A Appendix

In this Appendix, we provide detailed derivations of the results above.

A.1 A "Local" Spatial Economic Model

In this section, we provide a micro-foundation for the "local" supply and demand equations (1) and (2).

Consider a region comprising N different locations embedded in a larger economy. Agents can move freely across locations (or between these locations and the rest of the economy). Agents both produce and consume goods wherever they choose to live. Suppose that each location produces a homogeneous variety of good (e.g. corn). This good is freely traded across locations, but as all regions produce only that good there is no trade. We instead normalize the price of that good to one, $p = 1$, so that it provides a reference price to determine wages across locations.

The consumer problem is very simple in this context. Consumers maximize welfare,

$$W_i = c_i \times u_i \tag{7}$$

where c_i is the consumption in region i and u_i is a (non-consumption) amenity of residing in location i. The budget constraint of the consumer is simply $p \times c_i = w_i \iff c_i = w_i$, i.e. consumers consume an amount of the reference good equal to their wage.

The producer uses labor and capital to produce the final good. The production function in location i is given by:

$$Y_i = A_i K_i^{\theta} L_i^{1-\theta},$$

where $\theta \in [0, 1)$. Factor markets are perfectly competitive, so the marginal productivity of labor equals the wage and the marginal productivity of capital equals the rental rate:

$$w_i = (1-\theta) A_i K_i^{\theta} L_i^{-\theta}, \; r_i = \theta A_i K_i^{\theta-1} L_i^{1-\theta}.$$

It is evident that given capital the marginal productivity of labor declines with higher population and thus the wage in a location decreases when the population increases in that location. Assuming capital is fully mobile across locations so that the rental rate is equalized across locations (i.e. $r_i = r$), we have:

$$\frac{K_i}{L_i} = \left(\frac{r}{\theta A_i}\right)^{\frac{1}{\theta-1}}$$

and replacing in the wage equation yields:

$$w_i = (1-\theta)\, A_i^{-\frac{1}{\theta-1}} \left(\frac{r}{\theta}\right)^{\frac{\theta}{\theta-1}} \tag{8}$$

This model abstracts from a number of potentially important mechanisms, including other factors of production (like land), the consumption of non-tradables (like housing), possible heterogeneous preferences of different agents for different locations (e.g. I like beach and you like the mountains), economies of scale in production, etc. It turns out that a simple extension of the framework above is able to incorporate any combination of these different forces. Suppose that the productivity of a worker in a location depends in part on the total number of workers in that location:

$$A_i = \bar{A}_i L_i^{\alpha}, \tag{9}$$

where α may be positive or negative. Similarly, suppose that the amenity an agent derives from residing in a location depends in part on the total number of residents in that location:

$$u_i = \bar{u}_i L_i^{\beta}, \tag{10}$$

where again β may be positive or negative. In the model above, we have implicitly assumed $\alpha = \beta = 0$, but there are many reasons to think that α and β may be non-zero. For example, α may be negative if there is a fixed factor of production (like land) so that the more workers in a location, the less land there is per worker, driving down worker productivity. Alternatively, α may be positive if there are economies of scale in production. Similarly, β may be negative if residents also consume a local non-tradeable (like housing) that is in fixed supply, so that rent is driven up as the number of residents in a location increases. Or perhaps β is positive if greater population density induces greater supply of amenities (e.g. better parks). Allen and Arkolakis (2014) provide various micro-foundations for α and β along these lines.

Combining equations (8) and (9), we obtain the following labor demand equation:

$$\ln w_i = \frac{\alpha}{1-\theta} \ln L_i + \ln \left((1-\theta) \left(\frac{r}{\theta}\right)^{\frac{\theta}{\theta-1}} \left(\bar{A}_i\right)^{\frac{1}{1-\theta}} \right),$$

which is a special case of equation (1).

Similarly, because labor is perfectly mobile across locations, welfare equalization is equalized, i.e. $W = w_i \times u_i$. Combining welfare equalization with equation (10) yields the following labor supply equation:

$$\ln w_i = -\beta \ln L_i + \ln W \bar{u}_i^{-1},$$

which is a special case of equation (2).

A.2 A "Global" Spatial Economic Model

The goal in this section is to offer the derivations to the four equations comprising the equilibrium of the global spatial economy, namely equations (4), (4), (5) and (6). To do so, we rely on the same micro-economic foundations as in Allen and Arkolakis (2014), although as we discuss below, there are a number of alternative micro-economic foundations that also yield these equations (see e.g. Allen, Arkolakis, and Takahashi (2020)).

Consider a region comprising N different locations embedded in a larger economy. Agents can move freely across locations (or between these locations and the rest of the economy). Agents both produce and consume goods wherever they choose to live. Suppose that each location produces a distinct variety of good (e.g. French wine, Swiss cheese, ...). This assumption is called the "Armington" assumption (Armington (1969)). While clearly simplistic, it both makes the following derivations simpler and turns out to be mathematically equivalent to more realistic (but more complicated) models (see e.g. Eaton and Kortum (2002)).

Let us first consider the demand problem. Suppose that consumers like to consume many different varieties of goods. This "love of variety" creates an incentive for regions to trade with each other. In particular, we will assume that each agent has "constant elasticity of substitution" (CES) preferences such that if the agent lives in j and consumes quantity $\{q_{ij}\}_{j=1}^{N}$ of the variety of good from each location j she gets welfare:

$$W_j = \left(\sum_{i=1}^{N} q_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} u_j,$$

where $\sigma \geq 0$ is the elasticity of substitution (where higher σ indicates the agent is more willing to substitute one variety of good for another) and u_j is a (non-consumption) amenity of residing in location j. It is straightforward to show (but good practice to check!) that a utility-maximizing agent living in j with budget e_j and facing prices $\{p_{ij}\}_{i=1}^{N}$ will choose to spend $\{x_{ij}\}_{i=1}^{N}$, where:

$$x_{ij} = \frac{p_{ij}^{1-\sigma}}{\sum_{k=1}^{N} p_{kj}^{1-\sigma}} e_j.$$

and will receive welfare $W_j = e_j u_j / \left(\sum_{k=1}^{N} p_{kj}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$. Note that the share an agent spends of her income on each good does not depend on the level of her income, i.e. CES demand is homothetic. Since all agents living in a location face the same set of prices and CES demand is homothetic, we can can calculate the total amount spent by all agents living in location j on goods from i as:

$$X_{ij} = \frac{p_{ij}^{1-\sigma}}{\sum_{k=1}^{N} p_{kj}^{1-\sigma}} E_j, \tag{11}$$

where E_j is the total expenditure in j.

Now let us consider the supply problem. Suppose that the production of the differentiated varieties uses only labor as a factor of production and that each worker in location i can produce A_i units of the variety. Suppose too that it requires $t_{ij} \geq 0$ units of labor to ship a good from i to j. Finally, suppose that there is perfect competition, so that the price of goods are equal to their marginal cost of production. We then have that the price of a differentiated variety produced in i and sold in j is:

$$p_{ij} = \tau_{ij} \left(\frac{w_i}{A_i} \right), \tag{12}$$

where $\tau_{ij} \equiv 1 + t_{ij} A_i$ is the iceberg trade cost. Combining equations (11) and (12) yields the following gravity equation for trade flows:

$$X_{ij} = \frac{\tau_{ij}^{1-\sigma} \times \left(\frac{w_i}{A_i} \right)^{1-\sigma}}{\sum_k \tau_{kj}^{1-\sigma} \times \left(\frac{w_k}{A_k} \right)^{1-\sigma}} E_j. \tag{13}$$

Equation (13) is known as a *trade gravity equation* because it says that trade between locations is (a) proportional to the economic "size" of the origin and location; and (b) inversely proportional to the economic "distance" between the origin and destination. These two properties of trade flows are even more obvious when you re-write the equation as:

$$X_{ij} = T_{ij} \times \left(\frac{Y_i}{MA_i^{out}} \right) \times \left(\frac{E_j}{MA_j^{in}} \right), \tag{14}$$

where $T_{ij} \equiv \tau_{ij}^{1-\sigma}$ is the measure of (inverse) economic distance and we call $MA_j^{in} \equiv \sum_k \tau_{kj}^{1-\sigma} \times \left(\frac{w_k}{A_k} \right)^{1-\sigma}$ the *inward market access* and $MA_i^{out} \equiv \left(\frac{w_i}{A_i} \right)^{\sigma-1} Y_i$ the *outward market access*. As discussed in the main text, consumers in locations with higher inward market access benefit by being closer to the sellers of the goods they consume, whereas producers in locations with higher outward market access benefit from being closer to the buyers of the goods they produce. The new variant of the gravity equation in (14) highlights that the appropriate measure of economic size combines both the total income or expenditure of a location and its market access.

As noted in the main text, the inward and outward market accesses are closely related. To see this, we introduce two accounting identities. First, the income Y_i of each location i is equal to its total sales, i.e. $Y_i = \sum_{j=1}^{N} X_{ij}$, which when combined with the gravity equation (14) yields:

$$MA_i^{out} = \sum_{j=1}^{N} T_{ij} \times \left(\frac{E_j}{MA_j^{in}} \right). \tag{15}$$

Second, the expenditure E_j of each location j is equal to its total purchases, i.e. $E_j = \sum_{i=1}^{N} X_{ij}$, which when combined with the gravity equation (14) yields:

$$MA_j^{in} = \sum_{i} T_{ij} \times \left(\frac{Y_i}{MA_i^{out}} \right). \tag{16}$$

Equations (15) and (16) correspond to equations (3) and (4) in the main text. One neat thing about equations (15) and (16) is that given observed data on income and expenditures and estimates of (inverse) economic distances T_{ij}, you can use the two equations to uniquely identify (up-to-scale) the equilibrium inward and outward market access for every location. Another neat thing about the equations is that if the (inverse) economic distance is symmetric, i.e. $T_{ij} = T_{ji}$ and income is equal to expenditure, i.e. $Y_i = E_i$, then the inward and outward market access are equal up to scale, i.e. $MA_j^{in} \propto MA_j^{out}$ (which may be why oftentimes there is talk of "market access" without specifying if it is "inward" or "outward".).

Equations (15) and (16) let you calculate the inward and outward market access given information on income and expenditure. But how you figure out the equilibrium income and expenditure in each location? To close the model, we impose three market clearing conditions. The first market clearing conditions has to do with the demand for labor in a location. We require that the the income earned in a location is paid out to labor, i.e. $w_i L_i = Y_i$. This is straightforward in this model, as labor is the only factor of production and there is perfect competition, although the condition would have to be modified in models with multiple factors of production or with market power and firm profits.

Combining this equilibrium condition with the definition of outward market access (i.e. $MA_i^{out} \equiv \left(\frac{w_i}{A_i}\right)^{\sigma-1} Y_i$) yields the following labor demand equation:

$$\ln w_i = -\frac{1}{\sigma} \ln L_i + \frac{\sigma - 1}{\sigma} \ln A_i + \frac{1}{\sigma} \ln MA_i^{out}, \tag{17}$$

which is a special case of equation(5).

The second market clearing condition has to do with the supply for labor in a location. We assume that workers are equally happy to live in all locations and that level of happiness is in turn equal to the happiness they would achieve by living elsewhere in the economy. This comes from the assumption that workers are freely mobile across different locations: if workers can move wherever, why would anyone live in a location that makes them less happy? Of course, in reality, there may be many reasons that workers may live in locations with low levels of happiness, e.g. idiosyncratic preferences for different locations (more on this below) or the cost of moving between locations (which requires extending the static framework here into a dynamic one, see e.g. Desmet, Nagy, and Rossi-Hansberg (2018); Caliendo, Parro, Rossi-Hansberg, and Sarte (2018); Allen and Donaldson (2020)). Let us suppose that the level of happiness W_i an agent gets from residing in location i depends on both her utility from consumption and from a local amenity u_i. Finally, let us normalize the level of happiness in the rest of the world to one. While it may seems like a consequential choice to treat our set of locations as a small region in a large global economy, it actually is not: the equilibrium distribution of economic activity (i.e. the relative populations and incomes in all locations) is identical to a setting where the aggregate population is fixed.

Combining this equilibrium condition with the definition of inward market access (i.e. $MA_i^{in} \equiv \sum_k \tau_{ki}^{1-\sigma} \times \left(\frac{w_k}{A_k}\right)^{1-\sigma}$) yields the following labor supply equation:

$$W_i = 1 \iff \ln w_i = -\ln u_i - \frac{1}{\sigma - 1} \ln MA_i^{in}, \tag{18}$$

which is a special case of equation (6), albeit one where the labor supply is perfectly elastic.

Finally, we impose that income in equal to expenditure, i.e. $E_i = Y_i$. This implies that the value of goods being sent out of each location is equal to the value of goods being sent into each location, i.e. that trade is balanced. Since the model is a static one, this makes sense (although it highlights that the model is not well suited to explaining trade deficits observed in the data, which presumably arise due to dynamic considerations). Together, the labor demand equation (17), the labor supply equation (18), and the market access equations (15) and (16) can be solved together to determine the equilibrium population and wages in all locations.[13]

The model above provides an explanation for the market access equations (3) and (4), as well as special cases of the general labor supply and demand equations (5) and (6). As in the Rosen-Roback framework described in Appendix A.1, we can incorporate the presence of productivity and amenity spillovers of the form given in equations (9) and (10) to derive a more general form of the supply and demand curves.

Substituting equation (9) into the labor demand equation (17) yields the following mod-

[13] Because the equilibrium holds for any choice of units of wages, one also must choose a numeraire. In the companion Matlab code, we impose that the average wage across locations is equal to one.

ified labor demand curve:

$$\ln w_i = -\frac{1}{\sigma}\left(1 - \alpha\left(\sigma - 1\right)\right)\ln L_i + \frac{\sigma - 1}{\sigma}\ln \bar{A}_i + \frac{1}{\sigma}\ln MA_i^{out}. \tag{19}$$

The more positive α, the flatter the downward sloping labor demand curve is, up to the point that $\alpha = \frac{1}{\sigma - 1}$, at which point further increases in α actually cause the labor demand curve to shift upward! (It is at this point that a "black hole" equilibrium becomes possible where all population is concentrated in a single location, see Fujita, Krugman, and Venables (1999)).

Similarly, substituting equation (10) into the labor supply equation (18) yields the following modified labor supply curve:

$$\ln w_i = -\beta \ln L_i - \ln \bar{u}_i - \frac{1}{\sigma - 1}\ln MA_i^{in}. \tag{20}$$

If β is negative (i.e. more people in a location reduce the amenity value of residing in that location), then the labor supply curve has a positive slope: to compensate perfectly mobile individuals for the amenity loss of the greater population, wages have to rise. As above, given the labor supply and demand equations along with the market access equations (15) and (16), we can solve the model to determine the equilibrium population and wages in all locations. But this has an interesting (and somewhat surprising conclusion): conditional on the slope of the labor supply and demand curves, the particular micro-foundation for a non-zero α and β do not matter for the equilibrium spatial distribution of economic activity. Or put another way, two different micro-foundations that both yield the same labor supply and demand curves will have the exact same implications for the equilibrium spatial distribution of economic activity.