Is There Too Much Benchmarking in Asset Management?

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Online Appendices

B Omitted Proofs

The Fund Investor's Problem in Terms of Exponential Utilities:

$$\max_{a,b,c} - E\exp\left\{-\gamma\left[x_{-1}^{F}p + r_x - (ar_x - br_{\mathbf{b}}) - c\right]\right\}$$

subject to the manager's incentive constraint (4) and her participation constraint

$$-E\exp\left\{-\gamma\left[ar_x - br_{\mathbf{b}} + c\right]\right\} \geq \hat{u}_0, \tag{B.1}$$

where $\hat{u}_0$ is the exponential-utility version of u_0.[39] It is well known that in settings with normally distributed returns, CARA utility can be rewritten in a mean-variance form, leading to the problem described in Section IIIB in the main text.

The Social Planner's Problem in Terms of Exponential Utilities:

$$\max_{a,b,c} - \tilde{\omega}_F E\exp\left\{-\gamma\left[x_{-1}^{F}p + r_x - (ar_x - br_{\mathbf{b}}) - c\right]\right\}$$
$$- \tilde{\omega}_D E\exp\left\{-\gamma\left[x_{-1}^{D}p + x^{D}(D-p)\right]\right\}$$

subject to (3), (4), (5), and (B.1), where $\tilde{\omega}_i$, $i = F, F$, are the modified Pareto weights.

From the FOC with respect to c it follows that the Lagrange multiplier on the participation constraint equals $\tilde{\omega}_F MU_F/MU_M$, where MU_i denotes the expected marginal utility of agent i. This value is the effective Pareto weight on the manager's utility given that the contract allows transfer between the fund investor and manager (through c). Similarly, if transfers between fund and direct investors were allowed, then $\tilde{\omega}_F MU_F/\lambda_M = \tilde{\omega}_D MU_D/\lambda_D$,

[39] In particular, if the manager's outside option is risk-free, then $\hat{u}_0 = -\exp(-\gamma u_0)$.

and the distribution effects is zero. Without transfers, the Pareto weights that cancel out the distribution effects (in the formulation with exponential utilities) are equal to inverse marginal utilities times the population weights, $\tilde{\omega}_F = \lambda_M/MU_F$ and $\tilde{\omega}_D = \lambda_D/MU_D$.

Rewriting the objective function and the participation constraint in the mean-variance form gives the problem described in

Lemma 6. The following inequality holds:

$$\frac{1-a^*}{a^*}\left[\frac{1}{a^*} - \left(\frac{\lambda_M}{a^*} + \lambda_D\right)\right] > \frac{1-a^{**}}{a^{**}}\left[\frac{1}{a^{**}} - \frac{\lambda_M/a^{**} + \lambda_D}{\lambda_M + \lambda_D}\right].$$

Proof. For expositional convenience, denote $a_1 = a^*$ and $a_2 = a^{**}$. Given that both sides of the above inequality are positive, it is equivalent to

$$\frac{(1-a_1)/a_1^2}{(1-a_2)/a_2^2}\frac{\lambda_M + a_1\lambda_D + (1-2a_1)\lambda_D}{(\lambda_M + a_2\lambda_D)\lambda_D/(\lambda_M+\lambda_D) + (1-2a_2)\lambda_D} > 1. \tag{B.2}$$

From (12) and (23) we have

$$\frac{1-a_1}{a_1^3(2a_1-1)} = \frac{1-a_2}{a_2^3(2a_2-1)}\frac{\lambda_D}{\lambda_M+\lambda_D}. \tag{B.3}$$

Substituting this in (B.2), obtain

$$\frac{a_1(2a_1-1)}{a_2(2a_2-1)}\frac{\lambda_D}{\lambda_M+\lambda_D}\frac{\lambda_M + a_1\lambda_D + (1-2a_1)\lambda_D}{(\lambda_M + a_2\lambda_D)\lambda_D/(\lambda_M+\lambda_D) + (1-2a_2)\lambda_D} > 1.$$

Since $a_1 > a_2$, it suffices to show that

$$\frac{\lambda_M + a_1\lambda_D + (1-2a_1)\lambda_D}{(\lambda_M + a_2\lambda_D)\lambda_D/(\lambda_M+\lambda_D) + (1-2a_2)\lambda_D} > \frac{\lambda_D+\lambda_M}{\lambda_D},$$

which is equivalent to

$$\frac{\lambda_M(2a_2-1)}{\lambda_D(a_1-a_2)} > 1. \tag{B.4}$$

To show (B.4), we will use equation (B.3). Rearranging (B.3) yields

$$\frac{1-a_1}{a_1^3(2a_1-1)}\frac{\lambda_M}{\lambda_D} = \frac{1-a_2}{a_2^3(2a_2-1)} - \frac{1-a_1}{a_1^3(2a_1-1)},$$

or, equivalently,

$$\frac{\lambda_M(2a_2-1)}{\lambda_D} = \frac{a_1^3}{1-a_1}\left[\frac{(1-a_2)(2a_1-1)}{a_2^3} - \frac{(1-a_1)(2a_2-1)}{a_1^3}\right].$$

The right-hand side of the above equation equals

$$\begin{aligned}
&\frac{-a_1^3+2a_1^4-2a_1^4a_2+a_2a_1^3-(-a_2^3+2a_2^4-2a_2^4a_1+a_1a_2^3)}{(1-a_1)a_2^3} \\
&= \frac{(a_1-a_2)}{(1-a_1)a_2^3}\left[-(1+2a_1a_2)(a_1^2+a_1a_2+a_2^2)+2(a_1+a_2)(a_1^2+a_2^2)+a_1a_2(a_1+a_2)\right].
\end{aligned}$$

Rearranging terms and doing some more algebra, yields

$$\frac{\lambda_M(2a_2-1)}{\lambda_D(a_1-a_2)} = \frac{(2a_1-1)a_1^2(1-a_2)+(2a_2-1)a_2^2(1-a_1)+(2a_1-1)a_1a_2+2a_1a_2^2(1-a_1)}{a_2^3(1-a_1)}.$$

Since $1/2 < a_2 < a_1 < 1$,

$$\frac{\lambda_M(2a_2-1)}{\lambda_D(a_1-a_2)} > \frac{(2a_1-1)a_1^2(1-a_2)+(2a_2-1)a_2^2(1-a_1)+(2a_1-1)a_1a_2+a_2^3(1-a_1)}{a_2^3(1-a_1)} > 1,$$

and thus (B.4) holds. □

Lemma 7. The fund investor's and social planner's second-order conditions are satisfied in the equilibria with privately and socially optimal contracts, respectively.

Proof of Lemma 7. Denote by $F_{b/a}$ and F_a the left-hand sides of the FOCs with respect to b/a and a, respectively. From the proofs of Lemmas 2 and 4, once we plug in the FOC with respect to b/a in the FOC with respect to a, the remaining terms only depend a. Thus we can write F_a in the following form: $F_a = g(a) + F_{b/a}h(a, b/a)$. The function $g(a)$

is given by (the right-hand sides of) equations (12) and (23) with privately and socially optimal contracts, respectively.

Differentiating F_a with respect to a and b/a,

$$F_{aa} = \frac{\partial F_a}{\partial a} = g'(a) + \underbrace{F_{b/a}}_{=0} \frac{\partial h(a, b/a)}{\partial a} + F_{b/a,a} h(a, b/a),$$
$$F_{a,b/a} = \frac{\partial F_a}{\partial (b/a)} = \underbrace{F_{b/a}}_{=0} \frac{\partial h(a, b/a)}{\partial (b/a)} + F_{b/a,b/a} h(a, b/a).$$

Notice that $g'(a) < 0$ (this follows from (12) with privately optimal contracts and from (23) with socially optimal contracts). Furthermore, $F_{b/a,b/a} < 0$. Indeed, in the privately optimal case, $F_{b/a,b/a} = -\gamma\sigma^2/a < 0$. Similarly, in the socially optimal case, $F_{b/a,b/a} = -\gamma\Lambda\lambda_M\sigma^2 - \gamma\sigma^2/a < 0$. Finally,

$$\det \begin{pmatrix} F_{b/a,b/a} & F_{a,b/a} \\ F_{a,b/a} & F_{a,a} \end{pmatrix} = \det \begin{pmatrix} F_{b/a,b/a} & F_{b/a,b/a} h \\ F_{b/a,b/a} h & g'(a) + F_{b/a,b/a} h^2 \end{pmatrix} = g'(a) F_{b/a,b/a} > 0.$$

This completes the proof. □

C Discussion on Value Added and Costs of Asset Management

This appendix elaborates on the assumptions we make regarding the costs and benefits of asset management.

As mentioned in the body of the paper, there are a variety of interpretations for alpha. In our formulation, alpha has nothing to do with superior information, which could be associated with stock-selection and market-timing abilities. Under this interpretation, direct investors who happen to buy the same assets or traded at the same time still do not earn the same returns as the managers. This interpretation has the advantage of being consistent with the vast literature (e.g., Fama and French, 2010) that casts doubt on the ability to generate abnormal returns by stock picking or market timing.

It is also consistent with a great deal of empirical evidence suggesting that savvy investors can augment their returns by lending securities, by conserving on transactions costs (e.g., from crossing trades in-house or by obtaining favorable quotes from brokers) or by providing liquidity (i.e., serving as a counterparty to liquidity demanders and earning a premium on such trades). For example, securities lending contributed 5% of total revenue of both BlackRock and State Street in 2017. While it has recently become possible for some retail investors to participate in securities lending, they earn lower returns for this activity and do not have the same opportunities as a large asset management firm. It is also well established that portfolio managers can profit from providing immediacy in trades, by either buying assets which are out of favor or selling ones that are in high demand.[40] It would be prohibitively expensive for retail investors to try to do this. Finally, Eisele et al. (2020) present evidence that trades crossed internally within a fund complex are executed more cheaply than comparable external trades.

The noise term ε in (1) captures the fact that the return-augmenting activities do not produce a certain return each period. For example, the demand for liquidity, the opportunities to lend shares and the possibility of crossing trades all fluctuate, so even a very alert and skilled manager will have some randomness in her returns. Also for securities that are lent, there is a risk that they will not be returned in a timely manner or potentially at all.

There is also considerable evidence to support our assumption that the manager must incur a private cost in order to deliver the abnormal returns. For instance, to successfully buy and sell at the appropriate times to provide liquidity, the manager has to be actively monitoring market conditions while markets are open. For securities lending, the manager would also have to decide whether to accommodate requests to borrow shares. In some cases, these demands arise because the entity borrowing the shares wants to vote them and the manager must decide whether to pass up that choice.[41]

[40] In a classic paper, Keim (1999) estimates an annual alpha of 2.2% earned by liquidity provision activities of a fund. Rinne and Suominen (2016) document that the top decile of liquidity providing mutual funds outperform the bottom decile by about 60 basis points per year. Anand, Jotikasthira and Venkataraman (2018) find similar estimates using a different sample of funds over a different time period.

[41] Most managers also incur some costs that are observable and can be passed on directly to fund

We could instead assume that the private cost arises because the manager needs to exert costly effort to generate the excess returns, as is often done in the contracting literature (e.g., Holmstrom and Milgrom, 1987, 1991). Incorporating effort makes the algebra much more involved.[42] However, under certain assumptions our main insights extend to this case. Importantly, it is the unobservability of the portfolio holdings and not the unobservability of effort that is central to our mechanism. To make this clear and to focus on the key friction, in our main model we do not include an effort choice. We analyze an extension that incorporates effort in Appendix E.1 and show that our main insights carry over.

It is also plausible that the benefits and costs associated with the return-augmenting activities are increasing in the size of the holdings.[43] For example, in terms of the liquidity provision and trade-crossing, the wider the range of securities in the portfolio and/or the more a fund holds on any particular security, the easier it would be to provide liquidity or more likely it would be that a trade can be offset. For securities lending, a larger portfolio opens up additional lending opportunities. As mentioned earlier, it is simplest to think of the costs as being tied to the time it takes to undertake the various activities. Thought of this way, if the opportunities to augment returns increase as the portfolio expands, then the costs of realizing them would naturally grow too.

investors. Examples would include custody, audit, shareholder reports, proxies and some external legal fees. Our main results continue to hold in a model in which some costs are observable.

[42]Our results trivially extend if effort is bounded from above (e.g., if there is a time constraint), and the optimal solution is at the upper bound.

[43]Implicit in our expressions for the return on the fund in (1) and the portfolio-management cost is that they scale linearly with the size of the portfolio. This is seemingly inconsistent with Berk and Green (2004) who assume that there are decreasing returns to scale in asset management, but it is not. Berk and Green explicitly attribute decreasing returns to scale to the price impact of fund managers. The bigger the portfolio invested in an alpha-opportunity, the smaller the return on a marginal dollar invested. Berk and Green's model is in partial equilibrium and their price impact is simply an exogenous function of fund size. Ours is a general-equilibrium model, in which the price impact *endogenously* arises from a higher aggregate demand of portfolio managers for the risky asset. Linearity allows us to solve the model in closed form, but what is important conceptually is that the cost is increasing in x. We show in Appendix E.1 that while the algebra is messier, under some assumptions our main analysis extends to the case of more general specifications of the return and cost.

D Achieving the Social Optimum with Taxes

This appendix analyzes how imposing taxes can implement the constrained socially optimal allocation and stock price in the equilibrium in which contracts are chosen by fund investors. There are multiple ways of doing that, and we consider two alternatives here—one with proportional income taxes (or subsidies) on the managers and fund investors, the other with an income tax on the managers and a cap on a.[44]

First, suppose there are proportional tax rates on the fund investors' and managers' incomes, denoted by t and t', respectively. The tax revenue—which is uncertain, given that the incomes are uncertain—is distributed to the fund investors as a lump-sum transfer T. Denote the constant and stochastic part of the transfer by τ_0 and τ so that $T = \tau_0 + \tau(\tilde{D} - p)$. How τ_0 and τ are determined is discussed later.

Since we want to implement the constrained optimal allocation, the taxes and the lump-sum transfer will be such that $y = (1-t')[ax - b]$ and $z = (1-t)[(1-a)x + b] + \tau$ are the same as in the constrained social optimum.

The utilities of the fund investor and manager with taxes can be written as

$$U^F = (1-t)(1-a)x\Delta + z(\mu - p) - c(1-t) + \tau_0 - \frac{\gamma}{2}\left[z^2\sigma^2 + (1-t)^2(1-a)^2\sigma_\varepsilon^2\right] + x_{-1}^F p,$$
$$U^M = (1-t')ax\Delta - x\psi + y(\mu - p) + c(1-t') - \frac{\gamma}{2}\left[y^2\sigma^2 + (1-t')^2a^2\sigma_\varepsilon^2\right].$$

The manager's demand function is

$$x^M = \frac{\Delta - \psi/[a(1-t')] + \mu - p}{\gamma\sigma^2 a(1-t')} + \frac{b(1-t')}{a(1-t')}. \tag{D.1}$$

To implement the social optimum, we need $a(1-t') = a^{**}$ and $b(1-t') = b^{**}$.

From the first-order condition with respect to c, the Lagrange multiplier on the man-

[44] As will become clear from the analysis, we need two tax rates to eliminate the differences in the two first-order conditions (with respect to b/a and a) in the private and social cases, and one tax rate is not enough.

ager's participation constraint is $\xi = (1-t)/(1-t')$. The fund investor maximizes

$$U^F + \xi U^M = [(1-t)x + \tau](\Delta + \mu - p) + \tau_0 - \frac{1-t}{1-t'}x\psi$$
$$- \frac{\gamma}{2}\left\{z^2\sigma^2 + \frac{1-t}{1-t'}y^2\sigma^2 + (1-t)\left[(1-t)(1-a)^2 + (1-t')a^2\right]\sigma_\varepsilon^2\right\}$$

subject to the manager's incentive constraint (D.1), $y = (1-t')[ax-b]$, and

$$z = (1-t)\left[\frac{1}{1-t'}\frac{1-a}{a}y + \frac{b}{a}\right] + \tau.$$

The first-order condition with respect to b/a is

$$(1-t)(\Delta + \mu - p - \gamma\sigma^2 z) - \frac{1-t}{1-t'}\psi = 0,$$
$$\Delta + \mu - p - \gamma\sigma^2 z - \frac{1}{1-t'}\psi = 0. \tag{D.2}$$

Recall that the planner's first-order condition with respect to b/a is

$$\Delta + \mu - p - \gamma\sigma^2 z - \psi\frac{\lambda_M/a^{**} + \lambda_D}{\lambda_M + \lambda_D} = 0.$$

To equate the two, we need $1 - t' = (\lambda_M + \lambda_D)(\lambda_M/a^{**} + \lambda_D)$, or

$$t' = \frac{\lambda_M}{\lambda_M/a^{**} + \lambda_D}\frac{1-a^{**}}{a^{**}}. \tag{D.3}$$

Intuitively, the positive tax on the manager's income inflates his costs relative to returns, which discourages him from investing in the risky asset.

The first-order condition with respect to a is

$$(1-t)\left[(1-t)(1-a) - (1-t')a\right]\gamma\sigma_\varepsilon^2 + (\Delta + \mu - p + \gamma\sigma^2 z)\frac{1-t}{1-t'}\frac{1-a}{a}\frac{\partial y}{\partial a} = 0.$$

Dividing by $1-t$ and using (D.2), $\partial y/\partial a = \psi/(\gamma\sigma^2 a^2(1-t'))$, and $a(1-t') = a^{**}$, the above condition can be rewritten as

$$[(1-t)(1-a)-(1-t')a]\,\gamma\sigma_\varepsilon^2 + \frac{1-a}{a^{**3}}\frac{\psi^2}{\gamma\sigma^2} = 0.$$

Recall that the planner's first-order condition with respect to a is

$$(1-2a^{**})\gamma\sigma_\varepsilon^2 + \frac{1-a^{**}}{a^{**3}}\frac{\psi^2}{\gamma\sigma^2}\frac{\lambda_D}{\lambda_M+\lambda_D} = 0.$$

To equate the two, we need

$$\frac{1-a}{(1-t)(1-a)-(1-t')a} = \frac{\lambda_D}{\lambda_M+\lambda_D}\frac{1-a^{**}}{1-2a^{**}}, \tag{D.4}$$

From $a = a^{**}/(1-t') = a^{**}(\lambda_M/a^{**}+\lambda_D)/(\lambda_M+\lambda_D)$, $1-a = (1-a^{**})\lambda_D/(\lambda_M+\lambda_D)$, and (D.4) simplifies to $(1-t)(1-a)-(1-t')a = 1-2a^{**}$, or

$$t(1-a)+t'a = 0. \tag{D.5}$$

Using the expression for t' given in (D.3) and $a = a^{**}/(1-t')$, we have

$$t = -\lambda_M/\lambda_D.$$

That is, in order to implement the constrained social optimum, the fund manager's income tax rate should be negative. Intuitively, in order to discourage the fund investor from setting a too high, the subsidy should be used so that the fund investor effectively retains a larger share of the return for himself. His after-tax share of the return equals $(1-t)(1-a) = 1-(1-t')a$. That is, it is as if he only has to give $(1-t')a$ instead of a to the manager. Thus the income tax rates t and t' considered here effectively translate into the tax rates of t' imposed directly on a and b such that $(1-t')a = a^{**}$ and $(1-t')b = b^{**}$.

Finally, the transfer to the fund investor that balances the budget is

$$\begin{aligned} T &= [t(1-a)+t'a]x(\Delta+\tilde{D}-p)+(t-t')[b(\tilde{D}-p)-c] \\ &= (t-t')[b(\tilde{D}-p)-c], \end{aligned}$$

where the last equality follows from (D.5), and so $\tau_0 = (t-t')c$ and $\tau = (t-t')b$. Note that while $t-t' < 0$, the expected lump-sum transfer $(t-t)'\,[b(\mu-p)-c]$ can be negative or positive depending on the value of the manager's outside option, which pins down c.

An alternative scheme that achieves the social optimum is a combination of the income tax rate t' given by (D.3) imposed on the manager together with a cap (an upper bound) on the sensitivity of the manager's compensation with respect to the fund performance, a, at $\bar{a} = a^{**}/(1-t')$, so that $a \leq \bar{a} = (\lambda_M + a^{**}\lambda_D)/(\lambda_M+\lambda_D)$. As before, the total amount of tax revenue should be paid to the fund investor as a lump-sum transfer.

E Extensions

E.1 Incorporating an Effort Choice by the Manager

In this appendix we extend the model in the main text to incorporate an effort choice by the manager. We will assume here that the effort choice is unobservable to the fund investor (the analysis of the case with observable effort is similar). We still assume, as in the main text, that the manager's portfolio choice is unobservable as well. We will demonstrate that our main insights extend in this case. In particular, the individual fund managers overestimate the effectiveness of incentive provision relative to the planner, which results in crowded trades.

Consider general functional forms so that the benefit function is $\tilde{\Delta}(x,e)$, the cost function is $\tilde{\psi}(x,e)$, and the variance of the noise term is $\tilde{\varepsilon}(x,e)$.

The manager's problem is

$$\max_{x,e}\; a\tilde{\Delta}(x,e) - \tilde{\psi}(x,e) + (ax-b)(\mu-p) - \frac{\gamma}{2}\sigma^2(ax-b)^2 - \frac{\gamma}{2}a^2\tilde{\varepsilon}(x,e) + c.$$

The first-order conditions with respect to e is

$$\frac{\partial\tilde{\Delta}}{\partial e}-\frac{1}{a}\frac{\partial\tilde{\psi}}{\partial e}-\frac{\gamma}{2}a\frac{\partial\tilde{\varepsilon}}{\partial e}=0. \tag{E.1}$$

Think of the optimal effort solving (E.1) as $e^*(x,a)$.

We impose the following assumptions.

Assumption 1. *Suppose that for each $a\in[1/2,1]$, the function*

$$a\tilde{\Delta}(x,e)-\tilde{\psi}(x,e)-\frac{\gamma a^2}{2}\left[x^2+\varepsilon(x,e)\right]$$

is concave in (x,e). Moreover, denote

$$\frac{df(x,e^*(x,a))}{dx}=\frac{\partial f}{\partial e}\frac{\partial e^*}{\partial x}+\frac{\partial f}{\partial x},$$

where function f is either $\tilde{\Delta}$, $\tilde{\psi}$, or $\tilde{\varepsilon}$, and $e^*(x,a)$ is implicitly defined by (E.1). Suppose that for each $a\in[1/2,1]$,

$$\frac{d\psi}{dx}>\frac{\gamma}{2}\left|\frac{d\varepsilon}{dx}\right|,\quad \frac{d^2\psi}{dx^2}\geq\frac{\gamma}{2}\left|-\frac{d^2\varepsilon}{dx^2}\right|.$$

The above inequalities require that the manager's private cost is sufficiently increasing and sufficiently convex in x (once the optimal effort choice is taken into account).

We now proceed with the analysis of the manager's problem. The manager's first-order condition with respect to x (taking into account the fact that x affects the optimal choice of effort according to $e^*(x,a)$) is

$$\mu-p-\gamma\sigma^2(ax-b)+\frac{d\tilde{\Delta}}{dx}-\frac{1}{a}\frac{d\tilde{\psi}}{dx}-\frac{\gamma}{2}a\frac{d\tilde{\varepsilon}}{dx}=0. \tag{E.2}$$

Assumption 1 implies that the second-order conditions are satisfied, in particular,

$$SOC_x\equiv-\gamma\sigma^2a+\frac{d^2\tilde{\Delta}}{dx^2}-\frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2}-\frac{\gamma a}{2}\frac{d^2\tilde{\varepsilon}}{dx^2}<0.$$

In what follows, we will use expressions for the effects of b and a on x that we derive below. Differentiating (E.2) with respect to b,

$$\gamma\sigma^2 + SOC_x \frac{\partial x}{\partial b} = 0,$$
$$\frac{\partial x}{\partial b} = -\frac{\gamma\sigma^2}{SOC_x} = \frac{\gamma\sigma^2}{\gamma\sigma^2 a - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}} > 0.$$

Denote $\frac{dx}{di} \equiv \frac{\partial x}{\partial i} + \frac{\partial x}{\partial p}\frac{\partial p}{\partial i}$, $i \in \{a, b\}$. Taking the total derivative of (E.2) with respect to b,

$$\gamma\sigma^2 - \frac{\partial p}{\partial b} + SOC_x \frac{dx}{db} = 0. \tag{E.3}$$

Differentiating the market-clearing condition $\lambda_M x + \lambda_D x^D = \bar{x}$ with respect to b (and using the expression for x^D in the main text),

$$\lambda_M \frac{dx}{db} + \lambda_D \frac{\partial x^D}{\partial p}\frac{\partial p}{\partial b} = \lambda_M \frac{dx}{db} - \lambda_D \frac{1}{\gamma\sigma^2}\frac{\partial p}{\partial b} = 0,$$
$$\frac{\partial p}{\partial b} = \gamma\sigma^2 \frac{\lambda_M}{\lambda_D}\frac{dx}{db}.$$

Substituting this into (E.3), yields

$$\frac{dx}{db} = \frac{\gamma\sigma^2}{\gamma\sigma^2 \frac{\lambda_M}{\lambda_D} - SOC_x} = \frac{\gamma\sigma^2}{\gamma\sigma^2\left(a + \frac{\lambda_M}{\lambda_D}\right) - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}}.$$

Notice that $\frac{dx}{db} \leq \frac{\partial x}{\partial b}$, with strict inequality if $\lambda_M > 0$.

Similarly, differentiating (E.2) with respect to a, gives

$$
\begin{aligned}
& -\gamma\sigma^2 x + \frac{1}{a^2}\frac{d\tilde{\psi}}{dx} - \frac{\gamma}{2}\frac{d\tilde{\varepsilon}}{dx} + SOC_x \frac{\partial x}{\partial a} = 0, \\
& \frac{\partial x}{\partial a} = \frac{1}{\gamma\sigma^2 a - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}} \left[\frac{1}{a^2}\frac{d\tilde{\psi}}{dx} - \frac{\gamma}{2}\frac{d\tilde{\varepsilon}}{dx}\right] - x\frac{\partial x}{\partial b}. \qquad \text{(E.4)}
\end{aligned}
$$

The last term captures the negative effect of a on x because the manager is exposed to too much aggregate risk—the effect which b offsets. There is a new effect that we did not have before—a larger a reduces x if $\tilde{\varepsilon}$ is increasing in x because it exposes the manager to more idiosyncratic risk, and this risk cannot be offset by an increase in b. Notice that without it (as in the main text), we would have $\partial x/\partial a + x\partial x/\partial b > 0$, which captures the fact with b offsetting the negative effect of a on x, we are only left with the positive effect that is coming from reducing the effective cost. We want to make sure that $\partial x/\partial a + x\partial x/\partial b > 0$. Notice that if this was not the case, it would not be optimal for the fund investor to use a for incentive provision purposes. Assumption 1 ensures that, and we have

$$
\frac{\partial x}{\partial a} + x\frac{\partial x}{\partial b} = \frac{\frac{1}{a^2}\frac{d\tilde{\psi}}{dx} - \frac{\gamma}{2}\frac{d\tilde{\varepsilon}}{dx}}{\gamma\sigma^2 a - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}} > 0.
$$

Similarly, we have

$$
\frac{dx}{da} + x\frac{dx}{db} = \frac{\frac{1}{a^2}\frac{d\tilde{\psi}}{dx} - \frac{\gamma}{2}\frac{d\tilde{\varepsilon}}{dx}}{\gamma\sigma^2\left(a + \frac{\lambda_D}{\lambda_M}\right) - \frac{d^2\tilde{\Delta}}{dx^2} + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}},
$$

which is smaller than $\partial x/\partial a + x\partial x/\partial b$.

We now turn to the analysis of the fund investor's problem. Denoting $y = ax - b$ and

$z = x - y$, this problem is

$$\max_{a,b,c,x} \quad (1-a)\tilde{\Delta}(x, e^*(x,a)) + z(\mu - p) - \frac{\gamma\sigma^2}{2}z^2 - \frac{\gamma(1-a)^2}{2}\tilde{\varepsilon}^2(x, e^*(x,a)) - c$$

subject to the manager's participation constraint and incentive constraint (E.2) (in which we substituted $e^*(x,a)$ implicitly defined by (E.1)).

The fund investor's first-order condition with respect to b is

$$\frac{d(U^F + U^M)}{db} = \frac{\partial U^F}{\partial x}\frac{\partial x}{\partial b} + \underbrace{\frac{\partial U^M}{\partial x}}_{=0}\frac{\partial x}{\partial b} + \frac{\partial (U^F + U^M)}{\partial b} = 0. \tag{E.5}$$

The last term captures how b directly affects the social welfare by linearly transferring from y to z. The first term captures the indirect effect of b on social welfare through its effect on the manager's demand x. Intuitively, notice that $\partial U^F/\partial x$ should be positive, otherwise b would not be positive. We will show that $\partial U^F/\partial x > 0$ formally below. The last term in (E.5) is

$$\frac{\partial (U^F + U^M)}{\partial b} = -\frac{\gamma\sigma^2}{2}\frac{\partial (y^2 + z^2)}{\partial b} = \gamma\sigma^2(y - z) = \gamma\sigma^2\left[(2a-1)x - 2b\right].$$

We will show below that this term is negative (notice that this term would be zero under perfect risk sharing $a = 1/2$ and $b = 0$.)

Using (E.2),

$$\begin{aligned}\frac{\partial U^F}{\partial x} &= (1-a)\left[\frac{d\tilde{\Delta}}{dx} + \mu - p - \gamma\sigma^2 z - \frac{\gamma}{2}(1-a)\frac{d\tilde{\varepsilon}}{dx}\right] \\ &= (1-a)\left[\gamma\sigma^2(y-z) + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right].\end{aligned}$$

Then the investor's first-order condition with respect to b becomes

$$(1-a)\left[\gamma\sigma^2(y-z) + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right]\frac{\partial x}{\partial b} + \gamma\sigma^2(y-z) = 0, \tag{E.6}$$

or equivalently

$$\frac{(1-a)\dfrac{\partial x}{\partial b}}{(1-a)\dfrac{\partial x}{\partial b}+1}\left[\frac{1}{a}\frac{d\tilde{\psi}}{dx}+\frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right]+\gamma\sigma^2(y-z)=0. \tag{E.7}$$

Notice that since the first term is strictly positive by Assumption 1, the second term is strictly negative. It then also follows that the term in the square brackets in E.6 must be strictly positive, that is, $\partial U^F/\partial x = \partial(U^F+U^M)/\partial x > 0$. Intuitively, it means that it is optimal for the fund investor to use contracts to provide incentives. It also then follows that $b > 0$. Indeed, notice that at $b = 0$ and $a \in [1/2, 1]$, the left-hand side of (E.7) is strictly positive given Assumption 1, and thus $b \leq 0$ cannot be optimal.

We will now compare the social planner's first-order condition with respect to b to that of an individual fund investor. The planner's first-order condition with respect to b (after canceling out the distributive effects, as in the main text) is the same as the corresponding first-order condition for an investor, but $\partial x/\partial b$ is being replaced with dx/db, namely

$$\frac{\partial U^F}{\partial x}\frac{dx}{db}+\frac{\partial(U^F+U^M)}{\partial b}=0,$$

or

$$\frac{(1-a)\dfrac{dx}{db}}{(1-a)\dfrac{dx}{db}+1}\left[\frac{1}{a}\frac{d\tilde{\psi}}{dx}+\frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right]+\gamma\sigma^2(y-z)=0.$$

Since $dx/db < \partial x/\partial b$ as long as $\lambda_M > 0$,

$$\frac{(1-a)\dfrac{dx}{db}}{(1-a)\dfrac{dx}{db}+1}<\frac{(1-a)\dfrac{\partial x}{\partial b}}{(1-a)\dfrac{\partial x}{\partial b}+1}.$$

It then follows that under Assumption 1, the additional terms in the planner's first-order condition relative to the investor's first-order condition are strictly negative.

Now consider the first-order condition with respect to a. In the privately optimal case, it is

$$\frac{d(U^F+U^M)}{da} = \frac{\partial U^F}{\partial x}\frac{\partial x}{\partial a} + \frac{\partial U^F}{\partial e}\frac{\partial e}{\partial a} + \frac{\partial (U^F+U^M)}{\partial a} = 0.$$

Rewrite this to get

$$\begin{aligned}
\frac{d(U^F+U^M)}{da} &= (1-a)\left[\gamma\sigma^2(y-z) + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right]\frac{\partial x}{\partial a} \\
&+ (1-a)\left[\frac{\partial\tilde{\Delta}}{\partial e} - \frac{\gamma}{2}(1-a)\frac{\partial\tilde{\varepsilon}}{\partial e}\right]\frac{\partial e}{\partial a} - \gamma\sigma^2(y-z)x - \gamma\varepsilon^2(2a-1). \\
&= (1-a)\left[\gamma\sigma^2(y-z) + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right]\frac{\partial x}{\partial a} \\
&+ (1-a)\left(\frac{1}{a}\frac{\partial\tilde{\psi}}{\partial e} + \frac{\gamma}{2}(2a-1)\frac{\partial\tilde{\varepsilon}}{\partial e}\right)\frac{\partial e}{\partial a} - \gamma\sigma^2(y-z)x - \gamma\varepsilon^2(2a-1) = 0.
\end{aligned}$$

where the second equality uses (E.1). Then using (E.6), we can rewrite the above condition as follows:

$$\begin{aligned}
&(1-a)\left[\gamma\sigma^2(y-z) + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right]\left(\frac{\partial x}{\partial a} + x\frac{\partial x}{\partial b}\right) \\
&+ (1-a)\left(\frac{1}{a}\frac{\partial\tilde{\psi}}{\partial e} + \frac{\gamma}{2}(2a-1)\frac{\partial\tilde{\varepsilon}}{\partial e}\right)\frac{\partial e}{\partial a} - \gamma\varepsilon^2(2a-1) = 0.
\end{aligned}$$

Using (E.7), the fund investor's first-order condition with respect to a becomes

$$\begin{aligned}
&\frac{(1-a)\left(\frac{\partial x}{\partial a} + x\frac{\partial x}{\partial b}\right)}{(1-a)\frac{\partial x}{\partial b} + 1}\left[\frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right] + (1-a)\left(\frac{1}{a}\frac{\partial\tilde{\psi}}{\partial e} + \frac{\gamma}{2}(2a-1)\frac{\partial\tilde{\varepsilon}}{\partial e}\right)\frac{\partial e}{\partial a} \\
&- \gamma\varepsilon^2(2a-1) = 0. \qquad \text{(E.8)}
\end{aligned}$$

Notice that we need $d\tilde{\psi}/dx > 0$ or $\partial\tilde{\psi}/\partial e > 0$, otherwise $a = 1/2$ is optimal. This is guaranteed by Assumption 1.

The social planner's first-order condition with respect to a is obtained from (E.8) by replacing

$$\frac{(1-a)\left(\frac{\partial x}{\partial a}+x\frac{\partial x}{\partial b}\right)}{(1-a)\frac{\partial x}{\partial b}+1}=\frac{\left(\frac{1}{a}-1\right)\left(\frac{1}{a}\frac{d\tilde{\psi}}{dx}-\frac{\gamma}{2}a\frac{d\tilde{\varepsilon}}{dx}\right)}{\gamma\sigma^2-\frac{d^2\tilde{\Delta}}{dx^2}+\frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2}+\frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}}$$

by a strictly smaller term,

$$\frac{(1-a)\left(\frac{dx}{da}+x\frac{dx}{db}\right)}{(1-a)\frac{dx}{db}+1}=\frac{\left(\frac{1}{a}-1\right)\left(\frac{1}{a}\frac{d\tilde{\psi}}{dx}-\frac{\gamma}{2}a\frac{d\tilde{\varepsilon}}{dx}\right)}{\gamma\sigma^2\left(1+\frac{\lambda_D}{\lambda_M}\right)-\frac{d^2\tilde{\Delta}}{dx^2}+\frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2}+\frac{\gamma}{2}a\frac{d^2\tilde{\varepsilon}}{dx^2}}.$$

Recall that the term in square brackets in (E.8) is strictly positive (by Assumption 1). Therefore in the socially optimal case, there are additional negative terms (or the positive terms are smaller) in the first-order condition with respect to a relative to that in the privately optimal case.

As in the main model in the text, the planner recognizes that incentive provision is weaker than individual fund investors perceive it to be. This is captured by additional negative terms in the first-order conditions for a and b. It is no longer straightforward to establish that the presence of these terms imply that both a and b in the socially optimal case are smaller than those in the privately optimal case. Doing so requires us to impose additional, hard to interpret, assumptions on the cross-derivatives and third derivatives of the functions $\tilde{\Delta}$, $\tilde{\psi}$ and $\tilde{\varepsilon}$. Intuitively, these assumptions are sufficient conditions to guarantee that a and b are complements.

We can still prove the crowded trades result, namely, $p^{**} < p^*$. Define $k = (a, b)$, $W(k,p) = U^F(k,p,x(k,p),e^*(k,x(k,p))) + U^M(k,p,x(k,p),e^*(k,x(k,p)))$. The fund investor's problem is to maximize $W(k,p)$ with respect to k taking p as given. Since we cancel out the distributive effects in the social planner's problem, it is equivalent to maximizing $W(k,p(k))$ with respect to k.

Denote the optimal solutions in the privately and socially optimal cases by k^* and k^{**}, respectively. Notice that

$$W(k^{**}, p(k^{**})) > W(k^{*}, p(k^{*})) > W(k^{**}, p(k^{*}))$$

implying

$$W(k^{**}, p(k^{**})) > W(k^{**}, p(k^{*})). \tag{E.9}$$

Differentiating W with respect to p (and canceling the distributive effects),

$$\frac{dW}{dp} = \frac{\partial U^F}{\partial x}\frac{dx}{dp} = (1-a)\left\{\gamma\sigma^2\left[(2a-1)x(p) - 2b\right] + \frac{1}{a}\frac{d\tilde{\psi}}{dx} + \frac{\gamma}{2}(2a-1)\frac{d\tilde{\varepsilon}}{dx}\right\}\frac{dx}{dp} < 0.$$

Differentiating with respect to p one more time,

$$\begin{aligned}\frac{d^2W}{dp^2} &= \frac{dW_s}{dx}\left(\frac{dx}{dp}\right)^2 + W_s\underbrace{\frac{d^2x}{dp^2}}_{=0} \\ &= \left[\gamma\sigma^2(2a-1)x + \frac{1}{a}\frac{d^2\tilde{\psi}}{dx^2} + \frac{\gamma}{2}(2a-1)\frac{d^2\tilde{\varepsilon}}{dx^2}\right]\left(\frac{dx}{dp}\right)^2 > 0\end{aligned}$$

by Assumption 1. Since $dW(k^{**}, p)/dp < 0$ at $p = p^{**}$, this implies that $W(k^{**}, p(k^{**})) < W(k^{**}, p)$ for $p < p(k^{**})$. Given inequality (E.9), it must be the case $p(k^{**}) < p(k^{*})$. It then also follows that $x(k^{**}) < x(k^{*})$. So the crowded trade results from the main text extends to the case with unobservable effort.

E.2 Endogenous Δ

In this appendix we consider the case in which Δ is determined in equilibrium in the market for securities lending. We include a new class of investors who seek to borrow the stock from fund managers so that they could sell them short. These investors therefore incur a borrowing cost of Δ per share, which allows the fund managers to earn revenue of Δ per share. Typical motives for shorting considered in the literature are (i) hedging and (ii) speculation. We choose the first one, so that the resulting model is not too far from our baseline setting. We believe that the insights of this appendix go through in alternative settings, so long as one is not adding market frictions together with additional classes of agents.

We consider a new group of agents, hedgers, H (measure λ_H), endowed with $e\tilde{D}$ units of consumption in period 1.[45] They engage in short selling in period 0 for hedging purposes. Their utility (converted into the mean-variance form) is

$$\max_x \ (x+e)\mu - xp + x\Delta \mathbf{1}_{x\leq 0} - \frac{\gamma}{2}(x+e)^2\sigma^2,$$

where Δ is the borrowing cost and it is incurred only when the hedgers' demand is negative. It is easy to show that the hedgers' portfolio demand is given by

$$x^H = \frac{\mu - p + \Delta}{\gamma\sigma^2} - e. \tag{E.10}$$

We focus on the case when e is large enough so that x^H.

In practice, a fund manager would not be permitted to lend out the entire portfolio and would lend out only a fraction of it. We assume that the number of shares lent out by the manager is ℓx^M, where $\ell \in (0,1]$ is exogenous. The fund's augmented return is now $\ell\Delta x^M$ and the manager's cost is $\ell\psi x^M$. The manager's portfolio is then

$$x^M = \frac{\mu - p + \ell\Delta - \ell\psi/a}{a\gamma\sigma^2} + \frac{b}{a}. \tag{E.11}$$

Substituting (E.10) and (E.11) into the securities-lending market-clearing condition,

$$\ell\lambda_M x^M + \lambda_H x^H = 0, \tag{E.12}$$

[45]Without loss of generality, we assume that the hedgers are endowed with zero shares at time zero.

leads to the following expression for $p - \ell\Delta$:

$$p - \ell\Delta = \mu - \frac{1}{\lambda_H + \ell\lambda_M/a}\left[\gamma\sigma^2\left(\lambda_H e - \ell\lambda_M\frac{b}{a}\right) + \ell\lambda_M\frac{\ell\psi}{a^2} - (1-\ell)\lambda_H\Delta\right]. \tag{E.13}$$

With the new class of agents, the market-clearing condition in the asset market becomes

$$\lambda_M x^M + \lambda_H x^H + \lambda_D x^D = \bar{x},$$

which, using (E.12), can be written as

$$(1-\ell)\lambda_M x^M + \lambda_D x^D = \bar{x}. \tag{E.14}$$

Substituting (3) and (E.11) and solving for $p - \ell\Delta$ yields

$$p - \ell\Delta = \mu - \frac{1}{\lambda_D + (1-\ell)\lambda_M/a}\left[\gamma\sigma^2\left(\bar{x} - (1-\ell)\lambda_M\frac{b}{a}\right) + \lambda_D\ell\Delta - \frac{(1-\ell)\lambda_M}{a}\ell\frac{\psi}{a}\right]. \tag{E.15}$$

Next, we compare the privately and socially optimal contracts. To do this, we consider first-order conditions with respect to b/a and a. The first-order condition for the privately optimal case with respect to b/a and a are

$$\ell\Delta - \ell\psi + \mu - p - \gamma\sigma^2 z = 0 \tag{E.16}$$

and

$$\begin{aligned} 0 &= -(2a-1)\gamma\sigma_\epsilon^2 + \frac{1-a}{a}(\ell\Delta + \mu - p - \gamma\sigma^2 z)\frac{\partial y}{\partial a} \\ &= -(2a-1)\gamma\sigma_\epsilon^2 + (1-a)\ell\frac{\psi^2\sigma^2}{\gamma a^3}, \end{aligned} \tag{E.17}$$

respectively.

Now consider the socially optimal case. Define $U^H = x^H(\Delta+\mu-p)+e\mu-\frac{\gamma}{2}\left(x^H+e\right)^2\sigma^2$.

The social planner's problem is

$$\max_{a,b,c} \ \omega_F U^F + \omega_D U^D + \omega_H U^H$$

subject to (3), (7), (E.10), (E.11), (E.13), and (E.15). Denote

$$y = ax^M - b = \frac{\mu - p + \ell\Delta - \ell\psi/a}{\gamma\sigma^2}.$$

The social planner's first-order condition with respect to b/a is

$$\begin{aligned} 0 = &\left[\omega_F\left(x^F_{-1} - x^M\right) + \omega_D\left(x^D_{-1} - x^D\right) - \omega_H x^H\right]\frac{\partial p}{\partial(b/a)} + \left[\omega_F \ell x^M + \omega_H x^H\right]\frac{\partial\Delta}{\partial(b/a)} \\ &+ \ell\Delta - \ell\psi + \mu - p - \gamma\sigma^2 z + \left(\ell\Delta + \mu - p - \gamma\sigma^2 z\right)\left[\frac{1}{a} - 1\right]\frac{\partial y}{\partial(p-\ell\Delta)}\frac{\partial(p-\ell\Delta)}{\partial(b/a)}. \end{aligned}$$

As in the main text, we choose the Pareto weights to eliminate the distributive effect. Specifically, if $\omega_F = \lambda_M$, $\omega_D = \lambda_D$, and $\omega_H = \lambda_H$, then the terms in the first line of (E.18) are zero by market clearing. Thus the planner's first-order with respect to b/a becomes

$$\ell\Delta - \ell\psi + \mu - p - \gamma\sigma^2 z + \left(\ell\Delta + \mu - p - \gamma\sigma^2 z\right)\left[\frac{1}{a} - 1\right]\frac{\partial y}{\partial(p-\ell\Delta)}\frac{\partial(p-\ell\Delta)}{\partial(b/a)} = 0. \tag{E.18}$$

Differentiating (E.13) and (E.15) with respect to b/a, we can solve for $\partial(p-\ell\Delta)/\partial(b/a)$:

$$\frac{\partial(p-\ell\Delta)}{\partial(b/a)} = \Gamma\gamma\sigma^2,$$

where

$$\Gamma = \frac{[\ell^2\lambda_D + (1-\ell)^2\lambda_H]\lambda_M}{\lambda_D\lambda_H + [\ell^2\lambda_D + (1-\ell)^2\lambda_H]\,\lambda_M/a} \in (0,1).$$

and using $\partial y/\partial(p-\ell\Delta) = -1/(\gamma\sigma^2)$, we can rewrite the social planner's first-order condition with respect to b/a as

$$\ell\Delta - \frac{\ell\psi}{1-(1/a-1)\,\Gamma} + \mu - p - \gamma\sigma^2 z = 0. \tag{E.19}$$

The planner's first-order condition with respect to a (after canceling out the distributive effect) is

$$\begin{aligned} 0 = &-(2a-1)\gamma\sigma_\epsilon^2 - \left[\ell\Delta - \ell\psi + \mu - p - \gamma\sigma^2 z\right]\frac{y}{a^2} \\ &+ \frac{1-a}{a}(\ell\Delta + \mu - p - \gamma\sigma^2 z)\left[\frac{\partial y}{\partial a} + \frac{\partial y}{\partial(p-\ell\Delta)}\frac{\partial(p-\ell\Delta)}{\partial a}\right], \end{aligned}$$

which, using (E.18), becomes

$$\begin{aligned} 0 = &-(2a-1)\gamma\sigma_\epsilon^2 \\ &+ \frac{1-a}{a}(\ell\Delta + \mu - p - \gamma\sigma^2 z)\left[\frac{\partial y}{\partial a} + \frac{\partial y}{\partial(p-\ell\Delta)}\frac{\partial(p-\ell\Delta)}{\partial a} + \frac{y}{a^2}\frac{\partial y}{\partial(p-\ell\Delta)}\frac{\partial(p-\ell\Delta)}{\partial(b/a)}\right]. \end{aligned}$$

As in the main model, differentiating (E.14) with respect to b/a and a, we can show that

$$\frac{\partial y}{\partial(p-\ell\Delta)}\frac{\partial(p-\ell\Delta)}{\partial a} + \frac{y}{a^2}\frac{\partial y}{\partial(p-\ell\Delta)}\frac{\partial(p-\ell\Delta)}{\partial(b/a)} = \frac{1}{a}\frac{\partial y}{\partial a}\frac{\partial y}{\partial(p-\ell\Delta)}\frac{\partial(p-\ell\Delta)}{\partial(b/a)} = -\frac{\Gamma}{a}\frac{\partial y}{\partial a}.$$

Thus the planner's first-order condition with respect to a is

$$\begin{aligned} 0 &= -(2a-1)\gamma\sigma_\epsilon^2 + \frac{1-a}{a}(\ell\Delta + \mu - p - \gamma\sigma^2 z)\left(1 - \frac{\Gamma}{a}\right)\frac{\partial y}{\partial a} \\ &= -(2a-1)\gamma\sigma_\epsilon^2 + (1-a)\ell\frac{\psi^2}{\gamma\sigma^2 a^3}\frac{1-\Gamma/a}{1-\Gamma/a+\Gamma}. \end{aligned} \tag{E.20}$$

Comparing (E.19) with (E.16) and (E.20) with (E.17), we can see that the benefit of incentive provision is lower for the planner than for private agents, just as in the main text. The same proofs as in the main model go through for this case and thus our main results continue to hold.

The intuition for why our results go through in this setting is the following. First, all the frictions from the main model are still present. Second, the addition of hedgers and the motive for short selling do not create any additional sources of inefficiency. In particular, adding the hedgers does not complicate the contracting problem. Just as with direct investors, contracts only affect hedgers through the distributive effect. The pecuniary externality occurs because prices (now both p and Δ) enter the manager's incentive constraint. So all the forces are the same as in the main model. The mechanism for alleviating the friction is the same as in the main text, i.e., it involves using skin-in-the-game and benchmarking. The comparison of the privately and socially optimal contracts is also the same.