Online Appendix to
"Ad clutter, time use, and media diversity"
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Our base model features ad congestion. In this online appendix, we allow for some viewers with unlimited attention. For tractability, we confine ourselves to symmetric platforms. We first analyze the effect of entry in the two models with either limited or unlimited attention under symmetry (so that $a_i^* = a^*$ and $\lambda_i^* = \lambda^*$) and then compare the solutions for a given number of platforms. We show that markets with congested viewers behave markedly different from those with uncongested viewers: platform entry leads to more advertising with congested viewers, while it leads to less advertising with uncongested viewers. As we then argue these findings continue for sufficiently asymmetric shares of congested and uncongested viewers.

Unlimited attention: the effect of entry

Profits without congestion are $\lambda_i a_i p(a_i) = \lambda_i R(a_i)$; see (5). Using symmetry, the first-order condition can be written as

$$\frac{R'(a^*)}{R(a^*)} = \frac{(1-\lambda^*)\tilde{\alpha}}{1-a^*}.$$

Equilibrium market share is

$$\lambda^* = \frac{[s(1-a^*)]^{\tilde{\alpha}}}{n[s(1-a^*)]^{\tilde{\alpha}} + v_0^{\tilde{\alpha}}}.$$

We have that $R'(a)/R(a) = (1-\varepsilon)/a$ where $\varepsilon = -ap'(a)/p$.

Under full coverage this becomes $\lambda^* = 1/n$ and the first-order condition simplifies to

$$\frac{1-\varepsilon}{a^*} = \frac{n-1}{n}\frac{\tilde{\alpha}}{1-a^*}. \tag{31}$$

This can be rewritten as

$$\frac{1-a^*}{a^*}(1-\varepsilon) = \tilde{\alpha}\frac{n-1}{n}. \tag{32}$$

The inverse price elasticity ε is upward sloping in a since $p(.)$ is assumed to be log-concave. This implies that the left-hand side is decreasing in a. The right-hand side is increasing in n. As a result the equilibrium ad level must be decreasing in n. The standard intuition of

the competitive bottleneck model applies: after entry there is fiercer competition for viewers' time on a platform leading to less ad nuisance.

Under partial coverage we have

$$\frac{1-\varepsilon}{a^*} = \left(\frac{n-1}{n} + \frac{v_0^{\tilde{\alpha}}}{[s(1-a^*)]^{\tilde{\alpha}}}\right)\frac{\tilde{\alpha}}{1-a^*}. \tag{33}$$

This can be rewritten as

$$\frac{1-a^*}{a^*}(1-\varepsilon) - \frac{\tilde{\alpha}v_0^{\tilde{\alpha}}}{[s(1-a^*)]^{\tilde{\alpha}}} = \tilde{\alpha}\frac{n-1}{n}.$$

Compared to (32) the left-hand side has an additional term. This term is also decreasing in a. As a result the equilibrium ad level with partial coverage also must be decreasing in n.

We now turn to the model with congestion in which platforms maximize $\lambda_i\phi a_i p(a_i)/A$. Under symmetry the first-order condition (9) simplifies to

$$\frac{1}{n} = 1 - \varepsilon\eta \tag{34}$$

which uniquely determines a^* as a function of n. Since ε and η are upward-sloping in a (see Assumption 2 and Lemma 2), the right-hand side is decreasing in a. An increase in n therefore implies that ad level a^* is increasing in n. This result is an implication of Proposition 2 which covers symmetric platforms as a special case. It illustrates our finding that entry has the opposite effect in the model with congestion compared to the standard media model without congestion. Our finding tells us that with limited attention (i.e. with ad congestion) there is a trade-off between media diversity and media quality. Such a trade-off does not exist with unlimited attention.

Limited vs. unlimited attention: comparison of ad levels

Recalling that $\eta = \frac{1-a_i}{1-(1+\tilde{\alpha})a_i}$, can write (34) as

$$\begin{aligned}
\varepsilon &= \frac{n-1}{n}\frac{1-(1+\tilde{\alpha})a^*}{1-a^*} \\
&= \frac{n-1}{n} - \frac{n-1}{n}\frac{a^*\tilde{\alpha}}{1-a^*}.
\end{aligned} \tag{35}$$

Rewriting (31), the inverse price elasticity ε must satisfy without congestion and with full coverage

$$\varepsilon = 1 - \frac{n-1}{n}\frac{a^*\tilde{\alpha}}{1-a^*}. \tag{36}$$

We observe that the right-hand side of (36) takes larger values than the right-hand side of (35) for all admissible values for a and thus there is less advertising with advertising congestion than without.[51]

This may not seem obvious because with congestion attention ϕ is a common property resource and multiple platforms will exploit it excessively. Without ad congestion, any watched ad raises the attention of viewers. This allows the platform to extract the surplus of the marginal advertiser $p(a_i)$. By contrast, with ad congestion, the platform can only extract $(\phi/A)p(a_i)$. A higher ad level puts further downward pressure on the ad price (through ϕ/A), and the platform has an incentive to set a lower ad level with congestion.

The platform's profit per time unit is $a_i p(a_i) = R(a_i)$ without congestion and $a_i(\phi/A)p(a_i)$ with congestion. For given viewing time λ_i, the platform would maximize these expressions with respect to a_i. Without congestion the solution satisfies $ap'(a_i) + p(a_i) = 0$ which is equivalent to $\varepsilon = 1$; with congestion it satisfies

$$\frac{\phi}{A}\left[ap'(a_i) + p(a_i)\right] - \frac{\lambda_i \phi}{A^2}a_i p(a_i) = 0$$

which can be written as $\varepsilon = 1 - \lambda_i a_i/A$. Since ε is upward sloping this shows that ad levels are lower with congestion than without congestion if we treat viewer numbers as exogenous. With congestion, the platform takes into account that a higher ad level increases the degree of congestion. The associated drop in the ad price reduces the incentive to increase the ad level.

Platforms of course do not maximize profits for a given viewing time but take into account that viewers allocate their viewing time depending on the net quality, $s_i(1 - a_i)$, of the platform. An ad-congested platform also takes into account that total ingestion A increases by less than λ_i as it marginally increases its ad level because its share λ_i decreases in the ad level, but this does not overturn the result for given viewing time.

Rewriting (31), the inverse price elasticity ε must satisfy without congestion and partial coverage

$$\varepsilon = 1 - \frac{n-1}{n}\frac{\frac{n}{n-1}(1-a^*) - a^*\tilde{\alpha}}{1-a^*} - \frac{v_0^{\tilde{\alpha}}}{[s(1-a^*)]^{\tilde{\alpha}}}\frac{a^*}{1-a^*}\tilde{\alpha}. \tag{37}$$

Since the right-hand side of (37) takes smaller values than (36) this is not clear with partial coverage. Here, the ad level with congestion is actually larger than without congestion if

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[51] The right-hand side of (35), (36), and (37) is downward sloping and therefore in all specifications any solution a^* must be unique.

and only if

$$\frac{1}{n} < \frac{v_0^{\tilde{\alpha}}}{[s(1-a^*)]^{\tilde{\alpha}}} \frac{a^*}{1-a^*} \tilde{\alpha}$$

which is equivalent to

$$\frac{1}{n^2} < \frac{\lambda_0}{1-\lambda_0} \frac{a^*}{1-a^*} \tilde{\alpha}.$$

A mix of consumers with limited and unlimited attention

It is possible to extend the model to allow for a fraction $1 - \kappa$ of viewers with unlimited attention. The profit function of media platform i is

$$\begin{aligned}
\Pi_i &= \kappa \frac{\lambda_i \phi a_i p(a_i)}{A} + (1-\kappa)\lambda_i a_i p(a_i) \\
&= \lambda_i R(a_i) \left(1 - \kappa + \kappa \frac{\phi}{A}\right).
\end{aligned}$$

Under full coverage and using symmetry, the first-order condition can be written as

$$\varepsilon = 1 - \frac{\kappa\phi}{n[(1-\kappa)a^* + \kappa\phi]} - \frac{n-1}{n}\frac{\tilde{\alpha}a^*}{1-a^*}. \tag{38}$$

In the special case $\kappa = 0$ we obtain (36) and in the special case $\kappa = 1$ we obtain (35). For a given number of platforms, the ad level is smaller for $\kappa \in (0,1)$ than when no viewer has limited attention ($\kappa = 0$). Regarding the comparative statics with respect to n we have to evaluate how the left-hand side varies with n for given a^*. By continuity, ad levels are increasing in the number of platforms for κ sufficiently close to 1 and decreasing for κ sufficiently close to zero.[52]

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[52]Here, we implicitly assume that there is a unique solution to the first-order condition and that this solution is an equilibrium.