Online Appendix

The Good, the Bad and the Complex: Product Design with Imperfect Information

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1 Robustness

In our baseline model, we assumed that complexity directly determined the information precision of the consumer and we also took the payoffs to the agents as given in order to isolate the key mechanisms driving our results. In this Appendix, we show that our results remain robust to alternative information acquisition technologies, such as costly information acquisition and rational inattention, and to endogenizing the agents' payoffs by introducing prices and production costs. We relegate detailed derivations to Section 2 of this Online Appendix.

1.1 Costly Information Acquisition

Our setting can be interpreted as one in which the consumer actively chooses how much information to acquire, and where the cost of information acquisition is increasing in the product's complexity, χ, determined as in our baseline model. Formally, we now suppose that after the designer proposes product (y, κ), the consumer observes the complexity of the product, χ, and acquires a signal $S \in \{b, g\}$ about the product's quality with noise z, where:

$$z \equiv \mathbb{P}(S = b|y = G) = \mathbb{P}(S = g|y = B) \in \left[0, \frac{1}{2}\right]. \tag{1}$$

The consumer can reduce the noise of the signal by exerting effort with associated cost $C(z, \chi)$, which is weakly decreasing in noise, z, with the properties that $C(\frac{1}{2}, \cdot) = 0$ and $C(z, \cdot) - C(z', \cdot)$ is increasing for all $z < z'$. As a result, it is more costly for the consumer to acquire information about products that are either naturally more complex (high η) or that have been purposefully complexified by the designer ($\kappa = \bar{\kappa}$). Finally, we assume that acquiring a perfectly informative signal is prohibitively costly for the consumer: $C(0, \chi(\eta, \kappa)) > \max\{w(G) - w_0, w_0 - w(B)\}$ with probability one for $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$.

The consumer's problem must now be adjusted to incorporate the decision of how much information to acquire. It can now be expressed in two steps, backwards. As in the baseline model, given her information set, as summarized by the posterior belief $\mu(s, z, \chi) \equiv \mathbb{P}(y =$

$G|s,z,\chi)$, the consumer makes an optimal acceptance decision:

$$W(s,z,\chi) \equiv \max_{a\in\{0,1\}} a\left[\mu(s,z,\chi)\, w(G) + (1-\mu(s,z,\chi))\, w(B)\right] + (1-a)\, w_0. \tag{2}$$

Next, in anticipation of her optimal acceptance decision and given her interim belief $\mu(\chi) \equiv \mathbb{P}(y = G|\chi)$, which incorporates the potential information contained in the product's complexity, χ, the consumer makes an optimal information acquisition decision:

$$\max_{z\in\left[0,\frac{1}{2}\right]} \sum_{s\in\{b,g\}} \mathbb{P}(S=s|z,\chi) W(s,z,\chi) - C(z,\chi), \tag{3}$$

where $\mathbb{P}(S=s|z,\chi) = \mathbb{P}(S=s|y=G)\mu(\chi) + \mathbb{P}(S=s|y=B)(1-\mu(\chi))$ and where $\mathbb{P}(S=s|y)$ is given by (1). We now denote the consumer's strategy by $\{z(\chi), a(s,\chi)\}_{s,\chi}$.

1.1.1 The Baseline Setup

Our baseline specification is obtained with the following information acquisition technology:

$$C(z,\chi) = \begin{cases} 0 & \text{if } z \geq \chi \\ \bar{C} & \text{if } z < \chi \end{cases} \tag{4}$$

where $\bar{C} > \max\{w(G) - w_0, w_0 - w(B)\}$. The reason is that such an information cost implies that it is free for the consumer to reduce the noise of the signal down to χ, but it becomes prohibitively costly to reduce it any further. As we have shown in our main analysis, this formulation is very convenient for obtaining sharp analytical results.

1.1.2 Convex Costs

Next, suppose that the consumer's cost of information acquisition is:

$$C(z,\chi) = \chi \cdot h\left(\frac{1}{2} - z\right), \tag{5}$$

where $\chi = \chi(\eta,\kappa) \in (0,\infty)$ and $h(\cdot)$ is continuously differentiable, increasing and convex, with $h'(0) = 0$ and $\lim_{x\to\frac{1}{2}} h'(x) = \infty$. These properties imply that the consumer's choice of information acquisition, $z(\chi)$, will be positive (i.e., information is always imperfect) and increasing in complexity, χ.

If the consumer chooses to acquire information, i.e., $z(\chi) < \frac{1}{2}$, it is because she will make her acceptance decision contingent on the received information: she will accept the product after observing signal g and reject it after observing signal b. Given this, the payoff from acquiring information is:

$$W^I(\chi,z) \equiv \max_{z\in[0,\frac{1}{2}]} w_0 + \mu(\chi)\cdot(1-z)\cdot(w(G)-w_0) + (1-\mu(\chi))\cdot z\cdot(w(B)-w_0) - C(z,\chi). \tag{6}$$

We can already see the main difference between this and our baseline model: the mapping between a product's complexity and the noise of the acquired information given by the solution to (6), $z(\chi)$, now also depends on the consumer's prior belief, μ. To highlight the main mechanisms of our paper, in the baseline model we chose a formulation for the cost function that eliminated this dependence. As we show next, although this dependence introduces complications, it does not change our main qualitative results.

If the consumer chooses not to acquire information, $z(\chi) = \frac{1}{2}$, her payoff is:

$$W^U(\chi) \equiv \max\left\{\mu(\chi) \cdot w(G) + (1 - \mu(\chi)) \cdot w(B), w_0\right\}. \tag{7}$$

It follows that the consumer acquires information and makes her decision contingent on the received information whenever $W^I(\chi, z(\chi)) > W^U(\chi)$. After some algebra, it follows that the consumer makes her decision conditional on information when:

$$z(\chi) < \begin{cases} \frac{(1-\mu(\chi))\cdot\omega}{(1-\mu(\chi))\cdot\omega+\mu(\chi)\cdot(1-\omega)} - \frac{h\left(\frac{1}{2}-z(\chi)\right)}{h'\left(\frac{1}{2}-z(\chi)\right)} & \text{if } \mu(\chi) \geq \omega \\ \frac{\mu(\chi)\cdot(1-\omega)}{(1-\mu(\chi))\cdot\omega+\mu(\chi)\cdot(1-\omega)} - \frac{h\left(\frac{1}{2}-z(\chi)\right)}{h'\left(\frac{1}{2}-z(\chi)\right)} & \text{if } \mu(\chi) < \omega \end{cases} \tag{8}$$

In contrast to our baseline model, the consumer now takes into account the cost of information acquisition when deciding whether to make her decision contingent on information. This is captured by the new term $\frac{h\left(\frac{1}{2}-z(\chi)\right)}{h'\left(\frac{1}{2}-z(\chi)\right)}$ on the right hand side of (8). As in our baseline analysis, we impose a regularity condition on the likelihood ratio $\frac{f(\cdot|\bar{\kappa})}{f(\cdot|\underline{\kappa})}$ to ensure that there is a unique threshold $\bar{\chi}$ such that the consumer makes her decision contingent on information if and only if $\chi < \bar{\chi}$. With this, we are able to prove the analogue of Lemma 1.

Given the consumer's optimal strategy, we proceed to the designer's problem. For this, we first compute the probability of having a product with attributes (y, κ) accepted, which is the same as in the baseline model, given by (11)-(13), except that now the noise of the signal is given by $z(\chi)$ rather than χ. We can then study the designer's complexification strategy. To do this, it is straightforward to prove Proposition 2, where $\widehat{\chi}$ is now defined by:

$$\int_0^{\widehat{\chi}} z(\chi) \cdot (f(\chi|\underline{\kappa}) - f(\chi|\bar{\kappa})) \cdot d\chi = 0. \tag{9}$$

Finally, it is also clear that the designer's optimal quality strategy continues to be characterized by Proposition 3. We have thus shown that the optimal strategies of the consumer and of the designer qualitatively coincide with those in the baseline model. As a final step, we show that an equilibrium with positive trade exists, and that it shares the same broad features as our baseline equilibrium. To do so, we impose a condition on the cost of information acquisition, which we explain below.

Recall that now the consumer's choice of information acquisition, $z(\chi)$, varies with prior belief μ. This adds an additional consideration that was absent in our baseline model; namely, that now not only the consumer's threshold, $\bar{\chi}$, but also the designer's threshold, $\widehat{\chi}$, given by (9), change with μ. Recall from the discussion following Proposition 4 that understanding how the ranking between $\bar{\chi}$ and $\widehat{\chi}$ depends on the prior belief μ is essential for characterizing the designer's complexification strategy that is consistent with an equilibrium belief μ. In the baseline model, monotonicity of $\bar{\chi} - \widehat{\chi}$ was ensured because $\bar{\chi}$ was monotonic in μ and $\widehat{\chi}$ was

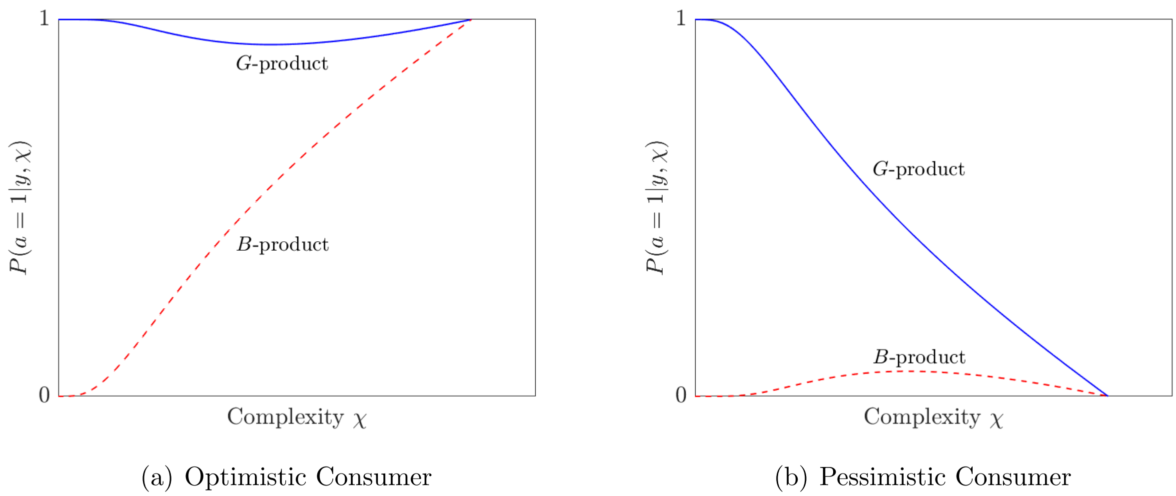


Figure 1: Illustrates the probability of acceptance of a y-product as a function of the product's complexity, χ.

independent of it. This monotonicity property does not come for free in the current setting, but we recover it by imposing a regularity condition on the cost function so that $\bar{\chi}$ is more sensitive to changes in μ than $\widehat{\chi}$. With this, we establish the following result.

Proposition 1.1 *An equilibrium with positive trade exists. In it, the designer produces a G-product with probability $\mu^* \in (0,1)$ and there exist thresholds $0 < \widetilde{\mu}_1 < \widetilde{\mu}_2 < \widetilde{\mu}_3 < \widetilde{\mu}_4 < 1$ such that:*

1. *If $\mu^* \in (0, \widetilde{\mu}_1]$, all products are simplified, $\sigma_G = \sigma_B = 0$.*
2. *If $\mu^* \in (\widetilde{\mu}_1, \widetilde{\mu}_2]$, G-products are simplified, $\sigma_G = 0$, and B-products complexified with probability $\sigma_B \in (0,1)$.*
3. *If $\mu^* \in (\widetilde{\mu}_2, \widetilde{\mu}_3]$, G-products are simplified, $\sigma_G = 0$, and B-products complexified, $\sigma_B = 1$.*
4. *If $\mu^* \in (\widetilde{\mu}_3, \widetilde{\mu}_4)$, G-products are complexified with probability $\sigma_G \in \{0, \widetilde{\sigma}, 1\}$ for some $\widetilde{\sigma} \in (0,1)$, and B-products complexified, $\sigma_B = 1$.*
5. *If $\mu^* \in [\widetilde{\mu}_4, 1)$, all products are complexified, $\sigma_G = \sigma_B = 1$.*

This result states that the structure of the equilibrium of the model with convex costs of information is effectively the same as that in our baseline model, as summarized by Propositions 4 and 5. The main difference is that we can no longer ensure uniqueness of equilibrium, which was useful for obtaining sharp comparative statics results (Section III).

1.2 Rational Inattention

We next consider an even more general information acquisition problem, by supposing that the consumer can choose how much uncertainty about the product quality to reduce, subject to an entropy-reduction cost, where entropy measures the consumer's uncertainty (Sims, 2003).

Thus, a product is more complex if it has a higher entropy-reduction cost. Although this approach allows for a more flexible information acquisition technology, it has the drawback that we can no longer obtain as sharp of an equilibrium characterization as in our baseline specification or as in the previous section. Nevertheless, we argue next that the model's main mechanisms remain robust to this alternative specification.

Since the consumer's action is binary, i.e., accept or reject, it is without loss of generality to focus on binary signals $S \in \{b, g\}$ (Woodford, 2009; Yang, 2015), where the consumer accepts the product if and only if she receives a g signal. Thus, the main difference from the analysis in Appendix 1.1 is that now the consumer's signal need not be symmetric, as the consumer may allocate "precision" optimally between the g and the b signals, trading off the costs of rejecting a G-product (type I error) with the costs of accepting a B-product (type II error).

Equipped with the optimal information structure (see equations (31) and (32)), we compute the probability of acceptance of a y-product, as it depends on the product's complexity, χ. These probabilities are depicted in Figure 1, which we can see closely resemble those in our baseline model (see Figure 3). When the product's complexity is low, the consumer extracts an informative signal and makes her decision contingent on its realization. Otherwise, the consumer accepts the product with probability one if she is optimistic, and she rejects it with probability one if she is pessimistic.

Although a full analytical characterization of the equilibrium set is difficult to obtain, we check that it resembles closely that of our baseline model. Figure 2(a) depicts the complexification strategy $\{\sigma_y\}$ of the designer that is consistent with an equilibrium prior belief μ. And, Figure 2(b) depicts the designer's net payoff from producing a G- versus a B- product as it depends on μ.[1] Thus, and in line with the results in Proposition 1.1, the right panel determines the expected product quality, $\mu = \psi$, whereas the left panel determines the equilibrium complexification of a y-product, given that the consumer's prior belief is $\mu = \psi$.

1.3 Prices and Production Costs

For some applications, it is natural to assume that a designer not only proposes a product to the consumer but that he also sets a price that is observable to the consumer. To analyze the role of such transfers in our environment, we modify the agents' payoff as follows. If a product is accepted, the designer's payoff is given by the price he charges the consumer minus the cost of production, $p - c(y)$.[2] In turn, the consumer's payoff from accepting a y-product is given by her valuation minus the price she pays, $\widetilde{w}(y) - p$. As before, the consumer's outside option is given by w_0. The following assumption replaces Assumption 1.

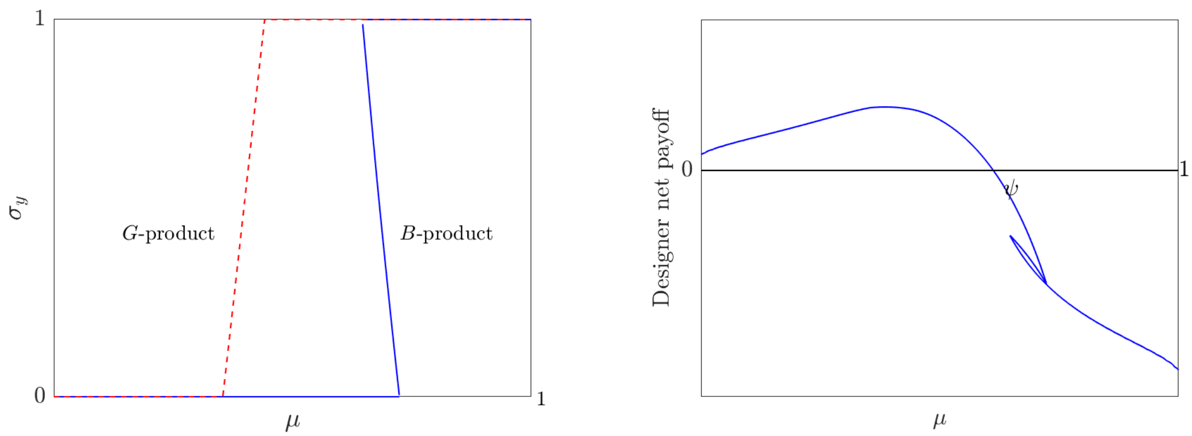
Assumption 1.1 *The payoffs satisfy the following properties:*

1. $\widetilde{w}(G) - c(G) > w_0 > \widetilde{w}(B) - c(B)$, *with* $w_0 \geq 0$.

2. $c(G) > c(B) \geq 0$.

[1] As in Figure 5, the kinks in Figure 2(b) arise due to a switch from separation on κ (i.e., $\sigma_G = 0$ and $\sigma_B = 1$) to pooling on κ (i.e., $\sigma_G = \sigma_B = 1$).

[2] We assume that the cost of production is incurred upon product acceptance in order to stay close to the payoff structure of our baseline model.



(a) Equilibrium complexification

(b) Equilibrium quality determination

Figure 2: The left panel illustrates how the complexification strategy of the designer who produces a y-product varies with equilibrium belief μ. The right panel illustrates the designer's net payoff from choosing the G-product, given belief μ. Note that equilibrium quality ψ is set so that the net payoff is equal to zero.

The first assumption states that G-products are efficient to produce, whereas B-products are not. The second assumption states that G-products are costlier to produce than B-products.

As prices are set by the designer, the consumer makes inferences not only from the product's complexity, χ, but also from its price, p; so the consumer's posterior belief is now denoted by $\mu(s, \chi, p)$. It is easy to see that the consumer will accept the product if and only if her posterior belief is greater than a price-adjusted relative outside option:

$$\mu(s, \chi, p) \geq \frac{\widetilde{w}_0 - \widetilde{w}(B) + p}{\widetilde{w}(G) - \widetilde{w}(B)}. \tag{10}$$

Given the consumer's acceptance strategy, the designer chooses $\{y, \kappa, p\}$ to maximize his expected payoff:

$$\mathbb{P}(a = 1|y, \kappa, p) \cdot (p - c(y)). \tag{11}$$

For simplicity, we focus on equilibria in which the designer has a pure strategy over the price. The following proposition summarizes the main results of this section.

Proposition 1.2 *In any positive trade equilibrium, the price set by the designer is independent of the product's quality. Moreover, any price $p^* \in (c(G), \widetilde{w}(G) - w_0)$ is consistent with equilibrium. The expected product quality and complexity are determined as in the baseline model with payoffs given by $w(y) \equiv \widetilde{w}(y) - p^*$ and $v(y) \equiv p^* - c(y)$.*

The result that separation through prices is not possible is intuitive. As a B-product is cheaper to produce, the designer of such a product is willing to set any price the G-product designer is willing to set. As a result, the B-product designer always mimics the pricing strategy of a G-product designer in order to avoid being identified. Due to the freedom in specifying off equilibrium beliefs, multiple prices can be supported as an equilibrium. The bounds on possible prices are due to the fact that in any positive trade equilibrium a G-product designer will not post a price below his cost of production, or a price high enough for the product to be rejected with probability one.

2 Derivations and Proofs

2.0.1 Convex Costs

Here, we provide derivations for Appendix 1.1. Let us begin with the consumer's optimal strategy. Recall that upon observing χ, and updating her belief to $\mu(\chi)$, the consumer decides whether to acquire information or not. Given the cost that the consumer must pay whenever information is acquired, it follows immediately that the consumer will acquire information only if her acceptance decision is made contingent on this information. If the consumer were to acquire information, the optimal noise would be:

$$z(\chi) = \arg\max_{z\in\left[0,\frac{1}{2}\right]} \ (1-z)\cdot\mu(\chi)\cdot(w(G)-w_0)+z\cdot(1-\mu(\chi))\cdot(w(B)-w_0)+w_0-\chi\cdot h\left(\frac{1}{2}-z\right), \tag{12}$$

or equivalently:

$$z(\chi) = \frac{1}{2} - h'^{-1}\left(\frac{\mu(\chi)\cdot(w(G)-w_0)-(1-\mu(\chi))\cdot(w(B)-w_0)}{\chi}\right), \tag{13}$$

where $h'^{-1}(x)$ is increasing in x. As the consumer accepts a Good product with probability $1-z(\chi)$ and a Bad product with probability $z(\chi)$, her expected payoff of acquiring information is:

$$W^I(\chi) = w_0+(1-z(\chi))\cdot\mu(\chi)\cdot(w(G)-w_0)+z(\chi)\cdot(1-\mu(\chi))\cdot(w(B)-w_0)-\chi\cdot h\left(\frac{1}{2}-z(\chi)\right). \tag{14}$$

Instead, if the consumer does not acquire information, she makes her optimal acceptance decision based on her interim belief alone, implying an expected payoff of:

$$W^U(\chi) = \max\left\{\mu(\chi)\cdot w(G)+(1-\mu(\chi))\cdot w(B),\ w_0\right\}. \tag{15}$$

Thus, after observing the product's complexity, the consumer acquires information if $W^I(\chi) > W^U(\chi)$. There are two cases to consider, depending on the consumer's outside option of not acquiring information, i.e. to accept or to reject the product with complexity χ:

If $\mu(\chi)\geq\omega$, the condition for acquiring information reduces to:

$$z(\chi)+\frac{\chi\cdot h\left(\frac{1}{2}-z(\chi)\right)}{(1-\mu(\chi))\cdot\omega+\mu(\chi)\cdot(1-\omega)} < \frac{(1-\mu(\chi))\cdot\omega}{(1-\mu(\chi))\cdot\omega+\mu(\chi)\cdot(1-\omega)}. \tag{16}$$

It is straightforward to show that if the likelihood ratio $\frac{f(\cdot|\bar{\kappa})}{f(\cdot|\underline{\kappa})}$ is not too steep, then (i) $z(\chi)$ defined by (13) is monotonically increasing in χ, i.e., the consumer's information gets noisier as the product gets more complex, and (ii) there is a unique threshold value for complexity, denoted by $\bar{\chi}^o$, such that inequality (16) holds if and only if $\chi < \bar{\chi}^o$. We assume this is the case from now on, which is analogous to assuming Condition II.1 holds in our baseline model.

If instead $\mu(\chi) < \omega$, then the condition for acquiring information reduces to:

$$z(\chi) + \frac{\chi \cdot h\left(\frac{1}{2} - z(\chi)\right)}{[(1-\mu(\chi)) \cdot \omega + \mu(\chi) \cdot (1-\omega)]} < \frac{\mu(\chi) \cdot (1-\omega)}{[(1-\mu(\chi)) \cdot \omega + \mu(\chi) \cdot (1-\omega)]}. \tag{17}$$

Here, again we can show that there is a unique threshold level of complexity, denoted by $\bar{\chi}^p$, such that inequality (17) holds if and only if $\chi < \bar{\chi}^p$.[3]

Moreover, as the consumer strictly prefers to acquire information when $\mu(\chi) = \omega$ (as $h'(0) = 0$), it must be that for all $\chi \geq \bar{\chi}$ for $\bar{\chi} \in \{\bar{\chi}^p, \bar{\chi}^o\}$, $\mu(\chi) \neq \omega$.

Given the above observations, the following lemma then follows immediately.

Lemma 2.1 *When the consumer is optimistic, i.e., $\lim_{\chi\to\infty} \mu(\chi) \geq \omega$, her acceptance strategy is:*

$$a(s,\chi) = \begin{cases} \mathcal{I}_{\{S=g\}} & \text{if } \chi \leq \bar{\chi} \\ 1 & \text{if } \chi > \bar{\chi} \end{cases}; \tag{18}$$

instead, when the consumer is pessimistic, i.e., $\lim_{\chi\to\infty} \mu(\chi) < \omega$, her acceptance strategy is:

$$a(s,\chi) = \begin{cases} \mathcal{I}_{\{S=g\}} & \text{if } \chi \leq \bar{\chi} \\ 0 & \text{if } \chi > \bar{\chi} \end{cases}; \tag{19}$$

where $\bar{\chi} = \begin{cases} \bar{\chi}^o & \text{if } \lim_{\chi\to\infty} \mu(\chi) > \omega \\ \infty & \text{if } \lim_{\chi\to\infty} \mu(\chi) = \omega \\ \bar{\chi}^p & \text{if } \lim_{\chi\to\infty} \mu(\chi) < \omega \end{cases}$.[4]

Note that Lemma 2.1 is the counterpart of Lemma 1 in our baseline model. It shows that the consumer's optimal acceptance decision depends crucially on whether she is optimistic or pessimistic, i.e., what she does when the signal that she would receive, conditional on acquiring information, becomes uninformative, which occurs as $\chi \to \infty$.

Using Lemma 2.1, we can compute the probability that a product (y, κ) proposed by the designer is accepted by the consumer:

$$\mathbb{P}(a=1|G,\kappa) = \int_0^{\bar{\chi}} (1-z(\chi)) \cdot f(\chi|\kappa)\, d\chi + \mathcal{I}_{\{\lim_{\chi\to\infty}\mu(\chi)\geq\omega\}} \cdot (1 - F(\bar{\chi}|\bar{\kappa})), \tag{20}$$

and

$$\mathbb{P}(a=1|B,\kappa) = \int_0^{\bar{\chi}} z(\chi) \cdot f(\chi|\kappa)\, d\chi + \mathcal{I}_{\{\lim_{\chi\to\infty}\mu(\chi)\geq\omega\}} \cdot (1 - F(\bar{\chi}|\bar{\kappa})). \tag{21}$$

Since $z(\chi)$ is increasing in χ, by the same reasoning as in the proof of Proposition 2, we obtain that the determinant of the designer's optimal choice of κ is whether the consumer is optimistic or pessimistic, and the threshold $\bar{\chi}$.

[3]Since in equilibrium $\mu(\chi)$ is weakly decreasing in χ, we do not need to impose additional conditions on the likelihood ratio $\frac{f(\cdot|\bar{\kappa})}{f(\cdot|\underline{\kappa})}$ to obtain this result.

[4]When $\lim_{\chi\to\infty} \mu(\chi) = \omega$, the consumer acquires information for any $\chi \in (0, \infty)$, so we set $\bar{\chi} = \infty$.

Lemma 2.2 *Fix $\mu \in (0,1)$, and let $\widehat{\chi} > 0$ denote the unique solution to $\int_0^{\widehat{\chi}} z(\chi) \cdot f(\chi|\underline{\kappa})\, d\chi = \int_0^{\widehat{\chi}} z(\chi) \cdot f(\chi|\bar{\kappa})\, d\chi$. Then, when the consumer is optimistic,*

$$\sigma_B = 1, \text{ and } \quad \sigma_G \begin{cases} = 1 & \bar{\chi} < \widehat{\chi} \\ \in [0,1] & \bar{\chi} = \widehat{\chi} \\ = 0 & \bar{\chi} > \widehat{\chi} \end{cases}, \tag{22}$$

whereas, when the consumer is pessimistic,

$$\sigma_B \begin{cases} = 0 & \bar{\chi} < \widehat{\chi} \\ \in [0,1] & \bar{\chi} = \widehat{\chi} \\ = 1 & \bar{\chi} > \widehat{\chi} \end{cases}, \text{ and } \quad \sigma_G = 0. \tag{23}$$

This result is essentially the same as Proposition 2, except that now the threshold $\widehat{\chi}$, which controls the designer's preference between complexification and simplification, also depends on the prior belief μ, as the latter affects the optimal information choice $z(\chi)$; we will come back to this dependence shortly. Finally, it should be clear that the designer's optimal choice of product quality is still given by Proposition 3.

We have thus shown that both the consumer's and the designer's optimal strategies remain qualitatively unchanged from our baseline model; we are therefore left to solve for the equilibrium. We again proceed in two steps. We first take the consumer's prior belief μ as given and find the designer's equilibrium complexification strategy by requiring that the consumer's interim belief, $\mu(\chi)$, be consistent with the designer's strategy and Bayes' rule.

Lemma 2.3 *Suppose that in equilibrium the consumer's prior belief is $\mu \in (0,1)$, then there exist thresholds $0 < \widetilde{\mu}_1 < \widetilde{\mu}_2 < \widetilde{\mu}_3 < \widetilde{\mu}_4 < 1$ such that:*

1. *If $\mu \in (0, \widetilde{\mu}_1]$, all products are simplified, $\sigma_G = \sigma_B = 0$.*
2. *If $\mu \in (\widetilde{\mu}_1, \widetilde{\mu}_2]$, G-products are simplified, $\sigma_G = 0$, whereas B-products are complexified with probability $\sigma_B \in (0,1)$.*
3. *If $\mu \in (\widetilde{\mu}_2, \widetilde{\mu}_3]$, G-products are simplified, $\sigma_G = 0$, whereas B-products are complexified, $\sigma_B = 1$.*
4. *If $\mu \in (\widetilde{\mu}_3, \widetilde{\mu}_4)$, G-products are complexified with probability $\sigma_G \in \{0, \widetilde{\sigma}, 1\}$ for some $\widetilde{\sigma} \in (0,1)$, whereas B-products are complexified, $\sigma_B = 1$.*
5. *If $\mu \in [\widetilde{\mu}_4, 1)$, all products are complexified, $\sigma_G = \sigma_B = 1$.*

Proof. Consider first the candidate equilibrium with $\sigma_G = \sigma_B = 0$. In this case, the consumer does not update upon observing complexity χ, i.e., $\mu(\chi) = \mu$ for all χ. For this to be an equilibrium, it must be that the consumer is pessimistic and $\bar{\chi}^p(\mu) \le \widehat{\chi}(\mu)$, where $\bar{\chi}^p(\mu)$ is given by the solution to:

$$z(\bar{\chi}^p) = \frac{\mu \cdot (1-\omega) - \bar{\chi}^p \cdot h\left(\frac{1}{2} - z(\bar{\chi}^p)\right)}{(1-\mu)\cdot\omega + \mu\cdot(1-\omega)}, \tag{24}$$

and where $z(\cdot)$ is given by (13). Thus, this equilibrium exists if and only if μ belongs to the set $\mathcal{M}_{P,\underline{\kappa}} \equiv \{\mu \in (0,1) : \bar{\chi}^p(\mu) \leq \widehat{\chi}(\mu)\}$.

Next, consider the candidate equilibrium with $\sigma_G = \sigma_B = 1$. In this case, also, the consumer does not update upon observing complexity χ, i.e., $\mu(\chi) = \mu$ for all χ. For this to be an equilibrium, it must be that the consumer is optimistic and that $\bar{\chi}^o(\mu) \leq \widehat{\chi}(\mu)$, where $\bar{\chi}^o(\mu)$ is given by the solution to:

$$z(\bar{\chi}^o) = \frac{(1-\mu)\cdot\omega - \bar{\chi}^o \cdot h\left(\frac{1}{2} - z(\bar{\chi}^o)\right)}{(1-\mu)\cdot\omega + \mu\cdot(1-\omega)}, \tag{25}$$

and where $z(\cdot)$ is given by (13). Thus, this equilibrium exists if and only if μ belongs to the set $\mathcal{M}_{P,\bar{\kappa}} \equiv \{\mu \in (0,1) : \bar{\chi}^o(\mu) \leq \widehat{\chi}(\mu)\}$.

Next, consider the candidate equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$. In this case, the consumer does update upon observing complexity χ, i.e., $\mu(\chi) = \frac{\mu}{\mu + (1-\mu)\frac{f(\chi|\kappa)}{f(\chi|\underline{\kappa})}}$ for all χ. There are two possibilities here, depending on whether the consumer is pessimistic or optimistic, which is equivalent to asking whether μ is greater than or smaller than $\widetilde{\mu} \equiv \lim_{\chi\to\infty} \frac{\omega}{\omega + (1-\mu)\frac{f(\chi|\kappa)}{f(\chi|\bar{\kappa})}}$.

If the consumer is pessimistic, i.e. $\mu < \widetilde{\mu}$, then for this to be an equilibrium, it must be that $\bar{\chi}^p(\mu) \geq \widehat{\chi}(\mu)$, where $\bar{\chi}^p(\mu)$ is given by the solution to:

$$z(\bar{\chi}^p) = \frac{\mu(\bar{\chi}^p)\cdot(1-\omega) - \bar{\chi}^p \cdot h\left(\frac{1}{2} - z(\bar{\chi}^p)\right)}{(1-\mu(\bar{\chi}^p))\cdot\omega + \mu(\bar{\chi}^p)\cdot(1-\omega)}, \tag{26}$$

and where $z(\cdot)$ is given by (13). Here, such an equilibrium exists if and only if μ belong to the set $\mathcal{M}_{S,a} \equiv \{\mu \in (0,\widetilde{\mu}) : \bar{\chi}^p(\mu) \geq \widehat{\chi}(\mu)\}$.

Instead, if the consumer is optimistic, i.e. $\mu \geq \widetilde{\mu}$, then for this to be an equilibrium, it must be that $\bar{\chi}^o(\mu) \geq \widehat{\chi}(\mu)$, where $\bar{\chi}^o(\mu)$ is given by the solution to:

$$z(\bar{\chi}^o) = \frac{(1-\mu(\bar{\chi}^o))\cdot\omega - \bar{\chi}^o \cdot h\left(\frac{1}{2} - z(\bar{\chi}^o)\right)}{(1-\mu(\bar{\chi}^o))\cdot\omega + \mu(\bar{\chi}^o)\cdot(1-\omega)}, \tag{27}$$

and where $z(\cdot)$ is given by (13). Here, such an equilibrium exists if and only if μ belongs to the set $\mathcal{M}_{S,b} \equiv \{\mu \in (\widetilde{\mu},1) : \bar{\chi}^o(\mu) \geq \widehat{\chi}(\mu)\}$. We therefore conclude that an equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$ exists if and only if $\mu \in \mathcal{M}_S \equiv \mathcal{M}_{S,a} \cup \mathcal{M}_{S,b}$.

Next, suppose that (i) $\bar{\chi}^p(\mu) - \widehat{\chi}(\mu)$ is increasing in μ in an equilibrium with $\sigma_G = \sigma_B = 0$, or with $\sigma_G = 0$, $\sigma_B = 1$ when $\mu < \widetilde{\mu}$; and (ii) $\bar{\chi}^o(\mu) - \widehat{\chi}(\mu)$ is decreasing in μ in an equilibrium with $\sigma_G = \sigma_B = 1$, or with $\sigma_G = 0$, $\sigma_B = 1$ when $\mu \geq \widetilde{\mu}$. These two conditions will hold, for example, if the function $h(\cdot)$ is convex enough so that $z(\cdot)$ is not too sensitive to changes in μ. We will assume this in what follows, in which case the three equilibrium regions can be represented as: $\mathcal{M}_{P,\underline{\kappa}} = (0,\widetilde{\mu}_1]$, $\mathcal{M}_S = (\widetilde{\mu}_2,\widetilde{\mu}_4)$, and $\mathcal{M}_{P,\bar{\kappa}} = (\widetilde{\mu}_3, 1)$ for some $0 < \widetilde{\mu}_1 < \widetilde{\mu}_2 < \widetilde{\mu}_3 < \widetilde{\mu}_4 < 1$; moreover, following similar arguments as in the proof of Proposition 4, we can construct the mixed strategy equilibria for μ in the intervals $(\widetilde{\mu}_1,\widetilde{\mu}_2]$ and $(\widetilde{\mu}_3,\widetilde{\mu}_4)$. ■

Finally, we are left to pin down the equilibrium prior belief μ^*. As before, in any positive trade equilibrium, the designer must be indifferent between producing either of the two products. With the results from Lemma 2.2 it easy to show that the correspondence $\Gamma(\mu)$ (defined just as in proof of Proposition 5), which consists of the designer's net payoffs γ from producing G- vs. B-products, is well defined and both upper and lower hemicontinuous. Then, by continuity, there must exist $\mu^* \in (0,1)$ such that the designer's net payoff from producing G- vs. B-product is equal to zero (i.e., $\gamma(\mu^*, \{\sigma_y\}_{\mu*}) = 0$).

2.0.2 Rational Inattention

We now provide derivations for Appendix 1.2. We adjust our baseline model to allow the consumer to optimally reduce her uncertainty about the product's quality, subject to an entropy-reduction cost, as in the literature on rational inattention (Sims, 2003). In this setting, the uncertainty faced by the consumer with belief $\tilde{\mu} = P(y = G)$ is measured by the entropy function:

$$H(\tilde{\mu}) = -\left(\tilde{\mu} \cdot log(\tilde{\mu}) + (1-\tilde{\mu}) \cdot log(1-\tilde{\mu})\right), \tag{28}$$

which reaches a minimum of zero at $\tilde{\mu} \in \{0,1\}$ and a maximum of $-log\left(\frac{1}{2}\right)$ at $\tilde{\mu} = \frac{1}{2}$.

As before, we let S denote the signal observed by the consumer and s denote its realization. The signal has a distribution conditional on the product's quality, $\pi(s|y) \equiv \mathbb{P}(S = s|y)$, which determines the consumer's posterior belief:

$$\tilde{\mu}(s) \equiv P(y = G|s) = \frac{\pi(s|G) \cdot \tilde{\mu}}{\pi(s|G) \cdot \tilde{\mu} + \pi(s|B) \cdot (1-\tilde{\mu})}. \tag{29}$$

The entropy associated with the posterior belief is $H(\tilde{\mu}(s))$.

We measure the amount of information that the consumer obtains from a particular information structure π as the expected reduction in entropy:

$$I(\pi) = H(\tilde{\mu}) - \int_s H(\tilde{\mu}(s)) \cdot \pi(s) \cdot ds, \tag{30}$$

and we assume that the consumer faces a cost $\chi \cdot I(\pi)$ of entropy-reduction, where $\chi \in (0,\infty)$ depends on the two components η and κ, with a conditional pdf $f(\chi|\kappa)$ that has full support and satisfies MLRP. Thus, when complexity is minimal, $\chi \to 0$, it is essentially costless for the consumer to find out the product's quality; instead, when complexity is maximal, $\chi \to \infty$, extracting any information about the product's quality becomes prohibitively costly.

Since the consumer's action is binary, i.e., she chooses to accept or reject the product, it is without loss of generality to restrict attention to information structures that consist of binary signals $S \in \{b, g\}$ such that the consumer accepts the product if and only if $S = g$ (Woodford, 2009; Yang, 2015). Let π_y denote the probability that the consumer accepts the product, conditional on the designer producing a y-product. Let $\mu(\chi)$ be the consumer's interim belief after observing the product's complexity χ. For a given χ, the consumer's problem is then reduced to choosing π_G and π_B in order to maximize her expected payoff:

$$\mu(\chi) \cdot \pi_G \cdot (w(G) - w_0) + (1-\mu(\chi)) \cdot \pi_B \cdot (w(B) - w_0) - \chi \cdot I(\pi) \tag{31}$$

where

$$I(\pi) = H\left(\mu(\chi) \cdot \pi_G + (1 - \mu(\chi)) \cdot \pi_B\right) - \mu(\chi) \cdot H(\pi_G) - (1 - \mu(\chi)) \cdot H(\pi_B). \tag{32}$$

Figure 1 illustrates the solution to this problem for a given prior belief $\mu \in (0,1)$, for the case where the consumer's interim belief satisfies $\mu(\chi) = \mu$, i.e., when the equilibrium features pooling on complexification. Under a condition on the likelihood ratio $f(\cdot|\bar{\kappa})/f(\cdot|\underline{\kappa})$ (akin to Condition II.1), there is a unique threshold value of complexity, $\bar{\chi}$, such that the consumer extracts an informative signal and makes her decision contingent on its realization if and only if $\chi < \bar{\chi}$. Otherwise, when complexity is high, the consumer makes her decision solely based on her interim belief. Finally, observe that when complexity is high enough, then the consumer either accepts the product with probability one or she rejects it with probability one. As in our baseline model, which of the two scenarios arises depends on whether the consumer is optimistic or pessimistic; that is, what she would do in the absence of an informative signal.[5]

Naturally, an equilibrium requires that the consumer's prior belief μ and her interim belief $\mu(\chi)$ be consistent with the designer's strategy $\{m, \sigma_G, \sigma_B\}$ and Bayes' rule. Although a full analytical characterization of the equilibrium set is now difficult to obtain, we are able to check numerically that the equilibrium set of the model with optimal information extraction resembles closely that of our baseline model. As we discussed in the text, Figure 2 is the analogue of the Figures 4 and 5. And, an equilibrium is found by requiring that the belief μ equals ψ, so that the designer is indifferent to producing a G- or a B-product, and then reading off the equilibrium complexification strategy of the designer from the left panel, given that the consumer's prior belief is $\mu = \psi$.

2.0.3 Prices and Production Costs

Proof of Proposition 1.2. Consider a positive-trade equilibrium in which the y-product has price p_y, with $p_G \neq p_B$. As the designer of a bad product would only offer a price $p_B \geq c(B)$, then the consumer would reject all products with price p_B with probability one, as $w_0 > \widetilde{w}(B) - p_B$. By the same argument $p_G \geq c(G) > c(B)$, but then the B-product designer would expect to make profits by deviating to offering price p_G, as the product has a positive probability of being accepted by the consumer. Thus, in any equilibrium with positive trade, it must be that $p_B = p_G$. As a result, prices do not convey information about product quality.

Consider now a candidate equilibrium in which p^* is the price set by the designer, which can be supported for example by an off-equilibrium belief that the designer has produced a B-product if he sets any other price. For any price $p^* \in (c(G), \widetilde{w}(G) - w_0)$, define payoffs $w(y) \equiv \widetilde{w}(y) - p^*$ and $v(y) \equiv p^* - c(y)$, and note that they satisfy Assumption 1. That such an equilibrium exists follows by Proposition 5, and its characterization is the same as that of our baseline model.

We also note that $p^* \leq c(G)$ cannot arise in a positive-trade equilibrium, since then only B-products would be produced (if any) and rejected with probability one. Similarly, $p^* > \widetilde{w}(G) - w_0$ would induce rejection with probability one by the consumer, as the product's payoff is now below the consumer's outside option even if the product is good. An equilibrium with $p^* =$

[5] As with convex costs of information acquisition in Appendix 1.1, the consumer is optimistic if $\lim_{\chi \to \infty} \mu(\chi) \geq \omega$, and she is pessimistic otherwise.

$\widetilde{w}(G) - w_0$ may exist, but it would require that the (indifferent) consumer accepts the product randomly, and in a manner that is correlated with the signal she acquires; moreover, such an equilibrium would unravel if we were to introduce an arbitrarily small cost of information acquisition in the region where information is costless in our baseline model. ■

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