Online Appendix to: A Simple Method to Measure Misallocation Using Natural Experiments

Not for Publication

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I. Proof of Proposition 4

First, we show that in an efficient economy with input/output linkages and capital stock K, labor L, total output is such that: $Y^* \propto K^{\alpha^*} L^{1-\alpha^*}$.

An efficiently allocated economy with capital stock K and labor L is defined by:

$$Y^*(K,L) = \left| \begin{array}{l} \max\limits_{(k_{is}),(l_{is}),(m_{ius})} Y = \prod\limits_{s=1}^{S} Y_s^{\phi_s} \\ Y_s = \underbrace{\left(\int_i q_{is}^{\theta_s}\right)^{\frac{1}{\theta_s}}}_{=Q_s} - \sum\limits_{u=1}^{S} \underbrace{\left(\int_i m_{ius}\right)}_{=M_{us}}, \quad q_{is} = e^{z_{is}} k_{is}^{s} l_{is}^{1-\alpha_s} \prod\limits_{u=1}^{S} m_{isu}^{\gamma_{su}} \\ \int_i k_{is} \leq K \quad (\lambda), \qquad \int_i l_{is} \leq L \quad (\mu) \end{array} \right.$$

The first-order conditions w.r.t. k_{is},l_{is} and m_{isu} are:

$$\begin{cases} \frac{\phi_s}{Y_s} \alpha_s \frac{q_{is}^{\theta_s}}{k_{is}} Q_s^{1-\theta_s} = \lambda \\ \frac{\phi_s}{Y_s} (1-\alpha_s) \frac{q_{is}^{\theta_s}}{l_{is}} Q_s^{1-\theta_s} = \mu \\ \frac{\phi_s}{Y_s} \gamma_{su} \frac{q_{is}^{\theta_s}}{m_{isu}} Q_s^{1-\theta_s} = \frac{\phi_u}{Y_u} \end{cases}$$

The first-order condition for m_{ius} can be written as:

$$\frac{\phi_u}{Y_u} \gamma_{us} q_{iu}^{\theta_u} Q_u^{1-\theta_u} = \frac{\phi_s}{Y_s} m_{ius}$$

Aggregate across all firms in industry u:

$$\frac{\phi_u}{Y_u} \gamma_{us} Q_u = \frac{\phi_s}{Y_s} M_{us}$$

Sum across all industries u:

$$\sum_{u=1}^{S} \frac{\phi_u}{Y_u} \gamma_{us} Q_u = \frac{\phi_s}{Y_s} (Q_s - Y_s)$$

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Let $\boldsymbol{Y} = (Y_s)_{s\in[1,S]}$. The previous equation across all industries s implies the following matrix equation:

$$(I-\Gamma')(\boldsymbol{\phi}\circ\boldsymbol{Q}\oslash\boldsymbol{Y}) = \boldsymbol{\phi} \Rightarrow \boldsymbol{\phi}\circ\boldsymbol{Q}\oslash\boldsymbol{Y} = (I-\Gamma')^{-1}\boldsymbol{\phi}$$

Aggregate the first-order condition w.r.t. k_{is} and l_{is} across all firms in industry s:

$$\alpha_s\frac{\phi_s Q_s}{Y_s} = \lambda K_s \quad \text{and} \quad (1-\alpha_s)\frac{\phi_s Q_s}{Y_s} = \mu L_s$$

Sum the previous equations across all industries s, in matrix form:

$$\lambda K = \boldsymbol{\alpha}'(I-\Gamma')^{-1}\boldsymbol{\phi} = \boldsymbol{\phi}'(I-\Gamma)^{-1}\boldsymbol{\alpha} = \alpha^* \quad \text{and} \quad \mu L = (1-\alpha^*)$$

Now, we derive q_{is} by combining all the first-order conditions:

$$q_{is} = e^{\frac{z_{is}}{1-\theta_s}}\left(\frac{\phi_s}{Y_s}\right)^{\frac{1}{1-\theta_s}} Q_s\left(\frac{\alpha_s}{\lambda}\right)^{\frac{\alpha_s}{1-\theta_s}}\left(\frac{\beta_s}{\mu}\right)^{\frac{\beta_s}{1-\theta_s}}\prod_{u=1}^{S}\left(\frac{\gamma_{su}Y_u}{\phi_u}\right)^{\frac{\gamma_{su}}{1-\theta_s}}$$

Define $Z_s = \left(\int_i e^{\frac{\theta_s}{1-\theta_s}z_i s}\right)^{\frac{1-\theta_s}{\theta_s}}$. Aggregating across all firms in the industry implies (after taking the power θ_s) leads to:

$$1 = Z_s\left(\frac{\phi_s}{Y_s}\right)\left(\frac{\alpha_s}{\lambda}\right)^{\alpha_s}\left(\frac{\beta_s}{\mu}\right)^{\beta_s}\prod_{u=1}^{S}\left(\frac{\gamma_{su}Y_u}{\phi_u}\right)^{\gamma_{su}}$$

Define $\mathbf{log}\,(\boldsymbol{Z}) = (\log(Z_s))_{s\in[1,S]}$, $\mathbf{log}\,(\boldsymbol{Y}) = (\log(Y_s))_{s\in[1,S]}$, $\mathbf{log}\,(\boldsymbol{\phi}) = (\log(\phi_s))_{s\in[1,S]}$. The previous equations implies across industries s imply the following matrix equation:

$$(I-\Gamma)\,\mathbf{log}\,(\boldsymbol{Y}) = (I-\Gamma)\,\mathbf{log}\,(\boldsymbol{\phi}) + \mathbf{log}\,(\boldsymbol{Z}) + \boldsymbol{\alpha}\circ\mathbf{log}\,(\boldsymbol{\alpha}) + \boldsymbol{\beta}\circ\mathbf{log}\,(\boldsymbol{\beta}) - \boldsymbol{\alpha}\log(\lambda) - \boldsymbol{\beta}\log(\mu)$$

Remember that: $\lambda K = \alpha^*$ and $\mu L = (1-\alpha^*)$. Therefore, define $\boldsymbol{\alpha}\,\mathbf{log}\,(\boldsymbol{\alpha}) = (\alpha_s\log(\alpha_s))_{s\in[1,S]}$ and $\boldsymbol{\beta}\,\mathbf{log}\,(\boldsymbol{\beta}) = (\beta_s\log(\beta_s))_{s\in[1,S]}$:

$$(I-\Gamma)\,\mathbf{log}\,(\boldsymbol{Y}) = (I-\Gamma)\,\mathbf{log}\,(\boldsymbol{\phi}) + \mathbf{log}\,(\boldsymbol{Z}) + \boldsymbol{\alpha}\log(K) + (1-\boldsymbol{\alpha})\log(L) + \text{cst},$$

where cst depends on the model's parameters and is independent of (K,L). Remember that, because of Cobb-Douglas production in the final good industry, total output is such that:

$$\log(Y) = \phi'\,\mathbf{log}\,(\boldsymbol{Y})$$

The last two equations imply that total output in the efficient economy is given by:

$$\log(Y^*(K,L)) = \text{cst} + \boldsymbol{\phi}'(I-\Gamma)^{-1}\,\mathbf{log}\,(\boldsymbol{Z}) + \alpha^*\log(K) + (1-\alpha^*)\log(L),$$

where cst does not depend on (K,L). Therefore, as in our baseline case, we define aggregate TFP: as $Y = \text{TFP}\times K^{\alpha^*}L^{1-\alpha^*}$, which corresponds to the output loss experienced in the actal economy relative to the efficient economy with the same amount of aggregate capital and labor than the actual economy.

Formula for Aggregate Output Because there is perfect competition in the final good market, the demand for industry s bundle coming from the final good market is given by:

$$\phi_s PY = P_s Y_s \Rightarrow Y_s \propto \frac{Y}{P_s},$$

where we have normalized the price of the final good market to 1 ($P=1$).

Perfect competition in the production of industry bundles leads to the following demand curve for product i in industry s:

$$P_s\left(\frac{q_{is}}{Q_s}\right)^{\theta_s-1} = p_{is}$$

The first-order condition in the profit of firm i in industry s w.r.t. bundles from industry $j\in[1,S]$ implies that:

$$P_s Q_s^{1-\theta_s}\theta_s\gamma_{sj}\,(q_{is})^{\theta_s} = P_j m_{isj}$$

As a result, the total demand for bundle j from firms in industry s simply comes from aggregating the previous equation across all firms i in industry s:

$$\theta_s \gamma_{sj} P_s Q_s = P_j \underbrace{\int_i m_{isj} di}_{=M_{sj}},$$

where M_{sj} corresponds to the demand for industry j's bundles coming from industry s.

As a result, the total demand for industry j bundles coming from intermediary inputs, $M_j = \sum_{s=1}^{S} M_{sj}$ is simply:

$$P_j M_j = \sum_{s=1}^{S} \theta_s \gamma_{sj} P_s Q_s$$

Remember that the demand for industry j bundles coming from the final good market is Y_j which satisfies $\phi_j Y = P_j Y_j$.

As a result, the total demand for industry j bundle is simply given by:

$$Q_j = M_j + Y_j = \frac{\sum_{s=1}^{S} \theta_s \gamma_{sj} P_s Q_s + \phi_j Y}{P_j} \Rightarrow P_j Q_j = \sum_{s=1}^{S} \theta_s \gamma_{sj} P_s Q_s + \phi_j Y$$

Note $\aleph = (\theta_s \gamma_{sj})_{(j,s)\in[1,S]^2}$, $\boldsymbol{P} = (P_s)_{s\in[1,S]}$, $\boldsymbol{Q} = (Q_s)_{s\in[1,S]}$ and $\boldsymbol{\phi} = (\phi_s)_{s\in[1,S]}$. $\circ$ denotes the Hadamard product of two matrixes and $\oslash$ the Hadamard division. The previous equation can be rewritten as:

$$(I - \aleph)\boldsymbol{P} \circ \boldsymbol{Q} = \boldsymbol{\phi} Y \Rightarrow \boldsymbol{P} \circ \boldsymbol{Q} = \left((I - \aleph)^{-1} \boldsymbol{\phi}\right) Y$$

Therefore, aggregate sales $P_s Q_s$ in each industry s are proportional to Y, although the coefficient is industry-specific.

Turning back to the optimisation problem, the labor first order condition for each firm leads to:

$$P_s Q_s^{1-\theta_s} \theta_s \beta_s (y_{is})^{\theta_s} = w l_{is}$$

Aggregating across firm i in industry s, then across industries leads to:

$$wL = \sum_{s=1}^{S} \theta_s \beta_s P_s Q_s$$

Note $\boldsymbol{\theta} = (\theta_s)_{s\in[1,S]}$ and $\boldsymbol{\beta} = (\beta_s)_{s\in[1,S]}$, we have:

$$wL = (\boldsymbol{\theta} \circ \boldsymbol{\beta})' (\boldsymbol{P} \circ \boldsymbol{Q}) = (\boldsymbol{\theta} \circ \boldsymbol{\beta})' \left((I - \aleph)^{-1} \boldsymbol{\phi}\right) Y$$

Given that labor supply is given by $L^s = \bar{L} \left(\frac{w}{\bar{w}}\right)^{\epsilon}$, we see directly that: $Y \propto w^{1+\epsilon}$, which is the first part of the equilibrium.

We now need to compute the equilibrium wage. First order conditions in labor, capital, and inputs are given by:

$$\begin{cases} k_{is} = \alpha_s \theta_s \dfrac{p_{is} q_{is}}{R(1+\tau_i)} \\ l_{is} = \beta_s \theta_s \dfrac{p_{is} q_{is}}{w} \\ m_{isj} = \gamma_{sj} \theta_s \dfrac{p_{is} q_{is}}{P_j} \end{cases}$$

We can use the last three equations to compute firm i's output:

$$p_{is} q_{is} = e^{\theta_s z_i} P_s Q_s^{1-\theta_s} (p_{is} q_{is})^{\theta_s} \left(\frac{\alpha_s \theta_s}{R(1+\tau_i)}\right)^{\alpha_s \theta_s} \left(\frac{\beta_s \theta_s}{w}\right)^{\beta_s \theta_s} \left[\prod_{j=1}^{S} \left(\frac{\gamma_{sj} \theta_s}{P_j}\right)^{\gamma_{sj} \theta_s}\right]$$

As a result:

$$p_{is}q_{is} = e^{\frac{\theta_s}{1-\theta_s}z_i} P_s^{\frac{1}{1-\theta_s}} Q_s \left(\frac{\alpha_s\theta_s}{R(1+\tau_i)}\right)^{\alpha_s \frac{\theta_s}{1-\theta_s}} \left(\frac{\beta_s\theta_s}{w}\right)^{\beta_s \frac{\theta_s}{1-\theta_s}} \left[\prod_{j=1}^{S}\left(\frac{\gamma_{sj}\theta_s}{P_j}\right)^{\gamma_{sj}\frac{\theta_s}{1-\theta_s}}\right]$$

We can aggregate the previous equation across all firms i industry s:

$$P_sQ_s = P_s^{\frac{1}{1-\theta_s}} Q_s \left(\frac{\alpha_s\theta_s}{R}\right)^{\alpha_s \frac{\theta_s}{1-\theta_s}} \left(\frac{\beta_s\theta_s}{w}\right)^{\beta_s \frac{\theta_s}{1-\theta_s}} \left[\prod_{j=1}^{S}\left(\frac{\gamma_{sj}\theta_s}{P_j}\right)^{\frac{\gamma_{sj}\theta_s}{1-\theta_s}}\right] \underbrace{\int \frac{e^{\frac{\theta_s}{1-\theta_s}z_i}}{(1+\tau_i)^{\alpha_s\frac{\theta_s}{1-\theta_s}}} di}_{=I_s}$$

The previous equation implies that the price of industry s bundles is proportional to:

$$P_s \propto w^{\beta_s}\left[\prod_{j=1}^{S}(P_j)^{\gamma_{sj}}\right] J_s^{-\frac{1-\theta_s}{\theta_s}}$$

Taking the logarithm of the previous equation, we get that:

$$\log(P_s) = \beta_s \log(w) - \frac{1-\theta_s}{\theta_s}\log(J_s) + \sum_{j=1}^{S}\gamma_{sj}\log(P_j) + cst,$$

With our parametric assumption on the joint-distribution of $(z_i, \log(1+\tau_i))$ in industry s, we know that:

$$\frac{1-\theta_s}{\theta_s}\log(I_s) = \mu_z(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\sigma_z^2(s) + \alpha_s\left(-\mu_\tau(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s\sigma_\tau^2(s) - 2\sigma_{z\tau}(s)\right)\right)$$

Define: $\log(\mathbf{A}) = \left(\alpha_s\left(-\mu_\tau(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s\sigma_\tau^2(s) - 2\sigma_{z\tau}(s)\right)\right) + \mu_z(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\sigma_z^2(s)\right)_{s\in[1,S]}$.

Then, the previous expression lead to the following matrix representation:

$$\log(\mathbf{P}) = (I-\Gamma)^{-1}(\boldsymbol{\beta}\log(w) - \log(\mathbf{A}))$$

Now, remember that because of Cobb-Douglas aggregation in the final good market, $\Pi_{s=1}^{S}P_s^{\phi_s} = \Pi_{s=1}^{S}\phi_s^{\phi_s}$. Hence, in log vector terms, we have that: $\boldsymbol{\phi}'\log\mathbf{P} = \mathbf{cst}$. Combining this with the above equation:

$$\log w = \frac{\boldsymbol{\phi'}(I-\Gamma)^{-1}\log(\mathbf{A})}{\boldsymbol{\phi'}(I-\Gamma)^{-1}\boldsymbol{\beta}} + cst,$$

which since $Y \propto w^{1+\epsilon}$ leads to the second-order approximation for output:

$$\log Y = (1+\epsilon)\frac{\boldsymbol{\phi'}(I-\Gamma)^{-1}\log(\mathbf{A})}{\boldsymbol{\phi'}(I-\Gamma)^{-1}\boldsymbol{\beta}} + cst,$$

Finally, note that $\alpha_s + \beta_s = 1 - \sum_{u=1}^{S}\gamma_{su}$, so that $\boldsymbol{\alpha} + \boldsymbol{\beta} = (I-\Gamma)E$, with $E = (1,1,\ldots,1)' \in [1,S]$. This implies:

$$\boldsymbol{\phi'}(I-\Gamma)^{-1}(\boldsymbol{\alpha}+\boldsymbol{\beta}) = \boldsymbol{\phi'}(I-\Gamma)^{-1}(I-\Gamma)E = \boldsymbol{\phi'}E = \sum_{s=1}^{S}\phi_s = 1$$

As a result:

$$\boldsymbol{\phi'}(I-\Gamma)^{-1}\boldsymbol{\beta} = 1 - \boldsymbol{\phi'}(I-\Gamma)^{-1}\boldsymbol{\alpha} = 1-\alpha^*,$$

where α^* is defined in Proposition 4.

Call $Y(1)$ (resp. $Y(0)$) the steady-state output in the economy with $(\Theta_s)_s = \left(\Theta_s^0 + d\Theta_s\right)_s$ (resp. with $\left(\Theta_s^0\right)_s$).

$$\Delta\log Y = (1+\epsilon)\frac{\boldsymbol{\phi'}(I-\Gamma)^{-1}(\log(\mathbf{A(1)}) - \log(\mathbf{A(0)}))}{1-\alpha^*}$$

With the assumption that the distribution of productivity is unaffected by the policy change:

$$\log(\mathbf{A}(\mathbf{1})) - \log(\mathbf{A}(\mathbf{0})) = \left(\alpha_s \left(-\Delta\mu_\tau(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s \Delta\sigma^2_\tau(s) - 2\Delta\sigma_{z\tau}(s)\right)\right)\right)_{s\in[1,S]}$$

So that the change in aggregate output is equal to:

$$\Delta \log Y = (1+\epsilon)\sum_{s=1}^{S}\frac{\alpha_s\phi^*_s}{1-\alpha^*}\left(-\Delta\mu_\tau(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s\Delta\sigma^2_\tau(s) - 2\Delta\sigma_{z\tau}(s)\right)\right)$$

We substitute $\Delta\sigma_{z\tau}(s)$ with $\Delta\sigma_{lMRPK,lpy}(s)$ so that:

$$\Delta \log Y = -(1+\epsilon)\sum_{s=1}^{S}\frac{\alpha_s\phi^*_s}{1-\alpha^*}\left(\Delta\mu_\tau(s) + \frac{1}{2}\frac{\alpha_s\theta_s}{1-\theta_s}\Delta\sigma^2_\tau(s) + \Delta\sigma_{lMRPK,lpy}(s)\right)$$

As we explained in Appendix A.A4, the statistics $\Delta\mu_\tau(s)$, $\Delta\sigma_{lMRPK,lpy}(s)$ and $\Delta\sigma^2_\tau(s)$ are approximated by $\widehat{\Delta\Delta\mu}(s)$, $\Delta\Delta\widehat{\sigma_{lMRPK,lpy}}(s)$ and $\widehat{\Delta\Delta\sigma^2}(s)$.

Formula for TFP $\log TFP = -\alpha^* \log\frac{K}{Y} - (1-\alpha^*)\log\frac{L}{Y}$. We start with the $\log\frac{L}{Y}$ term. Knowing that $wL_s = \alpha_s\theta_s P_s Q_s$ and that P_sQ_s are fixed fractions of Y, we obtain that $\frac{L}{Y}$ is proportional fo $\frac{1}{w}$, hence, given our final derivation for log output:

$$\log\frac{L}{Y} = -\frac{\boldsymbol{\phi'}\left(I-\Gamma'\right)^{-1}\log(\mathbf{A})}{1-\alpha^*} + cst$$

Again, let ϕ^*_s the sth element of $(I-\Gamma)^{-1}\boldsymbol{\phi}$, be the linkage-adjusted industry share. Then:

$$\Delta\log\frac{L}{Y} = \sum_s\left(\frac{\alpha_s\phi^*_s}{1-\alpha^*}\right)\left(\Delta\mu_\tau(s) - \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s\Delta\sigma^2_\tau(s) - 2\Delta\sigma_{z\tau}(s)\right)\right)$$

We now compute the second-term. Start with the fact that:

$$\frac{Y}{K} = \sum_{s=1}^{S}\frac{K_s}{K}\frac{Y_s}{K_s}$$

We need to calculate K_s. Note that:

$$p_{is}q_{is} = \frac{P_sQ_s}{J_s}\frac{e^{\frac{\theta_s}{1-\theta_s}z_i}}{(1+\tau_i)^{\frac{\alpha_s\theta_s}{1-\theta_s}}}$$

so that capital demand is given by:

$$k_{is} = \theta_s\alpha_s\frac{P_sQ_s}{RJ_s}\frac{e^{\frac{\theta_s}{1-\theta_s}z_i}}{(1+\tau_i)^{1+\frac{\alpha_s\theta_s}{1-\theta_s}}}$$

so that the industry level capital stock is:

$$K_s = \theta_s\alpha_s\frac{P_sQ_s}{RJ_s}\underbrace{\int_{i\in S}\frac{e^{\frac{\theta_s}{1-\theta_s}z_i}}{(1+\tau_i)^{1+\frac{\alpha_s\theta_s}{1-\theta_s}}}di}_{J_s}$$

Using the fact that $\boldsymbol{P}\circ\boldsymbol{Q} = \left((I-\aleph)^{-1}\boldsymbol{\phi}\right)Y$, we obtain

$$\frac{K_s}{Y} = \frac{\eta_s}{R}\frac{J_s}{I_s}$$

where $\eta_s = \boldsymbol{\alpha}\circ\boldsymbol{\theta}\circ\left((I-\aleph)^{-1}\boldsymbol{\phi}\right)$ is a function of parameters ($\boldsymbol{\alpha}$, $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$). Hence:

$$\log\frac{K}{Y} = \log\left(\sum_s\eta_s\frac{J_s}{I_s}\right) + cst$$

Define K^1, Y^1 (resp. K^0, Y^0) the capital stock and output in economy 1 (resp. economy 0). Using our parametric assumption and the fact that the experiment does not affect productivity, we have:

$$\Delta\log\left(\frac{J_s}{I_s}\right) = -\Delta\mu_\tau(s) + \frac{1}{2}\left(\left(1+2\frac{\alpha_s\theta_s}{1-\theta_s}\right)\Delta\sigma^2_\tau(s) - 2\frac{\theta_s}{1-\theta_s}\Delta\sigma_{z\tau}(s)\right) \ll 1$$

We can now decompose the change in the aggregate capital to output ratio:

$$\begin{aligned}
\log\frac{K^1}{Y^1} &= \log\left(\sum_s \eta_s \frac{J^1_s}{I^1_s}\right) + cst \\
&= \log\left(\sum_s \eta_s \frac{J^0_s}{I^0_s} e^{\Delta\log\left(\frac{J_s}{I_s}\right)}\right) + cst \\
&= \underbrace{\log(\sum_s \eta_s \frac{J^0_s}{I^0_s})}_{\frac{K^0}{Y^0}} + \log\left(\sum_s \underbrace{\frac{\eta_s\frac{J^0_s}{I^0_s}}{\sum_{s'}\eta_{s'}\frac{J^0_{s'}}{I^0_{s'}}}}_{=\kappa_s} e^{\Delta\log\left(\frac{I_s}{J_s}\right)}\right),
\end{aligned}$$

where $\kappa_s = \frac{K^0_s}{K^0}$ is the capital share of each industry in the economy 0. As a result, to a first-order approximation:

$$\begin{aligned}
\Delta\log\frac{K}{Y} &\approx \log\left(1+\sum_s \kappa_s\Delta\log\left(\frac{J_s}{I_s}\right)\right) \\
&\approx \sum_s \kappa_s\Delta\log\left(\frac{J_s}{I_s}\right) \\
&\approx \sum_s \kappa_s\left(-\Delta\mu_\tau(s) + \frac{1}{2}\left(\left(1+2\frac{\alpha_s\theta_s}{1-\theta_s}\right)\Delta\sigma^2_\tau(s) - 2\frac{\theta_s}{1-\theta_s}\Delta\sigma_{z\tau}(s)\right)\right)
\end{aligned}$$

This leads to the TFP formula:

$$\begin{aligned}
\Delta\log TFP \approx{}& -\alpha^*\sum_s \kappa_s\left(-\Delta\mu_\tau(s) + \frac{1}{2}\left(\left(1+2\frac{\alpha_s\theta_s}{1-\theta_s}\right)\Delta\sigma^2_\tau(s) - 2\frac{\theta_s}{1-\theta_s}\Delta\sigma_{z\tau}(s)\right)\right) \\
&+ \sum_s \alpha_s\phi^*_s\left(-\Delta\mu_\tau(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s\Delta\sigma^2_\tau(s) - 2\Delta\sigma_{z\tau}(s)\right)\right)
\end{aligned}$$

We can re-organize this last equation into:

$$\Delta\log TFP \approx -\frac{\alpha^*}{2}\sum_s \kappa_s\left(1+\frac{\alpha_s\theta_s}{1-\theta_s}\right)\Delta\sigma^2_\tau(s) + \alpha^*\sum_s\left(\frac{\alpha_s}{\alpha^*}\phi^*_s - \kappa_s\right)\left(-\Delta\mu_\tau(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s\Delta\sigma^2_\tau(s) - 2\Delta\sigma_{z\tau}(s)\right)\right)$$

We substitute $\Delta\sigma_{z\tau}(s)$ with $\Delta\sigma_{lMRPK,lpy}(s)$ so that:

$$\Delta\log TFP \approx -\frac{\alpha^*}{2}\sum_s \kappa_s\left(1+\frac{\alpha_s\theta_s}{1-\theta_s}\right)\Delta\sigma^2_\tau(s) - \sum_s\left(\alpha_s\phi^*_s - \alpha^*\kappa_s\right)\left(\Delta\mu_\tau(s) + \frac{1}{2}\frac{\alpha_s\theta_s}{1-\theta_s}\Delta\sigma^2_\tau(s) + \Delta\sigma_{z\tau}(s)\right)$$

As we explained in Appendix A.A4, the statistics $\Delta\mu_\tau(s)$, $\Delta\sigma_{lMRPK,lpy}(s)$ and $\Delta\sigma^2_\tau(s)$ are approximated by $\widehat{\Delta\Delta\mu}(s)$, $\Delta\Delta\widehat{\sigma_{lMRPK,lpy}}(s)$ and $\widehat{\Delta\Delta\sigma^2}(s)$

II. Proof of Proposition 5

We consider here S heterogeneous industries. The setup is similar to Section II.D, except that the final good market produces by combining industry outputs according to a CES production function:

$$Y = \left(\sum_{\sigma=1}^{S} \phi_s Y_s^{\psi} \right)^{\frac{1}{\psi}}, \text{ with } \sum_{s=1}^{S} \phi_s = 1$$

We assume that the capital share is constant across industries: $\alpha_s = \alpha$:

$$y_{is} = e^{z_{is}} k_{is}^{\alpha} l_{is}^{1-\alpha}, \qquad Y_s = \left(\int_i y_{is}^{\theta_s} di \right)^{\frac{1}{\theta_s}}$$

We first show how to define aggregate TFP in this environment. An efficiently allocated economy with capital stock K and labor L is defined by:

$$Y^*(K,L) = \left| \begin{array}{l} \max\limits_{(k_{is}),(l_{is})} Y = \left(\sum_{s=1}^{S} \phi_s Y_s^{\psi} \right)^{\frac{1}{\psi}} \\ Y_s = \left(\int_i y_{is}^{\theta_s} \right)^{\frac{1}{\theta_s}}, \quad y_{is} = e^{z_{is}} k_{is}^{\alpha} l_{is}^{1-\alpha} \\ \int_i k_{is} \leq K \quad (\lambda), \quad \int_i l_{is} \leq L \quad (\mu) \end{array} \right.$$

The first-order conditions w.r.t. k_{is} and l_{is} are:

$$\begin{cases} \phi_s Y_s^{\psi-1} \alpha y_{is}^{\theta_s} Y_s^{1-\theta_s} Y^{1-\psi} = \lambda k_{is} \\ \phi_s Y_s^{\psi-1} (1-\alpha) y_{is}^{\theta_s} Y_s^{1-\theta_s} Y^{1-\psi} = \mu l_{is} \end{cases}$$

Aggregate the first-order condition w.r.t. k_{is} across all firms in industry s:

$$\phi_s Y_s^{\psi-1} \alpha Y_s^{\theta_s} Y_s^{1-\theta_s} Y^{1-\psi} = \lambda K_s \Leftrightarrow \alpha \phi_s Y_s^{\psi} Y^{1-\psi} = \lambda K_s$$

Similarly, the first-order condition w.r.t. l_{is} delivers:

$$(1-\alpha) \phi_s Y_s^{\psi} Y^{1-\psi} = \mu L_s$$

Summing up these equations across industries:

$$\alpha Y = \lambda K \quad \text{and} \quad (1-\alpha) Y = \mu K$$

From the first-order conditions, we can write production for firm i:

$$y_{is} = e^{z_{is}} \left(\frac{\alpha}{\lambda} \right)^{\alpha} \left(\frac{1-\alpha}{\mu} \right)^{1-\alpha} \phi_s Y_s^{\psi-1} y_{is}^{\theta_s} Y_s^{1-\theta_s} Y^{1-\psi}$$

So that:

$$y_{is}^{\theta_s} = e^{\frac{\theta_s}{1-\theta_s} z_{is}} \left(\frac{\alpha}{\lambda} \right)^{\alpha \frac{\theta_s}{1-\theta_s}} \left(\frac{1-\alpha}{\mu} \right)^{(1-\alpha)\frac{\theta_s}{1-\theta_s}} \phi_s^{\frac{\theta_s}{1-\theta_s}} Y_s^{\theta_s \frac{\psi-\theta_s}{1-\theta_s}} Y^{(1-\psi)\frac{\theta_s}{1-\theta_s}}$$

Integrate over firms in industry s:

$$Y_s^{\theta_s} = \left(\int_i e^{\frac{\theta_s}{1-\theta_s} z_{is}} di \right) \left(\frac{\alpha}{\lambda} \right)^{\alpha \frac{\theta_s}{1-\theta_s}} \left(\frac{1-\alpha}{\mu} \right)^{(1-\alpha)\frac{\theta_s}{1-\theta_s}} \phi_s^{\frac{\theta_s}{1-\theta_s}} Y^{(1-\psi)\frac{\theta_s}{1-\theta_s}} Y_s^{\theta_s \frac{\psi-\theta_s}{1-\theta_s}}$$

As a result:

$$Y_s^{1-\theta_s} = Z_s \left(\frac{\alpha}{\lambda} \right)^{\alpha} \left(\frac{1-\alpha}{\mu} \right)^{(1-\alpha)} \phi_s Y^{(1-\psi)} Y_s^{\psi-\theta_s},$$

with: $Z_s = \left(\int_i e^{\frac{\theta_s}{1-\theta_s} z_{is}} di\right)^{\frac{1-\theta_s}{\theta_s}}$ The last expression allows us to write:

$$Y_s^{\psi} = Z_s^{\frac{\psi}{1-\psi}} \left(\frac{\alpha}{\lambda}\right)^{\alpha \frac{\psi}{1-\psi}} \left(\frac{1-\alpha}{\mu}\right)^{(1-\alpha)\frac{\psi}{1-\psi}} \phi_s^{\frac{\psi}{1-\psi}} Y^{\psi}$$

Multiply the previous expression by ϕ_s and sum across industries:

$$Y^{\psi} = \left(\sum_{s=1}^{S} \phi_s^{\frac{1}{1-\psi}} Z_s^{\frac{\psi}{1-\psi}}\right) \left(\frac{\alpha}{\lambda}\right)^{\alpha \frac{\psi}{1-\psi}} \left(\frac{1-\alpha}{\mu}\right)^{(1-\alpha)} Y^{\psi}$$

Since $\alpha Y = \lambda K$ and $(1-\alpha)Y = \mu K$, we see that:

$$Y = \left(\sum_{s=1}^{S} \phi_s^{\frac{1}{1-\psi}} Z_s^{\frac{\psi}{1-\psi}}\right)^{\frac{1-\psi}{\psi}} K^{\alpha} L^{(1-\alpha)}$$

And aggregate TFP is therefore defined by:

$$\log(TFP) = \log(Y) - \alpha \log(K) - (1-\alpha)\log(L)$$

We now find the equilibrium wage in the economy. Profit maximization in the final good market gives the demand for industry s output:

$$\frac{P_s}{P} = \phi_s \left(\frac{Y_s}{Y}\right)^{\psi-1}$$

Similarly, profit maximization in industry s gives the demand for firm i in industry s:

$$\frac{p_{is}}{P_s} = \left(\frac{y_{is}}{Y_s}\right)^{\theta_s-1}$$

Labor demand for firm i comes from:

$$\max_{l_{is}} \{p_{is} y_{is} - w l_{is}\} = \max_{l_{is}} \left(\phi_s P \left(\frac{Y_s}{Y}\right)^{\psi-1} Y_s^{1-\theta_s}\right) y_{is}^{\theta_s} - w l_{is}$$

$$\implies l_{is} = \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{1}{1-(1-\alpha)\theta_s}} \left(\phi_s P \left(\frac{Y_s}{Y}\right)^{\psi-1} Y_s^{1-\theta_s}\right)^{\frac{1}{1-(1-\alpha)\theta_s}} e^{\frac{\theta_s}{1-(1-\alpha)\theta_s} z_{is}} k_{is}^{\frac{\alpha\theta_s}{1-(1-\alpha)\theta_s}}$$

we have, for each firm in industry s: $(1-\alpha)\theta_s p_{is} y_{is} = w l_{is}$. Replacing above yields:

$$p_{is} y_{is} = \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\theta_s}{1-(1-\alpha)\theta_s}} \left(\phi_s P \left(\frac{Y_s}{Y}\right)^{\psi-1} Y_s^{1-\theta_s}\right)^{\frac{1}{1-(1-\alpha)\theta_s}} e^{\frac{\theta_s}{1-(1-\alpha)\theta_s} z_{is}} k_{is}^{\frac{\alpha\theta_s}{1-(1-\alpha)\theta_s}}$$

The first-order condition for firm i capital is simply: $(1+\tau_{is})R = \alpha\theta_s \frac{p_{is} y_{is}}{k_{is}}$. Combining with the labor first-order condition, we obtain:

$$k_{is} = \left(\phi_s \left(\frac{Y}{Y_s}\right)^{1-\psi} Y_s^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}} \left(\frac{\alpha\theta_s}{(1+\tau_{is})R}\right)^{\frac{1-(1-\alpha)\theta_s}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\theta_s}{1-\theta_s}} e^{\frac{\theta_s}{1-\theta_s} z_{is}}$$

Firm i output can be written as:

$$p_{is} y_{is} = \left(\phi_s \left(\frac{Y}{Y_s}\right)^{1-\psi} Y_s^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\theta_s}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\theta_s}{1-\theta_s}} \left(\frac{e^{z_{is}}}{(1+\tau_{is})^{\alpha}}\right)^{\frac{\theta_s}{1-\theta_s}}$$

We can combine the demand equation for firm i and industry s output to write firm i production in the following way:

$$y_{is} = \left(\phi_s \left(\frac{Y}{Y_s}\right)^{1-\psi} Y_s^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)}{1-\theta_s}} \left(\frac{e^{z_{is}}}{(1+\tau_{is})^{\alpha}}\right)^{\frac{1}{1-\theta_s}}$$

Aggregating within the industry, and since $Y_s = (\int_i y_{is}^{\theta_s})^{\frac{1}{\theta_s}}$:

$$\left(\frac{Y_s}{Y}\right)^{(1-\psi)\frac{\theta_s}{1-\theta_s}} = \phi_s^{\frac{\theta_s}{1-\theta_s}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\theta_s}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\theta_s}{1-\theta_s}} \underbrace{\left(\int_i \left(\frac{e^{z_{is}}}{(1+\tau_{is})^\alpha}\right)^{\frac{\theta_s}{1-\theta_s}} di\right)}_{I_s}$$

Since $Y = \left(\sum_{s=1}^S \phi_s Y_s^\psi\right)^{\frac{1}{\psi}}$, we can sum across industries s to pin down the wage:

$$1 = \sum_{s=1}^S \phi_s^{\frac{1}{1-\psi}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\psi}{1-\psi}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\psi}{1-\psi}} I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}$$

$$\implies w = \left[\sum_{s=1}^S \phi_s^{\frac{1}{1-\psi}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\psi}{1-\psi}} \left((1-\alpha)\theta_s\right)^{\frac{(1-\alpha)\psi}{1-\psi}} I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}\right]^{\frac{1-\psi}{(1-\alpha)\psi}}$$

First, consider the difference in the capital stock between two steady-states. Call κ_s the capital share of industry s in the initial economy:

$$\begin{aligned}
\Delta\log(K) &= \log\left(\frac{K^1}{K^0}\right) = \log\left(\sum_s \frac{K_s^1}{K^0}\right) \\
&= \log\left(\sum_s \frac{K_s^0}{K^0} e^{\Delta\log(K_s)}\right) \\
&\approx \log\left(1 + \sum_s \kappa_s \Delta\log(K_s)\right) \quad \text{since the policy change is assumed small } d\Theta \ll 1 \\
&\approx \sum_s \kappa_s \Delta\log(K_s)
\end{aligned}$$

Similarly:

$$\Delta\log(L) \approx \sum_s \chi_s \Delta\log(L_s),$$

where χ_s is the employment share of industry s in the initial economy.

We start from the first-order condition in capital:

$$k_{is} = \left(\chi_s \left(\frac{Y}{Y_s}\right)^{1-\psi} Y_s^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}} \left(\frac{\alpha\theta_s}{(1+\tau_{is})R}\right)^{\frac{1-(1-\alpha)\theta_s}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\theta_s}{1-\theta_s}} e^{\frac{\theta_s}{1-\theta_s}z_{is}}$$

Aggregating across firms in industry s

$$K_s = \left(\chi_s \left(\frac{Y}{Y_s}\right)^{1-\psi} Y_s^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{1-(1-\alpha)\theta_s}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\theta_s}{1-\theta_s}} \underbrace{\int_{i\in s} \frac{e^{\frac{\theta_s}{1-\theta_s}z_{is}}}{(1+\tau_{is})^{1+\frac{\alpha\theta_s}{1-\theta_s}}} di}_{=J_s}$$

Remember that:

$$\left(\frac{Y_s}{Y}\right)^{(1-\psi)\frac{\theta_s}{1-\theta_s}} = \chi_s^{\frac{\theta_s}{1-\theta_s}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\theta_s}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)\theta_s}{1-\theta_s}} \underbrace{\left(\int_i \left(\frac{e^{z_{is}}}{(1+\tau_{is})^\alpha}\right)^{\frac{\theta_s}{1-\theta_s}} di\right)}_{I_s},$$

which we can rewrite as:

$$Y_s = Y\chi_s^{\frac{1}{1-\psi}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha}{1-\psi}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)}{1-\psi}} I_s^{\frac{1-\theta_s}{\theta_s(1-\psi)}}$$

$$Y_s^{\frac{\psi-\theta_s}{1-\theta_s}} = Y^{\frac{\psi-\theta_s}{1-\theta_s}} \chi_s^{\frac{\psi-\theta_s}{(1-\psi)(1-\theta_s)}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha(\psi-\theta_s)}{(1-\psi)(1-\theta_s)}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)(\psi-\theta_s)}{(1-\psi)(1-\theta_s)}} I_s^{\frac{\psi-\theta_s}{\theta_s(1-\psi)}}$$

$$Y^{\frac{1-\psi}{1-\theta_s}} Y_s^{\frac{\psi-\theta_s}{1-\theta_s}} = Y \chi_s^{\frac{\psi-\theta_s}{(1-\psi)(1-\theta_s)}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha(\psi-\theta_s)}{(1-\psi)(1-\theta_s)}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{(1-\alpha)(\psi-\theta_s)}{(1-\psi)(1-\theta_s)}} I_s^{\frac{\psi-\theta_s}{\theta_s(1-\psi)}}$$

Plugging this expression into K_s above yields:

$$K_s = Y\left(\chi_s\theta_s\right)^{\frac{1}{1-\psi}} \left(\frac{\alpha}{R}\right)^{1+\frac{\alpha\psi}{1-\psi}} \left(\frac{1-\alpha}{w}\right)^{\frac{(1-\alpha)\psi}{1-\psi}} I_s^{\frac{\psi-\theta_s}{\theta_s(1-\psi)}} J_s$$

So that, summing across industries:

$$\frac{K}{Y} = \sum_{s=1}^{S} \frac{K_s}{Y} = \left(\frac{\alpha}{R}\right)^{1+\frac{\alpha\psi}{1-\psi}} \left(\frac{1-\alpha}{w}\right)^{\frac{(1-\alpha)\psi}{1-\psi}} \left(\sum_{s=1}^{S} \left(\chi_s\theta_s\right)^{\frac{1}{1-\psi}} I_s^{\frac{\psi-\theta_s}{\theta_s(1-\psi)}} J_s\right)$$

Now, we use the f.o.c. w.r.t. labor:

$$l_{is} = \left(\chi_s\left(\frac{Y}{Y_s}\right)^{1-\psi} Y_s^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}} \left(\frac{\alpha\theta_s}{(1+\tau_{is})R}\right)^{\frac{\alpha\theta_s}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{1-\alpha\theta_s}{1-\theta_s}} e^{\frac{\theta_s}{1-\theta_s} z_{is}}$$

Aggregating across firms in the industry:

$$L_s = \left(\chi_s\left(\frac{Y}{Y_s}\right)^{1-\psi} Y_s^{1-\theta_s}\right)^{\frac{1}{1-\theta_s}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\theta_s}{1-\theta_s}} \left(\frac{(1-\alpha)\theta_s}{w}\right)^{\frac{1-\alpha\theta_s}{1-\theta_s}} I_s$$

Note that:

$$L_s = K_s \frac{I_s}{J_s} \frac{\left(\frac{(1-\alpha)\theta_s}{w}\right)}{\left(\frac{\alpha\theta_s}{R}\right)},$$

so that:

$$L_s = Y\left(\chi_s\theta_s\right)^{\frac{1}{1-\psi}} \left(\frac{\alpha}{R}\right)^{\frac{\alpha\psi}{1-\psi}} \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha\psi}{1-\psi}} I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}},$$

and:

$$\frac{L}{Y} = \left(\frac{\alpha}{R}\right)^{\frac{\alpha\psi}{1-\psi}} \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha\psi}{1-\psi}} \sum_{s=1}^{S} \left(\chi_s\theta_s\right)^{\frac{1}{1-\psi}} I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}},$$

Remember that:

$$w = \left[\sum_{s=1}^{S} \chi_s^{\frac{1}{1-\psi}} \left(\frac{\alpha\theta_s}{R}\right)^{\frac{\alpha\psi}{1-\psi}} \left((1-\alpha)\theta_s\right)^{\frac{(1-\alpha)\psi}{1-\psi}} I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}\right]^{\frac{1-\psi}{(1-\alpha)\psi}}$$

But we know from the f.o.c. in labor:

$$(1-\alpha)\theta_s p_{is} y_{is} = w l_{is} \Rightarrow (1-\alpha)\theta_s P_s Y_s = w L_s$$

So that:

$$P_s Y_s = Y \chi_s^{\frac{1}{1-\psi}} \theta_s^{\frac{\psi}{1-\psi}} \left(\frac{\alpha}{R}\right)^{\frac{\alpha\psi}{1-\psi}} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)\frac{\psi}{1-\psi}} I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}} \Rightarrow \frac{P_s Y_s}{Y} = \frac{\chi_s^{\frac{1}{1-\psi}} \theta_s^{\frac{\psi}{1-\psi}} I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}}{\sum_{s'=1}^{S} \chi_{s'}^{\frac{1}{1-\psi}} \theta_{s'}^{\frac{\psi}{1-\psi}} I_{s'}^{\frac{\psi}{1-\psi}\frac{1-\theta_{s'}}{\theta_{s'}}}}$$

γ_s is the output share of industry s in the initial economy: $\gamma_s = \frac{P_s^0 Y_s^0}{Y^0}$. We have:

$$\Delta\log(w) \approx \frac{1}{(1-\alpha)} \sum_{s=1}^{S} \gamma_s \frac{1-\theta_s}{\theta_s} \Delta\log(I_s)$$

Going back to the expression for industry capital and employment:

$$\Delta\log(K_s) = \Delta\log(Y) - \frac{(1-\alpha)\psi}{1-\psi}\Delta\log(w) + \frac{\psi-\theta_s}{\theta_s(1-\psi)}\Delta\log(I_s) + \Delta\log(J_s)$$

So that:

$$\Delta\log(L_s) = \Delta\log(Y) - \frac{1-\alpha\psi}{1-\psi}\Delta\log(w) + \frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}\Delta\log(I_s)$$

$$\Delta\log(TFP) = \Delta\log(Y) - \alpha\Delta\log(K) - (1-\alpha)\Delta\log(L)$$

$$\Delta\log(TFP) = \frac{1-\alpha}{1-\psi}\Delta\log(w) - \sum_{s=1}^{S}\alpha\kappa_s\Delta\log(K_s) - \sum_{s=1}^{S}(1-\alpha)\chi_s\Delta\log(L_s)$$

Note that:

$$\Delta\log(J_s) = \frac{1}{2}(1+\frac{\alpha\theta_s}{1-\theta_s})\Delta\sigma_\tau^2(s) + (1+\frac{1-\theta_s}{\alpha\theta_s})\Delta\log(I_s)$$

So that:

$$\Delta\log(K_s) = \Delta\log(Y) - \frac{(1-\alpha)\psi}{1-\psi}\Delta\log(w) + \frac{1}{2}(1+\frac{\alpha\theta_s}{1-\theta_s})\Delta\sigma_\tau^2(s) + \left(\frac{1}{\alpha}+\frac{\psi}{1-\psi}\right)\frac{1-\theta_s}{\theta_s}\Delta\log(I_s)$$

$$\begin{aligned}\Delta\log(TFP) &= \sum_{s=1}^{S}\underbrace{\left(\frac{\gamma_s}{1-\psi} - \kappa_s(1+\frac{\alpha\psi}{1-\psi}) - \frac{(1-\alpha)\psi}{1-\psi}\chi_s\right)}_{=\omega_s}\frac{1-\theta_s}{\theta_s}\Delta\log(I_s) \\ &\quad - \frac{\alpha}{2}\sum_{s=1}^{S}\kappa_s(1+\frac{\alpha\theta_s}{1-\theta_s})\Delta\sigma_\tau^2(s)\end{aligned}$$

We can rewrite using the statistics defined in Section II.D:

$$\Delta\log(TFP) = -\frac{\alpha}{2}\sum_{s=1}^{S}\kappa_s(1+\frac{\alpha\theta_s}{1-\theta_s})\Delta\sigma_\tau^2(s) - \alpha\sum_{s=1}^{S}\omega_s\left(\Delta\mu(s) + \frac{1}{2}\frac{\alpha\theta_s}{1-\theta_s}\Delta\sigma^2(s) + \Delta\sigma_{lMPRK,lpy}\right)$$

As we explained in Appendix A.A4, the statistics $\Delta\mu_\tau(s)$, $\Delta\sigma_{lMRPK,lpy}(s)$ and $\Delta\sigma_\tau^2(s)$ are approximated by $\widehat{\Delta\Delta\mu}(s)$, $\Delta\Delta\sigma_{\widehat{lMRPK,lpy}}(s)$ and $\widehat{\Delta\Delta\sigma^2}(s)$

We now move on to the output formula. Remember that:

$$\frac{L}{Y} = \left(\frac{\alpha}{R}\right)^{\frac{\alpha\psi}{1-\psi}}\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha\psi}{1-\psi}}\sum_{s=1}^{S}(\chi_s\theta_s)^{\frac{1}{1-\psi}}I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}.$$

Combined with the aggregate labor supply curve, this equation implies:

$$\Delta\log(Y) = \left(\epsilon + \frac{1-\alpha\psi}{1-\psi}\right)\Delta\log(w) - \Delta\log\left(\sum_{s=1}^{S}(\chi_s\theta_s)^{\frac{1}{1-\psi}}I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}\right)$$

Remember that $L_s = Y(\chi_s\theta_s)^{\frac{1}{1-\psi}}\left(\frac{\alpha}{R}\right)^{\frac{\alpha\psi}{1-\psi}}\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha\psi}{1-\psi}}I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}$. As a result:

$$\Delta\log\left(\sum_{s=1}^{S}(\chi_s\theta_s)^{\frac{1}{1-\psi}}I_s^{\frac{\psi}{1-\psi}\frac{1-\theta_s}{\theta_s}}\right) \approx \frac{\psi}{1-\psi}\sum_{s=1}^{S}\chi_s\frac{1-\theta_s}{\theta_s}\Delta\log(I_s)$$

Since $\Delta\log(w) \approx \frac{1}{(1-\alpha)}\sum_{s=1}^{S}\gamma_s\frac{1-\theta_s}{\theta_s}\Delta\log(I_s)$, we obtain:

$$\Delta\log(Y) \approx \left(\frac{1+\epsilon}{1-\alpha}\right)\sum_{s=1}^{S}\gamma_s\frac{1-\theta_s}{\theta_s}\Delta\log(I_s) + \frac{\psi}{1-\psi}\sum_{s=1}^{S}(\gamma_s-\chi_s)\frac{1-\theta_s}{\theta_s}\Delta\log(I_s)$$

Using the expression for $\Delta\log(I_s)$, we obtain the formula for the change in aggregate output in Proposition 5:

$$\Delta \log Y = -\sum_{s=1}^{S} \left(\frac{\alpha \gamma_s}{1-\alpha}(1+\epsilon) + \frac{\psi}{1-\psi}\left(\gamma_s - \chi_s\right)\alpha \right) \left(\widehat{\Delta\Delta\mu}(s) + \frac{1}{2}\frac{\alpha\theta_s}{1-\theta_s}\widehat{\Delta\Delta\sigma^2}(s) + \Delta\Delta\sigma_{\widehat{lMRPK,lpy}}(s) \right)$$

III. Robustness Analysis

Table O.A. 1—: Variance of lMRPK distribution and banking deregulation: Robustness

	Var(log-MRPK)					
	(1)	(2)	(3)	(4)	(5)	(6)
Exposure × Post	-.81***	-.98***		-.53***	-.48**	
	(.2)	(.32)		(.14)	(.21)	
Q2 Exposure × Post			-.087			-.07
			(.064)			(.054)
Q3 Exposure × Post			-.17***			-.11**
			(.063)			(.052)
Q4 Exposure × Post			-.25***			-.11**
			(.073)			(.054)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry-trends	No	Yes	Yes	No	Yes	
Observations	7,917	7,917	7,917	7,915	7,915	7,915
Adj. R^2	0.56	0.60	0.60	0.57	0.61	0.61

Note: We estimate the following model:

$$X_{st} = \delta_t + \eta_s + b_X.\lambda_s \times POST_t + \mu_s \times t + \epsilon_{st},$$

where X_{st} is one of the three moments of the log-MRPK distribution mentioned above. δ_t is a year fixed-effect and η_s is an industry fixed-effect. λ_s is the industry-level measure of exposure to banking deregulation, and $\mu_s \times t$ are industry-specific trends. Finally, $POST_t$ is a dummy variable for the post-reform period (1985-1992). The dependent variable is the cross-sectional variance of log-MRPK in an industry-year. In column (1)-(3), the capital stock is defined as the average of the gross book value of assets at the beginning and end of the fiscal year (as opposed to the net value). In columns (4)-(6), we trim the log-MRPK distribution at the 1% level instead of windsorizing it. All columns include year and industry fixed-effects. Columns (2), (3), (5) and (7) include industry-specific trends. Standard errors are clustered at the industry level.