Online Appendix For: "Monopolistic Competition and Efficiency under Firm Heterogeneity and Non-Additive Preferences"

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This Online Appendix contains remaining proofs for the extensions explored in Section IV.

Appendix F: Comparative statics with respect to η

We consider here the comparative statics of the first best and market equilibrium solutions to changes in η under our maintained parameter restrictions which ensure that selection occurs in both solutions.

We begin with the first best solution. Referring to (17)-(19), we see that the first best cost cutoff and multiplier, c_D^{FB} and λ^{FB}, respectively, are determined as the solutions to the following system:

$$F^1(c_D, \lambda; \eta) \equiv \frac{1}{(k+1)f_E} - \frac{\gamma(k+1)(c_M)^k}{\eta} \frac{(\alpha - \lambda c_D)}{\lambda (c_D)^{k+1}} = 0$$

$$F^2(c_D, \lambda; \eta) \equiv (c_D)^{k+2} - \frac{\gamma(k+1)(k+2)f_E(c_M)^k}{\lambda} = 0.$$

In Appendix B, we establish that this system has a unique solution satisfying $c_D^{FB} \in (0, c_M)$ and $\lambda^{FB} > 0$. Applying the implicit function theorem to this system, we obtain that

$$\frac{\partial c_D^{FB}}{\partial \eta} = -\frac{\frac{\partial F^1}{\partial \eta}\frac{\partial F^2}{\partial \lambda}}{|J|} \text{ and } \frac{\partial \lambda^{FB}}{\partial \eta} = \frac{\frac{\partial F^2}{\partial c_D}\frac{\partial F^1}{\partial \eta}}{|J|},$$

where $|J| \equiv \frac{\partial F^1}{\partial c_D}\frac{\partial F^2}{\partial \lambda} - \frac{\partial F^1}{\partial \lambda}\frac{\partial F^2}{\partial c_D}$ and all terms are evaluated at $c_D = c_D^{FB}$ and $\lambda = \lambda^{FB}$.

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Calculations reveal the following relationships:

$$\begin{aligned}
\frac{\partial F^1}{\partial \eta} &= \frac{(k+1)\gamma(c_M)^k}{\eta^2}\frac{(\alpha-\lambda^{FB}c_D^{FB})}{\lambda^{FB}(c_D^{FB})^{k+1}} > 0 \\
\frac{\partial F^1}{\partial c_D} &= \frac{(k+1)\gamma(c_M)^k}{\eta}\frac{1}{\lambda^{FB}}\left\{\frac{\alpha(k+1)-\lambda^{FB}kc_D^{FB}}{(c_D^{FB})^{k+2}}\right\} > 0 \\
\frac{\partial F^1}{\partial \lambda} &= \frac{(k+1)\gamma(c_M)^k}{\eta}\frac{\alpha}{(\lambda^{FB})^2(c_D^{FB})^{k+1}} > 0,
\end{aligned}$$

where the first inequality follows since $F^1(c_D^{FB}, \lambda^{FB}; \eta) = 0$ ensures $\alpha - \lambda^{FB}c_D^{FB} > 0$. This inequality likewise ensures that the second inequality follows, since the bracketed expression is then positive when $k = 1$ and increasing in k. The third inequality is immediate. Calculations similarly reveal that

$$\begin{aligned}
\frac{\partial F^2}{\partial \eta} &= 0 \\
\frac{\partial F^2}{\partial c_D} &= (k+2)(c_D^{FB})^{k+1} > 0 \\
\frac{\partial F^2}{\partial \lambda} &= \frac{(k+1)(k+2)\gamma f_E(c_M)^k}{(\lambda^{FB})^2} > 0,
\end{aligned}$$

where all inequalities follow immediately.

Further calculations reveal that the Jacobian determinant for the equation system may be written as

$$\begin{aligned}
|J| &\equiv \frac{\partial F^1}{\partial c_D}\frac{\partial F^2}{\partial \lambda} - \frac{\partial F^1}{\partial \lambda}\frac{\partial F^2}{\partial c_D} \\
&= -\frac{(k+1)^2(k+2)f_E\gamma^2(c_M)^{2k}}{\eta(\lambda^{FB})^3(c_D^{FB})^{k+2}}\left(\alpha + \lambda^{FB}kc_D^{FB}\right) < 0,
\end{aligned}$$

where the equality follows after using $\lambda^{FB}(c_D^{FB})^{k+2} = \gamma(k+1)(k+2)f_E(c_M)^k$ as ensured by $F^2(c_D^{FB}, \lambda^{FB}; \eta) = 0$.

Using the results derived above, we may now conclude that

$$\frac{\partial c_D^{FB}}{\partial \eta} = -\frac{\frac{\partial F^1}{\partial \eta}\frac{\partial F^2}{\partial \lambda}}{|J|} > 0 \ \text{ and } \ \frac{\partial \lambda^{FB}}{\partial \eta} = \frac{\frac{\partial F^2}{\partial c_D}\frac{\partial F^1}{\partial \eta}}{|J|} < 0,$$

where again all terms are evaluated at $c_D = c_D^{FB}$ and $\lambda = \lambda^{FB}$.

As discussed in the text, since N_E^{FB} is independent of η, it now follows that the number of available varieties $N^{FB} = G(c_D^{FB})N_E^{FB}$ rises as η increases. We consider next the impact of η on the aggregate quantity in the first best solution,

Q^{FB}. We note that

$$
\begin{aligned}
Q^{FB} &= N_E^{FB} \int_0^{c_D^{FB}} \frac{\lambda^{FB}}{\gamma}(c_D^{FB} - c)dG(c) \\
&= \frac{1}{f_E(k+1)} \frac{(c_M)^{-k}(c_D^{FB})^{k+1}\lambda^{FB}}{\gamma(k+1)} = \frac{(k+2)}{(k+1)c_D^{FB}},
\end{aligned}
$$

where the second equality uses the Pareto distribution (1) and the third equality uses $\lambda^{FB} = \gamma(k+1)(k+2)f_E(c_M)^k(c_D^{FB})^{-(k+2)}$ as ensured by $\digamma^2(c_D^{FB}, \lambda^{FB}; \eta) = 0$. Since as established above c_D^{FB} rises as η increases, it follows that Q^{FB} falls as η increases.

Finally, we may evaluate the impact of η on the first best quantity allocation function $q^{FB}(c)$. We know from (20) that $q^{FB}(c) = \frac{\lambda^{FB}}{\gamma}(c_D^{FB} - c), \forall c \in [0, c_D^{FB}]$, where $q^{FB}(c) = 0, \forall c \in [c_D^{FB}, c_M]$. Using the results derived above, we find for $c \in [0, c_D^{FB}]$ that

$$
\frac{\partial q^{FB}(c)}{\partial \eta} = \frac{1}{\gamma} \frac{\frac{\partial \digamma^1}{\partial \eta}}{|J|} \left\{ -\frac{\gamma(k+1)(k+2)f_E(c_M)^k}{\lambda^{FB}} + (c_D^{FB} - c)(k+2)(c_D^{FB})^{k+1} \right\}.
$$

Using $\frac{\partial \digamma^1}{\partial \eta} > 0 > |J|$ when all terms are evaluated at $c_D = c_D^{FB}$ and $\lambda = \lambda^{FB}$, we find that (i) $\frac{\partial q^{FB}(c)}{\partial \eta} > 0$ when $c = c_D^{FB}$, (ii) $\frac{\partial q^{FB}(c)}{\partial \eta} < 0$ when $c = 0$ and (iii) $\frac{\partial^2 q^{FB}(c)}{\partial \eta \partial c} > 0 > \frac{\partial q^{FB}(c)}{\partial c}$ for $c \in (0, c_D^{FB})$. These findings indicate that the quantity allocation function $q^{FB}(c)$ falls less quickly with c as η increases, with a threshold cost $c_T^{FB} \in (0, c_D^{FB})$ existing such that the quantity produced falls (rises) (is unchanged) for $c < (>)(=)\ c_T^{FB}$.

Of the three itemized properties just listed, properties (i) and (iii) are immediate. To confirm property (ii), we observe that

$$
\begin{aligned}
\frac{\partial q^{FB}(c)}{\partial \eta}|_{c=0} &= \frac{1}{\gamma} \frac{\frac{\partial \digamma^1}{\partial \eta}}{|J|} \left\{ -\frac{\gamma(k+1)(k+2)f_E(c_M)^k}{\lambda^{FB}} + (k+2)(c_D^{FB})^{k+2} \right\} \\
&= \frac{\frac{\partial \digamma^1}{\partial \eta}}{|J|} \left\{ \frac{(k+1)^2(k+2)f_E(c_M)^k}{\lambda^{FB}} \right\} < 0,
\end{aligned}
$$

where the second equality follows after using $(c_D^{FB})^{k+2} - \frac{\gamma(k+1)(k+2)f_E(c_M)^k}{\lambda^{FB}}$ as ensured by $\digamma^2(c_D^{FB}, \lambda^{FB}; \eta) = 0$.

The market equilibrium solution can be analyzed similarly. Referring to (43)-(45), we see that the market equilibrium cost cutoff and multiplier, c_D^{mkt} and λ^{mkt}, respectively, are determined as the solutions to the following system:

$$M^1(c_D, \lambda; \eta) \equiv \frac{1}{(k+1)f_E} - \frac{2\gamma(k+1)(c_M)^k}{\eta}\frac{(\alpha - \lambda c_D)}{\lambda(c_D)^{k+1}} = 0$$

$$M^2(c_D, \lambda; \eta) \equiv (c_D)^{k+2} - \frac{2\gamma(k+1)(k+2)f_E(c_M)^k}{\lambda} = 0.$$

In Appendix E, we establish that this system has a unique solution satisfying $c_D^{mkt} \in (0, c_M)$ and $\lambda^{mkt} > 0$. Notice that the equation system for the market equilibrium solution takes exactly the same form as that analyzed above for the first best solution, except that each "γ" in the first best equation system is replaced by "2γ" in the market equilibrium equation system. Since the magnitute of γ plays no role in the comparative statics analysis of the first best equation system, we conclude that the comparative statics results extend in the obvious way to the market equilibrium solution.

Appendix G: Bounded Pareto Distribution

We consider here the extension of the model to a setting in which the distribution function takes the form of a bounded Pareto distribution. We derive the respective equation systems for the first best and market equilibrium solutions and explore the efficiency properties of the latter. In this Online Appendix section, we use the same notation as in the paper for the first best and market equilibrium solution variables. It is understood, however, that variables take different values here, due to the different distribution assumption. We provide additional clarification where needed.

First best solution. — For our analysis of the first best solution, we follow the steps in the paper. As confirmed in Appendix A and reported in Section I of the paper, we can use variational techniques and standard calculus tools to characterize the optimality conditions for our candidate solution in terms of the Euler equation for $q(c)$ as given by (8), the first order condition for N_E as given by (9), the conjectured boundary condition $q(c_D) = 0$ as given by (10), and the resource constraint as given by (6). These conditions are stated for general distribution functions but must be modified to reflect the new support, $[c_L, c_U]$.

As in the paper, we initially impose the assumption that we have found a solution $(q^*(c), N_E^*)$ to the planner's restricted problem such that the associated triplet $(q^*(c), c_D^*, N_E^*)$ and λ^* together satisfy $N_E^* \in (0, 1/f_E), c_D^* \in (c_L, c_M), \lambda^* \geq 0$, the optimality conditions (8)-(10) and the resouce constraint (6), again with the modified support. Following the arguments in the paper, it then follows from (12) that $q^*(c) = \frac{\lambda^*}{\gamma}(c_D^* - c), \forall c \in [c_L, c_D^*]$ and that $\lambda^* > 0$.

To further characterize this solution, we now assume that the distribution function is a uniform distribution over the interval $[c_L, c_U]$, where $c_U > c_L > 0$. Thus, we have that

$$G(c) = \frac{c - c_L}{c_U - c_L} \tag{B1}$$

for $c \in [c_L, c_U]$. The uniform distribution with $c_L > 0$ corresponds to a bounded Pareto distribution with $k = 1$. It is convenient to note that

$$\overline{c}^* \equiv \frac{\int_{c_L}^{c_D^*} c dG(c)}{G(c_D^*)} = \frac{c_D^* + c_L}{2} \text{ and } \widehat{c}^* \equiv \frac{\int_{c_L}^{c_D^*} c^2 dG(c)}{G(c_D^*)} = \frac{(c_D^*)^2 + c_D^* c_L + (c_L)^2}{3}.$$

With modifications for the new support, we now use (12) and the uniform distribution (B1) to rewrite the resource constraint (6) and the Euler equation (8), respectively, as

$$N_E^* \left[\frac{\lambda^*}{\gamma} \frac{(c_D^* - c_L)^2 (2c_L + c_D^*)}{6(c_U - c_L)} + f_E \right] = 1 \tag{B2}$$

$$N_E^* = \frac{2\gamma(c_U - c_L)}{\eta} \frac{(\alpha - \lambda^* c_D^*)}{\lambda^* (c_D^* - c_L)^2}. \tag{B3}$$

Similarly, using (12) and the uniform distribution (B1) along with the rewritten Euler equation (B3), we can rewrite the optimality condition (9) for the number of entrants as

$$c_D^* - c_L = \left[\frac{6 f_E (c_U - c_L)\gamma}{\lambda^*} \right]^{\frac{1}{3}}. \tag{B4}$$

Following the steps taken in the paper, we next substitute (B4) into (B2). Specifically, we isolate $(c_D^* - c_L)^3$ from (B4), multiply the numerator and denominator of the fraction in (B2) by $(c_D^* - c_L)$, and substitute the solution for $(c_D^* - c_L)^3$ from (B4) into (B2). We thereby obtain

$$f_E N_E^* [\frac{c_L + 2c_D^*}{c_D^* - c_L}] = 1. \tag{B5}$$

We note that $(c_D^*, N_E^*, \lambda^*)$ are determined by the equation system (B3), (B4) and (B5) with $q^*(c)$ then given by $q^*(c) = \max\{\frac{\lambda^*}{\gamma}(c_D^* - c), 0\}, \forall c \in [c_L, c_U]$.

As in the paper, the candidate solution $(q^{FB}(c), N_E^{FB})$ and associated multiplier λ^{FB} for the planner's problem can now be constructed from $(c_D^*, N_E^*, \lambda^*)$ and $q^*(c)$ as just determined by using the translation under which $q^{FB}(c) = q^*(c)$ for $c \in [c_L, c_D^*]$ and $q^{FB}(c) = 0$ for $c \in (c_D^*, c_U]$, $N_E^{FB} = N_E^*$, and $\lambda^{FB} = \lambda^*$. We again define $c_D^{FB} \equiv c_D^*$. From here, we may follow similar arguments to those given in Appendix B. Specifically, under the parameter restrictions given in the paper (which correspond to the current setting when $k = 1, c_L = 0$ and $c_M = c_U$), we can establish that there exists an upper bound $\bar{c}_L^* > 0$ such that for all $c_L \in (0, \bar{c}_L^*)$ the above system of equations has a unique solution satisfying $c_D^{FB} \in (c_L, c_U)$. We can then uniquely determine $N_E^{FB} \in (0, 1/f_E)$ by using (B5), with $\lambda^{FB} > 0$ thus uniquely determined by (B4). From here, $q^{FB}(c)$ is uniquely determined as $q^{FB}(c) = \max\{\frac{\lambda^{FB}}{\gamma}(c_D^{FB} - c), 0\}, \forall c \in [c_L, c_U]$. Finally, we may argue as in the paper to confirm that the candidate solution solves the planner's problem.

Market equilibrium solution. — The market equilibrium solution under the uniform distribution (B1) may be determined using the same steps as in Section II of the paper. Here, we identify novel expressions and then state the associated equation system that determines the market equilibrium solution for the new setting.

The consumer's problem is unaltered. For given $p^{\max}$, the firm's problem is also unaltered. Assuming for now that $c_U > c_D$, we find that $\bar{c}$ and $\bar{p}$ now take the following respective forms:

$$\bar{c} = \frac{c_D + c_L}{2} \text{ and } \bar{p} = \frac{3c_D + c_L}{4}.$$

The new expression for $\bar{p}$ in turn implies that (34) now takes the following form:

$$N = \frac{4\gamma}{\eta} \frac{(\alpha - \lambda c_D)}{\lambda(c_D - c_L)}.$$

Using $N_E = \frac{N}{G(c_D)}$, we find that (37) now takes the following form:

$$N_E = \frac{4\gamma\,(c_U - c_L)}{\eta} \frac{(\alpha - \lambda c_D)}{\lambda\,(c_D - c_L)^2}. \tag{B6}$$

Labor market clearing again ensures that the resource constraint is binding and given by (38). We simplify further by using (32) for $c \in [c_L, p^{\max}]$, $p^{\max} = c_D$, (B6) and the uniform distribution (B1). Proceeding thus, we find that the resource constraint (38) can be written as

$$\frac{3\eta}{\gamma \cdot \widetilde{\phi}} = \frac{(\alpha - \lambda c_D)}{\lambda\,(c_D - c_L)^2} \left(\frac{(2c_L + c_D)\,(c_D - c_L)^2\,\lambda}{\gamma\widetilde{\phi}} + 1 \right), \tag{B7}$$

where $\widetilde{\phi} \equiv 12 f_E (c_U - c_L)$.

Finally, the Free Entry (FE) condition is again defined by (40). We represent this condition equivalently as

$$c_D - c_L = (\frac{\gamma\widetilde{\phi}}{\lambda})^{\frac{1}{3}}, \tag{B8}$$

where we use (32) and (33) for $c \in [c_L, p^{\max}]$, set $p^{\max} = c_D$ and use the uniform distribution (B1).

Following the steps taken in the paper, we next substitute (B8) into (B7). Specifically, we isolate $(c_D - c_L)^3$ from (B8), multiply the numerator and denominator of the bracketed fraction in (B7) by $(c_D - c_L)$, and substitute the solution for $(c_D - c_L)^3$ from (B8) into (B7). We thereby obtain

$$f_E N_E[\frac{c_L + 2c_D}{c_D - c_L}] = 1. \tag{B9}$$

Following the paper, given $q(c) = \lambda\,(c_D - c)\,/(2\gamma)$ for all $c \in [c_L, c_D]$ and a triplet (c_D, N_E, λ) solving (B6), (B8) and (B9), the market equilibrium solution $(q^{mkt}(c), N_E^{mkt})$ and associated multiplier λ^{mkt} can be constructed by using the translation under which $c_D^{mkt} = c_D$, $q^{mkt}(c) = q(c)$ for $c \in [c_L, c_D^{mkt}]$ and $q^{mkt}(c) = 0$ for $c > c_D^{mkt}$, $N_E^{mkt} = N_E$, and $\lambda^{mkt} = \lambda$. From here, we may follow similar arguments to those given in Appendix E. Specifically, under the parameter restrictions given in the paper (which correspond to the current setting when $k = 1, c_L = 0$ and $c_M = c_U$), we can establish that there exists an upper bound $\overline{c}_L^{mkt} > 0$ such that for all $c_L \in (0, \overline{c}_L^{mkt})$ the above system of equations indeed has a unique solution satisfying $c_D^{mkt} \in (c_L, c_U)$. We can then uniquely determine $N_E^{mkt} \in (0, 1/f_E)$ by using (B9), with $\lambda^{mkt} > 0$ thus uniquely determined by (B8). From here, $q^{mkt}(c)$ is uniquely determined as $q^{mkt}(c) = \max\{\frac{\lambda^{FB}}{\gamma}(c_D^{FB} - c), 0\}, \forall c \in [c_L, c_U]$.

Comparison. — Having established the existence of the respective solutions, we now show that $c_D^{mkt} > c_D^{FB}$ and $N_E^{mkt} > N_E^{FB}$. To do this, we extend (A5) and (A43) from Appendices B and E, respectively, to the current setting and then compare the resulting expressions.

For the first best setting, we solve for λ^{FB} from (B4) and for N_E^{FB} from (B5), where we impose the translation of the "$*$" solutions to the "FB" solutions as described above. We then plug these solutions into (B3), obtaining that

$$\frac{3\eta}{2c_D^{FB} + c_L} + \frac{6c_D^{FB}\gamma f_E(c_U - c_L)}{(c_D^{FB} - c_L)^3} = \alpha. \tag{B10}$$

Similarly, for the market setting, we solve for λ^{mkt} from (B8) where $\widetilde{\phi} \equiv 12 f_E(c_U - c_L)$ and for N_E^{mkt} from (B9). We then plug these solutions into (B6), obtaining that

$$\text{(B11)} \qquad \frac{3\eta}{2c_D^{mkt}+c_L}+\frac{12c_D^{mkt}\gamma f_E(c_U-c_L)}{(c_D^{mkt}-c_L)^3}=\alpha.$$

Under our parameter restrictions, we establish above that (B10) has a unique solution satisfying $c_D^{FB} \in (c_L, c_U)$ and likewise that (B11) has a unique solution satisfying $c_D^{mkt} \in (c_L, c_U)$. The LHS's of these equations are also strictly decreasing in c_D^{FB} and c_D^{mkt}, respectively, over the (c_L, c_U) domain. Comparing the LHS's, it is now straightforward to see that $c_D^{mkt} > c_D^{FB}$. Referring to (B5) and (B9), we may now conclude that $N_E^{mkt} > N_E^{FB}$. Notice that this conclusion would not follow if $c_L = 0$.

Appendix H: Market Size

We consider here the extension of the model to the L-economy setting in which there are $L > 0$ consumers. We derive the equation system for the market equilibrium solution as presented in Section IV.D of the paper, and we confirm that the cost cutoff falls while consumer welfare rises with market size. We also discuss other comparative statics results. In this Online Appendix section, we use the same notation as in the paper for certain functions used at intermediate steps. The functions are defined more generally here, due to the relaxation of the assumption that $L = 1$, and the functions are explicitly defined so that the meaning is clear from context.

Market equilibrium solution. — The market equilibrium solution for the L-economy may be determined using the same steps as in Section II of the paper. Here, we identify novel expressions and then state the associated equation system that determines the market equilibrium solution for the L-economy setting.

The consumer's problem is unaltered. For given $p^{\max}$ as defined in (30), the demand of a variety for an individual consumer takes the form $(\lambda/\gamma)(p^{\max} - p)$ so that the (aggregate) demand facing a firm is $(L\lambda/\gamma)(p^{\max} - p)$. The (aggregate) inverse demand function that confronts a firm is thus $p^{\max} - \frac{\gamma}{\lambda}\frac{q}{L}$. A profit-maximizing firm with cost c thus produces

$$\text{(B12)} \qquad q(c) = \lambda L(p^{\max} - c)/(2\gamma)$$

units in aggregate. The induced price is again $p(c) = [p^{\max} + c]/2$, and the firm enjoys the aggregate profit $\pi(c) = \lambda L(p^{\max} - c)^2/(4\gamma)$.

With $\pi(c)$ thus defined, the ZCP condition is again defined by $\pi(c_D) = 0$ where $p^{\max} = c_D$. For a given value of c_D, the expressions for $\overline{c}$ and $\overline{p}$ are unaltered from those given in Section II of the paper. Following the same steps, we can confirm that (34), (35) and (37) continue to hold, where N, c_D and N_E again represent the number of varieties, the cost cutoff, and the number of entrants. For convenience, we reproduce (37) here:

$$N_E = \frac{2\,(k+1)\,\gamma\,(c_M)^k}{\eta}\frac{(\alpha - \lambda c_D)}{\lambda\,(c_D)^{k+1}} \tag{B13}$$

For the L-economy, the resource constraint (38) takes the following form:

$$N_E\left(\int_0^{c_D} c \cdot q\,(c)\,dG\,(c) + f_E\right) = L,$$

where we recall from (B12) the (aggregate) quantity allocation function $q(c)$ now depends on L. Following the steps taken in the paper, we may rewrite the resource constraint. The corresponding expression (39) now takes the following form:

$$\frac{\eta\,(2+k)}{\gamma \cdot \phi / L} = \frac{(\alpha - \lambda c_D)}{\lambda\,(c_D)^{k+1}}\left(\frac{(c_D)^{k+2}\,k\lambda}{\gamma \cdot \phi/L} + 1\right), \tag{B14}$$

where as before $\phi = 2\,(k+1)\,(k+2)\,f_E\,(c_M)^k$.

Finally, the FE condition for a firm is again represented by (40), where we recall again from (B12) that the quantity allocation function $q(c)$ now depends on L. Using as well $p(c) = [p^{\max} + c]/2$, the Pareto distribution (1) and $p^{\max} = c_D$, we may rewrite (40) with the resulting expression now taking the following form:

$$c_D = \left[\frac{2\gamma(k+1)(k+2)f_E(c_M)^k}{\lambda L}\right]^{\frac{1}{k+2}}. \tag{B15}$$

Following the steps in the paper, we next solve the FE condition (B15) for $\lambda \cdot (c_D)^{k+2}$ and insert that solution into the resource constraint (B14). We then solve the resulting expression for $(\alpha - \lambda c_D)/[\lambda\,(c_D)^{k+1}]$ and plug this solution into (B13). Following these steps, we thus find that (42) extends to the L-economy as

$$N_E = \frac{L}{(k+1)f_E}. \tag{B16}$$

We now summarize our findings regarding the market equilibrium solution. Given $q(c) = \lambda L(c_D - c)/(2\gamma)$ for all $c \in [0, c_D]$ and a triplet (c_D, N_E, λ) solving (B13), (B15) and (B16), the *market equilibrium solution for the L-economy* $(q^{mktL}(c), N_E^{mktL})$ and associated multiplier λ^{mktL} can be constructed by using the translation under which $c_D^{mktL} = c_D$, $q^{mktL}(c) = q(c)$ for $c \in [0, c_D^{mktL}]$ and $q^{mktL}(c) = 0$ for $c > c_D^{mktL}$, $N_E^{mktL} = N_E = L/[(k+1)f_E]$, and $\lambda^{mktL} = \lambda$. We thus conclude that the market equilibrium solution $(q^{mktL}(c), N_E^{mktL})$ and associated multiplier λ^{mktL} are determined by the following system:

$$N_E^{mktL} = \frac{2\,(k+1)\,\gamma\,(c_M)^k}{\eta} \frac{\left(\alpha - \lambda^{mktL} c_D^{mktL}\right)}{\lambda^{mktL}\left(c_D^{mktL}\right)^{k+1}} \tag{B17}$$

$$c_D^{mktL} = \left[\frac{2\gamma(k+1)(k+2)\frac{f_E}{L}(c_M)^k}{\lambda^{mktL}}\right]^{\frac{1}{k+2}} \tag{B18}$$

$$N_E^{mktL} = \frac{1}{(k+1)\frac{f_E}{L}} \tag{B19}$$

$$\frac{q^{mktL}(c)}{L} = \max\{\frac{\lambda^{mktL}}{2\gamma}(c_D^{mktL} - c), 0\}, \forall c \in [0, c_M], \tag{B20}$$

which matches exactly the equation system provided in Section IV.D of the paper.

As noted in Section IV.D of the paper, (B17)-(B19) determine $(N_E^{mktL}, c_D^{mktL}, \lambda^{mktL})$ with the quantity allocation function then determined by (B20). Comparing this system with the previous ($L = 1$) system (43)-(46), we see that the only change in the first three equations is that f_E is now replaced by $\frac{f_E}{L}$. We may likewise directly recover the per-capita consumption for the L-economy, $q^{mktL}(c)/L$, from $q^{mkt}(c)$ by replacing f_E with $\frac{f_E}{L}$ in (43)-(46). As discussed in Section IV.D of the paper, our maintained parameter restriction is more more easily satisfied when $L > 1$ than when $L = 1$, since $\frac{f_E}{L}$ falls as L rises. Our previously derived results about the existence and uniqueness of the market equilibrium solutions thus carry over to the L-economy setting. In particular, following the arguments in Appendix E, $c_D^{mktL} \in (0, c_M)$ and $\lambda^{FB} > 0$ are uniquely determined, with $N_E^{mktL} \in (0, L/f_E)$ uniquely determined by (B19) and $q^{mktL}(c)$ then uniquely given by (B20).

Welfare analysis. — We first establish that the cost cutoff falls with L. We then show that welfare rises with L.

To establish that c_D^{mktL} falls with L, we use (B17) and (B19) to obtain

$$\frac{\left(\alpha - \lambda^{mktL} c_D^{mktL}\right)}{\lambda^{mktL}\left(c_D^{mktL}\right)^{k+1}} = \frac{L\eta}{2\gamma(k+1)^2 f_E\,(c_M)^k}. \tag{B21}$$

Using (B18), we also have that

$$\text{(B22)} \qquad \lambda^{mktL}(c_D^{mktL})^{k+2} = \frac{2\gamma(k+1)(k+2)f_E(c_M)^k}{L}.$$

Equations (B21) and (B22) determine λ^{mktL} and c_D^{mktL}.

Rearranging (B21), we obtain

$$\lambda^{mktL}c_D^{mktL} = \frac{\alpha}{1+(c_D^{mktL})^k \frac{L\eta}{2\gamma(k+1)^2 f_E(c_M)^k}}.$$

Plugging this expression into (B22) and simplifying, we obtain an expression that determines c_D^{mktL}:

$$\text{(B23)} \qquad (c_D^{mktL})^k[c_D^{mktL} - \frac{\eta(k+2)}{\alpha(k+1)}] = \frac{2(k+1)(k+2)(c_M)^k f_E\gamma}{\alpha L}.$$

Viewed as a function of c_D^{mktL}, the LHS of (B23) is negative for $c_D^{mktL} \in (0, \frac{\eta(k+2)}{\alpha(k+1)})$, zero for $c_D^{mktL} \in \{0, \frac{\eta(k+2)}{\alpha(k+1)}\}$ and positive for $c_D^{mktL} > \frac{\eta(k+2)}{\alpha(k+1)}$. As expected, $c_D^{mktL} > 0$ is thus confirmed, since the LHS increases without bound as $c_D^{mktL} > \frac{\eta(k+2)}{\alpha(k+1)}$ rises and the RHS of (B23) is positive. Further, given our restriction that $c_M > \frac{\eta(k+2)}{\alpha(k+1)}$, we may also confirm as expected that $c_D^{mktL} < c_M$ when f_E/L is sufficiently small (as the RHS approaches zero as f_E/L approaches zero). We may now confirm from (B22) that $\lambda^{mktL} > 0$ follows as well.

Taking the derivative of the LHS, we can also confirm that the LHS is minimized when $c_D^{mktL} = \frac{k\eta(k+2)}{\alpha(k+1)^2}$, with the LHS falling (rising) for $c_D^{mktL} < (>) \frac{k\eta(k+2)}{\alpha(k+1)^2}$. Since $\frac{k\eta(k+2)}{\alpha(k+1)^2} < \frac{\eta(k+2)}{\alpha(k+1)}$, we conclude that the LHS is positive and increasing for $c_D^{mktL} > \frac{\eta(k+2)}{\alpha(k+1)}$. Notice finally that the LHS is independent of L while the RHS falls with L. Together, these results establish the following result:

$$\text{(B24)} \qquad \frac{dc_D^{mktL}}{dL} < 0.$$

Thus, a larger market size leads to a lower cost cutoff in the L-economy model.

We may argue similarly to establish how c_D^{mktL} responds to changes in η and α, respectively. As η increases, the LHS of (B23) falls while the RHS is unchanged. We thus conclude that c_D^{mktL} rises as η increases. An increase in α increases the LHS and lowers the RHS of (B23). These effects both lead to a lower value for c_D^{mktL}.

Finally, we can assess the impact of an increase in η on dc_D^{mktL}/dL. For this purpose, it is useful to provide an explicit expression for dc_D^{mktL}/dL. Using (B23),

we find that
$$\frac{dc_D^{mktL}}{dL} = \frac{-2(k+2)(c_M)^k f_E \gamma}{\alpha L^2 (c_D^{mktL})^{k-1}[c_D^{mktL} - \frac{k\eta(k+2)}{\alpha(k+1)^2}]} < 0,$$

where the inequality follows since $c_D^{mktL} > \frac{\eta(k+2)}{\alpha(k+1)} > \frac{k\eta(k+2)}{\alpha(k+1)^2}$. Using this finding, we obtain that

$$\frac{d}{d\eta}\frac{dc_D^{mktL}}{dL} = \frac{2(k+2)^3(c_M)^k f_E \gamma k \eta}{\alpha^3 L^2 (k+1)^4 (c_D^{mktL})^{k-1}[c_D^{mktL} - \frac{k\eta(k+2)}{\alpha(k+1)^2}]^3} > 0.$$

Thus, the reduction in c_D^{mktL} that is induced by an increase in market size L as reported in (B24) is weakened as η increases. As discussed in the paper, the comparative statics results provided in this and the preceding paragraphs for changes in η and α are not present in the original MO model. These novel relationships emerge in the one-sector model due to the impact of η and α on the endogenous marginal utility of income λ^{mktL}.

We turn next to welfare analysis. To this end, we represent the welfare function when evaluated at the market equilibrium solution for the L-economy. Using (B12), $p^{\max} = c_D^{mktL}$, the Pareto distribution (1) and (B17), we find that

$$\begin{aligned} N_E^{mktL}\int_0^{c_M}\frac{q(c)}{L}dG(c) &= N_E^{mktL}\int_0^{c_D^{mktL}}\frac{q(c)}{L}dG(c) = \frac{\alpha - \lambda^{mktL}c_D^{mktL}}{\eta} \\ N_E^{mktL}\int_0^{c_M}(\frac{q(c)}{L})^2 dG(c) &= N_E^{mktL}\int_0^{c_D^{mktL}}(\frac{q(c)}{L})^2 dG(c) = \frac{(\alpha - \lambda^{mktL}c_D^{mktL})\lambda^{mktL}c_D^{mktL}}{\eta(2+k)\gamma}. \end{aligned}$$

At the market equilibrium solution, the welfare function for an individual consumer takes the form

$$U^{mktL} = N_E^{mktL}\left[\alpha\int_0^{c_M}\frac{q(c)}{L}dG(c) - \frac{\gamma}{2}\int_0^{c_M}(\frac{q(c)}{L})^2 dG(c) - \frac{\eta}{2}N_E^{mktL}\left(\int_0^{c_M}\frac{q(c)}{L}dG(c)\right)^2\right].$$

Substituting, we obtain that consumer welfare at the market equilibrium solution is given as

(B25) $$U^{mktL} = \frac{(\alpha - \lambda^{mktL}c_D^{mktL})}{2\eta}(\alpha + \frac{k+1}{k+2}\lambda^{mktL}c_D^{mktL})$$

as reported in Section IV.D of the paper. As noted in the paper, Bagwell and Lee (2021) derive this representation when $L = 1$ for general values of N_E.

Welfare at the market equilibrium solution depends on market size through $\lambda^{mktL}c_D^{mktL}$. We thus now explore how $\lambda^{mktL}c_D^{mktL}$ varies with L. Multiplying

(B21) and (B22), we obtain

$$\left(\alpha - \lambda^{mktL} c_D^{mktL}\right) c_D^{mktL} = \frac{\eta(k+2)}{k+1}.$$

Rearranging, we find

$$\lambda^{mktL} c_D^{mktL} = \alpha - \frac{\eta(k+2)}{(k+1)c_D^{mktL}}.$$

Differentiating this expression with respect to L, we obtain

$$\text{(B26)} \qquad \frac{d(\lambda^{mktL} c_D^{mktL})}{dL} = \frac{\eta(k+2)}{(k+1)(c_D^{mktL})^2} \frac{dc_D^{mktL}}{dL} < 0,$$

where the inequality follows from (B24).

Referring to (B25), we find that

$$\frac{dU^{mktL}}{dL} = -\frac{\alpha + 2(k+1)\lambda^{mktL} c_D^{mktL}}{2(2+k)\eta} \frac{d(\lambda^{mktL} c_D^{mktL})}{dL} > 0,$$

where the inequality follows from (B26). Thus, and as reported in Section IV.D of the paper, a larger market generates greater consumer welfare. As in the original MO model with an outside good and as reported in Section IV.D of the paper, this welfare gain reflects the lower average price and greater product variety that are attributable to a lower cost cutoff. We confirm these and other comparative statics next.

Comparative statics. — We now briefly report comparative statics results for other economic measures of interest. We relate these findings to those reported by Melitz and Ottaviano (2008) for the original MO model with an outside good.

Using $p^{\max} = c_D^{mktL}$ at the market equilibrium, the profit-maximizing price for a firm with cost c is represented as $p(c) = [c_D^{mktL} + c]/2$. Given (B24), it follows immediately that active firms set lower prices in larger markets. Likewise, the markup for an active firm with cost c is lower in larger markets, whether we measure the markup as $p(c)/c$, $p(c) - c$ or $(p(c) - c)/c$.

As in Section II, we have that $\overline{c}^{mktL} \equiv E(c|c \leq c_D^{mktL}) = \left(\frac{k}{k+1}\right) c_D^{mktL}$ and $\overline{p}^{mktL} \equiv E(p(c)|c \leq c_D^{mktL}) = \left(\frac{c_D^{mktL} + \overline{c}^{mktL}}{2}\right) = \left(\frac{2k+1}{2(k+1)}\right) c_D^{mktL}$ in the L-economy setting. If we follow Melitz and Ottaviano (2008) and define the markup as the difference between price and cost, then we find that $\overline{\mu}^{mktL} \equiv \overline{p}^{mktL} - \overline{c}^{mktL} =$

$\left(\frac{1}{2(k+1)}\right) c_D^{mktL}$. Given (B24), it thus follows immediately that

$$\begin{aligned}\frac{d\overline{c}^{mktL}}{dL} &= \left(\frac{k}{k+1}\right)\frac{dc_D^{mktL}}{dL} < 0\\ \frac{d\overline{p}^{mktL}}{dL} &= \left(\frac{2k+1}{2(k+1)}\right)\frac{dc_D^{mktL}}{dL} < 0\\ \frac{d\overline{\mu}^{mktL}}{dL} &= \left(\frac{1}{2(k+1)}\right)\frac{dc_D^{mktL}}{dL} < 0.\end{aligned}$$

Hence, as in the original MO model, the conditional expected cost, price and markup are all lower in larger markets. These results match those reported by Melitz and Ottaviano (2008, p. 301). (While the exact expression for $\frac{dc_D^{mktL}}{dL}$ depends on the model, the sign of the comparative statics depends only on the sign of $\frac{dc_D^{mktL}}{dL}$ which is common across the two models.) We note that if the markup were instead defined as the price-cost ratio, then the conditional expected markup would be independent of market size.

We can also show that the number of consumed varieties in the market equilibrium solution is higher in larger markets. To see this, note that the number of consumed varieties, N^{mktL}, satisfies $N^{mktL} = G(c_D^{mktL})N_E^{mktL}$. Thus, using (1) and (B19), we find that $N^{mktL} = (\frac{c_D^{mktL}}{c_M})^k \frac{L}{(k+1)f_E}$. Referring to (B24), it thus follows that $N^{mktL} = (\frac{c_D^{mktL}}{c_M})^k \frac{L}{(k+1)f_E} = \frac{2(k+2)\gamma}{\alpha[c_D^{mktL} - \frac{\eta(k+2)}{\alpha(k+1)}]}$. Hence, we have that

$$\frac{dN^{mktL}}{dL} = -\frac{2(k+2)\gamma}{\alpha[c_D^{mktL} - \frac{\eta(k+2)}{\alpha(k+1)}]^2}\frac{dc_D^{mktL}}{dL} > 0,$$

which establishes the desired conclusion.

We likewise calculate conditional expected output as $\overline{q}^{mktL} \equiv E(q(c)|c \leq c_D^{mktL}) = (\frac{L}{2\gamma(k+1)})(\lambda_D^{mktL} c_D^{mktL})$, where we use (B12), (1) and $p^{\max} = c_D^{mktL}$. Using (B22), we find that $\overline{q}^{mktL} = (k+2)(c_M)^k f_E (c_D^{mktL})^{-(k+1)}$. Similarly, we express conditional expected sales as $\overline{r}^{mktL} \equiv E(p(c)q(c)|c \leq c_D^{mktL}) = (\frac{L}{2\gamma(k+2)})(\lambda_D^{mktL}(c_D^{mktL})^2)$. Using (B22), we find that $\overline{r}^{mktL} = (k+1)(c_M)^k f_E (c_D^{mktL})^{-k}$. Finally, we write conditional average profit as $\overline{\pi}^{mktL} \equiv E(\pi(c)|c \leq c_D^{mktL}) = (\frac{L}{2\gamma(k+1)(k+2)})(\lambda_D^{mktL}(c_D^{mktL})^2)$. Using (B22), we find that $\overline{\pi}^{mktL} = (c_M)^k f_E (c_D^{mktL})^{-k}$. The final expressions for $\overline{q}^{mktL}$, $\overline{r}^{mktL}$ and $\overline{\pi}^{mktL}$ take the same form as in Melitz and Ottaviano (2008, p. 301), although c_D^{mktL} itself differs across the two models. Nevertheless, since c_D^{mktL} falls with L in both model settings, we have the same comparative statics results:

$$\begin{aligned}\frac{d\overline{q}^{mktL}}{dL} &= -(k+1)(k+2)(c_M)^k f_E (c_D^{mktL})^{-(k+2)} \frac{dc_D^{mktL}}{dL} > 0 \\ \frac{d\overline{r}^{mktL}}{dL} &= -k(k+1)(c_M)^k f_E (c_D^{mktL})^{-(k+1)} \frac{dc_D^{mktL}}{dL} > 0 \\ \frac{d\overline{\pi}^{mktL}}{dL} &= -k(c_M)^k f_E (c_D^{mktL})^{-(k+1)} \frac{dc_D^{mktL}}{dL} > 0.\end{aligned}$$

Hence, as in the original MO model, the conditional expected output, sales and profit are all higher in larger markets. We also observe that, as in the original MO model, average industry profitability $\overline{\pi}^{mktL}/\overline{r}^{mktL}$ does not vary with market size:

$$\frac{d(\overline{\pi}^{mktL}/\overline{r}^{mktL})}{dL} = 0.$$

Finally, we can also explore how the conditional variance of economic measures responds to changes in market size. In the market equilibrium solution for the L-economy, the variance of cost, conditional on a firm being active, is

$$\sigma^2_{c^{mktL}} = \frac{\int_0^{c_D^{mktL}} (c - \overline{c}^{mktL})^2 dG(c)}{G(c_D^{mktL})} = \frac{k(c_D^{mktL})^2}{(k+1)^2(k+2)},$$

where we use the Pareto distribution (1) and our finding above that $\overline{c}^{mktL} = \left(\frac{k}{k+1}\right) c_D^{mktL}$. Likewise, using the Pareto distribution (1), we calculate the conditional variance of price and markup (defined as the price-cost ratio) as $\sigma^2_{p^{mktL}} \equiv k(c_D^{mktL})^2/[4(k+1)^2(k+2)] = \sigma^2_{\mu^{mktL}}$, where we use our findings above that $\overline{p}^{mktL} = \left(\frac{2k+1}{2(k+1)}\right) c_D^{mktL}$ and $\overline{\mu}^{mktL} = \left(\frac{1}{2(k+1)}\right) c_D^{mktL}$, respectively.

Given (B24), it thus follows immediately that

$$\begin{aligned}\frac{d\sigma^2_{c^{mktL}}}{dL} &= \left(\frac{2kc_D^{mktL}}{(k+1)^2(k+2)}\right) \frac{dc_D^{mktL}}{dL} < 0 \\ \frac{d\sigma^2_{p^{mktL}}}{dL} &= \left(\frac{kc_D^{mktL}}{2(k+1)^2(k+2)}\right) \frac{dc_D^{mktL}}{dL} < 0 \\ \frac{d\sigma^2_{\mu^{mktL}}}{dL} &= \left(\frac{kc_D^{mktL}}{2(k+1)^2(k+2)}\right) \frac{dc_D^{mktL}}{dL} < 0.\end{aligned}$$

Thus, as in the original MO model, the conditional variance of cost, price and markup are all lower in larger markets. These results match those reported by Melitz and Ottaviano (2008, p. 312). (As noted previously, while the exact expression for $\frac{dc_D^{mktL}}{dL}$ depends on the model, the sign of the comparative statics depends only on the sign of $\frac{dc_D^{mktL}}{dL}$ which is common across the two models.)

Finally, we can similarly calculate the conditional variance of output and sales as $\sigma^2_{q^{mktL}} = k(k+2)\left((c_M)^k f_E\right)^2 (c_D^{mktL})^{-2(k+1)}$ and $\sigma^2_{r^{mktL}} = k\left((k+1)(c_M)^k f_E\right)^2 (c_D^{mktL})^{-2k}/(k+4)$. These expressions for the one-sector model do not take the same form as in the MO model with an outside good, as may be verified by comparison with the expressions in Melitz and Ottaviano (2008, p. 301). Nevertheless, since c_D^{mktL} falls with L in both model settings, the sign of the comparative statics are common across the two model settings. Specifically, we find that

$$\frac{d\sigma^2_{q^{mktL}}}{dL} = -2(k+1)k(k+2)\left((c_M)^k f_E\right)^2 (c_D^{mktL})^{-2k-3}\frac{dc_D^{mktL}}{dL} > 0$$
$$\frac{d\sigma^2_{r^{mktL}}}{dL} = \frac{-2k^2\left((k+1)(c_M)^k f_E\right)^2 (c_D^{mktL})^{-2k-1}}{k+4}\frac{dc_D^{mktL}}{dL} > 0$$

Thus, as in the original MO model, the conditional variance of output and sales are both greater in larger markets.

Appendix I: Short Run Comparison

We consider here a short run version of the model in which $N_E \in (0, 1/f_E)$ is fixed. We compare the second best and market solutions when N_E is fixed at any level that is sufficiently high so that selection ($c_D < c_M$) occurs in both solutions.

Second best solution. — The second best solution for this setting is already characterized in Proposition 1 of the paper, which is proved in Appendix C. To facilitate comparison with the short run market solution, we reproduce a portion of this proof here.

As Proposition 1 states, for $N_E > \widetilde{N}_E$, the planner's fixed-N_E problem is solved by $\widehat{q}(c)$ with cost cutoff value $\widehat{c}_D \in (0, c_M)$ and associated multiplier $\widehat{\lambda} > 0$, where $(\widehat{q}, \widehat{c}_D, \widehat{\lambda})$ satisfies the resource constraint (6), the Euler condition (8) and $\widehat{q}(c) = \max\{\frac{\widehat{\lambda}}{\gamma}(\widehat{c}_D - c), 0\}$ for all $c \in [0, c_M]$. As also discusssed below, the proof shows further that there exists $\widetilde{N}_E \in (0, 1/f_E)$ such that $\widehat{c}_D < c_M$ if and only if $N_E > \widetilde{N}_E$.

To confirm that $\widehat{\lambda} > 0$ and $\widehat{c}_D \in (0, c_M)$, we note in Appendix C that $\widehat{c}_D > 0$ follows from the resource constraint (6). To establish that $\widehat{\lambda} > 0$ and $\widehat{c}_D < c_M$, we follow the steps used to derive (13) and (14) and represent the resource constraint and Euler condition, respectively, as

$$N_E\left[\frac{(\widehat{c}_D)^{k+2}(c_M)^{-k}k\widehat{\lambda}}{(k+1)(k+2)\gamma} + f_E\right] = 1 \tag{B27}$$

(B28) $$N_E = \frac{\gamma(k+1)(c_M)^k}{\eta}\frac{(\alpha - \widehat{\lambda}\widehat{c}_D)}{\widehat{\lambda}(\widehat{c}_D)^{k+1}}.$$

We may use (B27) and (B28) to solve for $\widehat{\lambda}$ and $\widehat{c}_D$.

Solving (B27) for $\widehat{\lambda}$, we obtain

(B29) $$\widehat{\lambda} = \frac{1 - N_E f_E}{N_E}\frac{\gamma(k+1)(k+2)(c_M)^k}{k(\widehat{c}_D)^{k+2}} > 0,$$

where the inequality follows from $N_E \in (0, 1/f_E)$ and $\widehat{c}_D > 0$. Thus, $\widehat{\lambda}$ is a valid multiplier. Next, plugging (B29) into (B28), we obtain

(B30) $$[\frac{1}{N_E} - f_E]\frac{\gamma(k+1)(k+2)(c_M)^k}{k(\widehat{c}_D)^{k+1}} + (1 - N_E f_E)\frac{\eta(k+2)}{k\widehat{c}_D} = \alpha.$$

As noted in Appendix C, the left hand side of (B30) is decreasing in N_E and $\widehat{c}_D$. Thus, for any given parameter specification, a unique and positive value for $\widehat{c}_D$ solves (B30); furthermore, a higher value for N_E generates in a lower value for the solution $\widehat{c}_D$. Using (B30), we may further verify that (i) $\widehat{c}_D$ approaches infinity as N_E approaches zero, and (ii) $\widehat{c}_D$ approaches zero as N_E approaches $1/f_E$. Thus, for a given c_M, there exists $\widetilde{N}_E \in (0, 1/f_E)$ such that $\widehat{c}_D < c_M$ if and only if $N_E > \widetilde{N}_E$.

As in Appendix C, we may now confirm that our candidate solution exists and is uniquely determined for any $N_E > \widetilde{N}_E$. As noted, a unique and positive value for $\widehat{c}_D$ solves (B30). It follows from (B29) that a unique and positive value for $\widehat{\lambda}$ is thus determined. Last, $\widehat{q}(c)$ is thus uniquely determined as $\widehat{q}(c) = \max\{\frac{\widehat{\lambda}}{\gamma}(\widehat{c}_D - c), 0\}$ for all $c \in [0, c_M]$.

Market solution. — We now derive a similar pair of equations to determine the market solution for given $N_E \in (0, 1/f_E)$. For comparison purposes, let us denote the market solution for given N_E as $(\widetilde{q}, \widetilde{c}_D, \widetilde{\lambda})$. Using (32) and $p^{\max} = c_D$, we have that

$$\widetilde{q}(c) = \frac{\widetilde{\lambda}(\widetilde{c}_D - c)}{2\gamma}$$

for all $c \in [0, \widetilde{c}_D]$. The labor market equilibrium condition again is represented as the resource constraint (38). With profit redistributed to consumers in equal shares, (38) then ensures that the consumer budget constraint binds. Plugging this quantity allocation function into the resource constraint (38), we get that

$$N_E\left[\frac{(\widetilde{c}_D)^{k+2}(c_M)^{-k}k\widetilde{\lambda}}{(k+1)(k+2)2\gamma}+f_E\right]=1, \tag{B31}$$

where $\widetilde{c}_D > 0$ clearly follows from (B31) given $N_E \in (0, 1/f_E)$. We also have from (37) that

$$N_E=\frac{2\gamma\,(k+1)\,(c_M)^k}{\eta}\frac{\left(\alpha-\widetilde{\lambda}\widetilde{c}_D\right)}{\widetilde{\lambda}\,(\widetilde{c}_D)^{k+1}}. \tag{B32}$$

We may use (B31) and (B32) to solve for $\widetilde{\lambda}$ and $\widetilde{c}_D$.

Solving (B31) for $\widetilde{\lambda}$, we obtain

$$\widetilde{\lambda}=\frac{1-N_Ef_E}{N_E}\frac{2\gamma(k+1)(k+2)(c_M)^k}{k(\widetilde{c}_D)^{k+2}}>0, \tag{B33}$$

where the inequality follows from $N_E \in (0, 1/f_E)$ and $\widetilde{c}_D > 0$. Thus, $\widetilde{\lambda}$ is a valid multiplier. Next, plugging (B33) into (B32), we obtain

$$[\frac{1}{N_E}-f_E]\frac{2\gamma(k+1)(k+2)(c_M)^k}{k(\widetilde{c}_D)^{k+1}}+(1-N_Ef_E)\frac{\eta(k+2)}{k\widetilde{c}_D}=\alpha. \tag{B34}$$

Proceeding as above for (B30), we note that the left hand side of (B34) is decreasing in N_E and $\widetilde{c}_D$. Thus, for any given parameter specification, a unique and positive value for $\widetilde{c}_D$ solves (B34); furthermore, a higher value for N_E generates in a lower value for the solution $\widetilde{c}_D$. Using (B34), we may further verify that (i) $\widetilde{c}_D$ approaches infinity as N_E approaches zero, and (ii) $\widetilde{c}_D$ approaches zero as N_E approaches $1/f_E$. Thus, for a given c_M, there exists $\widetilde{N}_E^{mkt} \in (0, 1/f_E)$ such that $\widetilde{c}_D < c_M$ if and only if $N_E > \widetilde{N}_E^{mkt}$.

Proceeding again as above, we may now confirm that the market solution exists and is uniquely determined for any $N_E > \widetilde{N}_E^{mkt}$. As noted, a unique and positive value for $\widetilde{c}_D$ solves (B34). It follows from (B33) that a unique and positive value for $\widetilde{\lambda}$ is thus determined. Last, $\widetilde{q}(c)$ is thus uniquely determined as $\widetilde{q}(c) = \max\{\frac{\widetilde{\lambda}}{2\gamma}(\widetilde{c}_D - c), 0\}$ for all $c \in [0, c_M]$.

Comparisons. — Comparing (B30) and (B34), we see that the expressions differ only in that the LHS of the latter has "2γ" where the LHS of the former has "γ." It thus follows that the LHS of the latter is higher than the LHS of the former for all $N_E \in (0, 1/f_E)$. We may thus conclude that $\widetilde{N}_E^{mkt} > \widetilde{N}_E$. Hence, our assumption that N_E is sufficiently large so that selection ($c_D < c_M$) occurs in both settings is met if and only if $N_E > \widetilde{N}_E^{mkt}$. In other words, selection is more difficult to ensure in the market setting.

Similarly, for a given $N_E > \widetilde{N}_E^{mkt}$, it follows that $\widetilde{c}_D > \widehat{c}_D$. In other words, the market solution provides too little selection and has too many varieties. To compare aggregate quantities, we find that the first best aggregate $\widehat{Q}$ for the fixed-N_E setting is given as

$$\widehat{Q} = N_E \int_0^{\widehat{c}_D} \widehat{q}(c) dG(c) = (1 - N_E f_E) \frac{k+2}{k} (\widehat{c}_D)^{-1},$$

where the final equality utilizes $\widehat{q}(c) = \frac{\widehat{\lambda}}{\gamma}(\widehat{c}_D - c)$, the solution for $\widehat{\lambda}$ given in (B29) and the Pareto distribution (1). Similarly, the market aggregate $\widetilde{Q}$ for the fixed-N_E setting is given as

$$\widetilde{Q} = N_E \int_0^{\widetilde{c}_D} \widetilde{q}(c) dG(c) = (1 - N_E f_E) \frac{k+2}{k} (\widetilde{c}_D)^{-1},$$

where the final equality utilizes $\widetilde{q}(c) = \frac{\widetilde{\lambda}(\widetilde{c}_D - c)}{2\gamma}$, the solution for $\widetilde{\lambda}$ given in (B33) and the Pareto distribution (1). Given $\widetilde{c}_D > \widehat{c}_D$, it follows immediately that $\widehat{Q} > \widetilde{Q}$. The aggregate quantity of the market is thus too low.

Finally, we may also compare the quantity allocation functions. Following the two steps in the proof of Proposition 6, we first divide (B33) by (B29) and obtain

$$\text{(B35)} \qquad \frac{\widetilde{\lambda}(\widetilde{c}_D)^{k+2}}{\widehat{\lambda}(\widehat{c}_D)^{k+2}} = 2.$$

Since $\widetilde{c}_D > \widehat{c}_D$, $\widehat{c}_D > 0$ and $k \geq 1$, we can also write

$$\text{(B36)} \qquad \left(\frac{\widetilde{c}_D}{\widehat{c}_D} \right)^{k+1} = \kappa$$

for some $\kappa > 1$. Dividing (B35) by (B36), we obtain

$$\frac{\widetilde{\lambda}\widetilde{c}_D}{\widehat{\lambda}\widehat{c}_D} = \frac{2}{\kappa} < 2.$$

We thus conclude that

$$\text{(B37)} \qquad \widetilde{\lambda}\widetilde{c}_D < 2\widehat{\lambda}\widehat{c}_D.$$

For the second step, we recall from above that

$$\widehat{q}(c) = \max\{\frac{\widehat{\lambda}}{\gamma}(\widehat{c}_D - c), 0\}, \forall c \in [0, c_M].$$

$$\widetilde{q}(c) = \max\{\frac{\widetilde{\lambda}}{2\gamma}(\widetilde{c}_D - c), 0\}, \forall c \in [0, c_M].$$

We can thus write

$$\widetilde{q}(0) - \widehat{q}(0) = \frac{\widetilde{\lambda}\widetilde{c}_D - 2\widehat{\lambda}\widehat{c}_D}{2\gamma} < 0,$$

where the inequality follows from (B37). We also know that

$$\widetilde{q}(\widehat{c}_D) - \widehat{q}(\widehat{c}_D) = \widetilde{q}(\widehat{c}_D) = \frac{\widetilde{\lambda}}{2\gamma}(\widetilde{c}_D - \widehat{c}_D) > 0,$$

where the inequality follows from $\widetilde{\lambda} > 0$ and $\widetilde{c}_D > \widehat{c}_D$.

Given these findings about the endpoints and that $\widehat{q}(c)$ and $\widetilde{q}(c)$ are linear functions for $c \in [0, \widehat{c}_D]$, it follows that $\widehat{q}(c)$ is steeper over this range (i.e., $\widehat{\lambda} > \widetilde{\lambda}/2$). We thus conclude that there exists a critical cost level $c_{SR} \in (0, \widehat{c}_D)$ such that $\widehat{q}(c) > \widetilde{q}(c)$ for $c \in [0, c_{SR})$, $\widehat{q}(c_{SR}) = \widetilde{q}(c_{SR})$, and $\widehat{q}(c) < \widetilde{q}(c)$ for $c \in (c_{SR}, \widetilde{c}_D]$.