Online Appendix: Keeping up with "The Joneses": reference dependent choice with social comparisons

By Alastair Langtry*

Keeping up with "The Joneses" matters. This paper examines a model of reference dependent choice where reference points are determined by social comparisons. An increase in the strength of social comparisons, even by only a few agents, increases consumption and decreases welfare for everyone. Strikingly, a higher marginal cost of consumption can increase welfare. In a labour market, social comparisons with co-workers create a big fish in a small pond *effect, inducing incomplete labour market sorting. Further, it is the skilled workers with the weakest social networks who are induced to give up income to become the* big fish.
JEL: D60, D85, D91, J24
Keywords: social comparisons, reference dependence, networks, labour market sorting

Appendix A: A Comment for Policy-makers

Consider a special case where agents' reference strengths are uncorrelated with the structure of the social network. Its key feature is that the structure of the social network will not have any impact on outcomes.

ASSUMPTION A.1 (Uncorrelated case): *The reference structure, g, is uncorrelated with the reference strength: $corr(g_{ij}, \alpha_j) = 0$ for all agents i, j.*

While Assumption A.1 is appears rather abstract, it covers the natural case where all agents have the same reference strength ($\alpha_i = \alpha$ for all i), so it is not vacuous. Under this assumption, the network effects drop out.

COROLLARY A.1: *Under Assumption A.1: x_i^* and u_i^* do not depend on the reference structure, g.*

Assumption A.1 requires that i's neighbours are, on average, "Mr and Mrs Average" in terms of how strongly they make social comparisons. In turn, their

* University of Cambridge and King's College, Cambridge CB2 1ST, UK (email: atl27@cam.ac.uk).
Leslie Marx was coeditor for this article. I am grateful to two anonymous referees, Matthew Elliott, Lukas Freund, Edoardo Gallo, Charles Parry and Gabriela Stockler for insightful comments and discussions, and to seminar participants at the University of Cambridge. This work was supported by the Economic and Social Research Council [award ES/P000738/1]. I declare that I have no relevant or material financial interests that relate to the research described in this paper Any remaining errors are the sole responsibility of the author.

neighbours are also, on average, "Mr and Mrs Average", and so on throughout the whole social network. There is no way for a global network effect to build – it will always get averaged out. The clearest case where this assumption holds is where all agents have the same reference strength. This shows how homogeneity of the local social comparisons shuts down the effect of heterogeneity in global network structure.

This simplification also allows us to make stronger claims about the impact of reference strengths. First, an equilibrium exists if and only if the average reference strength is less than one, and Bonacich centralities collapse to a simple function of the reference strengths (in particular, $C_i^b = 1 + \frac{\alpha_i}{1-\overline{\alpha}}$ for all i).

The tight link between an agent's reference strength and their optimal consumption makes it straightforward to see how changes in the distribution of these reference strengths feeds through into the distribution of consumption. First [resp. second] order stochastically dominant shifts in the distribution of the the references strengths causes a first [resp. second] order stochastically dominant shift in the distribution of equilibrium consumption.[1] Inequality in social comparisons drives consumption inequality, even in a setting where agents are otherwise identical.

Implications for policy-making. Measuring networks in practice – especially the weighted networks present in my model – is typically difficult and resource intensive. In a perfect world with unlimited resources and attention, we should of course examine whether this assumption holds in a network, and proceed accordingly. In reality, however, corollary A.1 presents a pragmatic approach for policy-makers and governments short on time and money, so long as the assumption is not violated too egregiously.

Proofs

PROOF OF COROLLARY A.1:

First, let $a \equiv \text{diag}(\alpha)$ for convenience. Using a Neumann Series representation, we can express $(I - ag)^{-1} = \sum_{k=1}^{\infty}(ag)^k$. Therefore, $C_i^b = \sum_j (I - ag)_j^{-1} = \sum_j \sum_{k=1}^{\infty}[(ag)^k]_{ij}$. We deal with the first two terms individually, and then all further terms by induction. Clearly $\sum_j \mathbf{1}\{i = j\} = 1$, and $\sum_j (ag)_{ij} = \alpha_i \sum_j g_{ij} = \alpha_i$ by construction. Induction: for $k = 2$;

$$\sum_j (ag)_{ij}^2 = \sum_j \sum_s (ag)_{is}(ag)_{sj} \ = \sum_j \sum_s \alpha_i g_{is} \alpha_s g_{sj} \ = \alpha_i \sum_s g_{is}\alpha_s \sum_j g_{sj}$$

[1]First Order Stochastic Dominance and Second Order Stochastic Dominance formalise the notions of (unambiguously) "bigger" and "more spread out" respectively. They are first due to Hadar and Russell (1969) and Rothschild and Stiglitz (1970). More modern coverage can be found in (Mas-Colell et al., 1995, Ch 6.D). I prove the claim regarding equilibrium outcomes below. The additional claims follow straightforwardly from this.

By Assumption A.1: $\sum_s g_{is}\alpha_s = \overline{\alpha}\sum_s g_{is}$. Then notice that $\sum_j g_{sj} = 1$ for any s. This yields: $\sum_j (ag)^2_{ij} = \alpha_i \cdot \overline{\alpha}$. Now assume that for $k = t > 2$, $\sum_j (ag)^t_{ij} = \alpha_i \cdot \overline{\alpha}^{t-1}$. Then show for $k = t+1$. By definition $\sum_j G^{t+1}_{ij} = \sum_j [(ag)\cdot(ag)^t]_{ij}$. For clarity of exposition, we let $G^t_{ij} = \mathcal{A}_{ij}$.

$$\sum_j (ag)^{t+1}_{ij} = \sum_j \sum_s \alpha_i g_{is}\mathcal{A}_{sj} \ = \alpha_i \sum_s g_{is} \sum_j \mathcal{A}_{sj}$$

Using the assumption for $k = t$, $\sum_j \mathcal{A}_{sj} = \alpha_s \overline{\alpha}^{t-1}$. Therefore, $\sum_j (ag)^{t+1}_{ij} = \alpha_i \cdot \overline{\alpha}^{t-1}\sum_s g_{is}\alpha_s$. Using Assumption A.1 and the fact that rows of g sum to 1 as before: $\sum_j (ag)^{t+1}_{ij} = \alpha_i \cdot \overline{\alpha}^t$. Therefore, $C^b_i = \sum_j \sum_{k=0}^{\infty} [(ag)^k]_{ij} = 1 + \alpha_i(1 + \overline{\alpha} + \overline{\alpha}^2 + \overline{\alpha}^3 + ...)$. Clearly this does not depend on the network. Finally, it follows from Remark 1 that x^*_i and u^*_i do not depend on the network structure, g.

REMARK A.1: *Under Assumption A.1: (i) an equilibrium exists if and only if* $\overline{\alpha} < 1$*, and (ii) when* $\overline{\alpha} < 1$*:* $C^b_i = 1 + \frac{\alpha_i}{1-\overline{\alpha}}$ *for all* i*, where* $\overline{\alpha} = \frac{1}{n}\sum_i \alpha_i$

PROOF:

From the proof to Corollary A.1, we have $C^b_i = 1 + \alpha_i/(1-\overline{\alpha})$. An equilibrium exists if and only if Bonacich centrality, C^b_i is well defined. Clearly this happens if and only if $\overline{\alpha} < 1$.

Appendix B: Non-linear sub-utility

The model in Section 1 assumes that the sub-utility is linear. This was a simplification compared to the Kőszegi and Rabin (2006) benchmark. However, this was only for clarity of exposition. It is easier to understand how the model works when more things are linear. It also allows the proofs to use simpler machinery that readers, especially those somewhat familiar wit networks, ought to be more comfortable with.

Here, I introduce non-linear sub-utility. The model is exactly the same as in Section 1, except that i's utility function is now:

$$u_i = f\left(m(x_i) - \alpha_i \sum_j g_{ij} m(x_j)\right) - cx_i + b_i \sum_j \alpha_i g_{ij} \tag{1*}$$

where $m(\cdot)$ is twice continuously differentiable, strictly increasing, strictly concave, and WLOG $m(0) = 0$. I will not provide intuition alongside the results because while the maths becomes more complex with this additional non-linear function, there is no new economic insight.

One disadvantage of introducing this non-linear sub-utility is that a closed form characterisation of equilibrium behaviour is no longer possible. This makes the network effects far harder to see. Nevertheless, existence and uniqueness are unaffected.

REMARK B.1: *Suppose $\alpha_i < 1$ for all i. There is a unique Nash Equilibrium.*

Proposition 1 then goes through unaffected.

PROPOSITION B.1: *If $\alpha_i < 1$ for all i: (i) x_i^* is weakly increasing, and (ii) u_i^* is weakly decreasing, in α_j for all i, j, and strictly so if $i = j$.*

Proposition 2 is subject to some minor modification because the threshold in part (ii) is difficult to characterise with non-linear $m(\cdot)$

PROPOSITION B.2: *If $\alpha_i < 1$ for all i: (i) x_i^* is strictly decreasing in c, (ii) u_i^* is strictly increasing in c if x_i^* is sufficiently sensitive to c (specifically, if $\frac{dx_i^*}{dc} < \min\{\frac{-x_i^*}{c}, \frac{m'(x_i^*)}{c \cdot m''(x_i^*)}\}$).*

Without a closed form solution for x_i^*, it is not possible to show the tight link between changes to the network structure and changes in equilibrium consumption (i.e. an analogue to Proposition 3). Nevertheless, simply looking at the First Order Condition is suggestive.

$$m(x_i) - \alpha_i \sum_j g_{ij} m(x_j) = F\left(\frac{c}{m'(x_i)}\right). \tag{B.1}$$

We can see that agents' actions are complements. If one of i's neighbours consumes more, this pushes up i's reference point, which in turn pushes her to consume more in an effort to keep up. Intuitively, if i shifts from comparing herself with low consumption friends to high consumption neighbours, then her reference point increases, and so she chooses to consume more. The more "The Joneses" consume, the more you need to do to keep up with them.

Proofs

PROOF OF REMARK B.1:

Consider the First Order Condition [FOC] (with some minimal rearranging) – Equation (B.1). LHS is strictly increasing in x_i. $m'(x_i)$ is strictly decreasing in x_i, and $F(\cdot)$ is strictly decreasing in its argument. So RHS is strictly decreasing in x_i. Hence there is a unique value of x_i that solves the First Order Condition. I use x_i^* to denote *equilibrium* consumption (i.e. when *all* FOCs hold simultaneously). For clarity, let $\hat{x}_i$ denote the value that solves just agent i's FOC, for given values of x_j, $j \neq i$.

Existence. Note that $\hat{x}_i$ is increasing in x_j. So if no agent wants to choose more than A when all other agents are choosing A, then there cannot exist an equilibrium where any agent chooses A or more. Suppose all other agents choose $x_j = A$. Then i wants to choose less than A if and only if: $m(A)[1 - \alpha_i] > F(c/m'(A))$.[2] Since LHS is increasing in A, and RHS is decreasing in A, there must exist some finite value of A such that this inequality is true for all i. Denote this $\hat{A}$. Given this, we only need to consider the *compact* space $[0, \hat{A}]^n$. The functions are continuous by assumption. Therefore Brouwer's Fixed Point Theorem (e.g. Border (1985)) guarantees existence.

Uniqueness. Proof by contradiction. Denote the equilibrium with the smallest action x^*. Suppose there is another equilibrium, $x^{*\prime} \equiv x^* + D$ (it is convenient and WLOG to write the second equilibrium as the initial one, plus some change). Consider the agent i whose action increases the most when moving from x^* to $x^{*\prime}$. That is, the agent i such that $D_i \geq D_j$ for all $j \neq i$. Solve the first order condition for this agent i, assuming that all other agents are playing the equilibrium $x^{*\prime}$. That is, we solve:

$$x_i - \alpha_i \sum_j g_{ij}[x_j^* + D_j]) = F\left(\frac{c}{m'(x_i)}\right)$$

[2]Note that after we pulled $m(A)$ out of the summation, $\sum_j g_{ij} = 1$, and so disappears.

WLOG let the solution takes the form $x_i^* + z_i$.

$$x_i^* + z_i - \alpha_i \sum_j g_{ij} x_j^* - \alpha_i \sum_j g_{ij} D_j = F\left(\frac{c}{m'(x_i^* + z_i)}\right)$$

Substituting in the solution to the FOC from the initial equilibrium x^* (and rearranging):

$$z_i - \alpha_i \sum_j g_{ij} D_j = F\left(\frac{c}{m'(x_i^* + z_i)}\right) - F\left(\frac{c}{m'(x_i^*)}\right)$$

Since $F(\cdot)$ is decreasing in its argument.

$$z_i - \alpha_i \sum_j g_{ij} D_j < 0$$

This actually assumes that $z_i > 0$. But if $z_i \leq 0$ then we have already reached a contradiction, since we require the increase in i's action to be larger than the increase in all other agents' actions (when moving from x^* to $x^{*\prime}$). Then applying the requirement that $D_i \geq D_j$ for all $j \neq i$:

$$z_i < \alpha_i \sum_j g_{ij} D_j \leq \alpha_i \sum_j g_{ij} D_i = \alpha_i D_i < D_i$$

Contradiction.

PROOF OF PROPOSITION B.1:

For some fixed parameters g, α, c, b and functions $f(\cdot)$, $m(\cdot)$ there is a unique equilibrium x^*. **(i)** Suppose that $\alpha_j \uparrow$ for some j. Then $\hat{x}_j \uparrow$ (i.e. j's optimal action rises *conditional on* all other agents' actions). In turn $\hat{x}_j \uparrow \implies \hat{x}_k \uparrow$ for all k s.t. $g_{kj} > 0$. In turn this increases x_i for all i (but only weakly so, since there is no guarantee that there exists a directly path from j to i).

Remark B.1 guarantees a unique equilibrium. So this process must eventually converge to an equilibrium. But clearly all equilibrium actions weakly rises, and strictly so for the "initial" agent (who experienced the increase in α_j). **(ii)** In equilibrium we have:

$$m(x_i^*) - \alpha_i \sum_j g_{ij} m(x_j^*) = F\left(\frac{c}{m'(x_i^*)}\right)$$

We can substitute this back into the utility function to find equilibrium utility.

$$u_i^* = f\left(F\left[\frac{c}{m'(x_i^*)}\right]\right) - cx_i^*$$

We know from (i) that $\alpha_j \uparrow$ weakly increases x_i^*. Finally, notice that u_i^* is strictly decreasing in x_i^*. To see this clearly, note that $x_i^* \uparrow \implies m'(x_i^*) \downarrow \implies F(c/m'(x_i^*)) \downarrow \implies f(F(c/m'(x_i^*))) \downarrow$.

PROOF OF PROPOSITION B.2:

For some fixed parameters g, α, c, b and functions $f(\cdot)$, $m(\cdot)$ there is a unique equilibrium x^*. **(i)** Suppose that $c \uparrow$. This decreases RHS of the FOC (Equation (B.1)). To restore equality, it must be that $\hat{x}_i \downarrow$ (as this decreases LHS and increases RHS). In turn $\hat{x}_i \downarrow \implies \hat{x}_j \downarrow$. Remark B.1 guarantees a unique equilibrium. So this process must eventually converge to an equilibrium. But all equilibrium actions strictly decrease. **(ii)** Equilibrium utility is:

$$u_i^* = f\left(F\left[\frac{c}{m'(x_i^*)}\right]\right) - cx_i^*.$$

This was derived in the proof to Proposition B.1. It is clear that a sufficient condition for u_i^* to be strictly increasing in c is that: (a) cx_i^* and (b) $c/m'(x_i^*)$ are strictly decreasing in c (recall that $f(F(\cdot))$ is decreasing in its argument)[3] Consider each condition separately. First,

$$\frac{d(cx_i^*)}{dc} = c \cdot \frac{dx_i^*}{dc} + x_i^*,$$

which is less that zero if and only if $\frac{dx_i^*}{dc} < -x_i^*/c$. Second,

$$\frac{d(c/m'(x_i^*))}{dc} = [m'(x_i^*)]^{-1} - cm''(x_i^*)[m'(x_i^*)]^{-2}\frac{dx_i^*}{dc}$$

which is less than zero if and only if $\frac{dx_i^*}{dc} < -m'(x_i^*)/[c \cdot m''(x_i^*)]$.

An equivalent sufficient condition for u_i^* to be strictly decreasing in c would be easy. The method for proving it would be similar to above. The condition would simply be $\frac{dx_i^*}{dc} > \max\{\frac{-x_i^*}{c}, \frac{m'(x_i^*)}{c \cdot m''(x_i^*)}\}$. Due to the inability to find closed form solutions for x_i^* when $m(\cdot)$ is nonlinear, I am not able to provide a tight link between network centrality and how welfare is affected by cost changes. Nevertheless, the flavour of Proposition 3 goes through – agent's whose consumption is

[3]So a decrease in $c/m'(x_i^*)$ increases $f(F(c/m'(x_i^*)))$.

highly sensitive to costs end up experiencing welfare gains when the marginal cost of consumption rises. This is again because the benefits (in terms of a lower reference point) of lower consumption by neighbours more than offsets the directly higher cost of consumption.

Appendix C: Loss Aversion

In Section 1 we assume that $u_i = f(x_i - \alpha_i \sum_j g_{ij} x_j)$, where $f(\cdot)$ is strictly increasing and concave. This captures references dependence and diminishing sensitivity, but *not* loss aversion. To capture loss aversion, we now assume that $f(\cdot)$ is strictly concave [resp. convex] in the positive [resp. negative] domain, and kinked at zero. This is in line with the Kahneman and Tversky's canonical setup (Tversky and Kahneman, 1979, 1991). Formally, we can write these properties as the following restrictions on the function $f(\cdot)$: (i) $f : a \to \mathbb{R}$ for $a \in \mathbb{R}$, (ii) $f'(a) > 0\ \forall a$, (iii) $f''(a) < 0\ \forall a > 0$, $f''(a) > 0\ \forall a < 0$, (iv) $\lim_{a \to 0^-} |f'(a)| > \lim_{a \to 0^+} |f'(a)|$, (v) $f''(a) \in \mathbb{R}\ \forall a \neq 0$, $f(0) = 0$.

Adding the functional form requirements for loss aversion does not affect Remark 1. This is because agents are always able to, and always choose to, consume above their reference point in this model. Therefore the convexity in the negative domain that is the core addition of loss averse preferences (over and above reference dependent preferences) has no bite.

Appendix D: Multiple Goods

In section 1 we assume that there is only 1 good. Here, consider the model with K goods (i.e. a K-dimensional consumption bundle), as in Kőszegi and Rabin (2006). This generalisation has no effect.

There are K goods, $x_1, ..., x_K$. Each agent i simultaneously chooses a consumption bundle $(x_{i1}, x_{i2}, ..., x_{iK}) \in \mathbb{R}^n_+$. Following Kőszegi and Rabin (2006), I assume that preferences are additively separable over goods. All other elements of the model are the same as in section 1. Therefore i's utility function is:

$$u_i = \sum_{k=1}^{K} \left[f\left(x_{ik} - \sum_j \alpha_i g_{ij} x_{jk} \right) - c_k x_{ik} \right] + b_i \sum_j \alpha_i g_{ij} \tag{D.1}$$

where $f(\cdot)$ has the same properties as in section 1. Now consider agents' First Order Conditions.

$$\frac{du_i}{dx_{ik}} = f'\left(x_{ik} - \sum_j \alpha_i g_{ij} x_{jk} \right) - c_k = 0 \quad \text{for all } i \quad \text{and for all } k \tag{D.2}$$

These First Order Conditions are clearly identical to those in the 1-good case.

Obviously there are now K First Order Conditions for each agent, but x_{ik} only appears in the First Order Conditions relating to good k, and never in any relating to $k' \neq k$. Therefore the solution for each good k is the same as it would be if k were the only good.

Appendix E: Network Structure

This section presents technical results regarding the effects of comparison shifts, and formalises claims made in section 2. The natural starting point is to characterise the exact impact of a single comparison shift.

LEMMA E.1: *The change in agent i's consumption due a comparison shift D, Δx_i^*, is equal to:*

$$\frac{\phi B_{ir}(C_u^b - C_d^b)}{1 - \phi(B_{ur} - B_{dr})} \cdot f'^{-1}(c)$$

This exact characterisation is not very user-friendly and is largely technical. However, it is useful because it forms the basis for a number of further results, most importantly Proposition 3. Its implications for the effects of a comparison shift on different agents are immediate.

COROLLARY E.1: *Given a comparison shift D, the change in optimal actions is:*
(i) in the same direction for all agents,
(ii) proportional to the amount that agent i compares herself to r (the subject of the comparison shift) prior to the shift.

It also allows us to examine the nature of the returns to the magnitude of a comparison shift. When an agent r moves from direct comparison with a lower centrality agent to direct comparison with a higher centrality agent (i.e. $C_u^b > C_d^b$), then there are increasing returns to the magnitude of a comparison shift if and only if agent r influences agent u more than she influences agent d, but the gap is not too large.

COROLLARY E.2: *Given a comparison shift D, if $C_u^b > C_d^b$, then for all i, x_i^* is: convex in ϕ if and only if $B_{ur} - B_{dr} \in (0, \frac{1}{\phi})$, and concave in ϕ otherwise.*

Now consider a *composite comparison shift*, $\widehat{D} = D_1 + ... + D_Z$, which is constructed by summing up $n \geq 2$ comparison shifts. The impact of a composite comparison shift is not equal to the sum of the effects of each comparison shift that form part of it. This is because the impact of any one comparison shift depends on the *whole* network immediately before it occurs.

COROLLARY E.3: *Consider a composite comparison shift $\widehat{D}$. The impact of $\widehat{D} = D_1 + ... + D_Z$ is not the same as the sum of the impacts of $D_1, ..., D_Z$.*

For a given network, it would be straightforward to calculate the change in actions due to a more complex change in the reference structure using an application of the formula provided by Chang (2006). However, we cannot obtain analytic results due to the interactions between the effects of each comparison shift. Even with a few comparison shifts, the outcome would be too complicated to yield any insight. However, if the comparison shifts are all of an equal size, for example Δ, then interactions between the are of the order Δ^2. Therefore, when considering small changes to the network (i.e. small $Z \cdot \Delta$) we can reasonably disregard the interactions – a naive summation is a close approximation for the actual aggregate effect.

COROLLARY E.4: *If the total change to the network, $Z \cdot \Delta$, is small,then the impact of a composite comparison shift $\widehat{D}$ approximately equal to the sum of the impacts of each comparison shift that makes up $\widehat{D}$.*

This approach works because we are able to take a linear approximation (i.e. ignore any terms that are $\mathcal{O}(\delta^2)$). The total change to agents' Bonacich centrality will be well approximated by simply adding up the effects of each comparison shift. However, we should be very wary of extrapolating from small changes to large ones. It is difficult to characterise the interactions and small changes may not be indicative of large ones. Past observations from small or localised changes may cease to be a useful guide in the face of large-scale social change happens.

Finally, we prove that assuming $\alpha_i < 1$ for all i is sufficient to guarantee existence, even after a comparison shift (something we claimed, but did not prove, in section 2).

COROLLARY E.5: *If $\alpha_i < 1 \; \forall i \in N$ then $(I-(ag+D))^{-1}$ exists for any network ag, and any comparison shift D.*

However, if $\alpha_i > 1$ for some i, then it is possible that a solution exists for some, *but not all*, reference structures g. In this instance it is necessary to check that a solution exists both before and after the shift. That is: $\lambda_1(G) < 1$ and $\lambda_1(G + D) < 1$. While it is possible for only one of these conditions to be met, the results cannot apply unless both hold.

Proofs

PROOF OF LEMMA E.1:

This is a restatement of Lemma 3 (which was needed to prove other results from the main text).

PROOF OF COROLLARY E.1:

Proposition E.1 characterises the change in agent i's optimal action, Δx_i^*. The only term in the expression for Δx_i^* that depends on i is B_{ir}. **(i)** Since $B_{ij} \geq 0$ for all i, j, Δx_i^* has the same sign for all i. **(ii)** Δx_i^* is equal to B_{ir} multiplied by some terms that do not depend on i. Therefore Δx_i^* is proportional to B_{ir} (i.e. the amount that i compares herself to r).

PROOF OF COROLLARY E.2:

For ease of exposition, let: $B_{ir}(C_u^b - C_d^b)f'^{-1}(c) \equiv y$ and $(B_{ur} - B_{dr}) \equiv z$. Let $\hat{x}_i^* \equiv x_i^* + \Delta x_i^*$ be the new equilibrium value of consumption. So $\hat{x}_i^* = x_i^* + y\phi(1 - z\phi)^{-1}$. Now consider the second derivative. $\frac{d^2\hat{x}_i^*}{d\phi^2} = 2yz(1 - z\phi)^{-3}$. We can partition values of z into three cases: (i) $z < 0$, (ii) $0 \leq z \leq \frac{1}{\phi}$, (iii) $z > \frac{1}{\phi}$. In case (ii), $\frac{d^2\hat{x}_i^*}{d\phi^2} \geq 0$. In cases (i) and (iii), $\frac{d^2\hat{x}_i^*}{d\phi^2} < 0$. These observations follow straightforwardly from the fact that $y > 0$ and $\phi > 0$. The second derivative determines convexity/concavity. Note that if $(C_u^b - C_d^b) < 0$ then $y < 0$ and so all results flip.

PROOF OF COROLLARY E.3:

W.L.O.G any composite comparison shift $\hat{D}$ can be expressed as the sum of comparison shifts $D_1, ..., D_Z$: $\hat{D} \equiv \sum_{i=1}^{Z} D_i$. It follows from Lemma 2 that H is a function of both the comparison shifts D_i and $(I - G)^{-1}$.

Trivially $(I - G - X)^{-1} \neq (I - G)^{-1}$ for any $X \neq 0$. Therefore the effect of a comparison shift D_i depends on the network immediately prior to the comparison shift: $H(D_i, (I - G - \sum_{j=1}^{i-1} D_i)^{-1}) \neq H(D_i, (I - G)^{-1})$. The effect of a composite comparison shift $\hat{D}$ is therefore not equal to the sum of the effects of individual comparison shifts: $\sum_{i=1}^{Z} H(D_i, (I - G - \sum_{j=1}^{i-1} D_i)^{-1}) \neq \sum_{i=1}^{Z} H(D_i, (I - G)^{-1})$.

PROOF OF COROLLARY E.4:

Let the composite comparison shift $\hat{D}$ be constructed from a series of comparison shifts $(D_1, ..., D_Z)$, each of a size Δ. $H(D_i, (I - G - X)^{-1})$ is the change in the optimal actions following a comparison shift D_i, when the starting network is $(G + X)$. From Lemma 2: $H(D_i, (I - G)^{-1}) = \Delta \cdot fn((I - G)^{-1}) = \mathcal{O}(\Delta)$. Then $H(D_i, (I - G - D_j)^{-1}) = \Delta \cdot fn((I - G - D_j)^{-1}) = \Delta \cdot fn((I - G)^{-1} + \mathcal{O}(\Delta)) = H(D_i, (I - G)^{-1}) + \mathcal{O}(\Delta^2)$. By a simple induction argument we can see that $H(D_i, (I - G - \sum_{j=1}^{J} D_j)^{-1}) = H(D_i, (I - G)^{-1}) + \mathcal{O}(\Delta^2)$. Therefore the effect of the earlier comparison shifts $D_1, ..., D_{i-1}$ on the change in optimal actions induced by D_i is on the order Δ^2. If $Z \cdot \Delta$ is sufficiently small, then we can ignore these interaction effects, which are collectively of the order $Z \cdot \Delta^2$.

PROOF OF COROLLARY E.5:

This follows trivially from Remark 1, which proves that $\alpha_i < 1$ for all i is a sufficient condition to ensure equilibrium existence for any network G. Since a comparison shift leaves α_i unchanged for all i (by definition), then Remark 1 continues to apply.

Appendix F: Chang (2006): A perturbation theorem

I restate Lemma 2 for clarity and then present a proof. This is a special case of Chang (2006), which simplifies the proof. I have also aligned the notation to match my paper.

LEMMA 2 (A perturbation theorem. Chang (2006)): If D is a comparison shift, then $(I-[G+D])^{-1}-(I-G)^{-1}=H$. Where:

$$H=\frac{\phi}{1-\phi(B_{ur}-B_{dr})}\begin{pmatrix}B_{1r}(B_{u1}-B_{d1}) & \cdots & B_{1r}(B_{un}-B_{dn})\\ \vdots & \ddots & \vdots\\ B_{nr}(B_{u1}-B_{d1}) & \cdots & B_{nr}(B_{un}-B_{dn})\end{pmatrix}$$

PROOF OF LEMMA 2:

This proof is a simplified version of Chang (2006). By the Woodbury Identity Matrix: if A and D are $n\times n$ matrices, and A is non-singular, then $(A-D)^{-1}=E+E(I-DE)^{-1}DE$, where $E\equiv A^{-1}$ [Woodbury (1950) and Sherman and Morrison (1949)]. Now partition the matrices D and E:

$$D=\begin{bmatrix}\overline{\underline{D}} & 0\\ 0 & 0\end{bmatrix}\qquad E=\begin{bmatrix}\overline{\underline{E}} & E_2\\ E_1 & E_3\end{bmatrix}\qquad \overline{E}=\begin{bmatrix}\overline{\underline{E}}\\ E_1\end{bmatrix}\qquad \underline{E}=\begin{bmatrix}\overline{\underline{E}} & B_E\end{bmatrix}$$

where $\overline{\underline{D}}$ is the smallest matrix that contains non-zero elements of D, and $\overline{\underline{E}}$ contains the transpose of the elements in $\underline{E}$.[4] Simple matrix algebra yields: $(A-D)^{-1}=E+\overline{E}(I-\overline{\underline{D}}\,\overline{\underline{E}})^{-1}\overline{\underline{D}}\,\underline{E}$.

Now recall that D is a 'comparison shift' (as per Definition 2). Therefore: $\overline{\underline{D}}=\begin{bmatrix}D_{ru} & D_{rd}\end{bmatrix}=\begin{bmatrix}\phi & -\phi\end{bmatrix}$, and so:

$$\overline{\underline{E}}=\begin{bmatrix}E_{ur}\\ E_{dr}\end{bmatrix}\ ,\ \overline{E}=\begin{bmatrix}E_{1r}\\ \vdots\\ E_{nr}\end{bmatrix}\ \text{and}\ \underline{E}=\begin{bmatrix}E_{u1} & \cdots & E_{un}\\ E_{d1} & \cdots & E_{dn}\end{bmatrix}$$

This yields;

$$(A-D)^{-1}=E+\begin{bmatrix}E_{1r}\\ \vdots\\ E_{nr}\end{bmatrix}\cdot\left(I-\begin{bmatrix}D_{ru} & D_{rd}\end{bmatrix}\begin{bmatrix}E_{ur}\\ E_{dr}\end{bmatrix}\right)^{-1}\cdot\begin{bmatrix}D_{ru} & D_{rd}\end{bmatrix}\cdot\begin{bmatrix}E_{u1} & \cdots & E_{un}\\ E_{d1} & \cdots & E_{dn}\end{bmatrix}$$

Now multiply out, substitute in $D_{ru}=\phi$ and $D_{rd}=-\phi$, noticing that the matrix

[4] So if $\overline{\underline{D}}$ consists of elements $\overline{\underline{D}}_{ij}$ for $i\in I$, $j\in J$, then $\overline{\underline{D}}$ consists of elements $\overline{\underline{D}}_{ji}$ for $i\in I$, $j\in J$.

inverse $(I - \underline{D}\ \underline{E})^{-1}$ is a scalar, and rearrange.

$$(A - D)^{-1} - A^{-1} = \frac{\phi}{1 - \phi(B_{ur} - B_{dr})} \begin{pmatrix} B_{1r}(B_{u1} - B_{d1}) & \cdots & B_{1r}(B_{un} - B_{dn}) \\ \vdots & \ddots & \vdots \\ B_{nr}(B_{u1} - B_{d1}) & \cdots & B_{nr}(B_{un} - B_{dn}) \end{pmatrix}$$

This result holds for any non-singular $n \times n$ matrix A. Letting $A = (I - G)$ yields the result.

Appendix G: Heterogeneous Costs

The cost parameter c_i reflects agents' underlying propensity/ability to take the action, x_i. In this section, we examine the implications of introducing cost heterogeneity. The key takeaway is that allowing for heterogeneous costs has relatively little impact on the insights of the main model.

With this generalisation, equilibrium play depends on the individual entries in the full matrix B, rather than only on Bonacich centralities (the row sums of B). Nevertheless, the existence condition is unaffected, and the solution takes a similar form. At this point, it is helpful to interpret individual elements of the matrix B and to introduce a notion of "generalised Bonacich centrality".

DEFINITION G.1 (Comparisons): *The comparison matrix is $B \equiv (I - G)^{-1}$, where B_{ij} captures how much i compares herself to j.*

Bonacich centrality captures an agent's connectedness to the network as a whole. The comparison matrix breaks this down to the individual agent level. An element B_{ij} measures the total weight of walks from i to j, and captures the extent to which i compares herself to j. This is a dis-aggregation of Bonacich centrality. We can then weight these individual level comparisons with a function of the cost parameters to obtain a generalised notion Bonacich centrality.[5]

DEFINITION G.2 (Generalised Bonacich Centrality): *The vector of Bonacich centralities for a network $G \equiv ag$ is $C^{gen} = B \cdot f'^{-1}(c)$ The Bonacich centrality of agent i is $C_i^{gen} = \sum_j B_{ij} f'^{-1}(c_j)$*

With this definition we can restate Remark 1, accounting for heterogeneous costs. The condition for existence depends only on the network and so is unaffected by cost heterogeneity. However, the optimal actions are now proportional to our new notion of generalised centrality, rather than the usual Bonacich centralities.

REMARK G.1 (Existence and Solution): *With heterogeneous costs, an equilibrium exists if and only if $\lambda_1 < 1$. If this condition is met, then there is a unique Nash Equilibrium: $x_i^* = C_i^{gen}$*

[5]It is clear that when $c_j = c$ for all j this collapses back to the original Bonacich centrality.

PROOF OF REMARK G.1:

This follows from the proofs to Remark 1, with the only change that $f'^{-1}(c_j)$ now depends on j, and so cannot be pulled out through the summation sign.

The other results from Section 2 also extend to this heterogeneous cost setting. The proofs here only provide the required extension from their homogeneous cost analogues.

PROPOSITION G.1 (Reference strength): *If $\lambda_1 < 1$: (i) x_i^* is weakly increasing, and (ii) u_i^* is weakly decreasing, in α_j for all i, j, and strictly so if $i = j$.*

PROOF OF PROPOSITION G.1:

(i) Proposition 1 shows that if a given α_j increases, then all elements of $(ag)^k$ weakly increase for any k. Consequently, all elements $(I - ag)^{-1}_{ij}$ increase. **(ii)** having found that x_i^* is increasing in α_k, the second part of the proof to Proposition 1 goes through unchanged.

PROPOSITION G.2 (Cost): *If $\lambda_1 < 1$: (i) x_i^* is strictly decreasing and convex in c_j for all j, (ii) u_i^* is strictly increasing in c_j for all $j \neq i$.*

PROOF OF PROPOSITION G.2:

(i) follows straightforwardly from Remark G.1 and the fact that $F(\cdot)$ is strictly decreasing and convex. (ii) equilibrium utility is $u_i^* = f(x_i^* - \sum_{k \neq i} G_{ik} x_k^*) - c_i x_i^*$. Differentiate with respect to c_j:

$$\frac{du_i^*}{dc_j} = \left(\frac{dx_i^*}{dc_j} - \sum_{k \neq i} G_{ik} \frac{dx_k^*}{dc_j} \right) f'(\cdot) - c_i \frac{dx_i^*}{dc_j}$$

Now recall that $f'(x_i^* - \sum_j x_j^*) - c_i = 0$ in equilibrium, and that $x_i^* = C_i^{gen}$ for all i, $\frac{dx_k^*}{dc_j} = \frac{dC_k^{gen}}{dc_j} = B_{kj} F'(c_j) < 0$. Substituting these in yields:

$$\frac{du_i^*}{dc_j} = -f'(\cdot) \sum_{k \neq i} G_{ik} B_{kj} F'(c_j) < 0$$

This result is somewhat different to the homogeneous cost version. Because the cost parameter is now agent-specific, the outcome is much simpler. An increase in an agent j's cost pushes down her consumption, relaxing the need for others to keep up with "The Joneses" (in this case, agent j). This increases welfare for all $i \neq j$. Since i has not experienced an increase in her own costs, there is no off-setting effect. The impact of someone else's cost parameter on your welfare is unambiguous.

PROPOSITION G.3: *The change in agent i's action due a comparison shift D is equal to;*

$$\frac{\phi B_{ir}(C_u^{gen} - C_d^{gen})}{1 - \phi(B_{ur} - B_{dr})}$$

PROOF OF PROPOSITION G.3:
This follows from Proposition E.1 but replacing $C_i^b \cdot f'^{-1}(c)$ with C_i^{gen}.

All other results concerning comparison shifts also follow as a result of this, including an analogue to Proposition 3. This is because they are also based on Lemma E.1 (i.e. Lemma 3) in the homogeneous cost case.

PROPOSITION G.4 (Endogenous network): *In all pairwise stable networks, if* $b_i \geq c_i f'^{-1}(c_i)$*, then* $G_{ij} > 0$ *only if* $\frac{b_i}{c_i} = \frac{b_j}{c_j}$.

PROOF OF PROPOSITION G.4:
This follows straightforwardly from the proof to Proposition 4, replacing c with the agent-specific version as appropriate.

*

REFERENCES

Border, Kim C. 1985. *Fixed Point Theorems with Applications to Economics and Game Theory.* Cambridge University Press.

Chang, Feng Cheng. 2006. "Inversion of a perturbed matrix." *Applied Mathematics Letters*, 19(2): 169–173.

Hadar, Josef, and William R Russell. 1969. "Rules for ordering uncertain prospects." *The American Economic Review*, 59(1): 25–34.

Kőszegi, Botond, and Matthew Rabin. 2006. "A model of reference-dependent preferences." *The Quarterly Journal of Economics*, 121(4): 1133–1165.

Mas-Colell, Andreu, Michael Dennis Whinston, Jerry R Green, et al. 1995. *Microeconomic Theory.* Oxford University Press.

Rothschild, Michael, and Joseph E Stiglitz. 1970. "Increasing risk: I. A definition." *Journal of Economic Theory*, 2(3): 225–243.

Tversky, Amos, and Daniel Kahneman. 1979. "Prospect theory: An analysis of decision under risk." *Econometrica*, 47(2): 263–291.

Tversky, Amos, and Daniel Kahneman. 1991. "Loss aversion in riskless choice: A reference-dependent model." *The Quarterly Journal of Economics*, 106(4): 1039–1061.