

# Appendix to “Monetary Policy when the Phillips Curve is Quite Flat”

Paul Beaudry, Chenyu Hou, Franck Portier

January 2023

## A Proof of Proposition 1

We assume that  $r_t = \rho_r^t r$ . Solving forward the Euler equation (3) with the marginal cost given by (1), we obtain

$$y_t = -\frac{\alpha_r}{1 - \rho_r \alpha_y} \rho_r^t r + \sum_{j=0}^{\infty} \alpha_y^j E_t d_{t+j},$$

which implies that

$$y_{t+j} = -\frac{\alpha_r}{1 - \rho_r \alpha_y} \rho_r^{t+j} r + \sum_{k=0}^{\infty} \alpha_y^k E_t d_{t+j+k}.$$

Take now the Phillips curve and solve forward to obtain

$$\pi_t = \underbrace{\kappa \gamma_y \sum_{j=0}^{\infty} \beta^j E_t y_{t+j}}_{\mathcal{A}} + \kappa \frac{\gamma_r}{1 - \rho_r \beta} \rho_r^t r + \sum_{j=0}^{\infty} \beta^j E_t \mu_{t+j},$$

with

$$\mathcal{A} = -\frac{\alpha_r}{(1 - \rho_r \alpha_y)(1 - \rho_r \beta)} \rho_r^t r + \sum_{j=0}^{\infty} \beta^j \left( \sum_{k=0}^{\infty} \alpha_y^k E_t d_{t+j+k} \right).$$

We therefore obtain the  $(\pi_t, r)$  equilibrium locus

$$\begin{aligned} \pi_t = & \kappa \frac{\rho_r^t}{1 - \rho_r \beta} \left( (\gamma_r - \alpha_r \gamma_y) - \frac{\rho_r}{1 - \rho_r \alpha_y} \alpha_y \alpha_r \gamma_y \right) r \\ & + \kappa \gamma_y \sum_{j=0}^{\infty} \beta^j \left( E_t \sum_{k=0}^{\infty} \alpha_y^k E_{t+j} d_{t+j+k} \right) + \sum_{j=0}^{\infty} \beta^j E_t \mu_{t+j}. \end{aligned}$$

## B The Patman Condition in Some Standard Cost Channel Models

Here we explore two simple models that are typical references for New Keynesian models with a cost channel, namely Ravenna and Walsh [2006] and the no-capital version of Rabanal [2007] proposed by Surico [2008]. Note that in those models, it is the nominal interest rate that enters the marginal cost and not the real interest rate. However, the T.E. Patman

condition is computed holding expectations fixed, so that real and nominal rates move as one. Also note that the T.E. Patman condition is a necessary condition for inflation to increase following a rise in the interest rate when expectations are not held constant. If the T.E. Patman condition does not hold, then the G.E. Patman condition will not hold either, so that inflation will never respond positively to monetary tightening when the monetary shock is persistent.

## B.1 Ravenna and Walsh [2006]

Firms must borrow the wage bill at the nominal interest rate. Preferences are  $\frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{N^{1+\eta}}{1+\eta}$ . Euler equation and Phillips curve are given by:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}),$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(\sigma + \eta)y_t + \kappa i_t.$$

The T.E. Patman condition writes  $\frac{\gamma_r}{\gamma_y} > \alpha_r$ , with  $\gamma_r = \kappa$ ,  $\gamma_y = \kappa(\sigma + \eta)$  and  $\alpha_r = \frac{1}{\sigma}$ . T.E. Patman condition implies  $\frac{1}{\sigma + \eta} > \frac{1}{\sigma}$ . It cannot hold as  $\eta \geq 0$ . Therefore, the T.E. Patman condition is never satisfied, and the G.E. Patman condition is not either.

## B.2 Surico [2008]

Here, only a fraction  $\theta$  of firms need to borrow the wage bill in advance. Euler equation and Phillips curve are given by:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}),$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(\sigma + \eta)y_t + \kappa \theta i_t.$$

The T.E. Patman condition writes  $\frac{\gamma_r}{\gamma_y} > \alpha_r$ , with  $\gamma_r = \theta \kappa$ ,  $\gamma_y = \kappa(\sigma + \eta)$  and  $\alpha_r = \frac{1}{\sigma}$ . The T.E. Patman condition implies  $\frac{1}{\sigma + \eta} > \frac{1}{\theta \sigma}$ . A lower bound of the right-hand side is attained at  $\theta = 1$ . In that case, the T.E. Patman condition cannot hold as  $\eta \geq 0$ . This implies that the T.E. Patman condition cannot hold for values of  $\theta$  lower than one. Therefore, the T.E. Patman condition is never satisfied, and the G.E. Patman condition is not either.

# C Model Microfoundations

## C.1 Discounted Euler Equation Specification

The derivation of the discounted Euler equation relies on two sets of assumptions. First, because of asymmetry of information and lack of commitment, individual households will face an upward sloping supply of funds when borrowing. To maintain tractability, we will consider an equilibrium in which agents never default, so that the income and wealth distributions will have a unique mass point. For exposition simplicity, we will derive the main features of the equilibrium in a two-period model and explain why the extension to an infinite horizon is trivial. Second, we will assume a particular timing of income and expenditure flows. Those two assumptions will allow us to derive a discounted Euler equation.

### C.1.1 A simple two-period model with asymmetric information and lack of commitment

We consider a deterministic model with two periods. There are two types of households and a zero-profit risk neutral representative bank that has access to an unlimited supply of funds at cost  $\bar{R}$ . Households receive no endowment in the first period, and  $\omega$  in the second period. The consumption good is the numéraire.

Some households (superscript  $c$ ) have access to commitment and always repay their debt while other households (superscript  $nc$ ) cannot commit to repay. Type is not observable. Because of this, the risk neutral bank will want to charge a risk premium on its loans. More specifically, the bank proposes to the households a schedule  $R(d)$  that is increasing in the level of debt  $d$ .

Preferences over consumption are given by  $u(c_1) + \beta u(c_2)$ . Households also bear an additively separable utility cost of defaulting  $\psi(d)$  which is an increasing and convex function of the amount of defaulted debt.

When households borrow (as they will always do under regularity conditions on preferences  $u$ ), they will consume  $(c_1, c_2)$  and their debt is  $d = c_1$ . Committed type households maximize their utility under the budget constraint  $c_2 = \omega - R(c_1)c_1$ . Their optimal choice for  $c_1$  satisfies

$$u'(c_1^c) = \beta \left( R(c_1^c) + R'(c_1^c)c_1^c \right) u'(\omega - R(c_1^c)c_1^c). \quad (\text{C.1})$$

The non-committed type households optimally decide whether they will default (superscript  $d$ ) or not (superscript  $nd$ ) in period 2, and this choice can be made in period 1 because there is no uncertainty in this example. If they repay (no default), non-committed households behave as the committed type, so that

$$c_1^{nc,nd} = c_1^c.$$

If they default, then they will borrow (in period 1) as much as they need to equalise marginal utility of consumption with marginal psychological cost of default. The optimal choice will then satisfy:

$$u'(c_1^{nc,d}) = \psi'(c_1^{nc,d}), \quad (\text{C.2})$$

while  $c_2^{nc,d} = \omega$ .

The optimal decision to default or not depends on the direction of the following inequality:

$$\underbrace{u(c_1^c) + \beta u(\omega - R(c_1^c)c_1^c)}_{\text{if no default}} \gtrless \underbrace{u(c_1^{nc-d}) + \beta u(\omega) - \psi(c_1^{nc,d})}_{\text{if default}}.$$

For given  $u(\cdot)$ ,  $\beta$  and  $\omega$ , there is always a psychological cost function  $\psi(\cdot)$  such that household of the non-committed type choose to behave as committed households. In this case, we have a pooling equilibrium in which all households behave the same and in which there are no defaults. From the bank's zero-profit condition, we should have  $R(c_1^c) = \bar{R}$  (as there is no default). This condition is the only restriction put on the  $R(\cdot)$  schedule, so that any off-equilibrium belief  $R'(\cdot) > 0$  is consistent with a no default pooling equilibrium.

**Extension to an infinite horizon model :** If we assume that past actions (default or not) are not observable, the logic of the two-period model still holds in a standard infinite horizon model. With asymmetric information on the household types (access or not to commitment), one can sustain an equilibrium with no default with the following properties: (i) households always make the same consumption and saving choices (no observed heterogeneity), (ii) there is no risk premium on the interest rate in equilibrium and (iii) households consistently face an upward sloping interest schedule  $R(b)$ . The interest of this modelling is the absence of observed heterogeneity that allows for a simple solving of the model.

### C.1.2 Household's problem with upward sloping interest schedule.

There is a measure one of identical households indexed by  $i$ . Each household chooses a consumption stream and labor supply to maximize discounted utility  $E_0 \sum_{t=0}^{\infty} \beta^t \zeta_{t-1} (U(C_{it}) - \nu(L_{it}))$ , where  $\zeta$  is a discount shifter.

We split each period into a morning and an afternoon. There is no difference in information between morning and afternoon. In the morning, household  $i$  must order and pay consumption expenditures  $P_t C_{it}$  and cannot use previous savings to do so. Household  $i$  must therefore borrow  $D_{it+1}^M = P_t C_{it}$  units of money (say dollars) at a nominal interest rate  $i_{it}^H$  that, for the reasons mentioned above, will depend on her total borrowing in period  $t$  (hence the subscript  $i$ ). In the afternoon, household  $i$  can borrow  $D_{it+1}^A$  for intertemporal smoothing motives, receives labor income  $W_t L_{it}$  and profits from intermediate firms  $\Omega_{it}$  and must repay principal and interest on the total debt inherited from the previous period  $(1 + i_{it-1}^H)(D_{it}^M + D_{it}^A)$ . The morning budget constraint is therefore given by:

$$D_{it+1}^M = P_t C_{it},$$

and the afternoon budget constraint writes:

$$D_{it+1}^A + W_t L_{it} + \Omega_{it} = (1 + i_{it-1}^H)(D_{it}^M + D_{it}^A).$$

Putting these together, we obtain the following budget constraint for period  $t$ :

$$D_{it+1}^A + W_t L_{it} + \Omega_{it} = (1 + i_{it-1}^H)D_{it}^A + (1 + i_{it-1}^H)P_{t-1}C_{it-1}.$$

As there is no new information between morning and afternoon, the interest rate  $i_{it}^H$  faced by household  $i$  is a function of the total real net debt subscribed in period  $t$ . We write it as a premium over the risk-free nominal rate:

$$1 + i_{it}^H = (1 + i_t) \left( 1 + \rho \left( \frac{D_{it+1}^M + D_{it+1}^A}{P_t} \right) \right) = (1 + i_t) \left( 1 + \rho \left( C_{it} + \frac{D_{it+1}^A}{P_t} \right) \right),$$

with  $\rho > 0$ ,  $\rho' > 0$  and  $\rho'' > 0$ .

The decision problem of household  $i$  is therefore given by:

$$\max \sum_{t=0}^{\infty} \beta^t \zeta_{t-1} E_0 [U(C_{it}) - \nu(L_{it})],$$

$$\begin{aligned} \text{s.t. } D_{it+1}^A + W_t L_{it} + \Omega_{it} &= (1 + i_{it-1}^H) D_{it}^A + (1 + i_{it-1}^H) P_t C_{it}, \\ 1 + i_{it}^H &= (1 + i_t) \left( 1 + \rho \left( C_{it} + \frac{D_{it+1}^A}{P_t} \right) \right). \end{aligned}$$

The first order conditions (evaluated at the symmetric equilibrium in which  $D_{it+1}^A = 0 \forall i$ ) associated with this problem are:

$$\begin{aligned} U'(C_t) &= \beta \frac{\zeta_t}{\zeta_{t-1}} E_t \left[ U'(C_{t+1}) (1 + i_t) (1 + \rho(C_t) + C_t \rho'(C_t)) \frac{P_t}{P_{t+1}} \right], \\ \frac{\nu'(L_t)}{U'(C_t)} &= \frac{W_t}{P_t}. \end{aligned}$$

Assuming that consumption utility is CRRA ( $U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$ ), the Euler equation can be log-linearized to obtain (omitting constant terms and using  $C_t = Y_t$ ):

$$y_t = \alpha_y E_t[y_{t+1}] - \alpha_r (i_t - E_t[\pi_{t+1}]) + d_t,$$

where  $y$  is the log of  $Y$  and with  $\alpha_y = \frac{\sigma}{\sigma + \varepsilon_\rho} \in ]0, 1[$ ,  $\alpha_r = \frac{1}{\sigma + \varepsilon_\rho} > 0$ ,  $\varepsilon_\rho = \frac{C(2\rho' + C\rho'')}{\rho + C\rho'} > 0$  and  $d_t = -\frac{1}{\sigma + \varepsilon_\rho} (\log \zeta_t - \log \zeta_{t-1})$ . This give us equation (EE) in the main text.

### C.1.3 Adding habit persistence

Assume that utility is  $\frac{(C_{it} - \gamma C_{it-1})^{1-\sigma}}{1-\sigma} - \nu(L_{it})$ . Note that we assume external habit. The first order conditions (evaluated at the symmetric equilibrium in which  $D_{it+1}^A = 0 \forall i$ ) become:

$$(C_t - \gamma C_{t-1})^{-\sigma} = \beta \frac{\zeta_t}{\zeta_{t-1}} E_t \left[ (C_{t+1} - \gamma C_t)^{-\sigma} (1 + i_t) (1 + \rho(C_t) + C_t \rho'(C_t)) \frac{P_t}{P_{t+1}} \right] \quad (\text{C.3})$$

$$\frac{\nu'(L_t)}{(C_t - \gamma C_{t-1})^{-\sigma}} = \frac{W_t}{P_t}, \quad (\text{C.4})$$

and the log-linearized Euler equation writes:

$$y_t = \alpha_{y,f} E_t[y_{t+1}] + \alpha_{y,b} y_{t-1} - \alpha_r (i_t - E_t[\pi_{t+1}]) + d_t.$$

## C.2 Derivation of the Augmented New Keynesian Phillips Curve

The introduction of the real interest rate in the marginal cost of firms is not new (Christiano, Eichenbaum, and Evans [2005], Ravenna and Walsh [2006]). However, the twists we introduce here allow for arbitrary elasticities of the marginal cost with respect to respectively the real wage and the real interest rate. In what follows, we present the derivation of the marginal cost, that can be done considering the static optimal choice of inputs.

### C.2.1 Production

Each monopolist produces a differentiated good using a basic input as the only factor of production, and according to a one to one technology. The marginal cost of production will

therefore be the price of that basic input. It is assumed that the basic input is produced by a representative competitive firm. The representative firm produces basic input  $Q_t$  with labor  $L_t$  and the final good  $M_t$  according to the following Leontief technology:

$$Q_t = \min(a\Theta_t L_t, bM_t).$$

For simplicity of the exposition, we assume that  $\Theta_t$  is constant and normalized to one. The optimal production plan implies  $Q_t = aL_t = bM_t$ , so that the optimal input demands are  $L_t = \frac{Q_t}{a}$  and  $M_t = \frac{Q_t}{b}$ . Denote by  $\mathcal{C}(Q_t) = W_t L_t + \Phi_t M_t$  the total cost of production, where the exact expression of  $\Phi_t$  will be derived later. Using the optimal input demands, we obtain:

$$\mathcal{C}(Q_t) = \left( \frac{W_t}{a} + \frac{\Phi_t}{b} \right) Q_t,$$

so that marginal cost is

$$\mathcal{C}'(Q_t) = \frac{W_t}{a} + \frac{\Phi_t}{b}.$$

Log-linearizing the above gives the following expression of the real marginal cost, where the variables are now in logs and where constant terms have been omitted:

$$mc_t = \left( \frac{\frac{W}{a}}{\frac{W}{a} + \frac{\Phi}{b}} \right) (w_t - p_t) + \left( \frac{\frac{\Phi}{b}}{\frac{W}{a} + \frac{\Phi}{b}} \right) (\phi_t - p_t).$$

### C.2.2 Derivation of the cost $\Phi_t$

The unit price of the final good that enters the production of basic input is  $P_t$ . We assume that, in the morning of each period, the basic input representative firm must borrow  $D_{t+1}^B$  at the risk-free nominal interest rate  $i_t$  to pay for the input  $M_t$ . In the afternoon, it produces, sells its production, pays wages, repays the debt contracted the previous period  $D_t^B$  and distributes all the profits  $\Omega_t^B$  as dividends. Those profits will be zero in equilibrium. The period  $t$  budget constraint of the firm is therefore:

$$D_{t+1}^B + \tilde{P}_t Q_t = W_t L_t + (1 + i_{t-1}) D_t^B + P_t M_t,$$

with  $D_{t+1}^B = P_t M_t$ . Period  $t$  profit writes:

$$\Omega_t^B = \tilde{P}_t Q_t - W_t L_t - (1 + i_{t-1}) P_{t-1} M_{t-1},$$

where  $\tilde{P}_t$  is the price of the basic input. Assuming that the firm maximizes the expected discounted sum of profits real profits  $\Omega_t^B/P_t$  with discount factor  $\beta$ , and using  $Q_t = aL_t = bM_t$ , we obtain the first order condition:

$$\tilde{P}_t = \left( \frac{1}{a} \frac{W_t}{P_t} + \frac{\beta}{b} E_t \left[ \frac{1 + i_t}{1 + \pi_{t+1}} \right] \right) P_t.$$

Therefore, the real marginal cost of the basic input firm will be given by:

$$mc_t = \frac{1}{a} \frac{W_t}{P_t} + \frac{\beta}{b} E_t \left[ \frac{1 + i_t}{1 + \pi_{t+1}} \right].$$

Note that  $\frac{1}{a} \frac{W_t}{P_t}$  can be expressed as  $\frac{b-1}{b} \frac{W_t L_t}{P_t(Q_t - Y_t)}$ , which is the labor share in total value added, so that a direct measure of the real marginal cost is

$$mc_t = \frac{b-1}{b} \times \text{labour share}_t + \frac{\beta}{b} E_t \left[ \frac{1+i_t}{1+\pi_{t+1}} \right].$$

The price of the basic input  $\tilde{P}_t$  is equal to the nominal marginal cost of the basic input firm and is also equal to the marginal cost of the intermediate input firm (which is the relevant one for pricing decisions). In logs, the real marginal cost will write (omitting constants):

$$mc_t = \underbrace{\left( \frac{\frac{1}{a} \frac{W}{P}}{\frac{1}{a} \frac{W}{P} + \frac{\beta}{b} \frac{1+i}{1+\pi}} \right)}_{\tilde{\gamma}_y} (w_t - p_t) + \underbrace{\left( \frac{\frac{\beta}{b} \frac{1+i}{1+\pi}}{\frac{1}{a} \frac{W}{P} + \frac{\beta}{b} \frac{1+i}{1+\pi}} \right)}_{\gamma_r} (i_t - E_t[\pi_{t+1}]).$$

### C.2.3 Pricing

As in the standard New Keynesian model, intermediate firms play a Calvo lottery to draw price setting opportunities. Except for the use of the basic input, the modelling is very standard. The optimal household labor supply, that we will derive later, will give us:

$$\frac{\nu'(L_t)}{U'(C_t)} = \frac{W_t}{P_t},$$

which writes in logs, using  $C_t = a \left( \frac{b-1}{b} \right) L_t$  and omitting constant terms:

$$w_t - p_t = \left( \frac{L\nu''(L)}{\nu'(L)} - \frac{CU''(C)}{U'(C)} \right) y_t.$$

As  $C_t = Y_t = aL_t$ , the marginal cost does not depend on the scale of production and is the same for all the intermediate input firms. It is written as

$$mc_t = \underbrace{\tilde{\gamma}_y \left( \frac{L\nu''(L)}{\nu'(L)} - \frac{CU''(C)}{U'(C)} \right)}_{\gamma_y} y_t + \gamma_r (i_t - E_t[\pi_{t+1}]).$$

The rest of the model is standard, and we obtain the New Keynesian Phillips curve:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa mc_t + \mu_t.$$

Plugging in the expression for the real marginal cost, we have:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \left( \gamma_y y_t + \gamma_r (i_t - E_t[\pi_{t+1}]) \right) + \mu_t.$$

This give us equation (PC) in the main text.

### C.2.4 Adding habit persistence

When habit persistence is added, labor supply depends on current and last period consumption (see section C.1.3). The Phillips curve writes :

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \left( \gamma_y y_t + \gamma_{y,b} y_{t-1} + \gamma_r (i_t - E_t[\pi_{t+1}]) \right) + \mu_t.$$

## D Determinacy with Cost Channel and Discounted Euler Equation

Here we show that if the economy is in the Patman regime, there is no need for a Taylor principle (of the type “monetary policy should be reacting aggressively enough to inflation”), but one should satisfy an “anti-Taylor” principle, in the sense that monetary policy should not be reacting too aggressively to inflation. Furthermore, being in the Patman regime is a sufficient condition for determinacy under a nominal or real interest rate peg policy.

It is more convenient to work with a continuous time version of the model, and to assume that the Taylor rule responds only to inflation. Adding the output gap in the Taylor rule and/or assuming discrete time would not change the qualitative results but makes the exposition more clumsy. The model is (assuming away shocks):

$$\begin{aligned} \dot{y}_t &= (1 - \alpha_y)y_t + \alpha_r(i_t - \pi_t) \\ \dot{\pi}_t &= (1 - \beta)\pi_t - \gamma_y y_t - \gamma_r(i_t - \pi_t) \end{aligned}$$

### D.1 Determinacy with a TAYLOR rule

The TAYLOR rule is specified as:

$$i_t = \phi_\pi \pi_t.$$

In that case, the model dynamics is given by

$$\begin{pmatrix} \dot{y}_t \\ \dot{\pi}_t \end{pmatrix} = \underbrace{\begin{bmatrix} 1 - \alpha_y & \alpha_r(\phi_\pi - 1) \\ -\gamma_y & (1 - \beta) - \gamma_r(\phi_\pi - 1) \end{bmatrix}}_A \begin{pmatrix} y_t \\ \pi_t \end{pmatrix}$$

The two eigenvalues of  $A$  solve

$$\lambda^2 - \underbrace{\left( (1 - \alpha_y) + (1 - \beta) - \gamma_r(\phi_\pi - 1) \right)}_{\lambda_1 + \lambda_2} \lambda + \underbrace{\left( \alpha_r \gamma_y (\phi_\pi - 1) + (1 - \alpha_y) \left( (1 - \beta) - \gamma_r(\phi_\pi - 1) \right) \right)}_{\lambda_1 \times \lambda_2} = 0$$

The necessary and sufficient condition condition for determinacy is  $S = \lambda_1 + \lambda_2 > 0$  and  $P = \lambda_1 \times \lambda_2 > 0$ . Let us look as some polar cases for intuition.

#### D.1.1 The case with no Euler discounting ( $\alpha_y = 1$ ) and no cost channel ( $\gamma_r = 0$ )

This is the standard New Keynesian model. In that case,  $S = 1 - \beta > 0$  and  $P = \alpha_r \gamma_y (\phi_\pi - 1) > 0$ , so that we have determinacy, if and only if

$$\phi_\pi > 1.$$

This is the TAYLOR principle for determinacy.



**D.1.2 The case with no Euler discounting ( $\alpha_y = 1$ ) and a cost channel ( $\gamma_r > 0$ )**

In that case,  $S = 1 - \beta - \gamma_r(\phi_\pi - 1) > 0$  if and only if

$$\phi_\pi < 1 + \frac{1 - \beta}{\gamma_r}$$

and  $P = \alpha_r \gamma_y (\phi_\pi - 1) > 0$  if and only if

$$\phi_\pi > 1.$$

Condition for determinacy is now that monetary policy  $\phi_\pi$  should satisfy the TAYLOR principle but not be too aggressive (because of the cost channel).

**D.1.3 The case with Euler discounting ( $\alpha_y < 1$ ) and no cost channel ( $\gamma_r = 0$ )**

In that case,  $S = (1 - \alpha_y) + (1 - \beta) > 0$  and  $P = \alpha_r \gamma_y (\phi_\pi - 1) + (1 - \beta)(1 - \alpha_y) > 0$  if and only if

$$\phi_\pi > 1 - \frac{(1 - \beta)(1 - \alpha_y)}{\alpha_r \gamma_y}.$$

Condition for determinacy with Euler discounting is looser than the TAYLOR principle ( $\phi_\pi$  can be smaller than one).

**D.1.4 The case with Euler discounting ( $\alpha_y < 1$ ) and cost channel ( $\gamma_r > 0$ )**

In the general case, results will depend on whether the economy is in the Patman regime or not. By Patman regime, we mean the case in which, in general equilibrium and for any persistence smaller than one, an increase in the real interest rate increases inflation. That condition, which we derive from Proposition 1, is

$$(1 - \alpha_y)\gamma_r - \alpha_r \gamma_y > 0.$$

Conditions for determinacy are then

- (i) If the economy is in the Patman regime, then there is an anti-Taylor principle. The condition for determinacy is only that the response to inflation should not be too strong:

$$\phi_\pi < 1 + \min \left\{ \frac{(1 - \alpha_y) + (1 - \beta)}{\gamma_r}, \frac{(1 - \alpha_y)(1 - \beta)}{(1 - \alpha_y)\gamma_r - \alpha_r \gamma_y} \right\}$$

- (ii) If the economy is not in the Patman regime, then  $\phi_\pi$  should be neither too small (loose Taylor principle) nor too strong):

$$1 - \frac{(1 - \alpha_y)(1 - \beta)}{\alpha_r \gamma_y - (1 - \alpha_y)\gamma_r} < \phi_\pi < 1 + \frac{(1 - \beta)(1 - \alpha_y)}{\gamma_r}$$

Monetary policy  $\phi_\pi$  can be looser than the TAYLOR principle, the more so when  $\alpha_y$  is small and  $\gamma_r$  is large. But it cannot be too aggressive, in particular when  $\alpha_y$  is small and  $\gamma_r$  is large.

## D.2 Nominal Interest Rate Peg

Policy is in that case (because we have not included shocks)

$$i_t = 0.$$

The model becomes

$$\begin{pmatrix} \dot{y}_t \\ \dot{\pi}_t \end{pmatrix} = \begin{bmatrix} 1 - \alpha_y & -\alpha_r \\ -\gamma_y & (1 - \beta) + \gamma_r \end{bmatrix} \begin{pmatrix} y_t \\ \pi_t \end{pmatrix}$$

Condition for determinacy is then

$$(1 - \alpha_y)(1 - \beta + \gamma_r) > \alpha_r \gamma_y.$$

A nominal interest rate peg will provide determinacy if Euler discounting and cost channel are large enough (meaning  $\alpha_y$  small and  $\gamma_r$  large). This condition is satisfied if the economy is in the Patman regime.

## D.3 Real Interest Rate Peg

Policy is in that case (because we have not included shocks)

$$i_t = \pi_t.$$

The model becomes

$$\begin{pmatrix} \dot{y}_t \\ \dot{\pi}_t \end{pmatrix} = \begin{bmatrix} 1 - \alpha_y & 0 \\ -\gamma_y & (1 - \beta) \end{bmatrix} \begin{pmatrix} y_t \\ \pi_t \end{pmatrix}$$

The two eigenvalues of  $A$  are  $(1 - \alpha_y)$  and  $(1 - \beta)$ . Both are always strictly positive as long as there is some Euler discounting. Therefore, the equilibrium is always determinate under a real rate peg. This condition is satisfied if the economy is in the Patman regime.

## E Equivalence of Different Forms of Policy Rules

We show below that two classes of policy rules can replicate the same allocations in a simple New Keynesian model without habit persistence and hybrid curve. The result can be easily extended to a model with these two features. Those two classes are a standard Taylor rule that satisfies the Taylor principle:

$$i_t = \phi_y y_t + \phi_\pi \pi_t + \nu_t, \tag{E.1}$$

and a real interest rate rule:

$$i_t = E_t[\pi_{t+1}] + \psi_d d_t + \psi_\mu \mu_t + \psi_\nu \nu_t. \tag{E.2}$$

We prove the equivalence result in the fully forward New Keynesian model, but the proof can be easily extended to the model with a backward component.

The Euler equation and Phillips curve of the simple sticky prices model can be written as:<sup>35</sup>

$$X_t = AE_t[X_{t+1}] + B(i_t - E_t[X_{t+1}]) + CS_t, \quad (\text{E.3})$$

where  $X_t = (y_t, \pi_t)'$ ,  $S_t = (d_t, \mu_t, \nu_t)'$  and each shock  $x \in \{d, \mu, \nu\}$  follows  $x_t = \rho_x x_{t-1} + \varepsilon_{xt}$ . Denote  $R$  the diagonal matrix with the persistence parameters  $\rho_x$  on the diagonal, with  $|\rho_x| < 1$ . Let's also define  $K = [0 \ 1]$  so that  $E_t[\pi_{t+1}] = KE_t[X_{t+1}]$ .

**Solution under a Taylor rule (E.1):** Note that policy rule (E.1) can be written:

$$i_t = \Phi X_t + JS_t \quad (\text{E.4})$$

with  $\Phi = (\phi_y, \phi_\pi)$  and  $J = [0 \ 0 \ 1]$ . Plugging (E.4) in (E.3), we obtain:

$$X_t = \underbrace{(I - B\Phi)^{-1}(A - BK)}_{\mathcal{A}} E_t[X_{t+1}] + \underbrace{(I - B\Phi)^{-1}(BJ + C)}_{\mathcal{B}} S_t \quad (\text{E.5})$$

We assume that the standard Taylor rule is restricted to give equilibrium determinacy, so that the eigenvalues of  $\mathcal{A}$  are inside the unit disk.

Solving forward, we obtain :

$$X_t = \underbrace{\left( \sum_{i=0}^{\infty} \mathcal{A}^i \mathcal{B} R^i \right)}_{F(\Phi)} S_t.$$

Under the assumption that the equilibrium is determinate,  $\sum_{i=0}^{\infty} \mathcal{A}^i \mathcal{B} R^i$  converges and  $F(\Phi)$  is well defined.

**Solution under the real interest rule (E.2):** The policy rule (E.2) can be written:

$$i_t - E_t[\pi_{t+1}] = \underbrace{[\psi_d \ \psi_\mu \ \psi_\nu]}_{\Psi} S_t. \quad (\text{E.6})$$

Plugging (E.6) in (E.3), we obtain:

$$X_t = AE_t[X_{t+1}] + \underbrace{(B\Psi + C)}_{\widehat{\mathcal{B}}} S_t. \quad (\text{E.7})$$

Solving forward, we obtain:

$$X_t = \underbrace{\left( \sum_{i=0}^{\infty} A^i \widehat{\mathcal{B}} R^i \right)}_{\widehat{F}(\Psi)} S_t,$$

with  $\Psi = (\psi_d, \psi_\mu, \psi_\nu)$ .  $\Psi$  is uniquely defined given that  $A$  has its eigenvalues inside the unit disk as long as  $|\alpha_y| < 1$ .

---

<sup>35</sup>This does not cover the case where  $\alpha_y$  is exactly 1. We can easily generalize the following analysis for this case.

**Equivalence:** Policy rules (E.1) and (E.2), which are respectively characterized by the parameters  $\Phi$  and  $\Psi$ , will give similar allocations if:

$$F(\Phi) = \widehat{F}(\Psi).$$

Given a standard Taylor rule with parameters  $\Phi$  that guarantees determinacy, the mapping  $\widehat{F}$  is typically invertible. One can recover the equivalent real interest rule with parameters  $\Psi$ , that will be given by  $\Psi = \widehat{F}^{-1}(F(\Phi))$ .

## F Data Definition and Sources

All series are final-vintage data.

**Inflation** : Headline CPI: Consumer Price Index for All Urban Consumers: All Items in U.S. City Average, Percent Change, Quarterly, Seasonally Adjusted, obtained from the FRED database, (CPIAUCSL\_PCH). Sample is 1947Q1–2017Q3.

**Inflation** : Consumer Price Index Retroactive Series, obtained from the BLS, U.S. city average, All items less food and energy, Monthly, Not Seasonally Adjusted, (R-CPI-U-RS). Sample is 1978M1–2020M12

**Domestic Producer Prices Index** : Manufacturing for the United States, Change from Year Ago, Index 2015=100, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (USAPPDMQINMEI\_CH1). Sample is 1961Q1–2021Q1.

**Expected Inflation** : Expected Change in Price During the Next Year, obtained from the Surveys of Consumers, University of Michigan. Transformed into annualized quarterly expected inflation. Sample is 1960Q1–2017Q4.

**Expected Inflation** : 10-year Expected Inflation, obtained from the Cleveland Fed. Sample is 1982M1–2021M9.

**Nominal interest rate** : Effective Federal Funds Rate, Percent, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (FEDFUNDS). Sample is 1954Q3–2017Q3.

**Nominal interest rate** : 3-Month Commercial Paper Rate, Percent, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (CP3M). Sample is 1971Q1–1997Q3.

**Nominal interest rate** : 3-Month AA Financial Commercial Paper Rate, Percent, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (CPF3M). Sample is 1907Q1–2021Q2.

**Gross output** : all industries, Millions of dollars, Annual, obtained from the BEA, Table TG0105-A. Sample is 1997–2020.

**Intermediate Inputs** : all industries, Millions of dollars, Annual, obtained from the BEA, Table TII105-A. Sample is 1997–2020.

**Labour share** : Nonfarm Business Sector, Index 2012=100, Quarterly, Seasonally Adjusted, obtained from the FRED database, (PR85006173). Sample is 1947Q1–2021Q1.

**Unemployment** : Civilian Unemployment Rate, Percent, Quarterly, Seasonally Adjusted, obtained from the FRED database, (UNRATE). Sample is 1948Q1–2017Q3.

**Unemployment** : Noncyclical Rate of Unemployment, Percent, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (NROU). Sample is 1949Q1–2017Q3.

**Unemployment gap** : constructed as UNRATE - NROU.

**Monetary Shocks** : obtained from Wieland and Yang [2020] who have followed the method in Romer and Romer [2004] on an extended sample. Sample is 1969Q1–2007Q4.

**International nominal interest rates** : The measure of the nominal interest rate is either the “Immediate interest rates, Call Money, Interbank Rate” or the “Short-term interest rates” depending on availability . Data are taken from the Oecd MEI database.

## G Transforming Year-to-Year Inflation Expectations into Quarter-to-Quarter Ones

In the Michigan Survey of Consumers, every month a representative sample of consumers are asked the following question: “*By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?*” The answer to this question is then the one-year-ahead inflation expectation  $E_t\pi_{t+4,t}$ . To keep consistency with the quarter-to-quarter inflation we use in the estimation, we rescale the one-year-ahead expected inflation in the following way.<sup>36</sup>

We first assume that realized quarter-to-quarter inflation follows an AR(1) process with persistence  $\rho_\pi$ :

$$\pi_{t+1,t} = \rho_\pi \pi_{t,t-1} + \epsilon_t \quad (\text{G.1})$$

Consumers may or may not have the correct belief on  $\rho_\pi$ . We assume they believe that persistence is  $\tilde{\rho}$ , so that the perceived law of motion of inflation is

$$\pi_{t+1,t} = \tilde{\rho} \pi_{t,t-1} + \epsilon_t \quad (\text{G.2})$$

Consumers observe a noisy signal on inflation:  $s_t = \pi_{t,t-1} + \eta_t$  where  $\eta_t$  is of mean zero, i.i.d., orthogonal to  $\epsilon_t$  and independent across time. Consumers will form quarter-to-quarter inflation expectation, denoted by  $E_t\pi_{t+1,t}$ , using a Kalman filter:

$$E_t\pi_{t+1,t} = \tilde{\rho} E_t\pi_{t,t-1} = \tilde{\rho}(1 - K)E_{t-1}\pi_{t,t-1} + \tilde{\rho}K\pi_{t,t-1} + \tilde{\rho}K\eta_t \quad (\text{G.3})$$

where  $K$  is the Kalman gain.

---

<sup>36</sup>For details of this approach extended to multi-variable joint learning environment, see Hou [2020].

Table G.1: Estimation of Equation (G.5)

OLS for:	$E_t\pi_{t+4,t} = \psi_1 E_{t-1}\pi_{t+3,t-1} + \psi_2\pi_{t,t-1} + \psi_2\eta_t$	
Sample:	1969-2007	1978-2007
$\psi_1$	0.45 (0.104)	0.64 (0.046)
$\psi_2$	1.53 (0.277)	1.02 (0.138)
Implied persistence:		
$\tilde{\rho}$	0.89	0.93

Notes: The constant term is omitted from the table. Newey-West standard errors reported in brackets. Measure of inflation in use is Headline CPI.

We do observe one-year-ahead expected inflation:

$$E_t\pi_{t+4,t} \equiv E_t(\pi_{t+4,t+3} + \pi_{t+3,t+2} + \pi_{t+2,t+1} + \pi_{t+1,t})$$

Using the perceived law of motion (G.2):

$$\begin{aligned} E_t\pi_{t+4,t} &= (1 + \tilde{\rho} + \tilde{\rho}^2 + \tilde{\rho}^3)E_t\pi_{t+1,t} \\ &= (1 + \tilde{\rho} + \tilde{\rho}^2 + \tilde{\rho}^3)(\tilde{\rho}(1 - K)E_{t-1}\pi_{t,t-1} + \tilde{\rho}K\pi_{t,t-1} + \tilde{\rho}K\eta_t) \end{aligned} \quad (\text{G.4})$$

We use the  $t - 1$  version of (G.4) and plug it in the above equation to obtain:

$$\begin{aligned} E_t\pi_{t+4,t} &= \underbrace{\tilde{\rho}(1 - K)}_{\psi_1} E_{t-1}\pi_{t+3,t-1} + \underbrace{(1 + \tilde{\rho} + \tilde{\rho}^2 + \tilde{\rho}^3)\tilde{\rho}K}_{\psi_2} \pi_{t,t-1} \\ &\quad + (1 + \tilde{\rho} + \tilde{\rho}^2 + \tilde{\rho}^3)\tilde{\rho}K\eta_t \end{aligned} \quad (\text{G.5})$$

We can estimate equation (G.5) with OLS because  $\eta_t$  is the i.i.d noise orthogonal to inflation. We need to use quarter-to-quarter (not annualized) inflation for  $\pi_{t,t-1}$  and year-ahead expected inflation and its lag from the Michigan Survey of Consumers. We consider Headline CPI as proxy for  $\pi_{t,t-1}$  here, but the implied estimates for  $\tilde{\rho}$  are very close to those obtained using Core CPI. We first use sample from 1969-2007 to guarantee it lines up with the sample in Table 1 and Table 2, and we use sample from 1978-2007 for Table 3.

Given the estimate on the perceived persistence of inflation, the quarter-to-quarter expected inflation is implied by equation (G.4):

$$E_t\pi_{t+1,t} = \frac{1}{1 + \tilde{\rho} + \tilde{\rho}^2 + \tilde{\rho}^3} E_t\pi_{t+4,t} \quad (\text{G.6})$$

## H Estimating the Phillips Curve Using Year-to-Year Inflation

We start by deriving a version of Equation (8) in the main text using four-quarter inflation, that we denote  $\pi_{t,t-4}$ . For periods  $t, t-1, t-2$  and  $t-3$ , Equation (8) writes

$$\begin{aligned}\pi_{t-j,t-j-1} &= \beta E_{t-j} \pi_{t-j+1,t-j} + \gamma_y x_{t-j} + \gamma_r (i_{t-j} - E_{t-j} \pi_{t-j+1,t-j}) + \mu_{t-j} \\ &= (\beta - \gamma_r) E_{t-j} \pi_{t-j+1,t-j} + \gamma_y x_{t-j} + \gamma_r i_{t-j} + \mu_{t-j} \quad \forall j = 0, 1, 2, 3.\end{aligned}\quad (\text{H.1})$$

Summing the above equation up for all  $j = 0, 1, 2, 3$ , we obtain

$$\begin{aligned}\pi_{t,t-4} &= \pi_{t,t-1} + \pi_{t-1,t-2} + \pi_{t-2,t-3} + \pi_{t-3,t-4} \\ &= (\beta - \gamma_r) (E_t \pi_{t+1,t} + E_{t-1} \pi_{t,t-1} + E_{t-2} \pi_{t-1,t-2} + E_{t-3} \pi_{t-2,t-3}) \\ &\quad + \gamma_y (x_t + x_{t-1} + x_{t-2} + x_{t-3}) + \gamma_r (i_t + i_{t-1} + i_{t-2} + i_{t-3}) \\ &\quad + \mu_t + \mu_{t-1} + \mu_{t-2} + \mu_{t-3}.\end{aligned}\quad (\text{H.2})$$

Taking expectation at time  $t-3$  and applying the law of iterated expectation, we obtain

$$\begin{aligned}E_{t-3}(\pi_{t,t-4}) &= (\beta - \gamma_r) E_{t-3} \pi_{t+1,t-3} + \gamma_y E_{t-3} (x_t + x_{t-1} + x_{t-2} + x_{t-3}) \\ &\quad + \gamma_r E_{t-3} (i_t + i_{t-1} + i_{t-2} + i_{t-3}) + \mu_{t-3}\end{aligned}\quad (\text{H.3})$$

Notice that  $\pi_{t,t-4}$  contains information (about shocks) from  $t-3$  up to  $t$ . Adding  $\pi_{t,t-4}$  and subtracting  $E_{t-3}(\pi_{t,t-4})$  from both sides, we obtain

$$\begin{aligned}\pi_{t,t-4} &= (\beta - \gamma_r) E_{t-3} \pi_{t+1,t-3} + \gamma_y E_{t-3} (x_t + x_{t-1} + x_{t-2} + x_{t-3}) \\ &\quad + \gamma_r E_{t-3} (i_t + i_{t-1} + i_{t-2} + i_{t-3}) + \mu_{t-3} + \underbrace{(\pi_{t,t-4} - E_{t-3} \pi_{t,t-4})}_{\epsilon_{t,t-3}}\end{aligned}\quad (\text{H.4})$$

In the above equation, the term  $\epsilon_{t,t-3}$  contains shocks realized after  $t-3$ , including the monetary shocks. Denote  $I_{t,t-3} = i_t + i_{t-1} + i_{t-2} + i_{t-3}$  and  $X_{t,t-3} = x_t + x_{t-1} + x_{t-2} + x_{t-3}$  to simplify notations. We add and subtract  $I_{t,t-3}$  and  $X_{t,t-3}$  to the right hand side of (H.4) to obtain:

$$\begin{aligned}\pi_{t,t-4} &= \beta E_{t-3} \pi_{t+1,t-3} + \gamma_y X_{t,t-3} + \gamma_r (I_{t,t-3} - E_{t-3} \pi_{t+1,t-3}) \\ &\quad + \underbrace{\mu_{t-3} + \epsilon_{t,t-3} - \gamma_y (X_{t,t-3} - E_{t-3} X_{t,t-3}) - \gamma_r (I_{t,t-3} - E_{t-3} I_{t,t-3})}_{\omega_{t,t-3}}\end{aligned}\quad (\text{H.5})$$

Now notice the error term  $\omega_{t,t-3}$  include time  $t-3$  cost-push shock  $\mu_{t-3}$ , and any shocks happening from time  $t-3$  to  $t$ . To estimate  $\beta$ ,  $\gamma_y$  and  $\gamma_r$ , we need to instrument with monetary shocks at time  $t-3$  and in earlier periods. Monetary policy shocks at  $t-3$  and earlier are indeed valid instruments because they are orthogonal to cost-push shocks at  $t-3$  and to any realized shocks between  $t-3$  and  $t$ .

# I Estimating an Hybrid Phillips Curve

Table I.1 shows estimates of a “hybrid” version of the Phillips curve of the type:

$$\pi_t = \beta_f \pi_{t+1}^e + \beta_b \pi_{t-1} + \gamma_y x_t + \gamma_r (i_t - \pi_{t+1}^e) + \mu_t, \quad (\text{I.1})$$

We find again an insignificant slope  $\gamma_y$  and a positive and significant at 1% cost channel parameter  $\gamma_r$ .

Table I.1: Estimation of the Hybrid Phillips Curve

$\pi$	Headline CPI		Core R-CPI	
	(1)	(2)	(3)	(4)
$\beta_f$	0.56 (0.095)	0.67 (0.103)	0.54 (0.067)	0.46 (0.042)
$\beta_b$	0.49 (0.058)	0.44 (0.075)	0.46 (0.067)	0.54 (0.041)
$\gamma_y$	0.04 (0.042)	-0.04 (0.060)	0.01 (0.005)	0.0138 <sup>†</sup> (-)
$\gamma_r$		0.13 (0.048)	0.03 (0.009)	0.02 (0.008)
Observations	150	150	118	118
J Test (jp)	3.585 (0.981)	4.765 (0.906)	10.069 (0.434)	8.123 (0.702)
Weak ID Test	1.495	1.473	7.237	61.945

*Notes: All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. The measure of inflation is BLS “Consumer Price Index retroactive series using current methods for all items less food and energy”, the measure of market tightness is the U.S. Congressional Budget Office unemployment gap. We use the Michigan Survey of Consumers to measure inflation expectations is the MSC columns, and assume Full Information Rational Expectations in the FIRE ones. Real oil price is added as a control in all the equations and all regressors are instrumented using six lags of Romer and Romer’s [2004] shocks (as extended by Wieland and Yang [2020]) and their squares as instruments. For  $\gamma_y$  and  $\gamma_r$ , estimates highlighted in grey are significant at 1% and not significant at 10% if not highlighted. Sample is 1969Q1–2007Q4.*

# J More Details on the Full Information Estimations

## J.1 The Simple Model

We refer to the simple model as the one without internal propagation mechanisms. It is given by the following three equations:

$$\begin{aligned} y_t &= \alpha_y E_t[y_{t+1}] - \alpha_r (i_t - E_t[\pi_{t+1}]) + d_t, \\ \pi_t &= \beta E_t[\pi_{t+1}] + \kappa (\gamma_y y_t + \gamma_r (i_t - E_t[\pi_{t+1}])) + \mu_t, \\ i_t &= E_t[\pi_{t+1}] + \phi_d d_t + \phi_\mu \mu_t + \nu_t, \end{aligned}$$



where  $d_t \mu_t$  and  $\nu_t$  are independent AR(1) processes. We estimate the model following a classical maximum likelihood method. We set  $\beta$  to .99 and  $\alpha_y$  to .99. Table J.1 presents the baseline estimation of our forward-looking sticky prices model.

Table J.1: Estimated Parameters, Simple Model

$\alpha_r$	0.01	$\gamma_y$	0.02	$\gamma_r$	0.04		
	(0.01)		(0.03)		(0.02)		
$\phi_d$	0.47	$\phi_\mu$	-0.57	$\sigma_d$	0.02	$\sigma_\mu$	0.63
	(0.11)		(0.11)		(0.01)		(0.10)
$\sigma_\nu$	0.26	$\rho_d$	0.94	$\rho_\mu$	0.38	$\rho_\nu$	0.99
	(0.12)		(0.03)		(0.09)		(0.01)
T.E. Patman condition						0.04	(0.02)
G.E. Patman condition						0.03	(0.01)

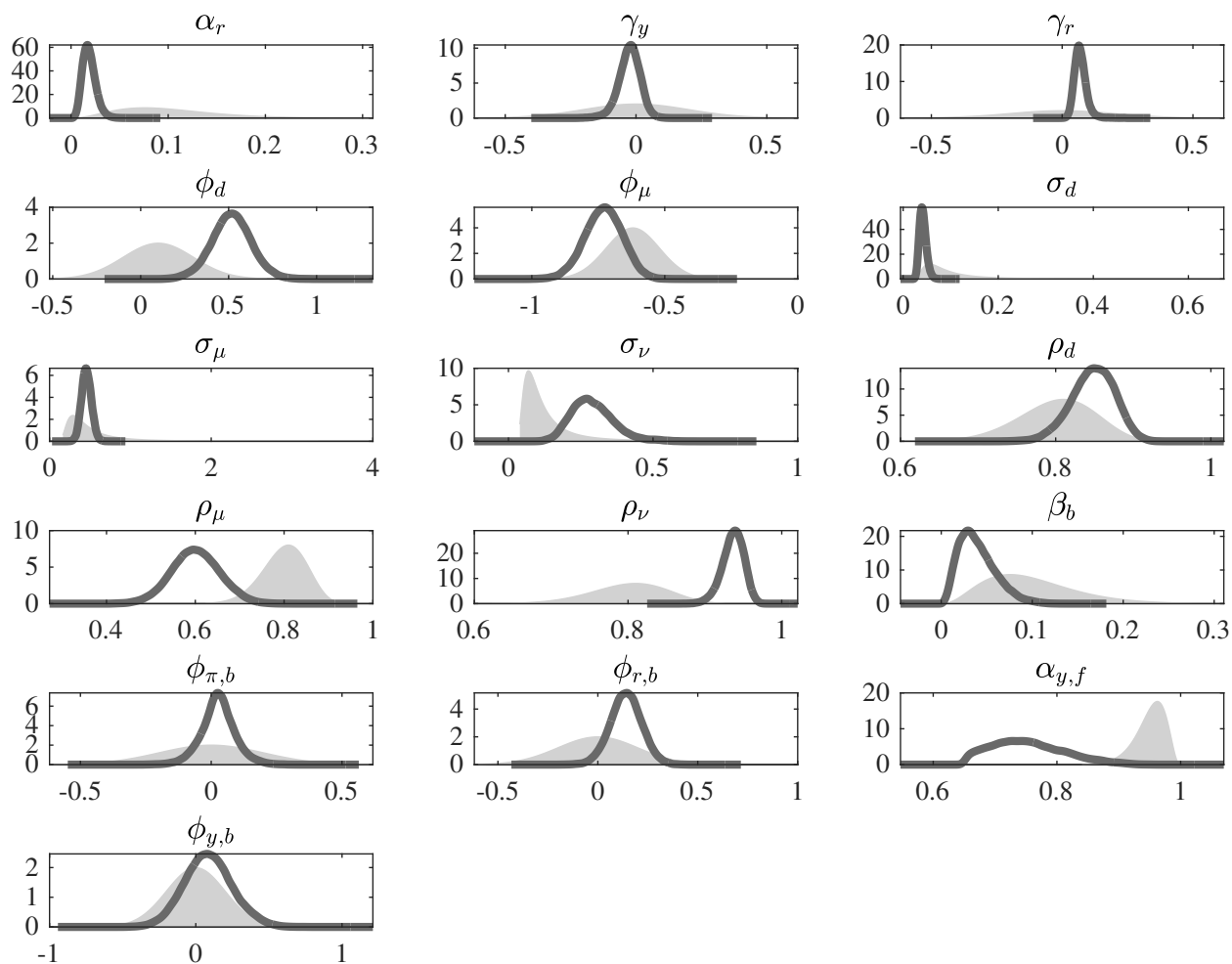
*Notes: this table shows the estimated coefficients of equations (EE), (PC) and (Policy) with unemployment gap, Core CPI Research Series. Parameters  $\beta$  and  $\alpha_y$  are not estimated and set to .99 and .99. Parameter  $\kappa$  is normalized to one. Standard errors are between parenthesis. Sample runs from 1978Q2 to 2007Q4. T.E. Patman condition corresponds to  $\gamma_r - \alpha_r \gamma_y$ , G.E. Patman condition is the impact response of inflation  $\pi$  to a one standard deviation monetary policy shock.*

In this estimation, parameters  $\rho_d$ ,  $\rho_\mu$  and  $\rho_\nu$  are restricted to be in the unit interval. We find that the Phillips curve slope ( $\gamma_y$ ) is not significantly different from zero and smaller than the real interest rate channel ( $\gamma_r$ ), which is positive and significant. Henceforth, the T.E. Patman condition is clearly satisfied. As this is only a necessary condition in this model with persistent shocks, we also compute the G.E. Patman condition, which is given by the impact response of inflation to a one standard deviation monetary policy shock. As it can be checked in Table J.1, that response is positive and significant at a 95% level.

## J.2 The Extended New Keynesian Model in the Baseline Case

We assume relatively dispersed priors. For the parameters that were estimated in the simple model, we center the prior distributions on the previously estimated value. For the new parameters, we center the priors around zero. Figure J.1 displays prior and posterior distributions for all the estimated parameters. One can check that all the parameters are indeed well identified. Table J.2 presents more details about the prior and posterior distributions.

Figure J.1: Prior and Estimated Posterior Distributions for Parameters, Extended New Keynesian Model, Baseline



Notes: this figure plots the prior (the light gray area) and posterior (the dark gray line) distributions for the extended model parameters. The posterior distribution is obtained using the Random Walk Metropolis algorithm, with two chains of 1,000,000 draws each and discarding the first 200,000 draws of each chain.

Table J.2: Detailed Results on Parameters Estimation, Extended New Keynesian Model, Baseline

Parameter	Prior distribution			Max. posterior		Posterior distribution MH			
	Type	$a$	$b$	Mode	s.d. (Hessian)	Mean	Med.	2.5%	97.5%
$\alpha_r$ : Euler coef. on real rate	Beta( $[a,b]$ )	0.10	0.05	0.02	0.04	0.02	0.02	0.01	0.03
$\gamma_y$ : Marginal cost loading to labour market	Normal( $[a,b]$ )	0.00	0.20	-0.03	0.05	-0.03	-0.02	-0.12	0.05
$\gamma_r$ : Marginal cost loading to the real interest rate	Normal( $[a,b]$ )	0.00	0.20	0.06	0.03	0.07	0.07	0.03	0.12
$\phi_d$ : Policy rule reaction to demand shock	Normal( $a,b$ )	0.10	0.20	0.53	0.14	0.51	0.52	0.31	0.76
$\phi_\mu$ : Policy rule reaction to markup shock	Normal( $a,b$ )	-0.62	0.10	-0.72	0.10	-0.73	-0.73	-0.88	-0.59
$\sigma_d$ : Demand shock s.d.	InvGamma( $a,b$ )	0.12	2.00	0.04	0.05	0.04	0.04	0.03	0.06
$\sigma_\mu$ : Markup shock s.d.	InvGamma( $a,b$ )	0.61	2.00	0.44	0.06	0.46	0.45	0.34	0.57
$\sigma_\nu$ : Monetary shock s.d.	InvGamma( $a,b$ )	0.15	2.00	0.23	0.07	0.29	0.28	0.16	0.42
$\rho_d$ : Demand shock persistence	Beta( $[a,b]$ )	0.80	0.05	0.86	0.03	0.85	0.85	0.79	0.90
$\rho_\mu$ : Markup shock persistence	Beta( $[a,b]$ )	0.80	0.05	0.61	0.06	0.60	0.60	0.49	0.71
$\rho_\nu$ : Monetary shock persistence	Beta( $[a,b]$ )	0.80	0.05	0.94	0.02	0.94	0.94	0.91	0.96
$\beta_b$ : Phillips curve inertia	Beta( $a,b$ )	0.10	0.05	0.03	0.02	0.04	0.04	0.01	0.09
$\phi_{\pi,b}$ : Past inflation in policy rule	Normal( $a,b$ )	0.00	0.20	0.07	0.16	0.03	0.03	-0.10	0.14
$\phi_{r,b}$ : Persistence in policy rule	Normal( $a,b$ )	0.00	0.20	0.17	0.08	0.14	0.14	-0.01	0.29
$\alpha_{y,f}$ : Habit persistence	Beta( $a,b$ )	0.95	0.03	0.75	0.03	0.75	0.75	0.67	0.88
$\phi_{y,b}$ : Past gap in policy rule	Normal( $a,b$ )	0.00	0.20	0.01	0.13	0.08	0.08	-0.30	0.38

Notes: this table shows the estimated coefficients of equations (EE), (PC') and (Policy) using unemployment gap, Core R-CPI and the sample is 1978Q2-2007Q4. Parameters  $\beta$  and  $\alpha_y$  are not estimated and set to .99 and .99. Parameter  $\kappa$  is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis algorithm, with two chains of 1,000,000 draws each and discarding the first 200,000 draws of each chains. "Med." is the median of the posterior distribution.

### J.3 Estimating with Phillips Curve (PC) Instead of (PC')

Here we repeat the benchmark estimation but we use Phillips curve (PC)

$$\pi_t = \beta((1 - \beta_b)E_t[\pi_{t+1}] + \beta_b\pi_{t-1}) + \kappa(\gamma_y y_t + \gamma_{y,b}y_{t-1} + \gamma_r(i_t - E_t[\pi_{t+1}])) + \mu_t, \quad (\text{PC})$$

instead of (PC').

$$\pi_t = \beta((1 - \beta_b)E_t[\pi_{t+1}] + \beta_b\pi_{t-1}) + \kappa(\gamma_y y_t + \gamma_r(i_t - E_t[\pi_{t+1}])) + \mu_t. \quad (\text{PC}')$$

Table J.3 shows that all the parameters are close to what was estimated in the benchmark case and the Patman condition is again satisfied. Table J.4 gives more details about the prior and posterior distributions.

Table J.3: Estimated Parameters, Extended New Keynesian Model with Phillips Curve (PC)

$\alpha_r$	0.02	$\gamma_y$	-0.11	$\gamma_r$	0.07	$\phi_d$	0.51
	[ 0.01, 0.03]		[ -0.40, 0.16]		[ 0.03, 0.11]		[ 0.31, 0.74]
$\phi_\mu$	-0.73	$\sigma_d$	0.04	$\sigma_\mu$	0.45	$\sigma_\nu$	0.29
	[ -0.87, -0.60]		[ 0.03, 0.06]		[ 0.34, 0.57]		[ 0.17, 0.44]
$\rho_d$	0.85	$\rho_\mu$	0.60	$\rho_\nu$	0.94	$\beta_b$	0.04
	[ 0.79, 0.90]		[ 0.50, 0.71]		[ 0.91, 0.96]		[ 0.01, 0.08]
$\phi_{\pi,b}$	0.02	$\phi_{r,b}$	0.14	$\alpha_{y,f}$	0.74	$\phi_{y,b}$	0.09
	[ -0.11, 0.16]		[ -0.01, 0.29]		[ 0.66, 0.87]		[ -0.23, 0.42]
$\gamma_{y,b}$	0.09						
	[ -0.17, 0.35]						
P.E. Patman condition			0.07	[ 0.03, 0.12]			
G.E. Patman condition			0.11	[ 0.08, 0.14]			

*Notes: this table shows the posterior mean estimates of the coefficients in equations (EE), (PC) and (Policy) using unemployment gap, Core CPI and the sample is 1978Q2-2007Q4. Parameters  $\beta$  and  $\alpha_y$  are not estimated and set to .99 and .99. Parameter  $\kappa$  is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis algorithm, with two chains of 1,000,000 draws each and discarding the first 200,000 draws of each chain. The numbers between brackets represent the 90% confidence band using the posterior distribution. The P.E. Patman condition corresponds to  $\gamma_r - \alpha_r\gamma_y$ , G.E. Patman condition is the impact response of inflation  $\pi$  to a one standard deviation monetary policy shock.*

Table J.4: Detailed Results on Parameters Estimation, Extended New Keynesian Model with Phillips Curve (PC)

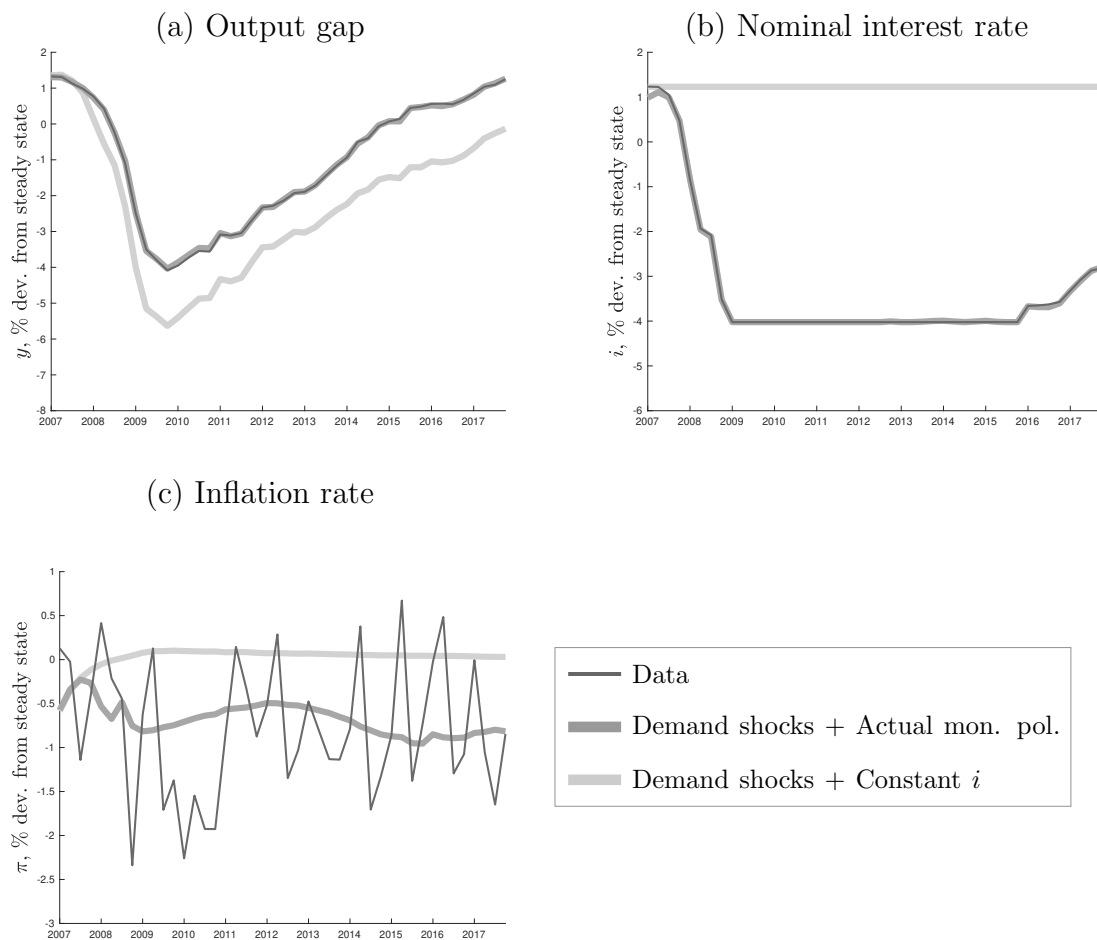
Parameter	Prior distribution			Max. posterior		Posterior distribution MH			
	Type	$a$	$b$	Mode	s.d. (Hessian)	Mean	Med.	2.5%	97.5%
$\alpha_r$ : Euler coef. on real rate	Beta( $[a,b]$ )	0.10	0.05	0.02	0.01	0.02	0.02	0.01	0.03
$\gamma_y$ : Marginal cost loading to labour market	Normal( $[a,b]$ )	0.00	0.20	-0.13	0.11	-0.11	-0.11	-0.40	0.16
$\gamma_r$ : Marginal cost loading to the real interest rate	Normal( $[a,b]$ )	0.00	0.20	0.06	0.02	0.07	0.07	0.03	0.11
$\phi_d$ : Policy rule reaction to demand shock	Normal( $a,b$ )	0.10	0.20	0.53	0.11	0.51	0.51	0.31	0.74
$\phi_\mu$ : Policy rule reaction to markup shock	Normal( $a,b$ )	-0.62	0.10	-0.72	0.07	-0.73	-0.73	-0.87	-0.60
$\sigma_d$ : Demand shock s.d.	InvGamma( $a,b$ )	0.12	2.00	0.04	0.01	0.04	0.04	0.03	0.06
$\sigma_\mu$ : Markup shock s.d	InvGamma( $a,b$ )	0.61	2.00	0.44	0.06	0.45	0.45	0.34	0.57
$\sigma_\nu$ : Monetary shock s.d.	InvGamma( $a,b$ )	0.15	2.00	0.24	0.07	0.29	0.28	0.17	0.44
$\rho_d$ : Demand shock persistence	Beta( $[a,b]$ )	0.80	0.05	0.85	0.03	0.85	0.85	0.79	0.90
$\rho_\mu$ : Markup shock persistence	Beta( $[a,b]$ )	0.80	0.05	0.61	0.05	0.60	0.60	0.50	0.71
$\rho_\nu$ : Monetary shock persistence	Beta( $[a,b]$ )	0.80	0.05	0.94	0.01	0.94	0.94	0.91	0.96
$\beta_b$ : Phillips curve inertia	Beta( $a,b$ )	0.10	0.05	0.03	0.02	0.04	0.04	0.01	0.08
$\phi_{\pi,b}$ : Past inflation in policy rule	Normal( $a,b$ )	0.00	0.20	0.07	0.06	0.02	0.02	-0.11	0.16
$\phi_{r,b}$ : Persistence in policy rule	Normal( $a,b$ )	0.00	0.20	0.18	0.07	0.14	0.14	-0.01	0.29
$\alpha_{y,f}$ : Habit persistence	Beta( $a,b$ )	0.95	0.03	0.72	0.04	0.74	0.74	0.66	0.87
$\phi_{y,b}$ : Past gap in policy rule	Normal( $a,b$ )	0.00	0.20	0.04	0.14	0.09	0.09	-0.23	0.42
$\gamma_{y,b}$ : Marginal cost loading to past labour market	Normal( $a,b$ )	0.00	0.20	0.09	0.11	0.09	0.09	-0.17	0.35

Notes: This table shows the estimated coefficients of equations (EE), (PC) and (Policy) using unemployment gap, Core CPI and the sample 1978Q2–2007Q4. Parameters  $\beta$  and  $\alpha_y$  are not estimated and set to .99 and .99. Parameter  $\kappa$  is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis algorithm, with two chains of 1,000,000 draws each and discarding the first 200,000 draws of each chain. “Med.” is the median of the posterior distribution.

## J.4 Counterfactual Simulations of Section 3.4 with Post-Volcker Estimation

We check here the robustness of our counterfactual simulations by re-estimating the extended model over 1983Q1–2007Q4. Parameters estimates are close to those obtained on a longer sample, although the cost channel is weaker. The mean of the posterior distribution is  $\gamma_y = -0.00$ , with 95% confidence interval  $[-0.02 \ 0.02]$  and  $\gamma_r = 0.02$ , with 95% confidence interval  $[-0.00 \ 0.05]$ . The slope of the Phillips curve is not different from zero and the cost channel is positive and significant. Furthermore, we find that the economy is in the Patman regime. With this new set of parameters, the counterfactual simulations are displayed below.

Figure J.2: Actual and Counterfactual Simulations Of the Extended New Keynesian Model during the ZLB Period with Post-Volcker Estimates



*Notes: This Figure is obtained from simulating the extended model from 2007Q1 on. The model parameters have been estimated in the post-Volcker pre-ZLB period (1983Q1–2007Q4). Then, the model with estimated parameters and a ZLB constraint is used over 2007Q1–2017Q4 to recover structural shocks. These shocks (or a subset) are then used for a simulations with an alternative monetary policy (fixed nominal interest rate at the 2007Q1 level).*