Online Appendix to "Consumption Heterogeneity: Micro drivers and macro implications", by Edmund Crawley and Andreas Kuchler

A Identification with Time Aggregation

In this section we show how to derive equations 3 and 5 for the variance of income growth and covariance of income and consumption growth.

From equation 2 we have

$$\Delta^N \bar{y}_T = \int_{T-1}^{T} (T-s)dP_s + (P_{T-1} - P_{T-N}) + \int_{T-N-1}^{T-N} (s-(T-2))dP_s + \Big(\int_{T-1}^{T} \int_{t-2}^{t} f(t-s)dQ_t dt - \int_{T-N-1}^{T-N} \int_{t-2}^{t} f(t-s)dQ_t dt \Big) \tag{8}$$

Making use of the independent increment property of P_t and Q_t, we get

$$\begin{aligned}\mathrm{Var}(\Delta^N \bar{y}_T) &= \int_{T-1}^{T} (T-s)^2 \sigma_P^2 ds + (N-1)\sigma_P^2 + \int_{T-N-1}^{T-N} (s-(T-2))^2 \sigma_P^2 ds \\ &\quad + \mathrm{Var}\Big(\int_{T-1}^{T} \int_{t-2}^{t} f(t-s)dQ_t dt \Big) + \mathrm{Var}\Big(\int_{T-N-1}^{T-N} \int_{t-2}^{t} f(t-s)dQ_t dt \Big) \\ &= (N - \frac{1}{3})\sigma_P^2 + 2\mathrm{Var}(\tilde{y}) \text{ for } n \geq 3 \end{aligned} \tag{9}$$

The equivalent of equation 2 for consumption is

$$\Delta^N \bar{c}_T = \int_{T-1}^{T} (T-s)\phi dP_s + \phi(P_{T-1} - P_{T-N}) + \int_{T-N-1}^{T-N} (s-(T-2))\phi dP_s + \Big(\int_{T-1}^{T} \int_{t-2}^{t} g(t-s)dQ_t dt - \int_{T-N-1}^{T-N} \int_{t-2}^{t} g(t-s)dQ_t dt \Big) \tag{10}$$

Again making use of the independent increment property, we can calculate the covariance of income and consumption growth:

$$\begin{aligned}\mathrm{Cov}(\Delta^N \bar{c_T}, \Delta^N \bar{y_T}) &= \int_{T-1}^{T} (T-s)^2 \phi \sigma_P^2 ds + \phi(N-1)\sigma_P^2 + \int_{T-N-1}^{T-N} (s-(T-2))^2 \phi \sigma_P^2 ds \\ &\quad + \mathrm{Cov}\Big(\int_{T-1}^{T} \int_{t-2}^{t} f(t-s)dQ_t dt, \int_{T-1}^{T} \int_{t-2}^{t} g(t-s)dQ_t dt \Big)\end{aligned}$$

$$+ \text{Cov}\Big(\int_{T-N-1}^{T-N}\int_{t-2}^{t} f(t-s)dQ_t dt, \int_{T-N-1}^{T-N}\int_{t-2}^{t} g(t-s)dQ_t dt\Big)$$

$$= \phi(N-\frac{1}{3})\sigma_p^2 + 2\text{Cov}(\tilde{c},\tilde{y}) \text{ for } N \geq 3 \tag{11}$$

B A Brief Introduction to the Time Aggregation Problem

An obvious question is why we have chosen not to use the well-known methodology of Blundell, Pistaferri, and Preston (2008), who achieve identification of transitory shocks from the facts that: (i) transitory income shocks in period t will mean-revert in period $t+1$; and (ii) permanent shocks in period t are uncorrelated with income changes in period $t+1$.[9] Unfortunately, as noted in Working (1960), in time aggregated data (ii) does not hold—a time aggregated random walk is positively autocorrelated. While macroeconomists have long been aware of the importance of time aggregation in time series regressions (see Campbell and Mankiw (1989) for a well-known example), the problem has been overlooked by the household finance and labor economics literature.[10] We will therefore briefly describe the problem here. For a more detailed account with particular attention to BPP, see Crawley (2020).

Time aggregation transforms an underlying, high-frequency time series, into a lower frequency time series. For example, we observe Danish tax returns at an annual frequency, while income in fact consists of paychecks arriving at a monthly, biweekly or irregular frequency. The observable income is the sum of all the income that was received by a household during the year. The key insight of Working (1960) is that even if there is no correlation between changes in income at the underlying frequency (it is a random walk), changes in the resulting time aggregated series will show positive autocorrelation. Figure B.1 shows how this autocorrelation is generated. The solid line shows the flow of income for a household that receives a permanent pay rise from \$50,000 to \$100,000 midway through the second year. The crosses show the income we would observe in annual tax data. During the second year, the household receives an annual \$50,000 salary for six months, followed by \$100,000 in the second six months, resulting in a reported income of

[9] Kaplan and Violante (2010) show in discrete time simulations that the methodology works reasonably well for standard calibrations of buffer-stock models and end up concluding, "The BPP insurance coefficients should become central in quantitative macroeconomics." However, some recent papers such as Commault (2021) and Hryshko and Manovskii (2018) have pointed to other potential problems of the methodology.

[10] For examples, see Moffitt and Gottschalk (2012); Meghir and Pistaferri (2004); Nielsen and Vissing-jorgensen (2004); Heathcote, Perri, and Violante (2010); and more recent quantile regression approaches such as Arellano, Blundell, and Bonhomme (2017).

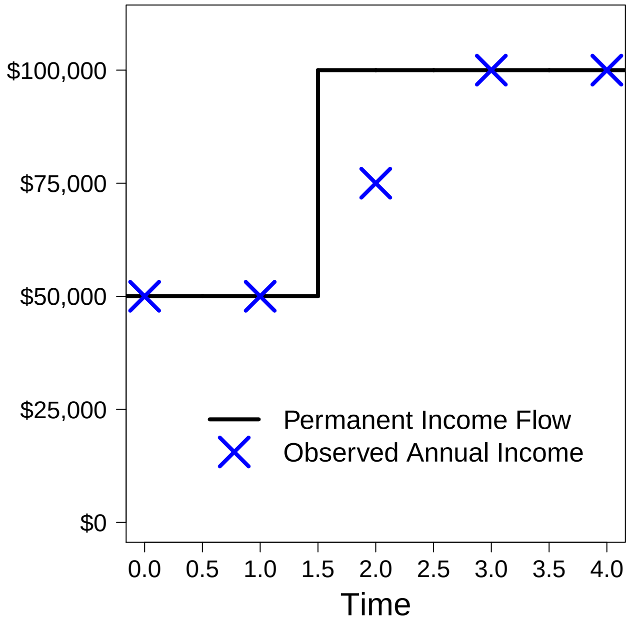

Figure B.1 The Time Aggregation Problem

$75,000 for the entire year. The single shock to income therefore appears in the observed data as two increases. In this way, if the underlying income process follows a random walk, shocks in one year result in observed income changes in that *and* the following year. The resulting autocorrelation—if the underlying process is in continuous time—is 0.25. Continuous time is a good approximation for quarterly or monthly underlying processes. The autocorrelation of an annual-time-aggregated quarterly random walk is 0.23. At a monthly or higher frequency the autocorrelation is almost indistinguishable from 0.25.[11]

While it would be possible to stick closely to the original BPP model and adjust the covariance restrictions to take account of the time aggregation problem,[12] we have found that in practice the underlying assumptions made by BPP (in particular that consumption follows a random walk) do not fit with the data. The random walk assumption was previously thought to be benign. Not only were the estimates of the consumption response to transitory shocks in BPP small and consistent with such an assumption, Kaplan and Violante (2010) show that without time aggregation, the BPP method correctly identifies the transitory consumption response in the period of

[11] If all permanent shocks to income occurred on January 1 each year, then this would not hold. Low, Meghir, and Pistaferri (2010) show that a significant portion of permanent income variance is explained by job mobility, which can occur at any point in the year.

[12] Crawley (2020) takes this more straightforward approach using the same PSID data as used in BPP.

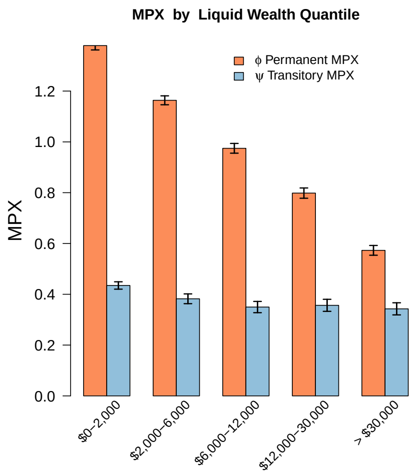

Figure C.1 Estimates by Liquid Wealth Quintile Obtained Using BPP's Methodology

the income shock regardless of the consumption dynamics going forward. This fact is again not robust to the time aggregation problem. With time aggregation taken into account, the estimates are highly sensitive to assumptions about short-term consumption dynamics—online appendix C shows how the random walk assumption affects MPX estimates. Therefore we have chosen to attain identification in a manner similar to Carroll and Samwick (1997), which allows us to be agnostic about the exact short-term dynamics of income and consumption.

C Comparison to Blundell, Pistaferri, and Preston (2008)

In this section we provide MPX estimates using the Danish data derived from the original estimation method from BPP.[13] The estimates are shown in figure C.1. These results look different to the baseline results of this paper: the BPP transitory estimates do not vary over liquid wealth quintile and the BPP permanent estimates are high, well above one in the lowest liquid wealth quintile.

Here we will show that different models of income and consumption dynamics, all plausible, can give rise to different MPX estimates under the BPP methodology. Note

[13] As in our baseline, we perform the estimation in levels, not logs, so that we can interpret our results as MPX provided the variance of permanent income change over the sample period is not large.

that the consumption data in the Danish administrative data is, like the income data, time-aggregated over the year. This contrasts with the consumption data in the PSID which provides a snapshot of consumption in the weeks before the survey is taken. The calculations we present below are therefore different to those found in Crawley (2020). These differences again serve to highlight the sensitivity of the BPP method to differences in the exact timing of income and consumption dynamics.

We will present three models of income and consumption dynamics: one in in which consumption follows a random walk, one in which the consumption response to transitory shocks is transitory, and one in which there is a durables response to permanent shocks.

1) Random walk consumption response. Here we show the estimates that the BPP methodology will produce, assuming the underlying model is exactly the same as in BPP, except for the fact that shocks are distributed uniformly through the calendar year. This is the same exercise carried out in Crawley (2020), but the formulae differ due to the timing of consumption in the Danish data. The true parameters are denoted $\sigma_P^2, \sigma_Q^2, \phi, \psi$, while the BPP estimates are denoted $\hat{\sigma}^2_{P,BPP}, \hat{\sigma}^2_{Q,BPP}, \hat{\phi}_{BPP}, \hat{\psi}_{BPP}$.

The observed, time-aggregated, moments are:

$$\begin{aligned}
\mathrm{Cov}(\Delta\bar{y}_T, \Delta\bar{y}_{T+1}) &= \frac{1}{6}\sigma_P^2 - \sigma_Q^2 && = -\hat{\sigma}^2_{Q,BPP} \\
\mathrm{Var}(\Delta\bar{y}_T) &= \frac{2}{3}\sigma_P^2 + 2\sigma_Q^2 && = \hat{\sigma}^2_{P,BPP} + 2\hat{\sigma}^2_{Q,BPP} \\
\mathrm{Cov}(\Delta\bar{c}_T, \Delta\bar{y}_{T+1}) &= \frac{1}{6}\phi\sigma_P^2 - \frac{1}{2}\psi\sigma_Q^2 && = -\hat{\psi}_{BPP}\hat{\sigma}^2_{Q,BPP} \\
\mathrm{Cov}(\Delta\bar{c}_T, \Delta\bar{y}_T) &= \frac{2}{3}\phi\sigma_P^2 && = \hat{\phi}_{BPP}\hat{\sigma}^2_{P,BPP} + \hat{\psi}_{BPP}\hat{\sigma}^2_{Q,BPP}
\end{aligned}$$

These equations can be used to derive the BPP estimate in terms of the underlying parameters:

$$\begin{aligned}
\hat{\sigma}^2_{P,BPP} &= \sigma_P^2 \\
\hat{\sigma}^2_{P,BPP} &= \sigma_Q^2 - \frac{1}{6}\sigma_P^2 \\
\hat{\phi}_{BPP} &= \frac{5}{6}\phi - \frac{1}{2}\psi\frac{\sigma_Q^2}{\sigma_P^2} \\
\hat{\psi}_{BPP} &= \frac{\frac{1}{2}\psi\sigma_Q^2 - \frac{1}{6}\phi\sigma_P^2}{\sigma_Q^2 - \frac{1}{6}\sigma_P^2}
\end{aligned}$$

At $\sigma_P^2 = 1$, $\sigma_Q^2 = 1$, $\phi = 0.8$, $\psi = 0.8$, this recovers $\hat{\phi}_{BPP} = 0.27$ and $\hat{\psi}_{BPP} = 0.32$. That is, if the true model is a random walk the BPP methodology underestimates both the permanent and transitory response coefficients.

2) Transitory consumption response to transitory income shocks. In this model, we assume the response to transitory income shocks is itself transitory and does not have any persistence. This close to what we find in our baseline estimates for the lowest quintile of liquid wealth. Under this model the moments are:

$$
\begin{aligned}
\mathrm{Cov}(\Delta\bar{y}_T, \Delta\bar{y}_{T+1}) &= \frac{1}{6}\sigma_P^2 - \sigma_Q^2 & &= -\hat{\sigma}_{Q,BPP}^2 \\
\mathrm{Var}(\Delta\bar{y}_T) &= \frac{2}{3}\sigma_P^2 + 2\sigma_Q^2 & &= \hat{\sigma}_{P,BPP}^2 + 2\hat{\sigma}_{Q,BPP}^2 \\
\mathrm{Cov}(\Delta\bar{c}_T, \Delta\bar{y}_{T+1}) &= \frac{1}{6}\phi\sigma_P^2 - \psi\sigma_Q^2 & &= -\hat{\psi}_{BPP}\hat{\sigma}_{Q,BPP}^2 \\
\mathrm{Cov}(\Delta\bar{c}_T, \Delta\bar{y}_T) &= \frac{2}{3}\phi\sigma_P^2 + 2\psi\sigma_Q^2 & &= \hat{\phi}_{BPP}\hat{\sigma}_{P,BPP}^2 + \hat{\psi}_{BPP}\hat{\sigma}_{Q,BPP}^2
\end{aligned}
$$

Leading to BPP estimates:

$$
\begin{aligned}
\hat{\sigma}_{P,BPP}^2 &= \sigma_P^2 \\
\hat{\sigma}_{P,BPP}^2 &= \sigma_Q^2 - \frac{1}{6}\sigma_P^2 \\
\hat{\phi}_{BPP} &= \frac{5}{6}\phi + \psi\frac{\sigma_Q^2}{\sigma_P^2} \\
\hat{\psi}_{BPP} &= \frac{\psi\sigma_Q^2 - \frac{1}{6}\phi\sigma_P^2}{\sigma_Q^2 - \frac{1}{6}\sigma_P^2}
\end{aligned}
$$

At $\sigma_P^2 = 1$, $\sigma_Q^2 = 1$, $\phi = 0.8$, $\psi = 0.8$, this recovers $\hat{\phi}_{BPP} = 1.47$ and $\hat{\psi}_{BPP} = 0.8$. That is, under this model with parameters similar to those found for the least liquid group, the BPP method significantly overestimates the consumption response to permanent shocks, while correctly estimating the consumption response to transitory shocks.

3) Durable splurge in response to permanent income shocks. In this final model, we make the same assumptions about the transitory consumption response as in **(2)**, but further assume that households make a one-off splurge on durable goods when their permanent income rises. This is what a simple theory would tell us households should do—they need to immediately raise their stock of durables to the new higher level—and our baseline model is robust to this type of behavior (see online appendix M.1). We denote this splurge as ϕ_d. The observed moments are now:

$$
\begin{aligned}
\mathrm{Cov}(\Delta\bar{y}_T, \Delta\bar{y}_{T+1}) &= \frac{1}{6}\sigma_P^2 - \sigma_Q^2 & &= -\hat{\sigma}_{Q,BPP}^2 \\
\mathrm{Var}(\Delta\bar{y}_T) &= \frac{2}{3}\sigma_P^2 + 2\sigma_Q^2 & &= \hat{\sigma}_{P,BPP}^2 + 2\hat{\sigma}_{Q,BPP}^2 \\
\mathrm{Cov}(\Delta\bar{c}_T, \Delta\bar{y}_{T+1}) &= \frac{1}{6}\phi\sigma_P^2 + \frac{1}{2}\phi_d\sigma_P^2 - \psi\sigma_Q^2 & &= -\hat{\psi}_{BPP}\hat{\sigma}_{Q,BPP}^2
\end{aligned}
$$

$$\text{Cov}(\Delta\bar{c}_T, \Delta\bar{y}_T) = \frac{2}{3}\phi\sigma_P^2 + 2\psi\sigma_Q^2 \qquad = \hat{\phi}_{BPP}\hat{\sigma}^2_{P,BPP} + \hat{\psi}_{BPP}\hat{\sigma}^2_{Q,BPP}$$

Leading to BPP estimates:

$$\hat{\sigma}^2_{P,BPP} = \sigma_P^2$$
$$\hat{\sigma}^2_{P,BPP} = \sigma_Q^2 - \frac{1}{6}\sigma_P^2$$
$$\hat{\phi}_{BPP} = \frac{5}{6}\phi + \frac{1}{2}\phi_d + \psi\frac{\sigma_Q^2}{\sigma_P^2}$$
$$\hat{\psi}_{BPP} = \frac{\psi\sigma_Q^2 - \frac{1}{6}\phi\sigma_P^2 - \frac{1}{2}\phi_d\sigma_P^2}{\sigma_Q^2 - \frac{1}{6}\sigma_P^2}$$

At $\sigma_P^2 = 1$, $\sigma_Q^2 = 1$, $\phi = 0.8$, $\psi = 0.8$, and $\phi_d = 0.5$, this recovers $\hat{\phi}_{BPP} = 1.72$ and $\hat{\psi}_{BPP} = 0.5$, close to that obtained empirically for the households in the lowest quintile of liquid wealth.

The three models above show that the BPP model can recover estimates far removed from the underlying parameters of the model following plausible changes to the model. The third model is one way to rationalize the high permanent and low transitory MPX BPP estimate we see for households in the lowest quintile of liquid wealth. If households in higher quintiles of liquid wealth purchase relatively fewer durable goods following a permanent income shock, this may explain the low correlation observed in the BPP estimates between liquid wealth and MPX.

A further set of questions arises as to how different the Danish data is the the PSID data. Unfortunately, our method is not directly applicable to the PSID data as we assume both time-aggregated income and consumption. To apply a similar method to the PSID data would require us to make far stricter assumptions about the path of consumption following a transitory shock. Better understanding the persistence of transitory income and consumption dynamics is ongoing work by the authors.

D Comparison to Commault (2021)

Commault (2021) suggests a robust estimator for the consumption response to transitory income shocks, using the change in income from $T+t$ to $T+t+1$ as an instrument for transitory shocks in period T. She shows that when transitory income shocks have no persistence then change with $t = 0$ provides an unbiased estimate even when consumption does not follow a random walk. This was also shown in Kaplan and Violante (2010). However, when transitory income shocks have persistence this is no longer true.

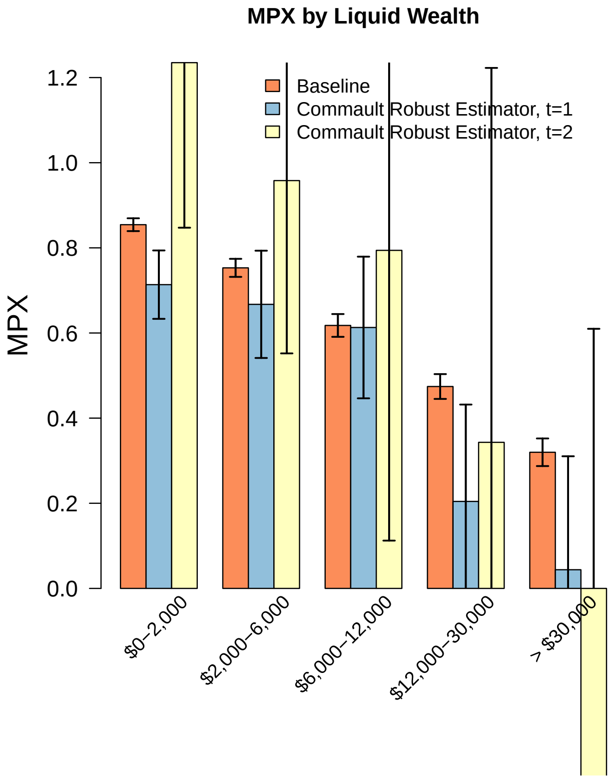

Figure D.1 Commault Robust Estimator for t=1 and 2

For persistence that lasts no more than one year, $t = 1$ provides an unbiased estimate, and for persistence that lasts between one and two years, $t = 2$ provides an unbiased estimate. These estimates with $t \geq 1$ are less subject to the time aggregation problem as the correlation in time-aggregated permanent shocks only persists for one year after the shock. See below for a brief discussion of time-aggregation in the context of Commault (2021).

In the PSID data, Commault (2021) finds $\text{Cov}(\Delta\bar{y_T}, \Delta\bar{y_{T+3}})$ to be statistically insignificant and therefore suggests $\frac{\text{Cov}(\Delta\bar{c_T}, \Delta\bar{y_{T+2}})}{\text{Cov}(\Delta\bar{y_T}, \Delta\bar{y_{T+2}})}$ as a robust estimator for the transitory consumption response ($t = 1$). In the Danish data, we find $\text{Cov}(\Delta\bar{y_T}, \Delta\bar{y_{T+3}})$ is statistically significant, and therefore we show results for both $t = 1$ and $t = 2$. The results are shown in figure D.1. In contrast the the BPP estimates, the results using the method from Commault (2021) are roughly in line with the baseline results from our paper. Furthermore, the estimates when $t = 1$ are somewhat lower than those in the baseline, which is what you would expect if there was some persistence in the transitory income shock beyond one year, which is implied by the statistical significance of $\text{Cov}(\Delta\bar{y_T}, \Delta\bar{y_{T+3}})$.

D.1 Time Aggregation in Commault (2021)

The Commault robust estimator is relatively robust to time-aggregation bias in data where both income and consumption is time-aggregated, such as the Danish administrative data used in this paper. For data, such as the PSID, in which income is time-aggregated but consumption is a snapshot around the time of the survey, changes to the transitory income dynamics can induce a large bias. In Commault (2021), it is assumed that transitory income arrives in two discrete lump sums, exactly one year apart. This assumption results in no time aggregation bias. Under an alternative assumption, that a transitory income shock increases the stream of income arriving for a period of one year, the response to transitory income shocks can be biased upward. For this alternative assumption, the relevant model moments are (ignoring permanent shocks and setting variance equal to one):

$$
\begin{aligned}
\mathrm{Cov}(\Delta y_T, \Delta y_{T+2}) &= \int_T^{T+1} (1-s) dQ_s \int_T^{T+1} -s dQ_s \\
&= \int_T^{T+1} -s(1-s) ds \\
&= -\frac{1}{6} \\
\mathrm{Cov}(\Delta c_T, \Delta y_{T+2}) &= \int_T^{T+1} \psi dQ_s \int_T^{T+1} -s dQ_s \\
&= \int_T^{T+1} -\psi s ds \\
&= -\frac{1}{2}\psi
\end{aligned}
$$

So the robust estimator gives $\frac{\mathrm{Cov}(\Delta c_T, \Delta y_{T+2})}{\mathrm{Cov}(\Delta y_T, \Delta y_{T+2})} = 3\psi$. This upward bias may explain why the estimate in Commault (2021) is more than twice as large as that in Crawley (2020).

E Sample Selection

We choose to look at households whose head is between the ages of 30 and 55 in 2008, which is driven by the desire to remove households for which the assumption that most of the income growth is unexpected is not likely to be fulfilled. For the old and the young, individual households will likely have a lot of information about their income path that is not available to the econometrician (for example, the year in which they plan to retire, or the fact that they are on a specific career track with set expectations of promotion and pay raises). We also want to remove households whose income volatility is increasing or decreasing sharply. Figures E.1 and E.2 show how our estimates of both income variance and MPX vary with age. The dots represent the point estimate for each age, while the lines are the centered moving averages over the five nearest age groups. The solid black line shows the total variance of income growth over one year. It should not be surprising that income growth for households with heads in their 20's is highly volatile. This volatility plateaus around the age of 35 and stays at a constant level until retirement, at which point it temporarily grows before falling to an even lower level. We can see that while both transitory and permanent shocks to income are high early in life, permanent income shocks are particularly high while individuals find their place in the workforce. From the ages of 30 to 55, both transitory and permanent shocks are approximately the same size and remarkably stable. At retirement, shocks to permanent income rise—not surprising, as the model sees retirement itself as a shock—even as transitory income variance declines.

As the model assumes the variance to permanent and transitory shocks to be constant in the observed period, interpretation of the numbers outside of the 30 to 55 age group needs to be treated with care. However, the figure clearly shows that within this age group the assumption of constant variance appears to be a reasonable one.

The dotted black line shows the variance of Δy, assuming no persistence in the transitory component. The fact that this line is slightly above the empirical variance of Δy is consistent with some persistence in the transitory component of income, justifying our decision to exclude growth over one and two years in our identification.

The level of both permanent and transitory shock variance for households aged 30 to 55 is approximately 0.0035, reflecting a standard deviation of 6%. Estimates using U.S. data are significantly higher, especially for the transitory shock variance (for example, Carroll and Samwick (1997) estimate 0.02 for permanent and 0.04 for transitory). This difference may be due to lower income inequality in Denmark, more progressive taxation, and more generous unemployment insurance. The lower transitory variance will also be due to significantly reduced measurement error relative to the survey-based U.S. data.

E.1 Quantile Selection

Throughout the paper, we first select a group of households by a characteristic such as income or liquid wealth quantile, and then we estimate the parameters for that group under the assumption they are constant over time. We select quantiles according to the mean level of liquid wealth/income/etc over the sample period. However, we may be

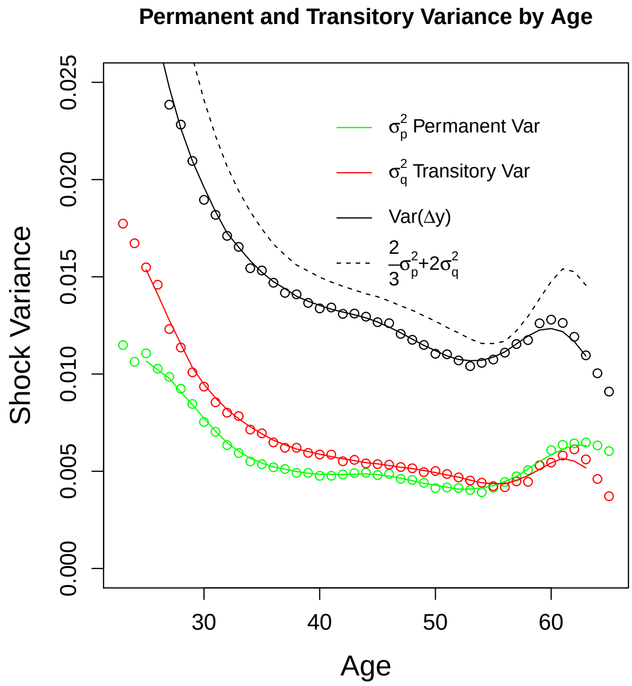

Figure E.1 Permanent and Transitory Shock Variance by Age

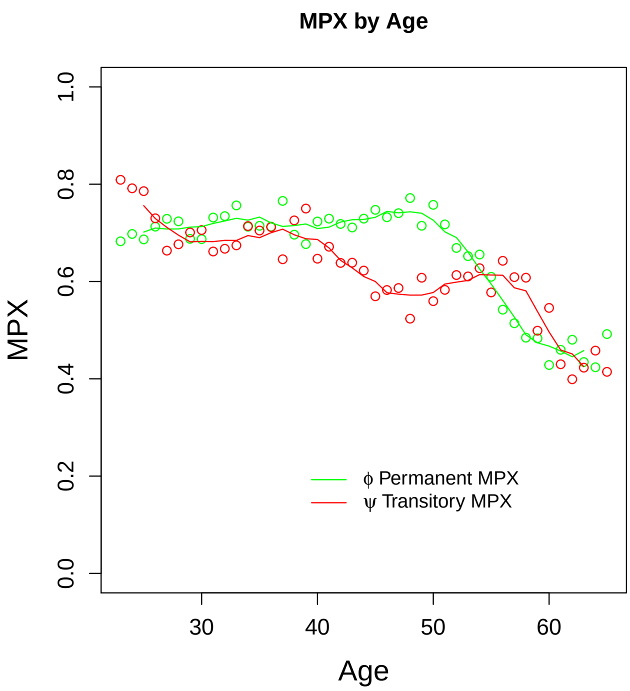

Figure E.2 MPX by Age

Table E.1 Short- and Long-term Liquid Wealth Quintile Probabilities

		Short-term Quintile 1	2	3	4	5
	1	66.45	30.22	3.11	0.21	0.00
Long-term	2	18.15	45.18	31.21	5.10	0.35
Quintile	3	6.95	17.56	42.66	29.49	3.34
	4	2.93	6.25	19.13	49.18	22.51
	5	0.97	1.77	4.93	18.93	73.40

Notes: Each row shows the percentage of time a household of that long run quintile will spend in the respective short-run quintile

Table E.2 Short- and Long-term Income Quintile Probabilities

		Short-term Quintile 1	2	3	4	5
	1	81.92	17.67	0.40	0.00	0.00
Long-term	2	11.12	72.16	16.32	0.39	0.02
Quintile	3	0.51	10.36	66.73	20.94	1.46
	4	0.01	0.48	14.59	65.79	19.13
	5	0.00	0.06	1.34	17.15	81.46

Notes: Each row shows the percentage of time a household of that long run quintile will spend in the respective short-run quintile

interested not in the MPX for households with the lowest average level of liquid wealth over the sample period, but instead in those with the lowest level of liquid wealth in a particular year. As households may move in out out of quantiles each year, these two measures are not identical and using one as a proxy for the other may introduce bias. We label the division into quantiles according to the average over the sample period as the "long-term quantile" and the division into quantiles at any particular snapshot in time as the "short-term" quantile and show how to quantify the difference between the two.

First, we empirically measure how often transitions between the quantiles occur over the sample period. Table E.1 shows for each long-term quantile, the fraction of years each household spends in each short-term quantile, on average. For example, households

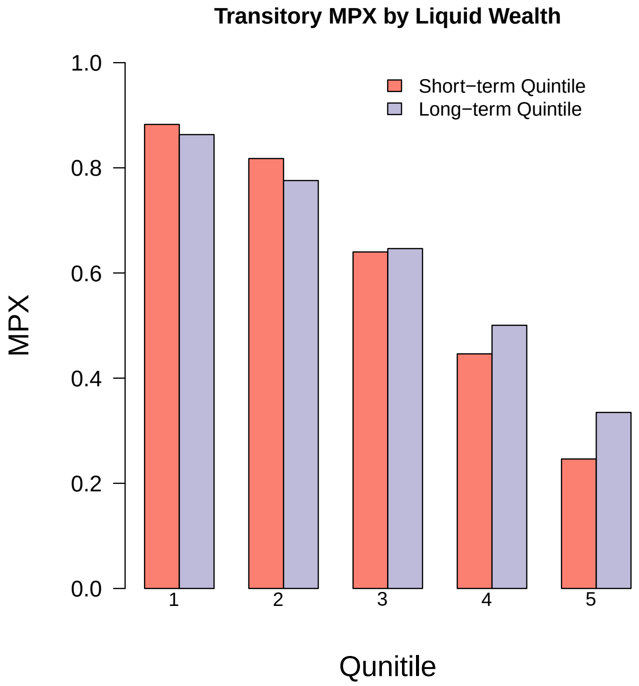

Figure E.3

that are in the highest quintile of average liquid wealth over the sample period spend, on average, 74 percent of years in the highest quintile of liquid wealth for that year, 19 percent of years in the second highest, and just a small fraction of years in lower quintiles. Somewhat surprisingly, of all the characteristics we look at, liquid wealth is the least 'sticky'—for other characteristics such as income (shown in table E.2) there is much less movement between quintiles. We then simulate a simple model to give an idea of how much bias is introduced by using the long-term quintiles as a proxy for the short-term quintiles. In our simple simulation, we assume that when a household is in a short-term quintile it has both the variance and the MPX for that quintile. For each long-term quintile, we then use the data from table E.1 to simulate transitions between short-term quintiles, assuming each period's short-term quintile is chosen independently. Using this simulated data, we the estimate the MPX for the long-term quintile using the method described in the paper.

Figure E.3 shows the results of this simulation. The variance of the short-term quintiles comes from the estimated values of the paper's baseline, while the short-term MPX are chosen such that the estimated long-term MPX are close to that of the baseline. The qualitative pattern of declining MPX with liquid wealth quintile is the same for both long- and short-term quintiles. However, the slope of the long-term quintile MPX is shallower than that of the short-term qunitles. This is a direct result of the fact that households who are in the top and bottom quintiles of long-term liquid wealth will not spend all their time in the top and bottom quintiles of short-term liquid wealth, so their MPX is shifted up and down respectively. The difference is most pronounced for the highest liquid wealth quintile.

A steeper slope would increase the size of the Auclert redistribution channels of monetary policy, so this exercise points toward the estimates we provide in the paper being a lower bound. However, such an exercise assumes a causal relationship between liquid wealth and MPX. It may be instead that the type of household that keeps a low balance of liquid wealth over a longer period is also the type of household that has a high MPX out of transitory income shocks. Furthermore, in practice it makes little difference to the results. Therefore, we have chosen not to include this analysis as part of our main results.

F Danish Household Balance Sheets

Table F.1 shows a comparison of liquid wealth and income in Denmark and the United States. Danish households on average hold more liquid wealth, both in absolute terms and as a percentage of their income.

F.1 The Danish Mortgage Market

Mortgage loans in Denmark are issued by specialized mortgage banks, which fully finance loans by issuing bonds. Interest rates are directly determined by sales prices at the bond market. That is, borrowers only pay the bond market interest rate plus a supplementary fee for the mortgage bank. Most loans are issued as 20- or 30-year loans, and households can only obtain loans from mortgage banks for up to 80% of the value at loan origination of properties used as permanent residences. The remaining (more insecure) part of the funding may be provided by commercial banks. The close link between loans and bonds, as well as fixed loan-to-value ratios, fast foreclosure procedures, full recourse, etc., mean that mortgage banks do not assume significant market risks. The status of Danish

Table F.1 Distribution of liquid wealth and income in Denmark and the United States

	Liquid wealth, USD		Income, USD		Liquid wealth, % of income	
Liquid wealth decile	U.S.	DK	U.S.	DK	U.S.	DK
1	15	43	24,303	39,557	0.1	0.1
2	300	1,120	29,366	43,974	1.1	2.4
3	800	2,080	37,468	52,282	2.1	4.0
4	1,500	3,358	48,607	66,562	3.1	5.0
5	2,950	5,267	55,695	82,711	4.9	6.4
6	4,900	8,232	74,935	90,824	6.5	9.2
7	8,000	13,063	84,049	95,872	9.6	13.8
8	14,100	21,542	93,163	101,125	15.5	21.7
9	30,500	38,711	117,466	106,937	25.1	37.0
10	90,900	94,137	193,414	117,609	53.7	86.9

Notes: The table shows medians of liquid wealth (measured in USD), pre-tax annual income and liquid wealth as a percentage of pre-tax income. Data for the US is based on the Survey of Consumer Finances for 2016 (Federal Reserve Board, 2016), and the measure of liquid wealth refers to the average balance of liquid wealth over the month before the survey date. Data for Denmark is averages of real values over 2009-2015 and refer to the balance at the end of the year. Only households aged between 30 and 55 years are included.

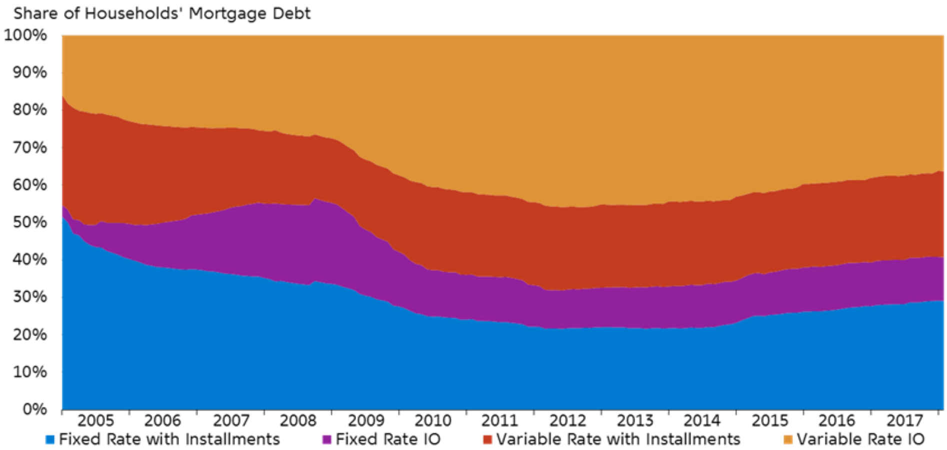

Figure F.1 Mortgage Debt by Type (All Households) *Source: Danmarks Nationalbank*

covered mortgage bonds as a safe asset class (AAA-rated by, e.g., S&P) implies that borrowers have access to cheap real estate funding.

The Danish mortgage system has been functioning for two centuries, but significant liberalization has taken place over the past 20 years. Variable interest loans were (re-)introduced in 1996, while interest only loans were introduced in 2003. These new loan characteristics are by now popular; see figure F.1. In contrast to the United States, where most mortgage debt is fixed rate, 40% of mortgage debt in Denmark is variable rate, with interest fixation periods mostly between six months and five years. Fixed-rate loans come with an option for early redemption, which implies that in practice, refinancing of fixed-rate mortgages often takes place, both when interest rates decrease and increase. The latter may be attractive because borrowers have the option to repay their loan by purchasing the corresponding amount of bonds. When interest rates increase, the bond value decreases, so the option to repay the loan by purchasing the corresponding amount of bonds in essence acts as an equity insurance.

Around one-fourth of the total loan balance is due to have interest rates reset over a 12-month period (see figure F.2). This figure only comprises loans that automatically will have a new interest rate and not active decisions to refinance or extract equity.

G Details on the Auclert Statistics

Table G.1 defines the five statistics calculated for Denmark in the paper.

In figure 5, we estimated MPX's for households with heads between the ages of 30 and 55, excluding the young and the old. Furthermore, some of the URE and NNP

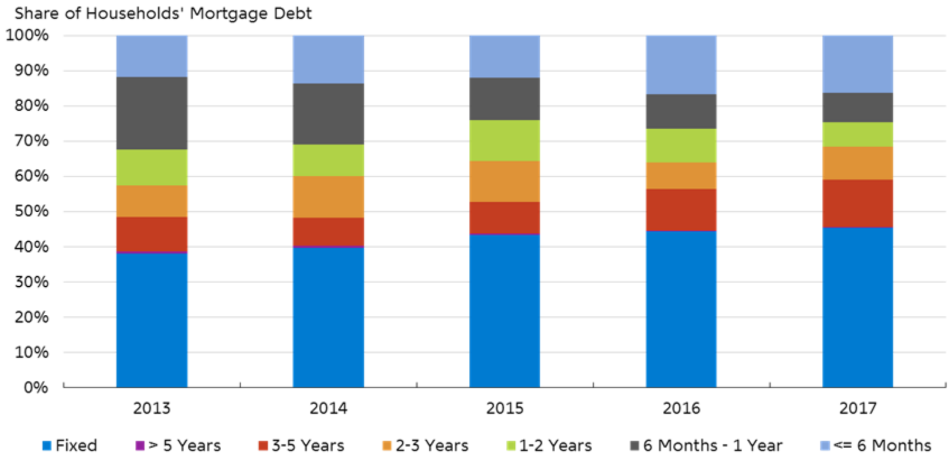

Figure F.2 Mortgage Debt by Maturity (All Households Excluding Self-employed)
Source: Danmarks Nationalbank

exposures are held indirectly on households' balance sheets through pension funds and corporations, or by the government and foreigners, so that the URE and NNP exposure in our sample does not aggregate to zero. We allocate the aggregate URE and NNP exposure from our sample into seven bins so that the total exposure across the economy is zero. These bins include households with (i) young (<30) and (ii) old (>55) heads, and exposures held by households indirectly through (iii) pension funds, (iv) government, (v) nonfinancial corporates, (vi) financials, and (vii) exposures held by the rest of the world. Within each of these bins we assume no heterogeneity so that the MPX with respect to these exposures is constant. This assumption is conservative, and likely underestimates the size of the heterogeneous agent channels. Our assumptions on the level of these MPXs can be seen in table G.2.

Table G.1 Sufficient Statistics Definitions

Statistic	Definition	Description
$\mathcal{M}$	$\frac{1}{C}\left[\sum_{i\in\text{Income deciles}} \text{MPX}_i Y_i + \sum_{j\in\{\text{young,old}\}} \text{MPX}_j Y_j\right]$	Income-weighted MPX
$\mathcal{E}_Y$	$\mathcal{M} - \overline{\text{MPX}}\frac{Y}{C}$	Redistribution elasticity for Y
$\mathcal{E}_P$	$\frac{1}{C}\left[\sum_{i\in\text{NNP deciles}} \text{MPX}_i \text{NNP}_i + \sum_{j\in\text{bins}} \text{MPX}_j \text{NNP}_j\right]$	Redistribution elasticity for P
$\mathcal{E}_R$	$\frac{1}{C}\left[\sum_{i\in\text{URE deciles}} \text{MPX}_i \text{URE}_i + \sum_{j\in\text{bins}} \text{MPX}_j \text{URE}_j\right]$	Redistribution elasticity for R
$\mathcal{S}$	$1-\frac{1}{C}\left[\sum_{i\in\text{Consumption deciles}} \text{MPX}_i C_i + \sum_{j\in\{\text{young,old}\}} \text{MPX}_j C_j\right]$	Hicksian scaling factor

Note: $\overline{\text{MPX}}$ is the mean MPX over all households in the economy. Y and C are aggregate household income and consumption respectively. Bins refers to the seven categories for which we have allocated URE and NNP exposures outside our estimation sample. {young,old} are the two bins that contain young and old households (the other five bins are only relevant for URE and NNP exposures as Y and C measure *household* income and consumption).

We define $\mathcal{E}_R$ as

$$\mathcal{E}_R = \frac{1}{C}\left[\sum_{i\in\text{URE deciles}} \text{MPX}_i \text{URE}_i + \sum_{j\in\text{bins}} \text{MPX}_j \text{URE}_j\right] \tag{12}$$

where i sums over the 10 deciles of URE, j over the seven bins defined above, and C is aggregate household expenditure in the economy. This method of dealing with the fact that aggregate exposure does not equal zero in the estimation sample is different than the approach taken by Auclert. He assumes the residual exposure is distributed equally across households in the sample. By making use of the national accounts, we believe we are able to get a better handle on the likely MPXs to attach to this residual exposure.

The assumptions we make about the MPX of the young and the old, as well as out of indirectly held URE and NNP exposures, are shown in table G.2. In each case we believe we have made conservative choices that will underestimate the size of the interest rate exposure channel of monetary policy. For the young we choose an MPX of 0.5, in line with the rest of the population. As the young have aggregate negative exposures, choosing an MPX on the low side is conservative. Similarly, for the old we choose an MPX of 0.5, which is on the high side for this age group. The assumption that there is no heterogeneity in MPX within these groups is also a conservative assumption.

Much of the URE and NNP exposure is not held directly on the balance sheet of households but instead indirectly through pension funds, corporates, and the government. There is significant evidence that the MPX out of shocks to the value of pension wealth, stocks, or the government balance sheet is substantially lower than the MPX

Table G.2 Aggregating Redistribution Elasticities

	MPX	NNP	URE	$\mathcal{E}_P$ component	$\mathcal{E}_R$ component
Sample	See Distribution	-204	-61	-0.82	-0.32
Young	0.5	-32	-15	-0.12	-0.06
Old	0.5	-23	6	-0.09	0.02
Pension Funds	0.1	137	37	0.10	0.03
Government	0.0	-85	-23	0.00	0.00
Non-financial Corp.	0.1	-49	-13	-0.04	-0.01
Financial Sector	0.1	223	61	0.17	0.05
Rest of World	0.0	33	9	0.00	0.00
Total		**0**	**-0**	**-0.80**	**-0.29**

Notes: NNP and URE numbers are in billions of 2015 USD. Pension Funds includes special saving such as children's savings accounts. See appendix H for detail.

from income. We choose to use the estimate from Maggio, Kermani, and Majlesi (2020) that households' MPX from changes in stock market wealth is about 10%. This choice is the most quantitatively important for the sufficient statistics, as the bin containing the most exposure is the financial sector, which is positively exposed to interest rate increases. This positive interest rate exposure may seem surprising because banks are typically thought to have long-term assets and short-term debt that would result in negative URE exposure. However, our findings are in line with those of Landier, Sraer, and Thesmar (2013), who find that the financial sector benefits from interest rate hikes overall. An important caveat is due here: we focus on the MPX out of changes in the assets indirectly held by households through the financial sector and do not assume any spending or lending response at the bank level. This assumption may be reasonable in good times when banks are not credit constrained, but may not hold during a banking crisis. Financial frictions could possibly result in monetary policy being much less effective during a banking crisis as the interest rate exposure channel to household spending is counterbalanced by a channel from bank balance sheet interest rate exposure to lending.[14]

We choose an MPX of zero for government and the rest of the world. There is no evidence that households respond in any significant way to changes in the government's balance sheet, and furthermore a low MPX is a conservative assumption for the size of the heterogeneous agent channels. As Denmark is a small part of the world economy, we assume that foreigners spend a negligible proportion of their wealth there.

[14] It should be noted that our analysis is all on the household side. Evidence suggests that firms are also sensitive to changes in cash flow; for example, see Blanchard, Lopez-de Silanes, and Shleifer (1994).

G.1 United States

A similar proceedure is followed for the United States, except that the MPX for all age groups is estimated using liquid wealth, and URE and NNP exposure not directely held by households is associated with an MPX of 0.1.

H Details on the Calculation of NNP and URE

The Net Nominal Position (NNP) and Unhedged Interest Rate Exposure (URE) for the various sectors in the Danish economy are calculated from our household-level dataset as well as the financial accounts from the national accounts statistics. All calculations are based on average values over the years 2009 to 2015, deflated by the consumer price index.

H.1 NNP and URE for Households

The NNP for households is calculated as financial assets minus liabilities. As financial assets, we include bank deposits as well as the market value of securities (excluding shares). Liabilities include all debt to financial institutions (including credit card debt) as well as publicly administered student debt, tax debt and other debt to government bodies. These data are reported to the tax authorities by financial institutions on behalf of the households.

URE is calculated as annual savings (i.e. after-tax income minus expenditure) plus maturing assets minus maturing liabilities. As maturing assets, we include all bank deposits, thereby assuming that they are floating rate. We assume a maturity of five years for securities held by households and therefore include 20% of the value of securities. Regarding liabilities, we assume that all bank debt is floating rate. According to the interest rate statistics collected by Danmarks Nationalbank since 2013, on average 95% of bank debt from households is floating rate, most of which is tied either to a market reference rate or to the Danmarks Nationalbank rate on certificates of deposit, with immediate adjustment. For mortgage debt, we have detailed information allowing us to calculate the stock of debt which is due to have interest rates reset over the coming 12 months, and assume that the new rate will only apply for half of the year. Voluntary refinancing of mortgage loans, with or without extraction of additional equity, takes place to a large extent in Denmark. Our measure of maturing liabilities only includes the loans that are contractually due to have their interest rates reset, and we do not attempt to estimate the amount of additional refinancing. For remaining liabilities, which constitute small amounts, we have no information regarding maturity, so we assume five years.

H.2 Other Sectors

NNP for the other sectors in the economy is obtained from the financial accounts statistics (Danmarks Nationalbank, 2021). To most closely resemble the definition used in the household-level data, we define NNP as net assets (i.e., assets minus liabilities) in the following categories: "Currency and deposits", "Securities other than shares", "Loans", and "Trade credits and other accounts receivable/payable".

NNP for the whole economy should, in principle, sum to 0. However, the household-level microdata on bank deposits that we have access to is exclusive of certain types of savings (specialized children's savings accounts as well as some forms of pension savings accounts administered by banks), which are included in the financial accounts statistics. For the age group included in our sample, these types of accounts can be assumed to be largely illiquid. We therefore group those deposits (33 billion USD) together with the assets of pension funds (see table G.2).[15]

URE for non-households is also based on the financial accounts. In the national accounts, we do not observe the maturity of different asset and liability classes. We hold household URE fixed at the values from the micro-level data and take advantage of the identity that total URE in the economy must be 0 to calibrate the maturity for the remaining sectors of the economy. This results in a maturity of assets and liabilities for non-households of 3.65 years.

I Persistent Consumption Response

Our estimation procedure makes the assumption that the consumption response to a transitory income shock decays to zero in a period of two years or less. A slower decay will lead to a downward bias in our estimates of the transitory MPX. Figure I.1 shows the results of our estimation procedure on simulated data under two different assumptions about the transitory consumption response.

[15] In practice, this amount is calculated as a residual, which may also reflect other minor differences between the household-level data and the national accounts statistics. For example, holdings of banknotes and coins are not observed in the microdata but are allocated based on certain assumptions in the financial accounts. For our exercise, the impact of such other differences is likely to be small.

The exponential decay line assumes that the consumption flow following a transitory shock decays exponentially.[16] We vary the decay rate to match a range of year 1 MPCs and assume that the entire transitory income is eventually consumed. For high MPCs, and especially those over 0.5, there is little bias. However, for MPCs significantly below 0.5 our method results in downward-biased estimates. This bias arises because low MPCs, combined with exponential consumption decay, result in a relatively stable consumption flow over the first few years that has not declined close to zero after two years.

Empirical evidence suggests that in fact the consumption response to a transitory shock decays quickly in the first few months and then more slowly after that.[17] The "Fagereng et al." line in figure I.1 shows the MPC estimate in simulated data in which the consumption response decays according to the estimates made in Fagereng, Holm, and Natvik (2021). In this case, the fast decay in the first few months results in a smaller bias than the exponential case for low MPCs, while the fact that the decay is slower following these first months results in a larger bias for high MPCs. Overall it seems likely that our assumption about the persistence of the consumption response leads to a slight downward bias across the range of MPCs.

We also show that our MPX estimates are not very sensitive to the choice of N (years of growth in our identification equations) between 3 and 6, which lends further support to the fact that assuming a two-year limit does not bias our results too much.[18]

I.1 Details on the simulations

For the simulations we divided each year into 20 sub-intervals. Both permanent and transitory shocks occur each period, and the transitory shocks have no persistence. At an annual frequency the variance of permanent and transitory shocks are equal. Households spend their permanent income each period, along with their consumption response to the history of transitory shocks. For the exponential decay model, this is

$$c_t = p_t + (1-\rho)\sum_{n=0}^{\infty} \rho^n \varepsilon_{t-n}$$

In Fagereng, Holm, and Natvik (2021) the T year MPC is estimated as a function:

$$\text{MPC}_T = \theta_1 T^{\theta_2}$$

[16] Standard buffer-stock models give rise to a consumption response that decays close to exponentially.

[17] Both Fagereng, Holm, and Natvik (2021) and Gelman (2021) provide evidence for this.

[18] Using N equal to 4 and 5 instead of 3, 4, and 5 allows us to extend the consumption response out to three years, at the expense of losing data and becoming more sensitive to misspecification of the income process.

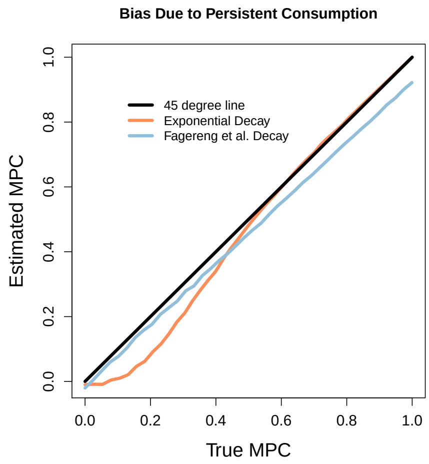

Figure I.1 Bias from Persistent Consumption

where θ_1 controls the size of the response and θ_2 the speed of decay. We vary θ_1 and choose $\theta_2 = 0.2142$ according to their estimate. In this model consumption in period t (measured in sub-intervals) is:

$$c_t = p_t + \theta_1 \sum_{n=0}^{\infty} \Big((\frac{n+1}{20})^{\theta_2} - (\frac{n}{20})^{\theta_2} \Big) \varepsilon_{t-n}$$

We then time aggregate both income and consumption over each 20-sub-interval period, choose a sample of 13 years, and run our estimation procedure with $N = 3, 4, 5$. The transitory MPC estimates are shown in figure I.1, and the permanent estimates are shown in figure I.2. The bias in permanent estimates is small across the range of transitory MPCs.

I.2 Estimates Using Different Values of N

Table I.1 ψ Estimates Using Different N

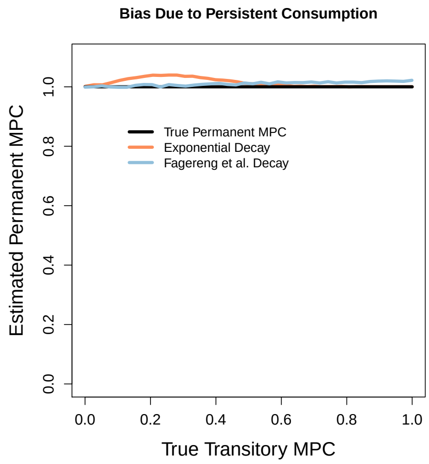

Figure I.2 Bias from Persistent Consumption

		n_2				
	1	**2**	**3**	**4**	**5**	**6**
1		0.55	0.56	0.57	0.58	0.58
2			0.62	0.62	0.63	0.63
n_1 **3**				0.63	0.64	0.65
4					0.67	0.66
5						0.65
6						

Table I.1 shows the estimates of the transitory MPX that we recover from our estimation sample when we just use $N = n_1, n_2$ in our identification equations 3 and 5. Remember in our main results we used GMM with $N = 3, 4, 5$ and we have circled $N = 3, 5$ to highlight where we get identification from in the paper. The purpose of this exercise is to show that the estimation results are not very sensitive to the values of N chosen, providing more evidence that the assumption we made that the transitory consumption response lasts less than two years is not biasing our results significantly. In fact, the results are not changed dramatically even when $N = 1, 2$, which suggests the majority of the transitory consumption response is short-lived.

J RIP or HIP Income Process?

J.1 RIP or HIP Income Process?

Our method makes strong assumptions on the income process—namely, that there is no persistent idiosyncratic component to income growth and that the process contains a random walk. Guvenen (2009) shows that it is empirically difficult to distinguish between a 'Restricted Income Profile' (RIP) like this and a 'Heterogenous Income Profile' (HIP) income process, in which (i) shocks to income are much less persistent (e.g., AR(1) with $\rho \approx 0.8$), and (ii) households have a persistent idiosyncratic growth component. The reason the RIP and HIP processes are difficult to tell apart is that the two features (i) and (ii) act in opposite directions on the cross-section variance of income growth. The less persistent income shocks lead the cross-sectional income growth variance to not grow as fast as the HIP model, while the persistent idiosyncratic growth component leads the same variance to grow at a faster rate. The result is that the increase in variance of income growth over three to four years is approximately the same as the increase from four to five years. To the extent that the consumption response to these semi-permanent shocks is similar to the response to the idiosyncratic persistent growth component,[19] our methodology will continue to provide reasonable estimates of the "permanent" MPX and the more familiar transitory MPX.

Both the Restricted Income Profile (RIP) and Heterogeneous Income Profile (HIP) processes can be described by the equations:

$$y_h^i = \beta^i h + z_h^i + \varepsilon_i^h$$
$$z_h^i = \rho z_{h-1]}^i + \eta_i^h$$

where i indexes the worker and h the years of experience. ε_i^h represents a transitory shock to income, while η_i^h is persistent. β^i represents an idiosyncratic persistent growth factor.

In the RIP model, $\beta^i = 0$ and ρ is usually estimated to be close to 1 (in this paper we assumed $\rho = 1$). In the HIP model, β^i has a cross-sectional variance $\sigma_\beta^2 > 0$, and ρ is normally estimated to be significantly lower than 1, around 0.8. The reason these are difficult to tell apart is because the theory does not give a strong indication in which model the cross-sectional variance of income growth over N years should grow

[19]See Guvenen (2007) for an example of why this might be the case: if households do not know their own idiosyncratic growth ex-ante, a Bayesian learning process will be slow, so households (at least initially) will react in similar ways to changes in income due to this persistent growth component as a true income shock.

faster. In the RIP model with $\rho = 1$, the cross-sectional variance of income growth increases linearly with N. In the RIP model with $\rho \approx 0.8$, the growth in the cross-sectional variance of income growth will decrease due to the low ρ but increase due to the idiosyncratic β^i.

Figure J.1 shows the empirical values for income growth variance and the covariance of income and expenditure growth over N years. We have also plotted the fitted values for these statistics that are implied by our model when fitted to $N = 3, 4, 5$ as we do in the paper. We see the empirical variance and covariance decline slightly below the model fitted line as N becomes large, which fits with the finding that ρ in the RIP model is usually slightly below 1.0, around 0.98 or 0.99. We also note that around the region where we achieve our identification ($N = 3, 4, 5$), there is little curvature in the empirical statistics, and the increase in both variance and covariance is close to linear.

While this linearity around $N = 3, 4, 5$ cannot help us distinguish between the RIP and HIP process, it does imply that our empirical methodology may be somewhat robust to misspecification along this dimension. If we assume that the expenditure response to a change in z_h^i and to the increase from the persistent idiosyncratic growth are equal to ϕ, and the response to a transitory shock is ψ, that is:

$$\Delta^N c_h^i \approx \phi \Delta^N (\beta^i h + z_h^i) + \psi \Delta^N \varepsilon_i^h$$

Then, the fact that $\mathrm{Var}\big(\Delta^N(\beta^i h + z_h^i)\big)$ grows approximately linearly with N means that our empirical method will correctly identify ϕ and ψ.

A full investigation of the implications of different income processes is beyond the scope of this paper but would be a useful exercise for future research.

K Time-Varying Risk

We have assumed that idiosyncratic risk remains constant over time. Given that our sample period covers the great recession, this may not be appropriate. Here we show how the variance of income growth has varied over time, peaking just after the crisis in 2010. In order to test how much this time-varying risk might bias our results, we simulated data with $\phi = 1$ and $\psi = 0.5$, with permanent variance equal to estimates from the data and transitory variance varying in order to match the time-varying income risk pattern observed in the data. When we run this simulation we find estimates of ϕ and ψ within 1% of their true values.

Figure K.1 shows how the standard deviation of income growth has changed over the

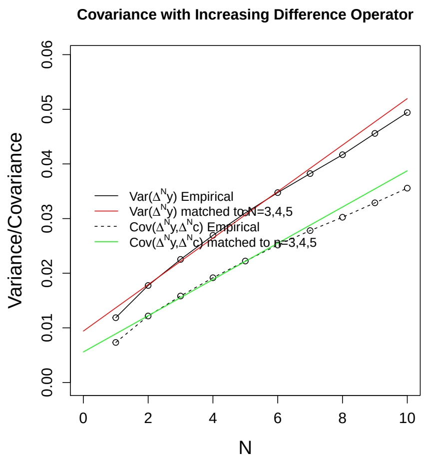

Figure J.1 Variance and Covariance with Years of Growth

sample period. From trough to peak, the standard deviation increases approximately 10%. In the simulation referred to in section K, we assume that both transitory income and transitory consumption response have no persistence. We divide each year into 20 sub-periods, choose the variance of permanent shocks to be 0.003, and allow time-varying transitory shocks to match the pattern in figure K.1. We choose values of $\phi = 1$ and $\psi = 0.5$ and apply our estimation procedure (that assumes constant variance) to the simulated data. We recover estimated values of ϕ and ψ to be 1.006 and 0.499, respectively.

L Robustness

L.1 Measurement Error

Our identification comes from estimating $\text{Var}(\Delta^N \bar{y})$ and $\text{Cov}(\Delta^N \bar{c}, \Delta^N \bar{y})$ using our observed data. For unbiased estimates of $\text{Var}(\Delta^N \bar{y})$ we require no measurement error in our observed changes in labor income. For unbiased estimation of $\text{Cov}(\Delta^N \bar{c}, \Delta^N \bar{y})$ we only require (further to no measurement error in income growth) that the measurement error in expenditure growth is uncorrelated with labor income growth. As our expenditure is imputed from income and changes in assets, this is potentially more of a concern than would be the case in survey data in which questions about consumption are not directly linked to those on income. We will examine potential sources of error in labor income and imputed consumption.

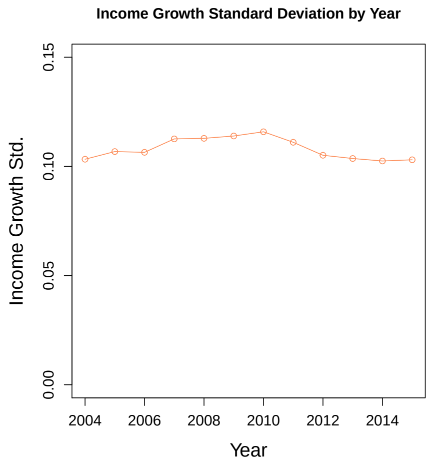

Figure K.1 Standard Deviation of Income Growth

L.1.1 Labor Income

For most workers, labor income is well measured. Third party reporting, along with a high level of trust in government institutions, means that underreporting is likely low. The black economy in Denmark is small, and to the extent that any growth in unreported income is uncorrelated with growth in reported income this will not bias our estimates.[20] In contrast to survey data, in which measurement error in income is likely to downwardly bias transitory MPX estimates, this is of little concern in our data.

L.1.2 Imputed Expenditure

Expenditure is calculated as the residual of total household income (including interest and dividends) after pension contributions and the change in net wealth have been deducted. For households with simple financial lives (which we believe fits most of the Danish population), this should work well. There are a few scenarios that merit further investigation.

- **Stock Market Capital Gains:** Only 10% of Danish households directly own stocks or mutual funds.[21] In online appendix L we show that the qualitative patterns we observe are unchanged even when we completely remove these households from the sample. For households that do own stocks, we assume the return they

[20]Such income may show up as a change in net wealth and hence expenditure, but measurement error in the change in expenditure uncorrelated with the change in labor income will not bias our MPX estimates.

[21]In our calculation we directly observe flows in and out of pension accounts, so these can be treated as off balance sheet in which capital gains do not affect our expenditure calculation.

receive is equal to a diversified portfolio of Danish stocks. Given that different households will have their own idiosyncratic portfolios, this methodology will result in significant measurement error. Baker, Kueng, Meyer, and Pagel (2021) show that the size of this measurement error is not only correlated with income and wealth, but also with the business cycle. Furthermore, Fagereng, Guiso, Malacrino, and Pistaferri (2020) show that some groups of investors consistently outperform the market, which would lead us to consistently underestimate their expenditure. Our concern, however, is that the *change* in measurement error of expenditure be correlated with the *change* in labor income. Consistently underestimating expenditure by the same amount is therefore not a problem for us. Furthermore, as we have removed all aggregate effects from the labor income residuals that we use in estimation, any measurement error correlated with the business cycle will be uncorrelated with our measure of changes in labor income. We see two potential ways in which mis-measuring stock returns may bias our results. First, if households have significantly invested in the stock of the firm they work for, which is likely only to be the case for high-level management. Second, to the extent that households invest their labor income gains halfway through the year, we will underestimate expenditure for those whose income increases, and overestimate it for those whose income decreases, leading us to underestimate the MPX. The size of this bias is limited by the size of excess expected returns, so our MPX estimate will be biased by no more than a few percentage points.

- **Family and Friends Transfers:** If a household receives a transfer of money from their parents, for example, imputed expenditure will be lower than true expenditure by this amount. Large transfers typically occur upon death of a parent, which is likely to be uncorrelated with the household head's labor income, or when purchasing a house—years that we have already excluded from our sample. However, to the extent that friends and family actively insure each other's labor income, our MPX estimates will be upward biased.

- **Off-Balance-Sheet Assets:** A larger concern is that some forms of saving may be hidden off balance sheet. Our imputation method would interpret off-balance-sheet saving as expenditure, so our estimate of the MPX would increase one-to-one for each percentage point of saving out of income shocks performed off balance sheet. All Danish banks and brokers are required to report their clients' holdings, so off-balance-sheet assets are likely to be either offshore or nonfinancial assets.

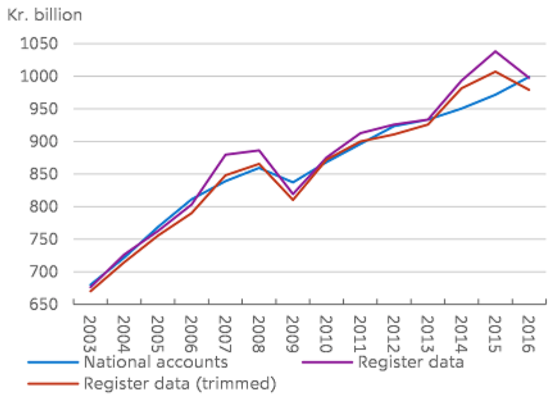

Figure L.1 Imputed Register Measure and National Account Measure of Expenditure. Source: Abildgren, Kuchler, Rasmussen, and Sorensen (2021)

Such off-balance-sheet saving would be a large concern if we were focused on the expenditure of the super wealthy 0.1%, but is less so when dividing the population into quintiles or deciles as we have done.

As would be clear from the main text, we have made a number of choices regarding both data and variable definitions as well as more methodological issues. In a series of graphs, this appendix presents a number of robustness checks aimed at assessing the extent to which our results are sensitive to the specific choices.

We begin with a number of robustness checks regarding our imputed expenditure measure, which may suffer from measurement error. In figure L.2, we compare our baseline estimates of the MPX to estimates based on different sample selection procedures. First, we exclude all households that own stocks corresponding to more than 10,000 USD (10% of households in our sample). Second, we do not remove households that have negative imputed expenditure. We remove those households in our baseline sample because negative expenditure is clearly not a good estimate of actual expenditure. However, for example, in the event that negative expenditure arises because of classical measurement error, removal of negative estimates may be asymmetric and introduce an upward bias in average imputed expenditure. Third, to check that large outliers do not drive our results, we remove observations in the top and bottom 2.5% in terms of level and change of income and expenditure. In the baseline calculations, we use only a 1% cutoff. Our results are qualitatively unchanged when using these alternative approaches to take account of measurement error. In terms of magnitudes of the estimated MPXs, the largest difference to the baseline results seems to be found when we include negative expenditure estimates. As expected, this makes the largest difference

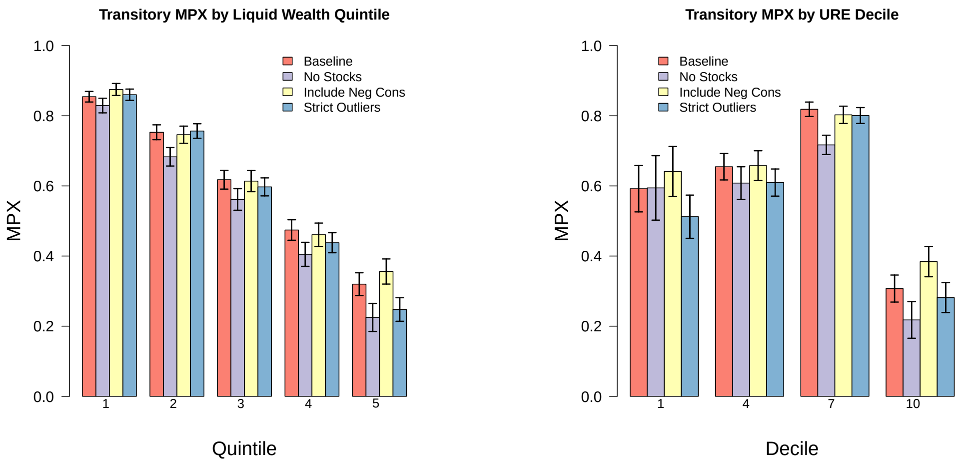

Figure L.2 Robustness of Liquid Wealth and URE Distributions

among the wealthier households. The specification of outliers also matters somewhat for the point estimates of MPX in certain groups of households, but differences are not large.

Another robustness check consists of specifying consumption and income in logs rather than in levels. The fundamental difference is that the log specification yields an elasticity rather than an MPX. Hence, some difference between level and log results must be expected for households that only spend a fraction of their annual income (typically wealthier households). Indeed, as expected, figure L.3 demonstrates that results hold qualitatively when specifying income and expenditure in logs rather than in levels, whereas estimated elasticities are higher than the MPXs for the wealthier households and those with high URE. Time-varying income risk may also potentially contribute to differences between results based on levels and logs. However, as shown in section K, this is not likely to be important in our setting.

As discussed in section A, we use total household income as our prime measure of income in line with previous consumption literature. The literature on idiosyncratic income processes tends to use income of the head of the household. Various mechanisms—e.g., intra-household income insurance—may give rise to differences between results based on income of the head of household and total household income. However, figure L.4 demonstrates that there is virtually no difference in our results between using total household income and only the household head's income. Online appendix Q briefly

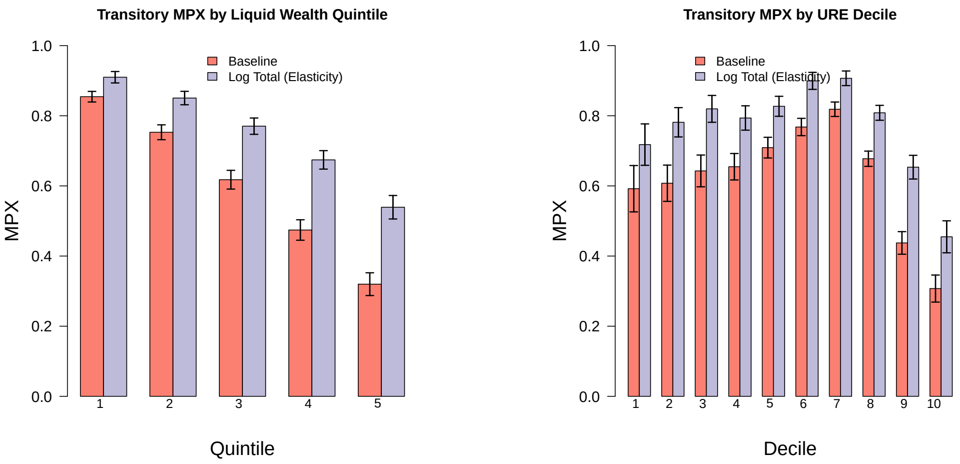

Figure L.3 Results Using Log Income and Expenditure

discusses the potential role that intra-household insurance may play, which we leave as an area for future research.

Finally, figure 2 shows the distribution of MPX by quintile of liquid wealth. It might be argued that the relevant level of liquid wealth is relative to income rather than in absolute terms. Figure L.5 demonstrates that results based on quintiles of liquid wealth divided by permanent income are similar. Also, results (not shown here) where deciles are based on a broader definition of liquid wealth—i.e., including stock and bond holdings—are similar to our baseline results.

M Durables

A critique of our empirical methodology is that it does not take account of durable goods, while our data include all spending (except on real estate) and therefore include large and durable goods such as cars and home improvements. The empirical model assumes that in response to a transitive income shock, expenditure increases temporarily for up to two years, which is entirely consistent with a model that includes durable goods. However, the model assumes that in response to a permanent shock to income, expenditure increases once to a new permanent level. A model that included durable goods would instead imply a large one-off expenditure on durable goods to get the households up to their desired flow of durable good services, followed by a decrease

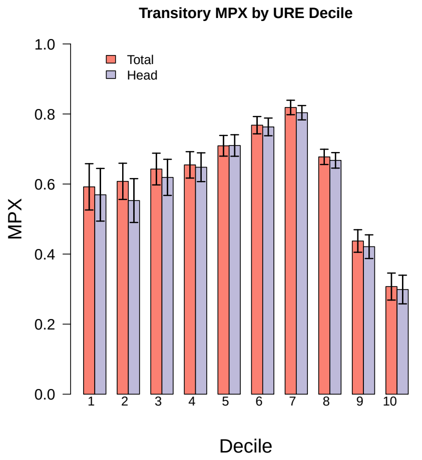

Figure L.4 Results Using Total Labor Income and Head Labor Income

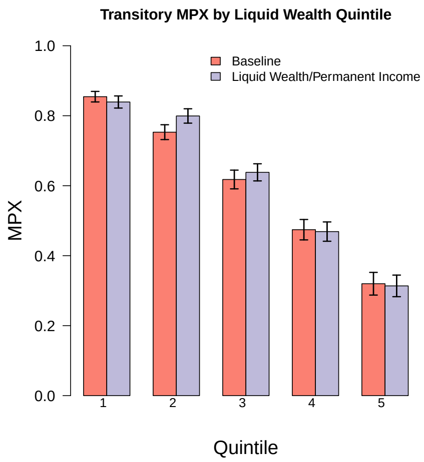

Figure L.5 Results Using Quintiles of Liquid Wealth over Permanent Income vs Liquid Wealth

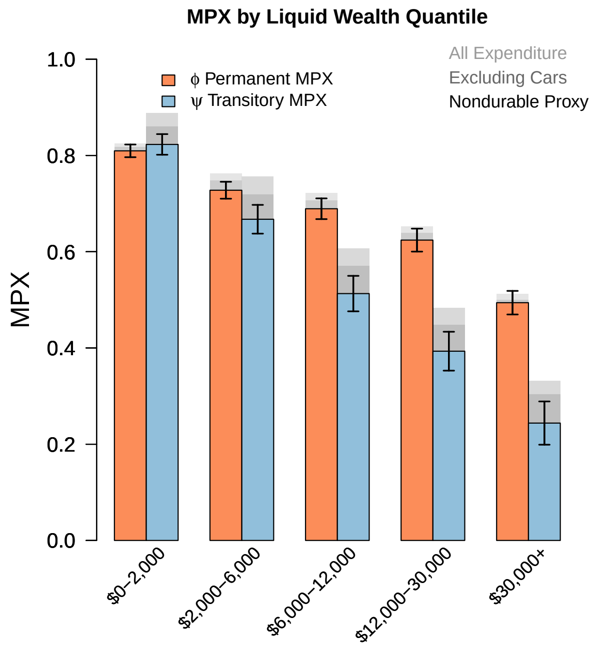

Figure M.1 MPX Removing Cars and Using the Nondurable Proxy Panel

back to a permanent level of spending that accounts for replenishing the higher level of depreciating durable goods.

We address this problem in two ways. First we show that our MPX estimates are unbiased in a simple model that includes durables, as long as we interpret the MPX out of transitory shocks to include durable expenditure (the correct definition for understanding aggregate demand) and the MPX out of permanent shocks to include only the consumption *flow* from durables. Second, we are able to construct a nondurable consumption proxy for each household using registry data on car purchases. This proxy has large measurement error, but will result in unbiased estimates of the MPC (excluding durables) to both permanent and transitory shocks. The estimated MPCs by liquid wealth quintile are shown in figure M.1. The figure shows the estimates using the nondurable proxy are, as expected, lower than those including all expenditures, although the change in magnitude is similar in size to the overall fraction of durable expenditure, suggesting durables do not play a special role in expenditures following transitory shocks. For the top quintile, durables do appear to play an outsized role, accounting for about a third of the expenditure response to transitory shocks.

M.1 Modeling Durables

It will help to write down a simple model. The model will show that our empirical methodology continues to estimate the consumption response to permanent and transitory shocks, but that these need to be interpreted carefully. The model uses the same income process as section A. Remembering the income process is made up of two martingale processes, P_t and Q_t, which may have jumps, instantaneous income is given by

$$dy_t = \Big(\int_0^t dP_s\Big)dt + dQ_t$$

while instantaneous expenditure now has both a durable and a nondurable component:

$$dc_t = \phi_{nd}\Big(\int_0^t dP_s\Big)dt + \phi_d dP_t + \psi dQ_s$$

Here we have assumed that the expenditure response to transitory shocks is instantaneous, but it would not change things to assume as before that the response decays to zero after two years. However, it is important that the durable component of the expenditure response to permanent shocks occurs instantaneously with the shock (or very soon after). Aggregating income and consumption annually gives

$$\begin{aligned}
\Delta^N \bar{y}_T = &\Big(\int_{T-N-1}^{T-N}(s-(T-N-1))dP_s + \int_{T-N}^{T-1} dP_s + \int_{T-1}^{T}(T-s)dP_s\Big)\\
&+ \Big(\int_{T-1}^{T} dQ_t - \int_{T-N-1}^{T-N} dQ_t\Big)\\
\Delta^N \bar{c}_T = \phi_{nd}&\Big(\int_{T-N-1}^{T-N}(s-(T-N-1))dP_s + \int_{T-N}^{T-1} dP_s + \int_{T-1}^{T}(T-s)dP_s\Big)\\
&+ \phi_d\Big(\int_{T-1}^{T} dP_t - \int_{T-N-1}^{T-N} dP_t\Big)\\
&+ \psi\Big(\int_{T-1}^{T} dQ_t - \int_{T-N-1}^{T-N} dQ_t\Big)
\end{aligned}$$

From this we can calculate the covariance:

$$\begin{aligned}
\mathrm{Cov}(\Delta^n \bar{c_T}, \Delta^n \bar{y_T}) = &\phi_{nd}\mathrm{Var}(\Delta^n \bar{y_T})\\
&+ \phi_d\left(\int_{T-1}^{T}(T-s)\sigma_P^2 dt - \int_{T-N-1}^{T-N}(s-(T-N-1))\sigma_P^2 dt\right)
\end{aligned}$$

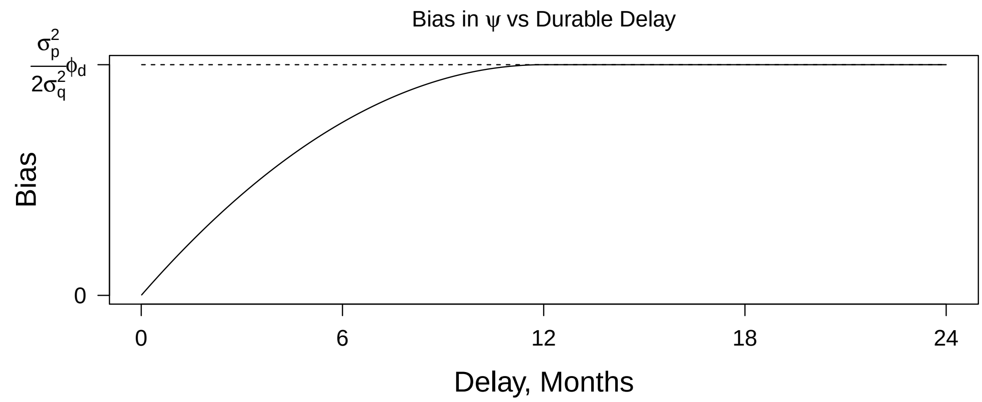

Figure M.2 Bias in Transitory MPX with Delay in Durable Goods Purchase

$$+\psi\left(\int_{T-1}^{T}\sigma_Q^2 dt+\int_{T-N-1}^{T-N}\sigma_Q^2 dt\right)$$
$$=\phi_{nd}(n-\frac{1}{3})\sigma_P^2+0+2\psi\sigma_Q^2$$

So the durable component of the covariance cancels out, and our identification method correctly identifies ϕ_{nd} and ψ but is unable to identify ϕ_d.

However, if there is some delay between the household receiving the permanent income shock and purchasing the durable goods, then this introduces an upward bias into the estimate of transitory MPX. The size of the bias grows with the number of months delay between the permanent income shock and the durable goods purchase, plateauing after 12 months at a level of $\frac{\sigma_p^2}{2\sigma_q^2}\phi_d$. Figure M.2 shows how this bias increases with the delay.

In order to quantify how large this bias may be in practice, we make use of the car registry data available in Denmark. Using data on the current value of cars owned by a household, we perform the same residual calculation to find the change in car value that is unpredictable with the household characteristics we are able to observe. We then construct two new expenditure panels: one in which we remove expenditures on cars, and one in which we make a proxy for non-durable consumption by removing expenditures on cars multiplied by $\frac{1}{0.421}$ (car purchases make up 42.1% of durable expenditure in Denmark):

$$C_T^{\text{nocar}}=C_T-\Delta\text{CarValue}$$
$$C_T^{\text{nondurable}}=C_T-\frac{1}{0.421}\Delta\text{CarValue}$$

The second, nondurable proxy consumption panel, can be modeled as the true non-durable consumption panel with classical measurement error added. This classical measurement error does not bias our estimates, so we can use this nondurable proxy panel to estimate an unbiased MPC out of transitory shocks, where the MPC does not include durable expenditures.

The results of this exercise can be seen in figure M.1. Even without bias, we would expect the nondurable proxy estimates to be lower than those including all expenditures, as the definition of transitory MPX changes over the three panels to exclude cars and then all durable goods. For the lower quintiles of liquid wealth it therefore looks as though the bias is likely small, as nondurable goods make up 10% of spending and the MPX estimates are smaller by approximately 10% in this region. For the top quintile of liquid wealth there seems to be some bias, with the estimate of MPX for all expenditures decreasing from 25% to an MPC for nondurable goods of 17%.

While there is some evidence that our results may be biased upward for those in the top quintiles of liquid assets, this bias will only have a small effect on our overall conclusions. As the relevant number for the monetary policy exercise is the MPX rather than the MPC, we have chosen not to adjust our baseline results using this method and accept that a small bias may exist in our data. It should be noted that such a bias will cause the heterogeneous channels of monetary policy to appear smaller than they actually are.

N Interpolating U.S. MPX

As we discussed in section B, liquid wealth holdings among U.S. households is lower than for Danish households, in absolute value and especially as a ratio of income. In our baseline results, we interpolate the MPX for U.S. households from Danish households according to the percentile of liquid wealth holding they are in. For example, a household in the 20th percentile of liquid wealth holdings in the United States is allocated the same MPX as a household in the 20th percentile of liquid wealth holdings in Denmark.

In this section we take two alternative approaches. The first is to interpolate based on the absolute level of liquid wealth. That is, a US. household with \$2,000 of liquid wealth is allocated the same MPX as a Danish household with \$2,000 of liquid wealth (equivalent in DKK). The second in to interpolate based on the ratio of liquid wealth to income.

The sufficient statistics under the three interpolation methods are shown in table N.1. This lower levels of liquid wealth and liquid wealth to income in the United States result in somewhat higher estimates of $\mathcal{M}$, the income-weighted MPX, for the two alternative

Table N.1 Auclert Sufficient Statistics for the U.S under different interpolation methods

	Baseline	Absolute Liquidity	Liquidy to Income Ratio
$\mathcal{M}$	0.44	0.47	0.63
$\mathcal{E}_Y$	-0.17	-0.20	-0.10
$\mathcal{E}_P$	-0.24	-0.28	-0.38
$\mathcal{E}_R$	0.01	0.01	-0.00

interpolation methods. Qualitatively, the results for the redistribution elasticities are the same as in the baseline, and the estimate for the interest rate exposure channel, $\mathcal{E}_R$ is little changed.

O MPX Heterogeneity over Liquid Wealth and Income

In this appendix we divide the population up into quintiles along two dimensions: liquid wealth and income. The step pattern of figure O.1, in which MPX steps down along the liquid wealth quintile, but is flat along the income quintile, demonstrates our finding that conditional on liquid wealth, the other dimensions we explore, such as income, have no further predictive power on MPX levels.

P Distribution of Permanent MPX by NNP, URE, and Income

Figure P.1 shows the distribution of both transitory and permanent MPX by NNP, URE and income decile. The transitory numbers are a repeat of figure 5.

Q Intra-household Income Insurance

As discussed in section A, we use labor income of the head of the household as our prime measure of income, in line with previous literature. Figure L.4 demonstrates that results based on total household income and income of the head of household are similar. However, MPXs from transitory shocks to the income of the spouse are lower

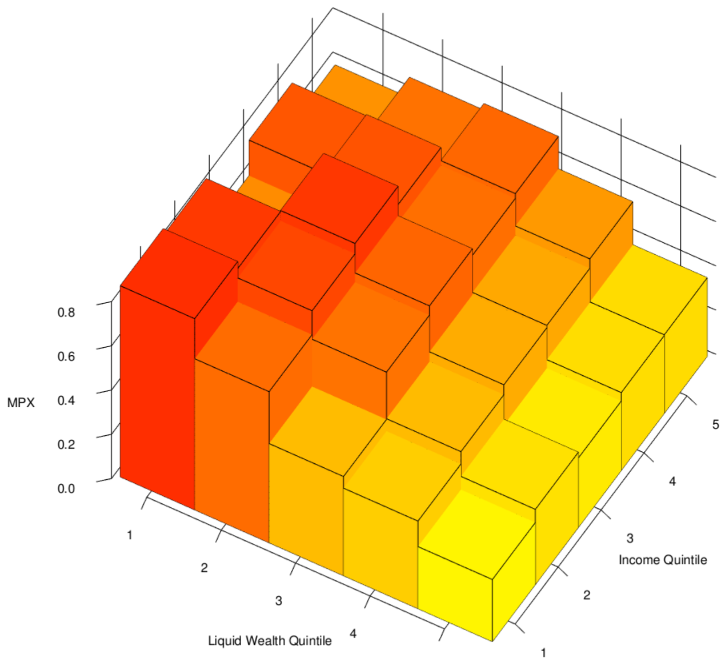

Figure O.1 Transitory MPX estimation by Liquid Wealth and Income Quintile

than MPXs from shocks to total income, in particular for the less wealthy households, as demonstrated in figure Q.1. This indicates heterogeneity in the role that intra-household income insurance plays across different groups of households. We leave this interesting topic for future research.

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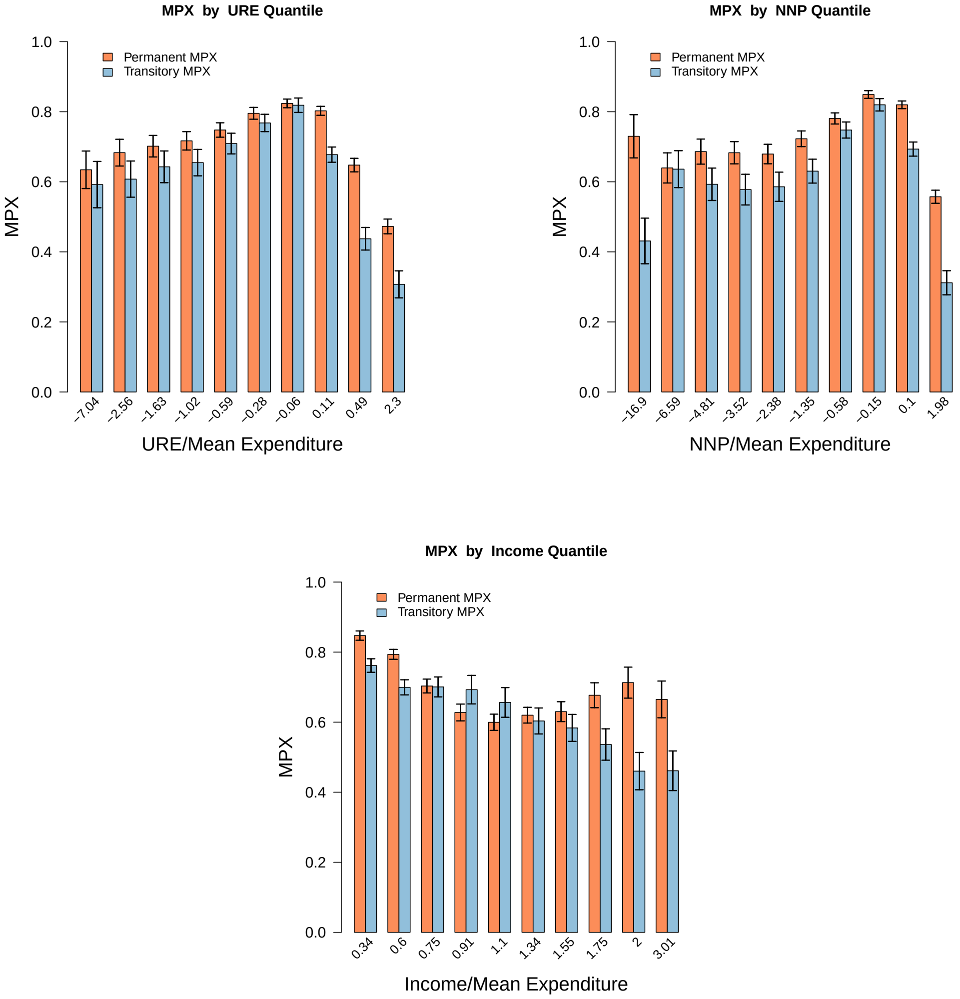
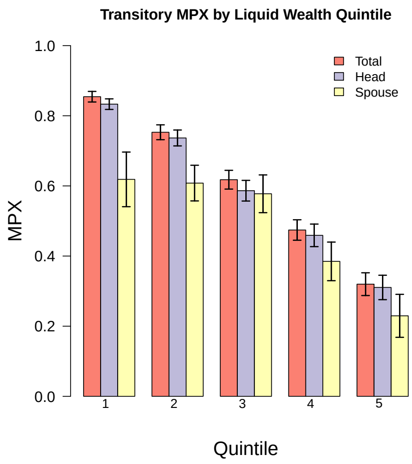


Figure P.1 MPX Distribution by URE, NNP, and Income



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