sides of this equation for $a \leq \bar{a}$, we have

$$\mu_\rho(\bar{a}) = w\mu_a(\bar{a}) + \frac{1-\beta}{\beta}\frac{f}{mG(\bar{a})}. \tag{A.1}$$

Substituting this result back into the $\rho(a)$ function then gives

$$\rho(a) = \beta wa + (1-\beta)\, w\mu_a(\bar{a}) + \frac{1-\beta}{\beta}\frac{f}{mG(\bar{a})}, \tag{A.2}$$

which is equation (4) in the main text. Next we use (5), the first-order condition for $\bar{a}$. This states

$$mw\mu_a'(\bar{a}) = \frac{fg(\bar{a})}{\beta G(\bar{a})^2}. \tag{A.3}$$

Note, however, that

$$\mu_a(\bar{a}) = \frac{1}{G(\bar{a})}\int_0^{\bar{a}} ag(a)da$$

and therefore

$$\mu_a'(\bar{a})\, G(\bar{a}) = g(\bar{a})\left[\bar{a} - \mu_a(\bar{a})\right]. \tag{A.4}$$

Substituting this into (A.3), we obtain

$$w\left[\bar{a} - \mu_a(\bar{a})\right] = \frac{f}{\beta mG(\bar{a})}. \tag{A.5}$$

Substituting (A.5) into (A.2) then yields equation (6),

$$\begin{aligned}\rho(a) &= \beta w\left[a - \mu_a(\bar{a})\right] + \beta w \mu_a(\bar{a}) + (1-\beta) w\bar{a} \\ &= \beta w a + (1-\beta) w\bar{a}.\end{aligned}$$

We next use the demand equation (3), the pricing equation (7), and (A.1) to compute operating profits. These profits are

$$\pi_o = x(p-c) - \frac{1-\beta}{\beta}\frac{f}{G(\bar{a})} - f_o,$$

where

$$p = \frac{\sigma}{\sigma-1}c,$$

$$x = X\left(\frac{p}{P}\right)^{-\sigma} = XP^{\sigma}\left(\frac{\sigma}{\sigma-1}c\right)^{-\sigma}, \tag{A.6}$$

and the aggregate cost of m units of the intermediate input is

$$w\mu_a(\bar{a})\, m + \frac{1-\beta}{\beta}\frac{f}{G(\bar{a})}.$$

Therefore,

$$\pi_o = XP^{\sigma}\frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}}c^{1-\sigma} - \frac{1-\beta}{\beta}\frac{f}{G(\bar{a})} - f_o, \tag{A.7}$$

where

$$c = c\left[w\mu_a(\bar{a})\right],$$

as stated in equation (8). By Shephard's Lemma, m is given by

$$m = XP^{\sigma}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma}}c^{-\sigma}c'. \tag{A.8}$$

A firm chooses $\bar{a}$ to maximize profits net of search costs, taking P and X as given. That is,

$$\begin{aligned}\bar{a} &= \arg\max_a XP^{\sigma}\frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}}c\left[w\mu_a(a)\right]^{1-\sigma} - \frac{1-\beta}{\beta}\frac{f}{G(a)} - \frac{f}{G(a)} - f_o \\ &= \arg\max_a XP^{\sigma}\frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}}c\left[w\mu_a(a)\right]^{1-\sigma} - \frac{f}{\beta G(a)} - f_o.\end{aligned}$$

For an interior solution, the first-order condition is

$$-XP^{\sigma}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma}}c\left[w\mu_a(\bar{a})\right]^{-\sigma}c'\left[w\mu_a(\bar{a})\right]w\mu_a'(\bar{a}) + \frac{fg(\bar{a})}{\beta G(\bar{a})^2} = 0,$$

which is the same as (5) in view of (A.8). Using Assumptions 1 and 2, this condition can be written as

$$-\alpha X P^{\sigma} \frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma}} \left(w \frac{\theta}{\theta+1} \bar{a}\right)^{-\alpha(\sigma-1)-1} \left(w \frac{\theta}{\theta+1}\right) + \theta \frac{f}{\beta \bar{a}^{\theta+1}} = 0.$$

Therefore the second-order condition for profit maximization is satisfied at the optimal choice of $\bar{a}$ if and only if $\theta > \alpha(\sigma-1)$, as stipulated in Assumption 3. This first-order condition can be expressed as

$$\bar{a}^{\theta-\alpha(\sigma-1)} X P^{\sigma} = \frac{\theta f}{\alpha \beta} \left(\frac{w\theta}{\theta+1}\right)^{\alpha(\sigma-1)} \left(\frac{\sigma}{\sigma-1}\right)^{\sigma}. \tag{A.9}$$

Substituting this expression into (A.7) yields

$$\pi_o - \frac{f}{G(\bar{a})} = \frac{\theta - \alpha(\sigma-1)}{\beta \alpha (\sigma-1)} f \bar{a}^{-\theta} - f_o.$$

The free entry condition is

$$\pi_o - \frac{f}{G(\bar{a})} = f_e,$$

which, together with the previous equation, yields equation (10):

$$\bar{a}^{\theta} = \frac{f}{f_o + f_e} \frac{\theta - \alpha(\sigma-1)}{\beta \alpha (\sigma-1)}. \tag{A.10}$$

The solution to this cutoff is interior if and only if

$$\frac{f}{f_o + f_e} \frac{\theta - \alpha(\sigma-1)}{\beta \alpha (\sigma-1)} < 1.$$

Substituting (A.10) and $XP^{\sigma} = P^{\sigma-\varepsilon}$ into (A.9) provides a solution for P. And substituting this equation into

$$P = \frac{\sigma}{\sigma-1} \left(w \frac{\theta}{\theta+1} \bar{a}\right)^{\alpha} n^{-\frac{1}{\sigma-1}} \tag{A.11}$$

provides a solution for n. Note that

$$\hat{n} = (\sigma-1)\left(\alpha \hat{\bar{a}} - \hat{P}\right),$$

where a hat over a variable represents a proportional rate of change, e.g., $\hat{y} = dy/y$. For an increase in the search cost f we have, from (A.9),

$$\hat{P} = \frac{\hat{f} - [\theta - \alpha(\sigma-1)]\hat{\bar{a}}}{\sigma - \varepsilon}$$

and from (A.10),

$$\hat{\bar{a}} = \frac{1}{\theta}\hat{f}.$$

Therefore,

$$\hat{P} = \frac{\alpha(\sigma-1)}{\theta(\sigma-\varepsilon)}\hat{f},$$

$$\hat{n} = \frac{\alpha(\sigma-1)}{\theta}\frac{1-\varepsilon}{\sigma-\varepsilon}\hat{f}.$$

These results are summarized in

Lemma A.1 *Suppose Assumptions 1-3 hold and*

$$\frac{f}{f_o+f_e}\frac{\theta-\alpha(\sigma-1)}{\beta\alpha(\sigma-1)} < 1.$$

Then lower search costs f lead to a lower cutoff $\bar{a}$ and a lower price index P. They also generate more variety n for $\sigma > \varepsilon > 1$.

Section 3 Unanticipated Tariffs

Section 3.1 Small Tariffs

In this case, the ex-factory price paid to a foreign supplier with inverse match productivity a is $\rho(a,\tau)$, which is the solution to

$$\rho(a,\tau) = \arg\max_{q}\left[\tau\mu_\rho[\bar{a}(\tau),\tau] + \frac{f}{m(\tau)G[\bar{a}(\tau)]} - \tau q\right]^{\beta}(q-wa)^{1-\beta}.$$

This f.o.b. price excludes the tariff levy. The first-order condition for this maximization problem is

$$\frac{1-\beta}{\rho(a,\tau)-wa} = \frac{\beta}{\mu_\rho[\bar{a}(\tau),\tau] + \frac{f}{\tau m(\tau)G[\bar{a}(\tau)]} - \rho(a,\tau)},$$

which yields

$$\rho(a,\tau) = \beta wa + (1-\beta)\mu_\rho[\bar{a}(\tau),\tau] + (1-\beta)\frac{f}{\tau m(\tau)G[\bar{a}(\tau)]}. \tag{A.12}$$

Taking conditional expectations on both sides of this equation for $a \leq \bar{a}(\tau)$, we find

$$\mu_\rho[\bar{a}(\tau),\tau] = w\mu_a[\bar{a}(\tau)] + \frac{1-\beta}{\beta}\frac{f}{\tau m(\tau)G[\bar{a}(\tau)]}. \tag{A.13}$$

Next, substituting this expression into (A.12), we obtain

$$\rho(a,\tau) = \beta wa + (1-\beta)w\mu_a[\bar{a}(\tau)] + \frac{1-\beta}{\beta}\frac{f}{\tau m(\tau)G[\bar{a}(\tau)]}, \tag{A.14}$$

which is equation (11) in the main text. As explained in the text, using the optimal search cutoff $\bar{a}(\tau)$ yields

$$w\{\bar{a}(\tau) - \mu_a[\bar{a}(\tau)]\} = \frac{f}{\beta\tau m(\tau)G[\bar{a}(\tau)]}. \tag{A.15}$$

Now substitute this equation into (A.14) to obtain

$$\rho(a,\tau) = \beta w a + (1-\beta) w \bar{a}(\tau). \tag{A.16}$$

Next note that it is cheaper to sources inputs from the original supplier a whenever

$$\tau\rho(a,\tau) \le \tau\mu_\rho[\bar{a}(\tau),\tau] + \frac{f}{m(\tau) G[\bar{a}(\tau)]}.$$

Using (A.13) and (A.15), the right-hand side of this inequality equals $\tau w_A \bar{a}(\tau)$. Therefore this inequality can be expressed as

$$a \le \bar{a}(\tau).$$

From this result, we have

Lemma A.2 *For a given $\bar{a}(\tau)$ the cost minimizing cutoff a_c is*

$$a_c = \min\{\bar{a}(\tau), \bar{a}\}.$$

As explained in the main text, the marginal cost of m is given by equation (15),

$$\phi^\tau = \beta\frac{G(a_c)}{G(\bar{a})}\tau w \mu_a(a_c) + \left[1-\beta\frac{G(a_c)}{G(\bar{a})}\right]\tau w \mu_a(\bar{a}^\tau)$$

and then optimal (mark-up) pricing implies

$$p^\tau = \frac{\sigma}{\sigma-1}c(\phi^\tau).$$

Using Assumption 2 and Lemma A.2, the marginal cost can be expressed as

$$\phi^\tau = \begin{cases} \frac{\theta}{\theta+1}\tau w \bar{a}^\tau & \text{for } \bar{a}^\tau < \bar{a} \\ \beta\frac{\theta}{\theta+1}\tau w \bar{a} + (1-\beta)\frac{\theta}{\theta+1}\tau w \bar{a}^\tau & \text{for } \bar{a}^\tau > \bar{a} \end{cases}. \tag{A.17}$$

This is the MM curve in Figure 2.

We next derive the NN curve, using the first-order condition for $\bar{a}^\tau$ in (A.15), Shephard's Lemma $m^\tau = x^\tau c'(\phi^\tau)$, the expression for the demand for variety ω in (A.6), and the expression for the price index, $P^\tau = p^\tau (n^\tau)^{-1/(\sigma-1)}$. This expression of the price index assumes that all firms, new and old, charge the same price p^τ, which we verify below. First, in the Pareto case (A.15) becomes

$$w\bar{a}(\tau)^{\theta+1} = \frac{f(\theta+1)}{\beta\tau m(\tau)}. \tag{A.18}$$

Second,

$$m^\tau = X^\tau \left(\frac{p^\tau}{P^\tau}\right)^{-\sigma} c'(\phi^\tau) \tag{A.19}$$
$$= X^\tau (n^\tau)^{-\frac{\sigma}{\sigma-1}} c'(\phi^\tau) = (P^\tau)^{-\varepsilon} (n^\tau)^{-\frac{\sigma}{\sigma-1}} c'(\phi^\tau)$$
$$= (p^\tau)^{-\varepsilon} (n^\tau)^{-\frac{\sigma-\varepsilon}{\sigma-1}} c'(\phi^\tau)$$

Combining these equations, we obtain

$$\frac{(\theta+1) f}{w\beta (\bar{a}^\tau)^{\theta+1}} = \tau (n^\tau)^{-\frac{\sigma-\varepsilon}{\sigma-1}} (p^\tau)^{-\varepsilon} c'(\phi^\tau) \ ,$$

which is equation (18) in the main text. Using $p^\tau = c(\phi^\tau)\sigma/(\sigma-1)$ and $c(\phi^\tau) = (\phi^\tau)^\alpha$, this equation becomes

$$\frac{(\theta+1) f}{w\beta (\bar{a}^\tau)^{\theta+1}} = \tau (n^\tau)^{-\frac{\sigma-\varepsilon}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} \alpha (\phi^\tau)^{\alpha(1-\varepsilon)-1} \ . \tag{A.20}$$

This implies that the NN curve is higher the greater is the tariff rate and that all along this curve,

$$\hat{\phi}^\tau = \frac{\theta+1}{1-\alpha(1-\varepsilon)} \hat{\bar{a}}^\tau .$$

The denominator is positive for all $\varepsilon > 0$, and since $\varepsilon < \sigma$ and $\theta > \alpha(\sigma-1)$, $\theta+1 > 1+\alpha(\varepsilon-1)$. Therefore the elasticity of the NN curve is larger than one. The upward shift of the curve in response to a rise in τ satisfies

$$\hat{\phi}^\tau = \frac{1}{1-\alpha(1-\varepsilon)} \hat{\tau}.$$

Therefore, ϕ^τ rises proportionately less for $\varepsilon > 1$. As a result, the marginal cost ϕ^τ rises, holding constant the number of firms.

We show at the end of this section that the NN curve is steeper than the MM curve for general distribution functions (i.e., not necessarily Pareto), as long as the choice of $\bar{a}$ that maximizes profits satisfies the second-order condition. In this event the above comparative static results also hold.

Next, consider the incentives for new firms to enter. For $\varepsilon > 1$, equations (16), (17) and (18) imply

$$\hat{\phi}^\tau = \frac{\theta+1-\gamma^\tau}{\theta+1-\gamma^\tau-\gamma^\tau\alpha(\varepsilon-1)} \hat{\tau} \tag{A.21}$$

and

$$\hat{\bar{a}}^\tau = \frac{\alpha(\varepsilon-1)}{\theta+1-\gamma^\tau-\gamma^\tau\alpha(\varepsilon-1)} \hat{\tau}, \tag{A.22}$$

where

$$\gamma^\tau = \frac{(1-\beta)\bar{a}^\tau}{\beta\bar{a}+(1-\beta)\bar{a}^\tau}.$$

Using (A.7) and (A.17), the objective function of a potential entrant is

$$\pi(\tau) = \max_a P(\tau)^{\sigma-\varepsilon} \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} [\tau w \mu_a(a)]^{\alpha(1-\sigma)} - \frac{f}{\beta G(a)} - f_o - f_e.$$

Therefore $\pi'(\tau) > 0$ if and only if $P(\tau)^{\sigma-\varepsilon} \tau^{\alpha(1-\sigma)}$ is rising in τ. However, using (A.21) and $\theta > \alpha(\sigma-1)$ we obtain

$$\begin{aligned}
\frac{(\sigma-\varepsilon)\hat{P}^\tau - \alpha(\sigma-1)\hat{\tau}}{\alpha\hat{\tau}} &= \frac{(\sigma-\varepsilon)\hat{\phi}^\tau - (\sigma-1)\hat{\tau}}{\hat{\tau}} \\
&= \frac{(\theta+1-\gamma^\tau)(\sigma-\varepsilon)}{\theta+1-\gamma^\tau-\gamma^\tau\alpha(\varepsilon-1)} - (\sigma-1) \\
&< \frac{[\alpha(\sigma-1)+1-\gamma^\tau](\sigma-\varepsilon)}{\alpha(\sigma-1)+1-\gamma^\tau-\gamma^\tau\alpha(\varepsilon-1)} - (\sigma-1) \\
&= -\frac{(1-\gamma^\tau)(\varepsilon-1)[\alpha(\sigma-1)+1]}{\alpha(\sigma-1)+1-\gamma^\tau-\gamma^\tau\alpha(\varepsilon-1)} < 0.
\end{aligned}$$

It follows that potential entrants face negative profits for all small tariff levels. Therefore, we have

Lemma A.3 *Suppose Assumptions 1-3 hold and $\sigma > \varepsilon > 1$. Then for small tariffs there is no entry of new final-good producers and prospective profits of potential entrants decline with the tariff rate.*

General Productivity Distribution and Cost Function

We now show that the NN curve is steeper than the MM curve for a general productivity distribution and cost function as long as the second-order condition for the choice of $\bar{a}$ is satisfied. We consider the case of a small tariff, so that the outside option is to search in country A. In this case

$$p^\tau = \frac{\sigma}{\sigma-1} c(\phi^\tau), \tag{A.23}$$

$$P^\tau = \frac{\sigma}{\sigma-1} c(\phi^\tau) n^{\frac{1}{1-\sigma}}, \tag{A.24}$$

$$x^\tau = (P^\tau)^{\sigma-\varepsilon} \left[\frac{\sigma}{\sigma-1} c(\phi^\tau)\right]^{-\sigma}, \tag{A.25}$$

$$m^\tau = (P^\tau)^{\sigma-\varepsilon} \frac{(\sigma-1)^\sigma}{\sigma^\sigma} c(\phi^\tau)^{-\sigma} c'(\phi^\tau). \tag{A.26}$$

These equations also apply to the case $\tau = 0$, i.e., the initial equilibrium. In the initial equilibrium $\phi = w\mu_a(\bar{a})$ and operating profits are (see (A.7))

$$\pi_o = (P)^{\sigma-\varepsilon} \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} c[w\mu_a(\bar{a})]^{1-\sigma} - \frac{1-\beta}{\beta} \frac{f}{G(\bar{a})} - f_o.$$

The choice of $\bar{a}$ maximizes operating profits minus search costs, $f/G(\bar{a})$, which yields the first-order condition

$$-(P^{\tau})^{\sigma-\varepsilon}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma}}c\left[w\mu_a(\bar{a}^{\tau})\right]^{-\sigma}c'\left[w\mu_a(\bar{a}^{\tau})\right]w\mu_a'(\bar{a}^{\tau})+\frac{1}{\beta}\frac{fg(\bar{a}^{\tau})}{G(\bar{a}^{\tau})^2}=0.$$

Since

$$\mu_a(\bar{a})=\frac{1}{G(\bar{a})}\int_0^{\bar{a}}ag(a)\,da,$$

$$\mu_a'(\bar{a})=\frac{g(\bar{a})}{G(\bar{a})}\left[\bar{a}^{\tau}-\mu_a(\bar{a}^{\tau})\right],$$

this first-order condition can be expressed as

$$-(P)^{\sigma-\varepsilon}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma}}c\left[w\mu_a(\bar{a})\right]^{-\sigma}c'\left[w\mu_a(\bar{a})\right]w\left[\bar{a}-\mu_a(\bar{a})\right]+\frac{1}{\beta}\frac{f}{G(\bar{a})}=0.$$

It follows that the second-order condition requires

$$\left\{G(\bar{a})\,c\left[w\mu_a(\bar{a})\right]^{-\sigma}c'\left[w\mu_a(\bar{a})\right]\left[\bar{a}-\mu_a(\bar{a})\right]\right\}'>0. \tag{A.27}$$

With a Pareto distribution and a Cob-Douglas (C-D) production function this holds if and only if

$$\left\{(\bar{a})^{\theta}(\bar{a})^{-\alpha\sigma}(\bar{a})^{\alpha-1}\bar{a}\right\}'>0,$$

which is satisfied if and only if $\theta>\alpha(\sigma-1)$. With C-D and a general distribution function, the second-order condition requires

$$\left\{G(\bar{a})\,\mu_a(\bar{a})^{-\alpha(\sigma-1)-1}\left[\bar{a}-\mu_a(\bar{a})\right]\right\}'>0.$$

Now consider the MM curve. It is represented by

$$\phi^{\tau}=\beta\frac{G(a_c)}{G(\bar{a})}\tau w\mu_a(a_c)+\left[1-\beta\frac{G(a_c)}{G(\bar{a})}\right]\tau w\mu_a(\bar{a}^{\tau}),$$

where

$$a_c=\min\{\bar{a}^{\tau},\bar{a}\}.$$

Therefore

$$\phi^{\tau}=\begin{cases}\tau w\mu_a(\bar{a}^{\tau}) & \text{for } \bar{a}^{\tau}\leq\bar{a}\\ \tau w\beta\mu_a(\bar{a})+\tau w(1-\beta)\mu_a(\bar{a}^{\tau}) & \text{for } \bar{a}^{\tau}\geq\bar{a}\end{cases}. \tag{A.28}$$

It is an increasing curve with a break in the slope at $\bar{a}^{\tau}=\bar{a}$, where the right-hand side slope is flatter than the left-hand side slope. The left-hand side slope equals $\tau w\mu_a'(\bar{a})$.

We next derive the NN curve, using the first-order condition (12) in the paper,

$$\tau w\left[\bar{a}^{\tau}-\mu_a\left(\bar{a}^{\tau}\right)\right]G\left(\bar{a}^{\tau}\right)=\frac{f}{\beta m^{\tau}}.$$

Using (A.24) and (A.26) above, this yields

$$\tau G\left(\bar{a}^{\tau}\right)c\left(\phi^{\tau}\right)^{-\varepsilon}c'\left(\phi^{\tau}\right)\left[\bar{a}^{\tau}-\mu_a\left(\bar{a}^{\tau}\right)\right]=\frac{f\sigma^{\varepsilon}}{w\beta n^{\frac{\sigma-\varepsilon}{1-\sigma}}\left(\sigma-1\right)^{\varepsilon}}. \tag{A.29}$$

With C-D and Pareto this is

$$\tau w\frac{\left(\bar{a}^{\tau}\right)^{\theta+1}}{\theta+1}=\frac{f}{\beta n^{\frac{\sigma-\varepsilon}{1-\sigma}}\frac{(\sigma-1)^{\varepsilon}}{\sigma^{\varepsilon}}\alpha\left(\phi^{\tau}\right)^{-\alpha(\varepsilon-1)-1}},$$

which is what we have in the paper.

The slope of the MM curve to the left of $\bar{a}^{\tau}=\bar{a}$, i.e., evaluated at $\tau=1$, equals $w\mu_a'\left(\bar{a}\right)$ (see (A.28)). From (A.29), the slope of the NN curve evaluated at $\bar{a}^{\tau}=\bar{a}$, i.e., at $\tau=1$, equals

$$s_{NN}=-\frac{\left\{G\left(\bar{a}\right)\left[\bar{a}-\mu_a\left(\bar{a}\right)\right]\right\}'c\left(\phi\right)^{-\varepsilon}c'\left(\phi\right)}{\left[c\left(\phi\right)^{-\varepsilon}c'\left(\phi\right)\right]'G\left(\bar{a}\right)\left[\bar{a}-\mu_a\left(\bar{a}\right)\right]} \tag{A.30}$$

However, the second-order condition (A.27) implies

$$\left\{G\left(\bar{a}\right)\left[\bar{a}-\mu_a\left(\bar{a}\right)\right]\right\}'c\left(\phi\right)^{-\varepsilon}c'\left(\phi\right)+\left[c\left(\phi\right)^{-\varepsilon}c'\left(\phi\right)\right]'G\left(\bar{a}\right)\left[\bar{a}-\mu_a\left(\bar{a}\right)\right]w\mu_a'\left(\bar{a}\right)>0,$$

or, using (A.30),

$$\left\{G\left(\bar{a}\right)\left[\bar{a}-\mu_a\left(\bar{a}\right)\right]\right\}'c\left(\phi\right)^{-\varepsilon}c'\left(\phi\right)\left[1-\frac{w\mu_a'\left(\bar{a}\right)}{s_{MM}}\right]>0.$$

Since $\left\{G\left(\bar{a}\right)\left[\bar{a}-\mu_a\left(\bar{a}\right)\right]\right\}'c\left(\phi\right)^{-\varepsilon}c'\left(\phi\right)>0$, this yields $w\mu_a'\left(\bar{a}\right)<s_{NN}$. That is, the NN curve is steeper than the MM curve at this point.

Next consider the upward shift in each one of the curves at $\bar{a}^{\tau}=\bar{a}$ in response to and increase in τ. The MM curve shifts proportionately to τ. The NN curve shifts less than proportionately if and only if the elasticity of $c\left(\phi\right)^{-\varepsilon}c'\left(\phi\right)$, which is negative, is smaller than -1 ($c''<0$ due to concavity of the cost function). In the C-D case this elasticity is $-\alpha\left(\varepsilon-1\right)-1<1$ and an increase in τ leads to $\bar{a}^{\tau}>\bar{a}$, as argued in the paper. This is true more generally, for cost functions whose elasticity of $c\left(\phi\right)^{-\varepsilon}c'\left(\phi\right)$ is smaller than -1.

Section 3.2 Large Tariffs

In this section, the outside option for buyers is to search for new suppliers in country B. The outside option is the same when a buyer bargains with a supplier in country A as when it bargains with one in country B. Since there are no tariffs on inputs purchased in country B, the bargaining game with a supplier in country B yields

$$\rho_B(b,\tau) = \arg\max_q \left[qm(\tau) - w_B bm(\tau)\right]^{1-\beta} \left[w_B \mu_b\left[\bar{b}(\tau)\right] m(\tau) + \frac{f}{\beta G\left[\bar{b}(\tau)\right]} - qm(\tau)\right]^{\beta}.$$

The first-order condition for this problem is

$$\frac{1-\beta}{\rho_B(b,\tau) - w_B b} = \frac{\beta}{w_B \mu_b\left[\bar{b}(\tau)\right] + \frac{f}{\beta m(\tau) G\left[\bar{b}(\tau)\right]} - \rho_B(b,\tau)},$$

and therefore

$$\rho_B(b,\tau) = \beta w_B b + (1-\beta) w_B \mu_b\left[\bar{b}(\tau)\right] + (1-\beta)\frac{f}{\beta m(\tau) G\left[\bar{b}(\tau)\right]}. \tag{A.31}$$

Taking the conditional mean of both sides of this equation for $b \leq \bar{b}(\tau)$, yields

$$\mu_{\rho_B}\left[\bar{b}(\tau)\right] = w_B \mu_b\left[\bar{b}(\tau)\right] + \frac{1-\beta}{\beta}\frac{f}{m(\tau) G\left[\bar{b}(\tau)\right]}. \tag{A.32}$$

Now use the first-order condition for $\bar{b}(\tau)$ that minimizes costs,

$$w_B\left\{\bar{b}(\tau) - \mu_b\left[\bar{b}(\tau)\right]\right\} = \frac{f}{\beta m(\tau) G\left[\bar{b}(\tau)\right]}, \tag{A.33}$$

to obtain

$$\rho_B(b,\tau) = \beta w_B b + (1-\beta) w_B \bar{b}(\tau). \tag{A.34}$$

Note that this cost of inputs depends on the tariff only through $\bar{b}(\tau)$ and it is the same for the original producers and new entrants.

Bargaining with suppliers in country A yields

$$\rho_A(a,\tau) = \arg\max_q \left[qm(\tau) - w_A am(\tau)\right]^{1-\beta} \left[w_B \mu_b\left[\bar{b}(\tau)\right] m(\tau) + \frac{f}{\beta G\left[\bar{b}(\tau)\right]} - \tau qm(\tau)\right]^{\beta}.$$

The first-order condition for this problem is

$$\frac{1-\beta}{\rho_A(a,\tau) - w_A a} = \frac{\beta\tau}{w_B \mu_b\left[\bar{b}(\tau)\right] + \frac{f}{\beta m(\tau) G\left[\bar{b}(\tau)\right]} - \tau\rho_A(a,\tau)}$$

and therefore

$$\tau\rho_A(a,\tau) = \beta\tau w_A a + (1-\beta) w_B \mu_b\left[\bar{b}(\tau)\right] + (1-\beta)\frac{f}{\beta m(\tau) G\left[\bar{b}(\tau)\right]}. \tag{A.35}$$

Substituting (A.32) and (A.33) into this equation we obtain

$$\rho_A(a,\tau) = \beta w_A a + (1-\beta) w_B \frac{\bar{b}(\tau)}{\tau}. \tag{A.36}$$

This negotiated price depends on τ through the ratio $\bar{b}(\tau)/\tau$. In these circumstances, it is cheaper to source an input a from country A if

$$\tau\rho_A(a,\tau) \leq \mu_{\rho_B}\left[\bar{b}(\tau)\right] + \frac{f}{m(\tau)\,G\left[\bar{b}(\tau)\right]}.$$

Using (A.32) and (A.33), the right-hand side of this inequality equals $w_B\bar{b}(\tau)$. Therefore this inequality can be expressed as

$$\tau w_A a \leq w_B\bar{b}(\tau).$$

From this result we have

Lemma A.4 *For given $\bar{b}(\tau)$, the cost minimizing cutoff a_B is*

$$a_B = \min\left\{\frac{w_B\bar{b}(\tau)}{\tau w_A}, \bar{a}\right\}. \tag{A.37}$$

Now consider the perceived marginal cost of the composite intermediate good for one of the original producers. From (A.31), we see that the average marginal cost of sourcing from country B is $w_B\mu_b\left[\bar{b}(\tau)\right]$, while from (A.35) we see that the average marginal cost of sourcing from country A is $\beta\tau w_A\mu_a(a_B) + (1-\beta)w_B\mu_b\left[\bar{b}(\tau)\right]$. Since an incumbent firm sources a fraction $G(a_B)/G(\bar{a})$ of its inputs from country A and the remaining fraction $1 - G(a_B)/G(\bar{a})$ from country B, its marginal cost of the intermediate input is

$$\begin{aligned}\phi^\tau &= \frac{G(a_B)}{G(\bar{a})}\left[\beta\tau w_A\mu_a(a_B) + (1-\beta)w_B\mu_b\left(\bar{b}^\tau\right)\right] + \left[1 - \frac{G(a_B)}{G(\bar{a})}\right] w_B\mu_b\left(\bar{b}^\tau\right) \\ &= \beta\frac{G(a_B)}{G(\bar{a})}\tau w_A\mu_a(a_B) + \left[1 - \beta\frac{G(a_B)}{G(\bar{a})}\right] w_B\mu_b\left(\bar{b}^\tau\right),\end{aligned}$$

where we have replace the function $\bar{b}(\tau)$ with the value of $\bar{b}$ at the tariff level τ, $\bar{b}^\tau$. Using (A.37) and properties of the Pareto distribution yields the equation for the MM curve,

$$\phi^\tau = \begin{cases} \frac{\theta}{\theta+1} w_B\bar{b}^\tau & \text{for } \bar{b}^\tau < \tau w_A\bar{a}/w_B \\ \frac{\theta}{\theta+1}\left[\beta\tau w_A\bar{a} + (1-\beta)w_B\bar{b}^\tau\right] & \text{for } \bar{b}^\tau > \tau w_A\bar{a}/w_B \end{cases}. \tag{A.38}$$

New entrants (if any exist) search for suppliers only in country B. Equation (A.32) implies that an entrant's marginal cost is

$$\phi^\tau_{new} = w_B\mu_b\left(\bar{b}^\tau\right) = \frac{\theta}{\theta+1} w_B\bar{b}^\tau. \tag{A.39}$$

For the tariff level $\tau = w_B/w_A$, the equilibrium values are $\bar{b}^\tau = \bar{a}$ and $\phi^\tau_{new} = \phi^\tau = \tau\phi = \frac{\theta}{\theta+1} w_B \bar{a}$.

We next derive the equation for the NN curve. We have (A.33). As we explained in the previous section, when all the firms are identical, m^τ, the volume of imported intermediate goods, is given by (see (A.19))

$$\begin{aligned} m^\tau &= (p^\tau)^{-\varepsilon} (n^\tau)^{-\frac{\sigma-\varepsilon}{\sigma-1}} c'(\phi^\tau) \\ &= \left[\frac{\sigma}{\sigma-1} c(\phi^\tau)\right]^{-\varepsilon} (n^\tau)^{-\frac{\sigma-\varepsilon}{\sigma-1}} c'(\phi^\tau) \\ &= \alpha \left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} (n^\tau)^{-\frac{\sigma-\varepsilon}{\sigma-1}} (\phi^\tau)^{\alpha(1-\varepsilon)-1}, \end{aligned} \tag{A.40}$$

where $n^\tau = n$ in the elastic case. Since higher tariffs do not raise profits when $\varepsilon > 1$, there is no entry of new firms. Substituting the expression for m^τ into (A.33) yields

$$\frac{(\theta+1) f}{w_B \beta \left(\bar{b}^\tau\right)^{\theta+1}} = n^{-\frac{\sigma-\varepsilon}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} \alpha (\phi^\tau)^{\alpha(1-\varepsilon)-1} \ , \tag{A.41}$$

which is the NN curve. It follows that the elasticity of the NN curve in this case is $(\theta+1)/[1-\alpha(1-\varepsilon)]$, which is larger than one under Assumption 3 for all $\varepsilon < \sigma$. From (A.38), the slope of the MM curve is smaller than one and therefore NN is steeper at the intersection point of the two curves, as drawn in Figure 3.

Now consider the response of ϕ^τ and $\bar{b}^\tau$ to tariff changes. First suppose that τ is such that $\bar{b}^\tau < \tau w_A \bar{a}/w_B$. In this case, there is sourcing from both countries and (A.38) and (A.41) imply that neither ϕ^τ nor $\bar{b}^\tau$ change as long as tariffs remain in the region with $\bar{b}^\tau < \tau w_A \bar{a}/w_B$. In contrast, consider an increase in the tariff when $\bar{b}^\tau > \tau w_A \bar{a}/w_B$. Then (A.38) and (A.41) imply

$$\begin{aligned} \hat{\phi}^\tau &= \gamma_B \hat{\bar{b}}^\tau + (1-\gamma_B)\hat{\tau}, \\ (\theta+1)\hat{\bar{b}}^\tau &= [1+\alpha(\varepsilon-1)]\hat{\phi}^\tau, \end{aligned}$$

where

$$\gamma_B = \frac{(1-\beta) w_B \bar{b}^\tau}{\beta\tau w_A \bar{a} + (1-\beta) w_B \bar{b}^\tau}.$$

Therefore,

$$\hat{\phi}^\tau = \frac{(\theta+1)(1-\gamma_B)}{\theta+1-\gamma_B-\gamma_B\alpha(\varepsilon-1)}\hat{\tau}, \tag{A.42}$$

$$\hat{\bar{b}}^\tau = \frac{[1-\alpha(1-\varepsilon)](1-\gamma_B)}{\theta+1-\gamma_B-\gamma_B\alpha(\varepsilon-1)}\hat{\tau}. \tag{A.43}$$

The numerators and the denominators of both equations are positive, implying that higher tariffs raise the cutoff and the marginal costs of intermediate inputs. Moreover, note from (A.43) that

$$\hat{\bar{b}}^\tau - \hat{\tau} = -\frac{(1-\gamma_B)[\theta-\alpha(\varepsilon-1)]}{\theta+1-\gamma_B-\gamma_B\alpha(\varepsilon-1)}\hat{\tau}.$$

The denominator on the right-hand side of this equation is positive. The numerator is negative under Assumption 3, because $\sigma > \varepsilon$. We conclude that the ratio $\bar{b}^{\tau}/\tau$ is declining with the tariff level.

As shown in the text, for $\tau \in (w_B/w_A, \tau_c)$ we have $\bar{b}^{\tau} > \tau w_A \bar{a}/w_B$, where τ_c is the tariff level at which $\tau_c w_A \bar{a} = w_B \bar{b}(\tau_c)$. For tariffs in this range, a higher tariff raises both ϕ^{τ} and $\bar{b}^{\tau}$ according to (A.42) and (A.43). In contrast, ϕ^{τ} and $\bar{b}^{\tau}$ are invariant to the tariff rate for all $\tau > \tau_c$. In this range, $a_B = w_B \bar{b}(\tau_c)/\tau w_A$ and $\bar{b}^{\tau} = \bar{b}(\tau_c)$, so we can express the weighted average of the foreign cost of the inputs using (A.34) and (A.36) as

$$\begin{aligned}
\rho^{\tau} &= \frac{G(a_B)}{G(\bar{a})}\left[\beta w_A \mu_a(a_B) + (1-\beta) w_B \frac{\bar{b}^{\tau}}{\tau}\right] + \left[1 - \frac{G(a_B)}{G(\bar{a})}\right]\left[\beta w_B \mu_b\left(\bar{b}^{\tau}\right) + (1-\beta) w_B \bar{b}^{\tau}\right] \\
&= \left(\frac{\tau_c}{\tau}\right)^{\theta} \frac{\theta+1-\beta}{\theta+1} \frac{w_B \bar{b}^{\tau}}{\tau} + \left[\left\{1 - \left(\frac{\tau_c}{\tau}\right)^{\theta}\right\}\right] \frac{\theta+1-\beta}{\theta+1} w_B \bar{b}^{\tau}.
\end{aligned}$$

The second line reveals the offsetting effects on the terms of trade: ρ^{τ} declines as a result of the decline in prices paid to suppliers in country A, but it rises with reallocation of supply from country A to country B, because net-of-tariff costs are higher in country B. The combined impact can be seen by rewriting the equation for ρ^{τ} as

$$\rho^{\tau} = \left\{1 - \frac{\tau-1}{\tau}\left(\frac{\tau_c}{\tau}\right)^{\theta}\right\} \frac{\theta+1-\beta}{\theta+1} w_B \bar{b}^{\tau}. \tag{A.44}$$

From this, we obtain

Lemma A.5 *Suppose $\varepsilon > 1$. Then for $\tau > \tau_c$, higher tariffs generate better terms of trade if and only if*

$$\tau < \frac{\theta+1}{\theta}.$$

Finally, we derive an equation for τ_c. From (A.5) we have

$$\frac{1}{\theta+1} w_A \bar{a} = \frac{f}{\beta m \bar{a}^{\theta}},$$

where m is the volume of intermediates in the free-trade equilibrium, before any tariff is imposed. From (A.33) we have

$$\frac{1}{\theta+1} w_B \bar{b}(\tau_c) = \frac{f}{\beta m(\tau_c) \bar{b}(\tau_c)^{\theta}}$$

when the tariff is τ_c. Therefore,

$$\frac{w_B \bar{b}(\tau_c)^{\theta+1}}{w_A \bar{a}^{\theta+1}} = \frac{m}{m(\tau_c)}.$$

However, from (A.40),

$$m = \alpha\left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} n^{-\frac{\sigma-\varepsilon}{\sigma-1}} \left(\frac{\theta}{\theta+1} w_A \bar{a}\right)^{\alpha(1-\varepsilon)-1},$$

$$m(\tau_c) = \alpha \left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} n^{-\frac{\sigma-\varepsilon}{\sigma-1}} \phi(\tau_c)^{\alpha(1-\varepsilon)-1}.$$

However, (A.38) implies that,

$$\phi(\tau_c) = \frac{\theta}{\theta+1} w_B \bar{b}(\tau_c) = \frac{\theta}{\theta+1} \tau_c w_A \bar{a}$$

and therefore,

$$\frac{w_B \bar{b}(\tau_c)^{\theta+1}}{w_A \bar{a}^{\theta+1}} = \left(\frac{w_A}{w_B}\right)^{\theta} (\tau_c)^{\theta+1} = \frac{m}{m(\tau_c)} = \frac{1}{(\tau_c)^{\alpha(1-\varepsilon)-1}}.$$

It follows that,

$$\tau_c = \left(\frac{w_B}{w_A}\right)^{\frac{\theta}{\theta-\alpha(\varepsilon-1)}}. \tag{A.45}$$

Since $\tau_c w_A \bar{a} = w_B \bar{b}(\tau_c)$, this implies

$$\bar{b}(\tau_c) = \left(\frac{w_B}{w_A}\right)^{\frac{\alpha(\varepsilon-1)}{\theta-\alpha(\varepsilon-1)}} \bar{a}. \tag{A.46}$$

We now consider tariffs that are large enough to induce exit. We denote by τ_{ex} the tariff rate at which the operating profits net of new search costs equal zero. To avoid taxonomy, we assume that $\tau_{ex} > \tau_c$; that profits drop to zero at a tariff rate that is high enough to induce surviving firms to switch suppliers from country A to country B.

For tariffs above τ_c the suppliers in country A that are replaced with suppliers from country B are all those with inverse productivity $a \in (a_B, \bar{a}]$, where

$$a_B = \frac{w_B \bar{b}^{\tau}}{\tau w_A} < \bar{a} \text{ for } \tau > \tau_c. \tag{A.47}$$

For these tariffs, the perceive marginal cost ϕ^{τ} and search cutoff $\bar{b}^{\tau}$ satisfy

$$\phi^{\tau} = \frac{\theta}{\theta+1} w_B \bar{b}^{\tau} \tag{A.48}$$

and

$$\frac{(\theta+1) f}{w_B \beta \left(\bar{b}^{\tau}\right)^{\theta+1}} = (n^{\tau})^{-\frac{\sigma-\varepsilon}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} \alpha (\phi^{\tau})^{\alpha(1-\varepsilon)-1}, \tag{A.49}$$

respectively. It follows, as we have already noted, that perceived marginal cost and the search cutoff are independent of the tariff rate for $\tau \in [\tau_c, \tau_{ex}]$ and that $n^{\tau} = n$ for all tariffs in this range.

We can write operating profits net of new search costs for the representative firm as a function of the number of active firms, n^{τ}, as follows:

$$\pi_{ex}^{\tau} = (P^{\tau})^{\sigma-\varepsilon} \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} (\phi^{\tau})^{\alpha(1-\sigma)} - \frac{(1-\beta) f}{\beta \left(\bar{b}^{\tau}\right)^{\theta}} - \left[1 - \left(\frac{w_B \bar{b}^{\tau}}{\tau w_A \bar{a}}\right)^{\theta}\right] \frac{f}{\left(\bar{b}^{\tau}\right)^{\theta}} - f_o. \tag{A.50}$$

The first term on the right-hand side represents revenue minus labor costs minus the variable costs of intermediate input. The second term represents payments to suppliers of intermediate inputs that do not depend on m^{τ}; these are the fixed payments that result from bargaining in the shadow of an outside option to search for a new supplier in country B. These fixed payments apply to all inputs, regardless of their source, because the outside option always involves search in country B when the tariff rate is large. The third term represents the new search costs incurred as a result of actual searches in country B to replace original suppliers in country A. These costs apply to the fraction of inputs with $a \in (a_B, \bar{a}]$ that are replaced after the tariff is introduced. Using (A.47), this fractions is $1 - \left(w_B \bar{b}^{\tau} / \tau w_A \bar{a}\right)^{\theta}$.

Note that

$$P^{\tau} = \frac{\sigma}{\sigma - 1} \left(\phi^{\tau}\right)^{\alpha} \left(n^{\tau}\right)^{-\frac{1}{\sigma-1}}. \tag{A.51}$$

It is apparent from (A.50) and (A.51) that, as long as the number of firms remains unchanged, and therefore ϕ^{τ} and $\bar{b}^{\tau}$ also do not change, operating profits net of new search costs decline with the tariff. Although revenues net of input costs are independent of the tariff rate, higher tariffs generate greater trade diversion to country B and thus greater expense on new searches. The critical tariff rate τ_{ex} that is large enough to induce exit is determined implicitly by

$$\pi_{ex}^{\tau} = \left(P_c^{\tau}\right)^{\sigma-\varepsilon} \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} \left(\phi_c^{\tau}\right)^{\alpha(1-\sigma)} - \frac{(1-\beta) f}{\beta \left(\bar{b}_c^{\tau}\right)^{\theta}} - \left[1 - \left(\frac{w_B \bar{b}_c^{\tau}}{\tau_{ex} w_A \bar{a}}\right)^{\theta}\right] \frac{f}{\left(\bar{b}_c^{\tau}\right)^{\theta}} - f_o = 0, \tag{A.52}$$

where ϕ_c^{τ} and $\bar{b}_c^{\tau}$ are the solution to (A.48) and (A.49) for $n^{\tau} = n$ and

$$P_c^{\tau} = \frac{\sigma}{\sigma-1} \left(\phi_c^{\tau}\right)^{\alpha} n^{-\frac{1}{\sigma-1}}.$$

Now consider the relationship between ϕ^{τ} and $\bar{b}^{\tau}$ and the tariff rate for $\tau \geq \tau_{ex}$. Substituting (A.51) into (A.50) yields the zero-profit condition,

$$\left(n^{\tau}\right)^{-\frac{\sigma-\varepsilon}{\sigma-1}} \frac{(\sigma-1)^{\varepsilon-1}}{\sigma^{\varepsilon}} \left(\phi^{\tau}\right)^{\alpha(1-\varepsilon)} - \frac{(1-\beta) f}{\beta \left(\bar{b}^{\tau}\right)^{\theta}} - \left[1 - \left(\frac{w_B \bar{b}^{\tau}}{\tau w_A \bar{a}}\right)^{\theta}\right] \frac{f}{\left(\bar{b}^{\tau}\right)^{\theta}} = f_o.$$

Next use (A.48) to rewrite (A.49) as

$$\frac{\theta f}{\beta \left(\bar{b}^{\tau}\right)^{\theta}} = \left(n^{\tau}\right)^{-\frac{\sigma-\varepsilon}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} \alpha \left(\phi^{\tau}\right)^{\alpha(1-\varepsilon)}. \tag{A.53}$$

These two equations imply

$$\frac{\theta}{\alpha(\sigma-1)} \frac{f}{\beta \left(\bar{b}^{\tau}\right)^{\theta}} - \frac{(1-\beta) f}{\beta \left(\bar{b}^{\tau}\right)^{\theta}} - \left[1 - \left(\frac{w_B \bar{b}^{\tau}}{\tau w_A \bar{a}}\right)^{\theta}\right] \frac{f}{\left(\bar{b}^{\tau}\right)^{\theta}} = f_o,$$

or

$$\frac{1}{\beta\left(\bar{b}^{\tau}\right)^{\theta}}\left[\frac{\theta}{\alpha\left(\sigma-1\right)}-1\right]+\left(\frac{w_B}{\tau w_A\bar{a}}\right)^{\theta}=\frac{f_o}{f}. \tag{A.54}$$

Assumption 3 ensures that the term in the square bracket is positive, implying that higher tariffs induce more selective search; i.e., lower values of $\bar{b}^{\tau}$. Moreover,

$$\widehat{\bar{b}^{\tau}}=-\xi^{\tau}\hat{\tau},\quad \xi^{\tau}=\frac{\beta\alpha\left(\sigma-1\right)}{\theta-\alpha\left(\sigma-1\right)}\left(\frac{w_B\bar{b}^{\tau}}{\tau w_A\bar{a}}\right)^{\theta}>0. \tag{A.55}$$

From (A.48), we see that ϕ^{τ} is proportional to $\bar{b}^{\tau}$ and therefore

$$\hat{\phi}^{\tau}=\widehat{\bar{b}^{\tau}}=-\xi^{\tau}\hat{\tau}.$$

Then (A.49) implies

$$\frac{\sigma-\varepsilon}{\sigma-1}\hat{n}^{\tau}=-\left[\theta-\alpha\left(\varepsilon-1\right)\right]\xi^{\tau}\hat{\tau}. \tag{A.56}$$

So the number of firms also declines. We therefore have

Proposition A.1 *Suppose Assumptions 1-3 hold and that $\tau\geq\tau_{ex}$. Then, the larger is the tariff, the smaller is ϕ^{τ}, $\bar{b}^{\tau}$, and n^{τ}.*

This proposition implies that, in the elastic case, the perceived marginal cost is a non-monotonic function of the size of the tariff. For tariffs in the range $\tau\in(1,\tau_c)$ perceived marginal cost rises with the tariff rate, in the range $\tau\in(\tau_c,\tau_{ex})$ it is independent of that rate, and in the range $\tau\geq\tau_{ex}$ it declines with τ. Since $\bar{b}^{\tau}$ follows the same non-monotonic pattern as ϕ^{τ}, and m^{τ} is decreasing in $\bar{b}^{\tau}$ from the equation that describes the optimal choice of $\bar{b}^{\tau}$ for a given m^{τ}, it follows that m^{τ} is also non-monotonic; it declines initially, remains constant for a range of tariffs, and then rises with τ when $\tau\geq\tau_{ex}$.

Next use (A.51) and (A.53) to obtain

$$\left(P^{\tau}\right)^{\sigma-\varepsilon}=\frac{\theta f}{\alpha\beta\left(\bar{b}^{\tau}\right)^{\theta}}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\phi^{\tau}\right)^{\alpha(\sigma-1)}.$$

Substituting (A.49) into this equation yields

$$\left(P^{\tau}\right)^{\sigma-\varepsilon}=\frac{\theta f}{\alpha\beta\left(\bar{b}^{\tau}\right)^{\theta-\alpha(\sigma-1)}}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\theta}{\theta+1}w_B\right)^{\alpha(\sigma-1)}. \tag{A.57}$$

Since $\bar{b}^{\tau}$ declines with the tariff, this implies that the price index is rising with the tariff in the range of large tariffs that induce exit. Moreover, (A.55) implies

$$\hat{P}^{\tau}=\frac{\theta-\alpha\left(\sigma-1\right)}{\sigma-\varepsilon}\xi^{\tau}\hat{\tau}.$$

Evidently, the price index rises with the tariff when $\tau\geq\tau_{ex}$ despite the decline in perceived marginal costs, because the variety reducing effect of exit dominates the effect on the price index of falling prices for brands that survive.

We can compute the size of the critical tariff, τ_{ex}., using (A.54) with $\bar{b}^{\tau} = \bar{b}_c^{\tau}$. Substituting (A.45) and (A.46) into (A.54), we find that τ_{ex} satisfies

$$\frac{\theta - \alpha(\sigma - 1)}{\beta\alpha(\sigma - 1)} + \left(\frac{\tau_c}{\tau_{ex}}\right)^{\theta} = \frac{f_o}{f}\bar{a}^{\theta}\left(\frac{w_B}{w_A}\right)^{\frac{\theta\alpha(\varepsilon-1)}{\theta-\alpha(\varepsilon-1)}}.$$

Now use the solution for $\bar{a}^{\theta}$ in (A.10) to obtain

$$\left(\frac{\tau_c}{\tau_{ex}}\right)^{\theta} = \frac{\theta - \alpha(\sigma - 1)}{\beta\alpha(\sigma - 1)}\left[\frac{f_o}{f_o + f_e}\left(\frac{w_B}{w_A}\right)^{\frac{\theta\alpha(\varepsilon-1)}{\theta-\alpha(\varepsilon-1)}} - 1\right]. \tag{A.58}$$

Clearly, this implies that, for $\tau_{ex} > \tau_c$, we need the term in the square brackets to be positive and the right-hand side to be smaller than one. These two conditions can be satisfied if and only if

$$\left(\frac{w_A}{w_B}\right)^{\frac{\theta\alpha(\varepsilon-1)}{\theta-\alpha(\varepsilon-1)}} < \frac{f_o}{f_o + f_e} < \frac{\theta - (1-\beta)\alpha(\sigma - 1)}{\theta - \alpha(\sigma - 1)}\left(\frac{w_A}{w_B}\right)^{\frac{\theta\alpha(\varepsilon-1)}{\theta-\alpha(\varepsilon-1)}}. \tag{A.59}$$

For every pair of wage rates w_A and w_B such that $w_B > w_A$ there exist fixed operating costs f_o and fixed entry costs f_e that satisfy these inequalities.

Section 4 Welfare Effects of Unanticipated Tariffs

Section 4.1 Increase in a Small Tariff

Consider the welfare effects of small tariffs. We showed in the main text that, apart from a constant, welfare can be expressed as

$$V(\tau) = U(X^{\tau}) - n^{\tau}\rho^{\tau}m^{\tau} - n^{\tau}\ell^{\tau} - n^{\tau}f\left[\frac{1}{G(a_c)} - \frac{1}{G(\bar{a})}\right].$$

In the elastic case, i.e., $\varepsilon > 1$, $a_c = \bar{a}$ and there are no additional search costs. Moreover, there is no entry, so that $n^{\tau} = n$. Therefore

$$V(\tau) = U(X^{\tau}) - n\rho^{\tau}m^{\tau} - n\ell^{\tau}$$

and

$$\frac{dV}{d\tau} = P^{\tau}\frac{dX^{\tau}}{d\tau} - n\frac{d\ell^{\tau}}{d\tau} - n\rho^{\tau}\frac{dm^{\tau}}{d\tau} - nm^{\tau}\frac{d\rho^{\tau}}{d\tau}.$$

The CES aggregator implies that

$$X^{\tau} = n^{\frac{\sigma}{\sigma-1}}z(\ell^{\tau}, m^{\tau})$$

and therefore

$$\begin{aligned}
P^\tau \frac{dX^\tau}{d\tau} &= n^{\frac{\sigma}{\sigma-1}} P^\tau \left(z_\ell \frac{d\ell^\tau}{d\tau} + z_m \frac{dm^\tau}{d\tau} \right) \\
&= n^{\frac{\sigma}{\sigma-1}} \frac{\sigma}{\sigma-1} \frac{P^\tau}{p^\tau} \left(\frac{d\ell^\tau}{d\tau} + \phi^\tau \frac{dm^\tau}{d\tau} \right) \\
&= n \frac{\sigma}{\sigma-1} \left(\frac{d\ell^\tau}{d\tau} + \phi^\tau \frac{dm^\tau}{d\tau} \right).
\end{aligned}$$

The second line is obtained from the first by noting that the marginal revenue generated by an increase in an input equals the input's marginal cost, which is one for labor and ϕ^τ for intermediate inputs. The third line is obtained from $P = pn^{-\frac{1}{\sigma-1}}$. Using this result, we obtain

$$\frac{dV}{d\tau} = n \frac{1}{\sigma-1} \frac{d\ell^\tau}{d\tau} + n \left(\frac{\sigma}{\sigma-1} \phi^\tau - \rho^\tau \right) \frac{dm^\tau}{d\tau} - nm^\tau \frac{d\rho^\tau}{d\tau}, \tag{A.60}$$

which is equation (28) in the main text.

Next, the assumption of a Cobb-Douglas technology implies

$$\ell^\tau = \frac{1-\alpha}{\alpha} \phi^\tau m^\tau$$

and therefore

$$\frac{1}{\sigma-1} \frac{d\ell^\tau}{d\tau} = \frac{1}{\sigma-1} \frac{1-\alpha}{\alpha} \frac{d(\phi^\tau m^\tau)}{d\tau}.$$

However, spending on intermediate inputs is a fraction α of spending on all inputs,

$$n\phi^\tau m^\tau = \alpha \frac{\sigma-1}{\sigma} P^\tau X^\tau, \tag{A.61}$$

and therefore

$$\begin{aligned}
n \frac{1}{\sigma-1} \frac{d\ell^\tau}{d\tau} &= n \frac{1}{\sigma-1} \frac{1-\alpha}{\alpha} \frac{d(\phi^\tau m^\tau)}{d\tau} = \frac{1-\alpha}{\sigma} \frac{d(P^\tau X^\tau)}{d\tau} \\
&= -\frac{1-\alpha}{\tau\sigma} (\varepsilon - 1) \alpha \left(\frac{d\phi^\tau}{d\tau} \frac{\tau}{\phi^\tau} \right) P^\tau X^\tau.
\end{aligned} \tag{A.62}$$

Using $P^\tau = (\phi^\tau)^\alpha n^{-\frac{1}{\sigma-1}} \sigma / (\sigma - 1)$, the last equality is obtained from

$$\frac{d(P^\tau X^\tau)}{d\tau} = \frac{d(P^\tau)^{1-\varepsilon}}{d\tau} = -(\varepsilon - 1) \alpha \left(\frac{d\phi^\tau}{d\tau} \frac{\tau}{\phi^\tau} \right) \frac{1}{\tau} P^\tau X^\tau.$$

Therefore, using (A.21),

$$n \frac{1}{\sigma-1} \frac{d\ell^\tau}{d\tau} = -\frac{1-\alpha}{\tau\sigma} (\varepsilon - 1) \alpha \frac{\theta + 1 - \gamma^\tau}{\theta + 1 - \gamma^\tau - \gamma^\tau \alpha (\varepsilon - 1)} P^\tau X^\tau.$$

This gives us the first term in (A.60). Since $\varepsilon > 1$, the tariff reduces employment and this has a negative (partial) effect on welfare.

To obtain the second term in (A.60), we again use (A.61) and (A.21), which gives

$$\begin{aligned} n\phi^{\tau}\frac{dm^{\tau}}{d\tau} &= \frac{\sigma-1}{\sigma}\alpha\frac{d\left(P^{\tau}X^{\tau}\right)}{d\tau} - nm^{\tau}\frac{d\phi^{\tau}}{d\tau} \\ &= -\frac{1-\alpha}{\tau\sigma}\left(\varepsilon-1\right)\alpha\frac{\theta+1-\gamma^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}P^{\tau}X^{\tau} - \frac{1}{\tau}nm^{\tau}\phi^{\tau}\frac{\theta+1-\gamma^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)} \\ &= -\frac{1}{\tau}\frac{\sigma-1}{\sigma}\alpha\left[\left(\varepsilon-1\right)\alpha+1\right]\frac{\theta+1-\gamma^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}P^{\tau}X^{\tau}. \end{aligned}$$

Now, (14) and (17) imply

$$\phi^{\tau} = \tau w\frac{\theta}{\theta+1}\left[\beta\bar{a}+\left(1-\beta\right)\bar{a}^{\tau}\right] \tag{A.63}$$

and

$$\rho^{\tau} = \beta w\frac{\theta}{\theta+1}\bar{a}+\left(1-\beta\right)w\bar{a}^{\tau}. \tag{A.64}$$

Therefore,

$$\begin{aligned} n\left(\frac{\sigma}{\sigma-1}\phi^{\tau}-\rho^{\tau}\right)\frac{dm^{\tau}}{d\tau} &= \left(\rho^{\tau}-\frac{\sigma}{\sigma-1}\phi^{\tau}\right)\frac{1}{\tau\phi^{\tau}}\frac{\sigma-1}{\sigma}\alpha\left[\left(\varepsilon-1\right)\alpha+1\right]\frac{\theta+1-\gamma^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}P^{\tau}X^{\tau} \\ &= \left(\frac{\theta+\gamma^{\tau}}{\theta}-\frac{\sigma}{\sigma-1}\tau\right)\frac{1}{\tau^{2}}\frac{\sigma-1}{\sigma}\alpha\left[\left(\varepsilon-1\right)\alpha+1\right]\frac{\theta+1-\gamma^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}P^{\tau}X^{\tau}. \end{aligned}$$

While the tariff reduces demand for the composite intermediate good, the welfare effect is ambiguous for the reasons discussed in the main text. This component of the welfare effect is positive if and only if

$$\frac{\theta+\gamma^{\tau}}{\theta} > \frac{\sigma}{\sigma-1}\tau.$$

This is the second term in (A.60).

To obtain the third term in the welfare formula, we use (A.64) and (A.22) to obtain

$$\begin{aligned} nm^{\tau}\frac{d\rho^{\tau}}{d\tau} &= wnm^{\tau}\left(1-\beta\right)\frac{d\bar{a}^{\tau}}{d\tau} \\ &= \frac{1}{\tau}nm^{\tau}\left(1-\beta\right)\frac{\alpha\left(\varepsilon-1\right)w\bar{a}^{\tau}}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}. \end{aligned}$$

Next, (A.61) and (A.63) imply

$$nm^{\tau} = \frac{1}{\tau w\frac{\theta}{\theta+1}\left[\beta\bar{a}+\left(1-\beta\right)\bar{a}^{\tau}\right]}\frac{\sigma-1}{\sigma}\alpha P^{\tau}X^{\tau}.$$

Therefore,

$$nm^{\tau}\frac{d\rho^{\tau}}{d\tau} = \frac{1}{\tau^{2}}\frac{\theta+1}{\theta}\gamma^{\tau}\frac{\sigma-1}{\sigma}\alpha\frac{\alpha\left(\varepsilon-1\right)}{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}P^{\tau}X^{\tau}.$$

So, in this case, $d\rho^{\tau}/d\tau > 0$; i.e., the terms of trade deteriorate.

Combining the three terms in the expression for the change in welfare, we have

$$\begin{aligned}
&\frac{\theta+1-\gamma^{\tau}-\gamma^{\tau}\alpha\left(\varepsilon-1\right)}{\theta+1-\gamma^{\tau}}\frac{\sigma\tau^{2}}{\alpha P^{\tau}X^{\tau}}\frac{dV}{d\tau}= \\
&-\tau\left(1-\alpha\right)\left(\varepsilon-1\right) \\
&+\left(\frac{\theta+\gamma^{\tau}}{\theta}-\frac{\sigma}{\sigma-1}\tau\right)\left(\sigma-1\right)\left[\left(\varepsilon-1\right)\alpha+1\right] \\
&-\frac{\theta+1}{\theta}\left(\sigma-1\right)\frac{\alpha\left(\varepsilon-1\right)\gamma^{\tau}}{\theta+1-\gamma^{\tau}}.
\end{aligned}\tag{A.65}$$

A marginal tariff raises welfare if and only if the right-hand side of this equation is positive. Since at free trade $\gamma(1)=1-\beta$, it follows that, starting with free trade, a very small tariff reduces welfare if and only if

$$\frac{\theta\varepsilon\left(\theta+\beta\right)}{\theta+\beta-\left(\varepsilon-1\right)\alpha\left(1-\beta\right)}>\left(\sigma-1\right)\left(1-\beta\right).$$

Next, note that, holding γ^{τ} constant, the right-hand side of (A.65) is declining in τ. Hence, any positive tariff must reduce welfare if

$$\frac{\theta\varepsilon\left(\theta+1-\gamma^{\tau}\right)}{\theta+1-\gamma^{\tau}-\left(\varepsilon-1\right)\alpha\gamma^{\tau}}>\left(\sigma-1\right)\gamma^{\tau}\quad\text{for all }\tau\geq 1.$$

Section 4.2 Increase in a Large Tariff

We now examine the welfare effects of tariffs for $\tau>w_B/w_A$. First, consider tariffs in the range $\tau\in(w_B/w_A,\tau_c)$. In this range, there are no new searches by any of the incumbent producers and country A continues to supply all intermediate inputs. As a result, tariffs are imposed on all imports, generating a revenue of $(\tau-1)\,\rho^{\tau}m^{\tau}$. Tariff revenue plus variable profits plus consumer surplus sum to

$$\begin{aligned}
V\left(\tau\right)&=T\left(\tau\right)+\Pi\left(\tau\right)+\Gamma\left(\tau\right)\\
&=\left(\tau-1\right)\rho^{\tau}m^{\tau}+\left[P^{\tau}X^{\tau}-\tau\rho^{\tau}nm^{\tau}-n\ell^{\tau}\right]+\left[U\left(X^{\tau}\right)-P^{\tau}X^{\tau}\right]\\
&=U\left(X^{\tau}\right)-\rho^{\tau}nm^{\tau}-n\ell^{\tau}.
\end{aligned}$$

Differentiating this equation gives

$$\begin{aligned}
\frac{1}{n}\frac{dV}{d\tau}&=\frac{1}{n}P^{\tau}\frac{dX^{\tau}}{d\tau}-\frac{d\ell^{\tau}}{d\tau}-\rho^{\tau}\frac{dm^{\tau}}{d\tau}-m^{\tau}\frac{d\rho^{\tau}}{d\tau}\\
&=\left(\frac{\sigma}{\sigma-1}-1\right)\frac{d\ell^{\tau}}{d\tau}+\left(\frac{\sigma}{\sigma-1}\phi^{\tau}-\rho^{\tau}\right)\frac{dm^{\tau}}{d\tau}-m^{\tau}\frac{d\rho^{\tau}}{d\tau}.
\end{aligned}$$

We have shown that, in this range, $\bar{b}^{\tau}$ is larger for larger tariffs whereas $\bar{b}^{\tau}/\tau$ is smaller for larger tariffs. The optimal choice of $\bar{b}^{\tau}$ for a given m^{τ}, equation (A.33), therefore implies that m^{τ} declines with the tariff, while (A.36) implies that ρ^{τ} declines. For these reasons, the change in the sourcing of intermediate inputs raises welfare if and only if

$$\frac{\sigma}{\sigma-1}\frac{\phi^{\tau}}{\rho^{\tau}}=\tau\frac{\sigma}{\sigma-1}\frac{\frac{\theta}{\theta+1}\left[\beta\tau w_A\bar{a}+(1-\beta)\,w_B\bar{b}^{\tau}\right]}{\frac{\theta}{\theta+1}\beta\tau w_A\bar{a}+(1-\beta)\,w_B\bar{b}^{\tau}}=\frac{\sigma}{\sigma-1}\frac{\theta\tau}{\theta+\gamma_B}<1.$$

Meanwhile, better terms of trade always contribute to higher welfare. Finally, since

$$n\ell^{\tau}=(1-\alpha)\,\frac{\sigma-1}{\sigma}P^{\tau}X^{\tau}$$

and ϕ^{τ} rises with the tariff level, it follows that $P^{\tau}X^{\tau}$ declines with the size of the tariff in the elastic case. As a result, ℓ^{τ} declines, which reduces welfare, all else the same. Clearly, in this case, a marginal increase in the tariff rate may increase or reduce welfare.

We next consider $\tau>\tau_c$. In this range, $d\ell^{\tau}/d\tau=dm^{\tau}/d\tau=dX^{\tau}/d\tau=dP^{\tau}/d\tau=0$, because neither ϕ^{τ} nor $\bar{b}^{\tau}$ vary with the size of the tariff. As a result,

$$\frac{dV}{d\tau}=-nm^{\tau}\frac{d\rho^{\tau}}{d\tau}-\frac{d\Sigma}{d\tau},$$

where $\Sigma(\tau)$ is the cost of the new searches that take place by incumbent producers. Using (A.33) and $a_B=\frac{w_B\bar{b}(\tau_c)}{\tau w_A}$, the cost of new searches amounts to

$$\begin{aligned}\Sigma&=n\left[1-\frac{G(a_B)}{G(\bar{a})}\right]\frac{f}{G\left[\bar{b}(\tau_c)\right]}\\&=nm^{t}\left[1-\left(\frac{\tau_c}{\tau}\right)^{\theta}\right]\frac{\beta}{\theta+1}w_B\bar{b}(\tau_c).\end{aligned}$$

Therefore, the variation in the search cost that results from a slightly higher tariff is

$$\frac{d\Sigma}{d\tau}=nm^{\tau}\frac{\theta}{\tau^{\theta+1}}(\tau_c)^{\theta}\frac{\beta}{\theta+1}w_B\bar{b}(\tau_c).$$

The terms of trade now are a weighted average of the cost of sourcing from country A and the cost of sourcing from country B,

$$\rho^{\tau}=\frac{G(a_B)}{G(\bar{a})}\left[\beta w_A\mu_a(a_B)+(1-\beta)\,w_B\frac{\bar{b}^{\tau}}{\tau}\right]+\left[1-\frac{G(a_B)}{G(\bar{a})}\right]w_B\left[\beta\mu_b\left(\bar{b}^{\tau}\right)+(1-\beta)\,\bar{b}^{\tau}\right].$$

The first term on the right-hand side represents the fraction of goods sourced from country A, $G(a_B)/G(\bar{a})$, times the average cost of goods sourced from that country, while the second term represents the fraction of goods sourced from country B times the average cost of those inputs.

Using $a_B = \frac{w_B \bar{b}(\tau_c)}{\tau w_A}$ and properties of the Pareto distribution, this equation becomes

$$\begin{aligned}\rho^\tau &= \left(\frac{\tau_c}{\tau}\right)^\theta \frac{\theta+1-\beta}{\theta+1} \frac{w_B \bar{b}(\tau_c)}{\tau} + \left[1 - \left(\frac{\tau_c}{\tau}\right)^\theta\right] \frac{\theta+1-\beta}{\theta+1} w_B \bar{b}(\tau_c) \\ &= \frac{\theta+1-\beta}{\theta+1} w_B \bar{b}(\tau_c) \left[1 - \frac{\tau-1}{\tau^{\theta+1}} (\tau_c)^\theta\right],\end{aligned}$$

$$\frac{d\rho^\tau}{d\tau} = \frac{\theta(\tau-1)-1}{\tau^{\theta+2}} (\tau_c)^\theta \frac{\theta+1-\beta}{\theta+1} w_B \bar{b}(\tau_c)$$

Since the right-hand side of the last equation is negative if and only if

$$\tau < \frac{\theta+1}{\theta},$$

it follows that the terms of trade improve if $\tau < (\theta+1)/\theta$ and deteriorate if $\tau > (\theta+1)/\theta$. Combining terms, we now have

$$\begin{aligned}\frac{1}{nm^\tau} \frac{dV}{d\tau} &= -\frac{d\rho^\tau}{d\tau} - \frac{1}{n^\tau m^\tau} \frac{d\Sigma}{d\tau} \\ &= w_B \bar{b}(\tau_c) \frac{\theta+1-\beta-\theta\tau}{\tau^{\theta+2}} (\tau_c)^\theta.\end{aligned}$$

Therefore, welfare rises with the tariff for $\tau > \tau_c$ if and only if

$$\tau < \frac{\theta+1-\beta}{\theta}.$$

When the label B denotes the home country, the social cost of inputs is

$$\rho^\tau = \frac{G(a_B)}{G(\bar{a})} \left[\beta w_A \mu_a(a_B) + (1-\beta) \frac{\bar{b}^\tau}{\tau}\right] + \left[1 - \frac{G(a_B)}{G(\bar{a})}\right] w_B \mu_b\left(\bar{b}^\tau\right),$$

where the second term now represents the cost of producing inputs at home. Using properties of the Pareto distribution and $a_B = \frac{w_B \bar{b}(\tau_c)}{\tau w_A}$, we have

$$\rho^\tau = \left(\frac{\tau_c}{\tau}\right)^\theta \frac{\theta+1-\beta}{\theta+1} \frac{w_B \bar{b}(\tau_c)}{\tau} + \left[1 - \left(\frac{\tau_c}{\tau}\right)^\theta\right] \frac{\theta}{\theta+1} w_B \bar{b}(\tau_c),$$

$$\begin{aligned}\frac{d\rho^\tau}{d\tau} &= -\frac{\theta+1}{\tau^{\theta+2}} (\tau_c)^\theta \frac{\theta+1-\beta}{\theta+1} w_B \bar{b}(\tau_c) + \frac{\theta}{\tau^{\theta+1}} (\tau_c)^\theta \frac{\theta}{\theta+1} w_B \bar{b}(\tau_c) \\ &= \frac{1}{(\theta+1)\tau^{\theta+2}} (\tau_c)^\theta \left[\tau\theta^2 - (\theta+1)(\theta+1-\beta)\right] w_B \bar{b}(\tau_c).\end{aligned}$$

In this case, the resource cost of inputs declines with the tariff if and only if

$$\tau < \frac{(\theta+1)(\theta+1-\beta)}{\theta^2}.$$

The effect of a higher tariff on social welfare can now be expressed as

$$\begin{aligned}\frac{1}{n^\tau m^\tau}\frac{dV}{d\tau} &= -\frac{d\rho^\tau}{d\tau} - \frac{1}{n^\tau m^\tau}\frac{d\Sigma}{d\tau} \\ &= -\frac{1}{(\theta+1)\,\tau^{\theta+2}}(\tau_c)^\theta\left[\tau\theta^2 - (\theta+1)(\theta+1-\beta)\right] w_B\bar{b}(\tau_c) \\ &\quad - \frac{\theta}{\tau^{\theta+1}}(\tau_c)^\theta \frac{\beta}{\theta+1} w_B\bar{b}(\tau_c) \\ &= w_B\bar{b}(\tau_c)\frac{-\tau\theta^2 + (\theta+1)(\theta+1-\beta) - \beta\theta\tau}{(\theta+1)\,\tau^{\theta+2}}(\tau_c)^\theta .\end{aligned}$$

Therefore, welfare rises with the tariff if and only if

$$\tau < \frac{(\theta+1)(\theta+1-\beta)}{\theta(\theta+\beta)}.$$

Finally, we turn to the welfare effects of tariffs that induce exit. Recall that the welfare components that might vary with the tariff are income from operating profits net of new search costs, tariff revenue, and consumer surplus. However, for $\tau \geq \tau_{ex}$ operating profits net of new search costs are zero, and we are left with tariff revenue and consumer surplus as the welfare components of interest, namely

$$V_{ex}(\tau) = T(\tau) + \Gamma(\tau).$$

Tariffs are collected on imports from country A only and are equal to

$$T(\tau) = \frac{G(a_B)}{G(\bar{a})}(\tau-1)\left[\beta w_A \mu_a(a_B) + (1-\beta)\frac{w_B\bar{b}^\tau}{\tau}\right] m^\tau .$$

Here, term in the square brackets represents the average ex-factory price paid for inputs from country A, while $G(a_B)/G(\bar{a})$ represents the fraction of inputs imported from A. Using (A.47), the revenue can be expressed as

$$T(\tau) = \frac{\theta+1-\beta}{\theta+1}\left(\frac{1}{w_A\bar{a}}\right)^\theta \left(\frac{w_B\bar{b}^\tau}{\tau}\right)^{\theta+1}(\tau-1)\,m^\tau .$$

In addition, the cost minimizing choice of $\bar{b}^\tau$ for a given m^τ implies

$$w_B\left(\bar{b}^\tau\right)^{\theta+1} = \frac{f(\theta+1)}{\beta m^\tau}$$

and therefore

$$T(\tau) = (\theta+1-\beta)\left(\frac{w_B}{w_A\bar{a}}\right)^\theta \frac{f}{\beta}\frac{\tau-1}{\tau^{\theta+1}}.$$

Again using (13), this can be written as

$$T(\tau) = \frac{(\theta+1-\beta)\,\alpha(\sigma-1)}{\theta-\alpha(\sigma-1)}\left(\frac{w_B}{w_A}\right)^\theta (f_o+f_e)\frac{\tau-1}{\tau^{\theta+1}}.$$

It follows that tariff revenue declines with τ for $\tau > \tau_{ex}$ if and only if $\tau > (\theta+1)/\theta$. Since the price index unambiguously rises with the size of the tariff, consumer surplus is inversely related to the tariff rate. Therefore, for $\tau > (\theta+1)/\theta$, higher tariffs in the range where exit occurs must result in lower welfare.

Section 5 Application to the Trump Tariffs

In this section we first show that our measure of welfare change relative to initial spending on differentiated products does not depend on the fixed costs (f_o, f_e, f). To this end we note that in all equilibria

$$p = \frac{\sigma}{\sigma-1}c = \frac{\sigma}{\sigma-1}\phi^{\alpha},$$

$$X = P^{-\varepsilon},$$

and therefore

$$x = X\left(\frac{p}{P}\right)^{-\sigma} = P^{\sigma-\varepsilon}\left(\frac{\sigma}{\sigma-1}\phi^{\alpha}\right)^{-\sigma}. \tag{A.66}$$

In the initial equilibrium, (A.9)-(A.11) provide a solution to $\bar{a}$, P and n. To emphasize the dependence on f, f_o and f_e, we express these equations as

$$\bar{a} = B_a\left(\frac{f}{f_o+f_e}\right)^{1/\theta}, \tag{A.67}$$

$$P = B_P\left(f_o+f_e\right)^{\frac{1}{\sigma-\varepsilon}}\bar{a}^{\frac{\alpha(\sigma-1)}{\sigma-\varepsilon}}, \tag{A.68}$$

$$n^{\frac{1}{\sigma-1}} = B_n\left(f_o+f_e\right)^{-\frac{1}{\sigma-\varepsilon}}\bar{a}^{-\frac{\alpha(\varepsilon-1)}{\sigma-\varepsilon}}, \tag{A.69}$$

where B_j, $j=a,P,n$ include neither f nor f_o or f_e. We also have from (A.16) and (A.17)

$$\phi = \frac{\theta}{\theta+1}w\bar{a}, \tag{A.70}$$

$$\rho = \left[\beta\frac{\theta}{\theta+1} + (1-\beta)\right]w\bar{a}. \tag{A.71}$$

Equation (A.68) implies

$$pxn = PX = P^{1-\varepsilon} = B_P^{1-\varepsilon}\left(f_o+f_e\right)^{\frac{1-\varepsilon}{\sigma-\varepsilon}}\bar{a}^{\frac{\alpha(\sigma-1)(1-\varepsilon)}{\sigma-\varepsilon}}. \tag{A.72}$$

Together with (A.69), it implies,

$$px = B_P^{1-\varepsilon}B_n^{1-\sigma}\left(f_o+f_e\right). \tag{A.73}$$

That is, px is proportional to (f_o+f_e) and independent of the search cost f. This implies that ℓ

is also proportional to $(f_o + f_e)$ and independent of the search cost f, i.e.,

$$\ell = B_\ell (f_o + f_e),$$

where B_ℓ is independent of f, f_o or f_e

Next, (A.8), (A.68) and (A.70) yield

$$\begin{aligned} m &= \alpha P^{\sigma-\varepsilon} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \phi^{\alpha(1-\sigma)-1} \\ &= B_m (f_o + f_e) \bar{a}^{-1}, \end{aligned}$$

where B_m is independent of f, f_o or f_e. Therefore, using (A.71),

$$\rho m = B_{\rho m} (f_o + f_e),$$

where $B_{\rho m}$ is independent of f, f_o or f_e.

Welfare is

$$V = U(X) - \rho m n - n\ell.$$

Therefore, using $X = P^{-\varepsilon}$,

$$\begin{aligned} V + \frac{\varepsilon}{\varepsilon-1} &= \frac{\varepsilon}{\varepsilon-1} X^{\frac{\varepsilon-1}{\varepsilon}} - \rho m n - n\ell \\ &= \frac{\varepsilon}{\varepsilon-1} P^{1-\varepsilon} - \rho m n - n\ell \\ &= \frac{\varepsilon}{\varepsilon-1} p x n - \rho m n - n\ell. \end{aligned} \tag{A.74}$$

It follows that

$$\frac{V + \frac{\varepsilon}{\varepsilon-1}}{pxn} = \frac{\varepsilon}{\varepsilon-1} - \frac{B_{\rho m} + B_\ell}{B_P^{1-\varepsilon} B_n^{1-\sigma}},$$

which is independent of f, f_o or f_e.

We now focus on the large tariff case $\tau > \tau_c > w_B/w_A$, which is relevant for the calibration. In this case (A.46) and (A.48) imply

$$\bar{b}^\tau = \bar{b}_c := B_b^\tau \bar{a}, \tag{A.75}$$

$$\phi^\tau = \phi_c := B_\phi^\tau \bar{a}, \tag{A.76}$$

where B_b^τ and B_ϕ^τ are independent of f, f_o or f_e. In other words, ϕ^τ and $\bar{b}^\tau$ are proportional to $\bar{a}$ for all $\tau \geq \tau_c$.

Consider the range $\tau \geq \tau_c$. In this range (A.40), (A.44) and (A.49) imply

$$m^\tau = \alpha \left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} n^{-\frac{\sigma-\varepsilon}{\sigma-1}} (\phi^\tau)^{\alpha(1-\varepsilon)-1} = \frac{(\theta+1) f}{w_B \beta \left(\bar{b}^\tau\right)^{\theta+1}}. \tag{A.77}$$

$$\rho^\tau m^\tau = \alpha\left[1-\frac{\tau-1}{\tau}\left(\frac{\tau_c}{\tau}\right)^\theta\right]\frac{\theta+1-\beta}{\theta+1}\frac{(\theta+1)f}{\beta\left(\bar{b}^\tau\right)^\theta}.$$

Using (A.67) and (A.75) then implies

$$\rho^\tau m^\tau = \left[1-\frac{\tau-1}{\tau}\left(\frac{\tau_c}{\tau}\right)^\theta\right]B^\tau_{\rho m}\left(f_o+f_e\right), \quad \text{for } \tau\geq\tau_c, \tag{A.78}$$

where $B^\tau_{\rho m}$ does not depend on f, f_o or f_e.

First, consider a tariff $\tau=\tau_c$. In this case, there are no new searches by any of the incumbent producers and country A continues to supply all intermediate inputs. As a result, tariffs are imposed on all imports, generating a revenue of $(\tau-1)\,\rho^\tau m^\tau$. Tariff revenue plus variable profits plus consumer surplus sum to

$$\begin{aligned} V^{\tau_c} &= T^{\tau_c}+\Pi^{\tau_c}+\Gamma^{\tau_c} \\ &= (\tau-1)\,\rho^{\tau_c}m^{\tau_c}+\left[P^{\tau_c}X^{\tau_c}-\tau\rho^{\tau_c}nm^{\tau_c}-n\ell^{\tau_c}\right]+\left[U\left(X^{\tau_c}\right)-P^{\tau_c}X^{\tau_c}\right] \\ &= U\left(X^{\tau_c}\right)-n\rho^{\tau_c}m^{\tau_c}-n\ell^{\tau_c}. \end{aligned}$$

In this case (A.78) implies

$$\rho^{\tau_c}m^{\tau_c}=\frac{1}{\tau_c}B^\tau_{\rho m}\left(f_o+f_e\right).$$

Labor employment is

$$n\ell^{\tau_c}=(1-\alpha)\frac{\sigma-1}{\sigma}P^\tau X^\tau=(1-\alpha)\frac{\sigma-1}{\sigma}\left(P^{\tau_c}\right)^{1-\varepsilon}. \tag{A.79}$$

In addition,

$$U\left(X^{\tau_c}\right)+\frac{\varepsilon}{\varepsilon-1}=\frac{\varepsilon}{\varepsilon-1}\left(X^{\tau_c}\right)^{\frac{\varepsilon-1}{\varepsilon}}=\frac{\varepsilon}{\varepsilon-1}\left(P^{\tau_c}\right)^{1-\varepsilon}.$$

Also,

$$\left(P^{\tau_c}\right)^{1-\varepsilon}=p^{\tau_c}x^{\tau_c}n=\left(P^{\tau_c}\right)^{\sigma-\varepsilon}\left(\frac{\sigma}{\sigma-1}\phi_c^\alpha\right)^{1-\sigma}n$$

and

$$P^{\tau_c}=\frac{\sigma}{\sigma-1}\phi_c^\alpha n^{\frac{1}{1-\sigma}}.$$

Therefore, (A.69) and (A.76) yield,

$$\left(P^{\tau_c}\right)^{1-\varepsilon}=C_P^{\tau_c}\left(f_o+f_e\right)n, \tag{A.80}$$

where $C_P^{\tau_c}$ does not vary with f, f_o or f_e. Together with (A.79), the last equation implies that ℓ^{τ_c} is proportional to (f_o+f_e).

Using (A.74) and (A.80), we now have

$$V^{\tau_c}+\frac{\varepsilon}{\varepsilon-1}=\frac{\varepsilon}{\varepsilon-1}\left(f_o+f_e\right)C_P^{\tau_c}n-\rho^{\tau_c}m^{\tau_c}n-n\ell^{\tau_c}.$$

It follows that

$$\frac{V^{\tau_c}+\frac{\varepsilon}{\varepsilon-1}}{pxn}=\frac{\frac{\varepsilon}{\varepsilon-1}C_P^{\tau_c}\left(f_o+f_e\right)-\rho^{\tau_c}m^{\tau_c}-\ell^{\tau_c}}{px}.$$

Here both the numerator and the denominator of the right-hand side are proportional to (f_o+f_e), and therefore the right-hand side does not depend on f, f_o or f_e.

For $\tau \geq \tau_c$ we have $P=P^{\tau_c}$, $X=X^{\tau_c}$ and $\ell=\ell^{\tau_c}$. There are now search costs, equal to (see Section 4.2 above)

$$\Sigma^\tau = nm^\tau\left[1-\left(\frac{\tau_c}{\tau}\right)^\theta\right]\frac{\beta}{\theta+1}w_B\bar{b}\left(\tau_c\right).$$

Using (A.67), (A.75) and (A.77) this yields

$$\Sigma^\tau = n\left[1-\left(\frac{\tau_c}{\tau}\right)^\theta\right]B_\Sigma^\tau\left(f_o+f_e\right),$$

where B_Σ^τ is independent of f, f_o and f_e. We can express the utility at $\tau \geq \tau_c$ as

$$V^\tau+\frac{\varepsilon}{\varepsilon-1}=V^{\tau_c}+\frac{\varepsilon}{\varepsilon-1}-\left(\rho^\tau m^\tau-\rho^{\tau_c}m^{\tau_c}\right)n-n\left[1-\left(\frac{\tau_c}{\tau}\right)^\theta\right]B_\Sigma^\tau\left(f_o+f_e\right).$$

Now use (A.78) to obtain

$$V^\tau+\frac{\varepsilon}{\varepsilon-1}=V^{\tau_c}+\frac{\varepsilon}{\varepsilon-1}-n\left[\frac{\tau_c-1}{\tau_c}-\frac{\tau-1}{\tau}\left(\frac{\tau_c}{\tau}\right)^\theta\right]B_{\rho m}^\tau\left(f_o+f_e\right)-n\left[1-\left(\frac{\tau_c}{\tau}\right)^\theta\right]B_\Sigma^\tau\left(f_o+f_e\right).$$

It follows that

$$\frac{V^\tau+\frac{\varepsilon}{\varepsilon-1}}{pxn}=\frac{V^{\tau_c}+\frac{\varepsilon}{\varepsilon-1}}{pxn}-\frac{\frac{\tau_c-1}{\tau_c}-\frac{\tau-1}{\tau}\left(\frac{\tau_c}{\tau}\right)^\theta}{px}B_{\rho m}^\tau\left(f_o+f_e\right)-\frac{1-\left(\frac{\tau_c}{\tau}\right)^\theta}{px}B_\Sigma^\tau\left(f_o+f_e\right),$$

which is independent of f, f_o and f_e. Finally, this implies that

$$\frac{V^\tau-V}{pxn}$$

is independent of f, f_o and f_e.

Section 5.1: Calibration Equations

In the remaining part of this appendix we describe the equations that are pertinent for the calibration. Condition $\tau > \tau_c$ requires (see (A.45))

$$\tau > \tau_c = \left(\frac{w_B}{w_A}\right)^{\frac{\theta}{\theta-\alpha(\varepsilon-1)}},$$

while condition $\tau_c < \tau < \tau_{ex}$ requires (see (A.59))

$$\left(\frac{w_A}{w_B}\right)^{\frac{\theta\alpha(\varepsilon-1)}{\theta-\alpha(\varepsilon-1)}} < \frac{f_o}{f_o+f_e} < \frac{\theta-(1-\beta)\alpha(\sigma-1)}{\theta-\alpha(\sigma-1)}\left(\frac{w_A}{w_B}\right)^{\frac{\theta\alpha(\varepsilon-1)}{\theta-\alpha(\varepsilon-1)}},$$

where (see (A.58))

$$\tau_{ex} = \tau_c \left[\frac{\beta\alpha(\sigma-1)}{\theta-\alpha(\sigma-1)}\right]^{\frac{1}{\theta}} \left[\frac{f_o}{f_o+f_e}\left(\frac{w_B}{w_A}\right)^{\frac{\theta\alpha(\varepsilon-1)}{\theta-\alpha(\varepsilon-1)}} - 1\right]^{-\frac{1}{\theta}}.$$

Free Trade Equilibrium

We solve for equilibrium sequentially, starting with the reservation productivity (see (A.10)):

$$\overline{a} = \left[\frac{f}{f_o+f_e}\frac{\theta-\alpha(\sigma-1)}{\beta\alpha(\sigma-1)}\right]^{\frac{1}{\theta}}.$$

Expected differentiated variety marginal cost is:

$$\phi = w_A\mu_a(\overline{a}) = w_A\frac{\theta}{\theta+1}\overline{a},$$

$$c(\phi) = \phi^{\alpha}.$$

Free entry requires

$$\pi_o = f_e + \frac{f}{G(\overline{a})} = f_e + \frac{f}{\overline{a}^{\theta}},$$

where operating profits, π_o, are (see (A.7))

$$\pi_o = \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}}P^{\sigma-\varepsilon}c(\phi)^{(1-\sigma)} - \frac{(1-\beta)f}{\beta\overline{a}^{\theta}} - f_o,$$

yielding the price index

$$P = \left\{\frac{1}{c(\phi)^{(1-\sigma)}}\frac{\sigma^{\sigma}}{(\sigma-1)^{\sigma-1}}\left[\pi_o + \frac{(1-\beta)f}{\beta\overline{a}^{\theta}} + f_o\right]\right\}^{\frac{1}{\sigma-\varepsilon}}.$$

Differentiated sector variety prices are

$$p = \frac{\sigma}{\sigma-1}\phi^{\alpha},$$

and the price index

$$P = n^{-\frac{1}{\sigma-1}}p$$

yields

$$n = \left(\frac{p}{P}\right)^{\sigma-1}.$$

From the optimal stopping rule (A.3) we have:

$$m = \frac{f(\theta+1)}{\beta w_A \overline{a}^{\theta+1}}. \tag{A.81}$$

Employment is (due to the Cobb-Douglas production function):

$$\ell = \frac{1-\alpha}{\alpha} m\phi.$$

Differentiated sector consumption index is:

$$X = P^{-\varepsilon}.$$

Quantity demanded of individual differentiated sector variety is:

$$x = X\left(\frac{p}{P}\right)^{-\sigma}.$$

Average price of differentiated sector imported intermediate inputs is:

$$\rho = \mu_{\rho}(\overline{a}) = w_A \overline{a}\left[\beta\frac{\theta}{\theta+1} + (1-\beta)\right], \tag{A.82}$$

where

$$\rho(a) = \beta w_A a + (1-\beta) w_A \overline{a}.$$

Aggregate value of differentiated sector imports is:

$$M = nm\rho. \tag{A.83}$$

Expected fixed costs are:

$$f_o + f_e + \frac{f}{\overline{a}^{\theta}}.$$

Expected variable costs are:

$$\rho m + \ell.$$

Free entry imposes:

$$\pi_o - f_e - \frac{f}{\overline{a}^{\theta}} = 0.$$

Share of profits in differentiated sector expenditure is:

$$\frac{n\pi_o}{P^{1-\varepsilon}}.$$

Share of imported input costs in differentiated sector expenditure is:

$$\frac{M}{P^{1-\varepsilon}}.$$

Welfare is (see (A.74)):

$$V = \frac{\varepsilon}{\varepsilon - 1}\left(X^{\frac{\varepsilon-1}{\varepsilon}} - 1\right) - n\rho m - n\ell.$$

Post-Tariff Equilibrium

We have the following system of simultaneous equations for $\overline{b}^{\tau}$ and ϕ^{τ} given n (see (A.38) and (A.41)):

$$\phi^{\tau} = \frac{\theta}{\theta+1} w_B \overline{b}^{\tau},$$

$$\frac{(\theta+1)\, f}{w_B \beta \left(\overline{b}^{\tau}\right)^{\theta+1}} = (n)^{-\frac{\sigma-\varepsilon}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-\varepsilon} \alpha \left(\phi^{\tau}\right)^{\alpha(1-\varepsilon)-1},$$

where we have used $n^{\tau} = n$. Substituting the first equation into the second equation, we obtain the following closed-form solution for $\overline{b}^{\tau}$:

$$\overline{b}^{\tau} = \left[\frac{(\theta+1)\, f}{\alpha\beta} (n)^{\frac{\sigma-\varepsilon}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{\varepsilon} \left(\frac{\theta}{\theta+1}\right)^{-\alpha(1-\varepsilon)+1} (w_B)^{-\alpha(1-\varepsilon)}\right]^{\frac{1}{\theta+\alpha(1-\varepsilon)}}.$$

Substituting this solution for $\overline{b}^{\tau}$ into the first of the two equations above, we recover ϕ^{τ}:

$$\phi^{\tau} = \frac{\theta}{\theta+1} w_B \overline{b}^{\tau}.$$

We can now solve for the rest of the post-tariff equilibrium sequentially. We start with a_B:

$$a_B = \frac{w_B \overline{b}^{\tau}}{\tau w_A}. \tag{A.84}$$

Average price of differentiated sector imported intermediate inputs is:

$$\rho^{\tau} = \left(\frac{a_B}{\overline{a}}\right)^{\theta}\left[\beta w_A \frac{\theta}{\theta+1} a_B + (1-\beta)\, w_B \frac{\overline{b}^{\tau}}{\tau}\right] + \left[1 - \left(\frac{a_B}{\overline{a}}\right)^{\theta}\right] w_B \left[\beta \frac{\theta}{\theta+1} \overline{b}^{\tau} + (1-\beta)\, \overline{b}^{\tau}\right].$$

Average price of differentiated sector imported intermediate inputs conditional on sourcing from Country A is:

$$\rho_A^{\tau} = \beta w_A \frac{\theta}{\theta+1} a_B + (1-\beta)\, w_B \frac{\overline{b}^{\tau}}{\tau}. \tag{A.85}$$

Average price of differentiated sector imported intermediate inputs conditional on sourcing from Country B is:

$$\rho_B^{\tau} = \beta w_B \frac{\theta}{\theta+1} \overline{b}^{\tau} + (1-\beta)\, w_B \overline{b}^{\tau}. \tag{A.86}$$

Differentiated sector variety prices are:

$$p^{\tau} = \frac{\sigma}{\sigma-1} \left(\phi^{\tau}\right)^{\alpha}.$$

Differentiated sector price index is:

$$P^{\tau} = n^{-\frac{1}{\sigma-1}} p^{\tau}.$$

Differentiated sector consumption index is:

$$X^{\tau} = (P^{\tau})^{-\varepsilon}$$

Quantity demanded of individual differentiated sector variety is:

$$x^{\tau} = X^{\tau} \left(\frac{p^{\tau}}{P^{\tau}}\right)^{-\sigma}.$$

Imports (from optimal stopping rule) of intermediate inputs per product are:

$$m^{\tau} = \frac{f}{\beta\left(\overline{b}^{\tau}\right)^{\theta} w_B} \frac{1}{\left[\overline{b}^{\tau} - \frac{\theta}{\theta+1}\overline{b}^{\tau}\right]} = \frac{(\theta+1)\, f}{\beta\left(\overline{b}^{\tau}\right)^{\theta+1} w_B}. \tag{A.87}$$

Employment (from Cobb-Douglas production function) is:

$$\ell^{\tau} = \frac{1-\alpha}{\alpha}\phi^{\tau} m^{\tau}, \qquad n\phi^{\tau} m^{\tau} = \alpha\frac{\sigma-1}{\sigma} P^{\tau} X^{\tau},$$

$$\Rightarrow \qquad \ell^{\tau} = (1-\alpha)\,\frac{\sigma-1}{\sigma}\frac{P^{\tau} X^{\tau}}{n}.$$

Aggregate value of imports of intermediate inputs from Country A is:

$$M^{A} = nm^{\tau}\left(\frac{a_B}{\overline{a}}\right)^{\theta}\rho_A^{\tau}. \tag{A.88}$$

Aggregate value of imports of intermediate input from Country B is:

$$M^{B} = nm^{\tau}\left[1 - \left(\frac{a_B}{\overline{a}}\right)^{\theta}\right]\rho_B^{\tau}. \tag{A.89}$$

Welfare is:

$$\begin{aligned} V^{\tau} &= \frac{\varepsilon}{\varepsilon-1}\left[(X^{\tau})^{\frac{\varepsilon-1}{\varepsilon}} - 1\right] - n\rho^{\tau} m^{\tau} - n\ell^{\tau} - n\left[1 - \frac{G(a_B)}{G(\overline{a})}\right]\frac{f}{G\left(\overline{b}^{\tau}\right)} \\ &= \frac{\varepsilon}{\varepsilon-1}\left[(X^{\tau})^{\frac{\varepsilon-1}{\varepsilon}} - 1\right] - n\rho^{\tau} m^{\tau} - n\ell^{\tau} - n\left[1 - \left(\frac{a_B}{\overline{a}}\right)^{\theta}\right]\frac{f}{\left(\overline{b}^{\tau}\right)^{\theta}}. \end{aligned}$$

Import and Price Responses to the Tariff

We now use these relationships from the free trade and tariff equilibria to derive expressions for (i) log changes in imports, (i) log changes in expected input prices from Country A, and (iii) the reallocation of imports towards Country B, in response to the tariff, for the empirically relevant range of the parameter space in which there is a partial reorganization of supply chains towards Other Asia ($\tau_c < \tau < \tau_{ex}$).

Country A Import Response: Combining equations (A.83), (A.84) and (A.88), the log growth of import values from Country A in response to the tariff is:

$$\log\left(\frac{M_A^\tau}{M_A}\right) = \log\left(\frac{m^\tau}{m}\right) + \log\left(\left(\frac{w_B\overline{b}^\tau}{\tau w_A\overline{a}}\right)^\theta\right) + \log\left(\frac{\rho_A^\tau}{\rho}\right).$$

Using equations (A.81), (A.82), (A.85) and (A.87), we can re-write this expression as follows:

$$\log\left(\frac{M_A^\tau}{M_A}\right) = \log\left(\left(\frac{w_B}{w_A}\right)^\theta \frac{1}{\tau^{\theta+1}}\right). \tag{A.90}$$

which depends on (τ, θ, w_B/w_A) alone. We can obtain the elasticity of imports from Country A with respect to the tariff by dividing through by $\log(\tau)$ on both sides of equation (A.90). Therefore, as commonly found for the Pareto distribution, this elasticity depends on the productivity dispersion parameter (θ) rather than the elasticity of substitution between varieties (σ).

Country A Price Response: From equations (A.82), (A.85) and (A.84), the log change in export prices from Country A following the tariff is:

$$\log\left(\frac{\rho_A^\tau}{\rho_A}\right) = \log\left(\frac{w_B\overline{b}^\tau}{\tau w_A\overline{a}}\right). \tag{A.91}$$

For $\tau \geq \tau_c$, we have:

$$a_B = \frac{w_B\overline{b}(\tau_c)}{\tau w_A}, \qquad \overline{b}^\tau = \overline{b}(\tau_c). \tag{A.92}$$

Additionally, τ_c solves:

$$\tau_c w_A\overline{a} = w_B\overline{b}(\tau_c). \tag{A.93}$$

Together these relationships imply:

$$\frac{a_B}{\overline{a}} = \frac{w_B}{w_A}\frac{\overline{b}^\tau}{\overline{a}}\frac{1}{\tau} = \frac{\tau_c}{\tau}.$$

Using this result in equation (A.91), the log change in export prices from Country A following the tariff can be re-written as:

$$\log\left(\frac{\rho_A^\tau}{\rho_A}\right) = \log\left(\frac{\tau_c}{\tau}\right), \tag{A.94}$$

which depends on (τ, θ, w_B/w_A, α, ε) alone, since

$$\tau_c = \left(\frac{w_B}{w_A}\right)^{\frac{\theta}{\theta-\alpha(\varepsilon-1)}}. \tag{A.95}$$

We can obtain the elasticity of Country A prices with respect to the tariff by dividing through by

$\log(\tau)$ on both sides of equation (A.94).

Import Reallocation from Country A to Country B: From equations (A.83), (A.88) and (A.89), the change in imports from Country B divided by the change in imports from Country A is:

$$\frac{M_B^\tau}{M_A^\tau - M_A} = \frac{\left[1 - \left(\frac{a_B}{\bar{a}}\right)^\theta\right] \frac{\rho_B^\tau}{\rho_A^\tau}}{\left(\frac{a_B}{\bar{a}}\right)^\theta - \frac{m}{m^\tau} \frac{\rho}{\rho_A^\tau}}. \tag{A.96}$$

Using equations (A.81), (A.82), (A.84), (A.85), (A.86) and (A.87), we can rewrite this import reallocation in equation (A.96) as:

$$\frac{M_B^\tau}{M_A^\tau - M_A} = \frac{\left[1 - \left(\frac{a_B}{\bar{a}}\right)^\theta\right] \tau}{\left(\frac{a_B}{\bar{a}}\right)^\theta - \tau \left(\frac{\bar{b}^\tau}{\bar{a}}\right)^\theta}.$$

Using the results in (A.92) and (A.93), we can further re-write this import reallocation as:

$$\frac{M_B^\tau}{M_A^\tau - M_A} = \frac{\left[1 - \left(\frac{\tau_c}{\tau}\right)^\theta\right] \tau}{\left(\frac{\tau_c}{\tau}\right)^\theta - \tau \left(\frac{w_A}{w_B}\tau_c\right)^\theta}, \tag{A.97}$$

which again depends on $(\tau,\, \theta,\, w_B/w_A,\, \alpha,\, \varepsilon)$ alone, since $\tau_c = \left(\frac{w_B}{w_A}\right)^{\frac{\theta}{\theta - \alpha(\varepsilon-1)}}$.

Parameter Calibration

Productivity Dispersion Parameter (θ): From equations (A.90) and (A.95), we can rewrite the log growth in import values from Country A as:

$$\log\left(\frac{M_A^\tau}{M_A}\right) = \log\left(\tau_c^{\theta - \alpha(\varepsilon-1)} \frac{1}{\tau^{\theta+1}}\right).$$

Using the log change in export prices from Country A from equation (A.94) to substitute for τ_c, we obtain:

$$\log\left(\frac{M_A^\tau}{M_A}\right) = \left(\frac{\rho_A^\tau}{\rho}\right)^{\theta - \alpha(\varepsilon-1)} \tau^{-(1+\alpha(\varepsilon-1))}.$$

We therefore obtain the following closed-form expression that we use to calibrate θ:

$$\theta = \frac{\log\left(\frac{M_A^\tau}{M_A}\right) + \left[1 + \alpha\left(\varepsilon - 1\right)\right] \log \tau}{\log\left(\frac{\rho_A^\tau}{\rho}\right)} + \alpha\left(\varepsilon - 1\right), \tag{A.98}$$

where we have event-study estimates for $\log\left(\frac{M_A^\tau}{M_A}\right)$ and $\log\left(\frac{\rho_A^\tau}{\rho}\right)$; we observe $\tau = 1.14$; we calibrate $\varepsilon = 1.19$; and we calibrate α to match the initial share of imports from China in U.S. manufacturing value-added.

Cost Disadvantage of Country B (w_B/w_A): From equations (A.94) and (A.95). we obtain the following closed-form expression that we use to calibrate w_B/w_A:

$$\frac{w_B}{w_A} = \left(\frac{\rho_A^\tau}{\rho}\tau\right)^{\frac{\theta-\alpha(\varepsilon-1)}{\theta}}, \tag{A.99}$$

where we have an event-study estimate for $\log\left(\frac{\rho_A^\tau}{\rho}\right)$; we observe $\tau = 1.14$; we calibrate $\varepsilon = 1.19$; we calibrate α to match the initial share of imports from China in U.S. manufacturing value-added; and we calibrate θ from equation (A.98) above.