Online Appendix: Separating Ownership and Information

Paul Voss and Marius Kulms

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Mixed Strategies

In this section, we show that mixed strategies cannot induce full separation among different bidder types realizing a takeover with positive probability in equilibrium. For ease of illustration suppose $J = 1$ and denote $\gamma_1 := \gamma$. Note that if two bidder types $\omega_E < \overline{\omega}_E$ with a positive takeover likelihood separate, we need to have $p_E(\omega_E) < p_E(\overline{\omega}_E)$ by monotonicity of $E$’s payoff. Further, any bidder type $\omega_E \in (\omega_E, \overline{\omega}_E)$ also needs to realize a takeover with positive probability because otherwise he could imitate $\omega_E$ and earn strictly positive profits because $\omega_E$ needs to make at least zero profits in equilibrium. Since every $\omega_E > \overline{\omega}_E$ needs to make strictly positive expected profits, $p_E(\omega_E) < \omega + \omega_E$ for any $\omega_E > \overline{\omega}_E$. By monotonicity of the bidder’s payoff, for any $\omega'_E, \omega''_E \in (\omega_E, \overline{\omega}_E)$ with $\omega'_E < \omega''_E$, it needs to hold for full separation that $p'_E < p''_E$ with $0 < \phi(p'_E) < \phi(p''_E) < 1$ where $\phi(p_E)$ is the outside shareholder’s mixing probability. $\phi(p''_E) < 1$ has to hold as $\overline{\omega}_E$ would have an incentive to imitate $\omega''_E$ if it was true that $\phi(p''_E) = 1$. Indifference of the outside shareholder at both $p'_E$ and $p''_E$ then requires that

$$\omega + \mathbb{E}[\omega_I] = \frac{\lambda}{1 - s} p'_E + (1 - \frac{\lambda}{1 - s})(\omega + \omega''_E) = \frac{\lambda}{1 - s} p'_E + (1 - \frac{\lambda}{1 - s})(\omega + \omega''_E),$$

(A.1)
where $\gamma = \frac{1}{1-s}$ follows from $p_E < \omega + \omega_E$ for any $\omega_E > \omega_E$. However, the latter equality cannot hold true since $\omega''_E > \omega'_E$ and $p''_E > p'_E$. Thus, the outside shareholder cannot be indifferent at $p'_E$ and $p''_E$, yielding a contradiction such that full separation among different bidder types realizing a takeover with positive probability in equilibrium cannot occur.

**Private Benefits of Control**

We consider the basic model (Section 1 in the main text) but simplify to $J = 1$, denote $\gamma_1 := \gamma$ and consider the following utility of $I$:

$$u_I = \begin{cases} 
  s [\omega + \omega_E], & \text{if takeover successful} \\
  s [\omega + \omega_I] + B_I, & \text{otherwise,}
\end{cases} \quad (A.2)$$

where $B_I \geq 0$ are the private benefits $I$ derives from being in charge. $B_I$ is common knowledge and the insider is now indifferent between a takeover and remaining in charge if $s\omega_I + B_I = s\mathbb{E} [\omega_E | p_E]$. Let $b_I := \frac{B_I}{s}$ denote $I$’s private benefit per share such that $I$’s indifference type is given by

$$\omega^{**}_I := \max \{ \mathbb{E} [\omega_E | p_E] - b_I; 0 \}. \quad (A.3)$$

As before, whenever $\omega_I \leq \omega^{**}_I$, the incumbent manager favors a takeover. The following proposition characterizes the analogous equilibrium to Theorem 1.

**Proposition A.** There exists a $b_I > 0$ such that for all $b_I \leq b_I$, there is an equilibrium in which $E$ fully reveals his type by posting

$$p^*_E(\omega_E) = \begin{cases} 
  \omega + \mathbb{E} [\omega_I | \omega_I \leq \omega^{**}_I (\omega_E)] + b_I, & \text{if } \omega_E \geq b_I \\
  \omega + \omega_E, & \text{otherwise.}
\end{cases}$$

1. If $\omega_I \leq \omega^{**}_I (\omega_E)$, then $m^*_I \in [0, \omega^{**}_I (\omega_E)]$, and a takeover occurs with probability one;
2. if $\omega_I > \omega^{**}_I (\omega_E)$, then $m^*_I \in (\omega^{**}_I (\omega_E), 1]$, and a takeover occurs with probability zero;
and $\gamma^*(m^*_I(\omega_I \leq \omega^*_I(\omega_E))) = \lambda$.

Further, the equilibrium improves expected firm value and overall welfare relative to any equilibrium where ownership and control are not separated.

Proposition A establishes that also with sufficiently small private benefits of control, there is an equilibrium with informative cheap talk in which the bidder fully reveals his type via his tender offer. Moreover, the expected firm value and welfare are higher than in any equilibrium under a unity of ownership and control.

The equilibrium only exists for small enough biases. Intuitively, if $b_I$ grows very large, that is, the private benefit $B_I$ is large relative to the share endowment $s$, the incumbent always prefers retaining control. Hence, her message is never informative, and there is also no scope to incentivize bidder separation. If the equilibrium exists, i.e., $b_I \leq \bar{b}_I$, there are some noteworthy differences relative to the basic model. Because the incumbent is now biased against a takeover by assumption, her indifference type has shifted downwards to $\omega^*_I = \max\{\omega_E - b_I; 0\}$. As a consequence, $I$ never recommends a takeover for bidder types $\omega_E \in [0, b_I)$. Since the outside shareholder still follows the message in equilibrium only bidder types above $b_I$ have a positive takeover probability.

The equilibrium tender offer $p^*_E$ is strictly increasing in $b_I$ because a large bias will make the incumbent less likely to endorse a takeover. Hence, takeovers become a scarcer opportunity such that the bidder is willing to ramp up his price offer.

Due to the bias, a takeover occurs if and only if $\omega_I \leq \max\{\omega_E - b_I; 0\}$ such that first-best is not attainable in equilibrium with $b_I > 0$. However, as $b_I$ converges to zero, $I$’s indifference type converges to $\omega_E$ resulting in the first-best allocation rule.

**Restoring First-Best: the Role of Golden Parachutes.** It is easy to see that efficiency can be restored if $I$ is provided with an adequate golden parachute if she
gives up control. With a golden parachute of size $G \in \mathbb{R}_+$, I's utility becomes

$$u_I = \begin{cases} s [\omega + \omega_E] + G, & \text{if takeover successful} \\ s [\omega + \omega_I] + B_I, & \text{otherwise.} \end{cases}$$

(A.4)

As a result, whenever the golden parachute is set equal to the private benefits of control, i.e., $G = B_I$, I's indifference type is

$$\omega^* = \max \{ \mathbb{E}[\omega_E|p_E] - \frac{B_I - G}{s}; 0 \} = \mathbb{E}[\omega_E|p_E].$$

Hence, it equals the indifference type from the baseline model such that there is an efficient equilibrium.

**Corollary A.** If $G = B_I$, first-best is attainable in equilibrium if ownership and control are separated.

**Omitted Proofs from Main Text**

**Proof of Proposition 2.** Step 0: The equilibrium from Theorem 1 and I never selling any shares together form an equilibrium in the extended game.

Suppose the statement is true. Then, if $\omega_I \leq \omega_I^*$, $\Gamma^* = \sum_{j=1}^I s_j \gamma_j = \lambda$ and $\eta^*(\omega_I \leq \omega_I^*, p^*_E) = 0$. $\eta^*(\omega_I \leq \omega_I^*, p^*_E) = 0$ is indeed a best response since any $\eta > 0$ would strictly reduce I's expected profits because $p^*_E(\omega_E) < \omega + \mathbb{E}[\omega_E|p_E(\omega_E)] = \omega + \omega_E$. Further, given $\eta^*(\omega_I \leq \omega_I^*, p^*_E) = 0$, it is optimal for any outside shareholder $j$ to tender sufficiently many shares to make the takeover successful if $m^*_I(\omega_I \leq \omega_I^*)$ since $\gamma_j^*(\omega + \mathbb{E}[\omega_I|\omega_I \leq \omega_E]) + (1 - \gamma_j^*)(\omega + \omega_E) \geq \omega + \mathbb{E}[\omega_I|\omega_I \leq \omega_E]$ for any $\gamma_j^* \in [0, 1]$.

If $m^*_I(\omega_I > \omega_I^*)$ and $\Gamma^* < \lambda$, it is optimal for $I$ to set $\eta^* = 0$. Either because $I$ cannot unilaterally change the takeover outcome (if $s < \Gamma^* - \lambda$), or if $s \geq \Gamma^* - \lambda$, $I$ does not want to make the takeover successful since

$$\eta(\omega + \mathbb{E}[\omega_I|\omega_I \leq \omega_E]) + (1 - \eta)(\omega + \omega_E) < \omega + \omega_I$$

(A.5)

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1We are by no means the first to consider the problem of golden parachutes or severance pay, see, e.g., Lambert and Larcker (1985); Knoeber (1986); Harris (1990); Almazan and Suarez (2003); Eisfeldt and Rampini (2008); Inderst and Müller (2010). However, none of these papers considers how golden parachutes influence management’s advisory role in takeovers.
which holds true for any \( \eta > 0 \) by full support and since \( \omega_I > \omega_I' = \omega_E \).

Last, we need to check that also if \( I \) can tender shares, it is not optimal for \( E \) to deviate to any \( p_E^* \notin [p_E^*(0), p_E^*(1)] \). Given the off-path beliefs defined in the proof of Theorem 1, a deviation to any \( p_E^{dev} > p_E^*(1) \) cannot be profitable because also if \( I \) can tender, a profitable deviation would require \( \Gamma^*(p_E^{dev}) > \lambda \) and, thus, \( p_E^{dev} \geq \omega + \mathbb{E}[\omega_E|p_E^{dev}] \), which is not consistent with off-path beliefs satisfying the intuitive criterion. Further, for any \( p_E^{dev} < p_E^*(0) = \omega \), \( I \) has no incentive to tender since we have specified off-path beliefs to assign probability one to \( \omega_E = 0 \) for any such deviation, and \( \omega + \omega_I > p_E^{dev} \) for all \( \omega_I \in [0,1] \).

**Step 1:** In any equilibrium, at most one bidder type posts a tender offer \( p_E^* = \omega + \mathbb{E}[\omega_E|p_E^*] \) with \( \mathbb{P}[\text{takeover}|p_E^*] > 0 \).

Suppose, on the way to a contradiction, that \( p_E^* = \omega + \mathbb{E}[\omega_E|p_E^*] \) but \( \mathbb{P}[\text{takeover}|p_E^*] > 0 \) for at least two types \( \omega_I' < \omega_I'' \) posting \( p_E^*(\omega_I') \) and \( p_E^*(\omega_I'') \). For any \( p_E^* \) such that \( p_E^* = \omega + \mathbb{E}[\omega_E|p_E^*] \), it either has to hold that there are bidder types making strictly negative profits (which yields a contradiction) or there is a unique \( \omega_I' \) such that \( p_E^*(\omega_I') = \omega + \omega_I' \). Therefore, we can conclude that it would need to hold that \( p_E^*(\omega_I') < p_E^*(\omega_I'') \) as \( \omega_I' < \omega_I'' \). But then, \( \omega_I'' \) can deviate to \( p_E^*(\omega_I') \) to make strictly positive profits since \( \mathbb{P}[\text{takeover}|p_E^*(\omega_I')]>0 \) by assumption. Thus, only the lowest type \( \omega_I' \) securing a takeover with strictly positive probability can make zero profits, and for all \( \omega_E < \omega_I' \) no takeover ever occurs.

**Step 2:** In any equilibrium in which the insider tenders shares during a successful takeover, not all value-increasing takeovers are realized.

Fix an equilibrium tender offer \( p_E^*(\omega_E) \) at which a takeover occurs and \( \eta^*(p_E^*(\omega_E)) > 0 \). First note that \( p_E^*(\omega_E) > \omega + \mathbb{E}[\omega_E|p_E^*(\omega_E)] \) cannot occur in equilibrium because it would imply strictly negative profits of some bidder types. Second, by step 1, \( p_E^*(\omega_E) = \omega + \mathbb{E}[\omega_E|p_E^*(\omega_E)] \) can only hold for at most one bidder type such that \( p_E^*(\omega_E) = \omega + \omega_E \). In this case, the efficient amount of takeovers may occur for this particular bidder type. However, this also implies, by step 1, that there is an interval
[0, \bar{\omega}_E] with \bar{\omega}_E > 0 such that no takeover ever occurs for any \omega_E \in [0, \bar{\omega}_E). \bar{\omega}_E > 0 holds because \mathbb{P}[\text{takeover}|p^*_E(\omega_E = 0)] = 0 as indifference requires p^*_E(\omega_E = 0) = \omega such that a takeover can only occur for \omega = 0. However, because F_i is continuous and, thus, has no atoms, we can conclude that \mathbb{P}[\text{takeover}|p^*_E(\omega_E = 0)] = F_i(0) = 0.

Last, consider p^*_E(\omega_E) < \omega + \mathbb{E}[\omega_E|p^*_E(\omega_E)]. Then, for any \omega_E at which a takeover occurs, \sum_{j=1}^{I} s_j \gamma_j + s \eta^* = \lambda such that I wants to tender \eta^* > 0 shares to make the takeover successful if and only if

\eta^* p^*_E + (1 - \eta^*) (\omega + \mathbb{E}[\omega_E|p^*_E]) \geq \omega + \omega_I

\iff \omega_I \leq \eta^* (p^*_E - \omega) + (1 - \eta^*) \mathbb{E}[\omega_E|p^*_E].

(A.6)

Together with \eta^* > 0, p^*_E(\omega_E) < \omega + \mathbb{E}[\omega_E|p^*_E(\omega_E)] implies that \eta^* (p^*_E - \omega) + (1 - \eta^*) \mathbb{E}[\omega_E|p^*_E(\omega_E)] < \omega_E. Hence, for any \omega_E at which p^*_E(\omega_E) < \omega + \mathbb{E}[\omega_E|p^*_E(\omega_E)] and \eta^* > 0, there is an interval (\omega_I, \bar{\omega}_I) \neq \emptyset such that a takeover occurs in the equilibrium of Theorem 1 but not in the equilibrium with \eta^* > 0. Thus, not all value-increasing takeovers are realized.

**Step 3:** Expected insider profits are maximal in the efficient equilibrium.

In the efficient equilibrium, expected insider profits are given by

\[ s(\omega + \mathbb{E}[1_{\omega_E > \omega_I} \omega_I] + \mathbb{E}[1_{\omega_E \geq \omega_I} \omega_E]) \]  

(A.7)

and, thus, maximize the value of I’s share endowment. In any equilibrium, \( p_E(\omega_E) \leq \omega + \omega_E \) and I can never buy additional shares by assumption. Hence, the efficient equilibrium also maximizes expected insider profits.

□

**Proof of Proposition 3. Step 1:** Suppose \( s_j < \lambda \ \forall j \). Then, there exists an equilibrium in which no takeover ever occurs.

We show by construction that the following equilibrium always exists provided no shareholder is pivotal on her own: \( \gamma^*_j(p_E, m_I) = 0 \ \forall j, p_E, m_I, \) and \( p^*_E = 0 \ \forall \omega_E, \) and
Given \( \gamma'_{j}(p_{E}, m_{I}) = 0 \), no shareholder \( j \) has an incentive to deviate as she cannot induce a takeover unilaterally \( (s_{j} < \lambda \ \forall \ j) \) and because the offer becomes void if fewer than \( \lambda \) shares are tendered. Further, since \( \gamma_{j}^* = 0 \) independent of \( m_{I} \), it is indeed a best response for \( I \) to send \( m_{I}^* = 1 \) for all \( \omega_{I} \in [0, 1] \). Because no price can induce a takeover, \( p_{E}^* = 0 \) is optimal for all \( \omega_{E} \in [0, 1] \). Off-path beliefs regarding \( \omega_{I} \) and \( \omega_{E} \) are irrelevant given the coordination failure.

**Step 2:** There exists an equilibrium with a cutoff price \( \hat{p}_{E} < \omega + 1 \) such that if \( \omega + \omega_{E} < \hat{p}_{E} \), a takeover occurs with probability zero. If \( \omega + \omega_{E} \geq \hat{p}_{E} \), \( E \) posts \( \hat{p}_{E} \), a takeover occurs with probability one, and \( T^*(\hat{p}_{E}) = \lambda \).

Consider a cutoff price \( \hat{p}_{E} < \omega + 1 \) such that all shareholders tender \( \gamma'_{j} = \gamma' = \frac{\lambda}{1-s} \) shares for each \( p_{E} \geq \hat{p}_{E} \). For \( p_{E} < \hat{p}_{E} \), each shareholder tenders zero shares. Now let \( \hat{p}_{E} \) be the price that makes shareholders exactly indifferent between tendering and not tendering given the proposed equilibrium, i.e.,

\[
\frac{\lambda}{1-s} \hat{p}_{E} + (1 - \frac{\lambda}{1-s})(\omega + \mathbb{E}[\omega_{E}|\omega_{E} \geq \hat{p}_{E} - \omega]) = \omega + \mathbb{E}[\omega_{I}]. \tag{A.8}
\]

This equilibrium is, for instance, supported by an off-path belief yielding posterior expected type of \( \mathbb{E}[\omega_{E}|\omega_{E} \leq p_{E} - \omega] \) for \( p_{E} < \hat{p}_{E} \) and of \( \mathbb{E}[\omega_{E}|\omega_{E} \geq p_{E} - \omega] \) for \( p_{E} > \hat{p}_{E} \).

By their symmetric tendering strategy \( \gamma' = \frac{\lambda}{1-s} \), each shareholder is pivotal at any \( p_{E} \geq \hat{p}_{E} \). Further, at \( \hat{p}_{E} \), each shareholder is indifferent between tendering \( \gamma' \) shares and not tendering thereby letting the takeover fail. Hence, it is (weakly) optimal for shareholders to tender exactly a fraction of \( \frac{\lambda}{1-s} \).

For any \( p_{E} > \hat{p}_{E} \), any shareholder strictly prefers a takeover to occur and tendering at least \( \gamma' \) shares. No shareholder has an incentive to tender more than \( \gamma' \) shares because off-path beliefs of \( \mathbb{E}[\omega_{E}|\omega_{E} \geq p_{E} - \omega] \) for \( p_{E} > \hat{p}_{E} \) ensure that expected security benefits strictly exceed the price for all \( p_{E} < \omega + 1 \). For \( p_{E} = \omega + 1 \), price equals expected security benefits, making \( \gamma'' = \frac{\lambda}{1-s} \) again a best response. Since \( \sum_{j} s_{j} = 1 - s \), it follows that \( \Gamma'' = \sum_{j} \gamma''_{j} = \lambda \). For any \( p_{E} < \hat{p}_{E} \), given the
proposed off-path beliefs and definition of $\hat{p}_E$, it holds for any $\gamma_j \in [0, 1]$ that

\[
\gamma_j p + (1 - \gamma_j)(\omega + \mathbb{E}[\omega_E | \omega_E \leq p - \omega]) < \gamma_j \hat{p} + (1 - \gamma_j)(\omega + \mathbb{E}[\omega_E | \omega_E \leq \hat{p} - \omega]) \leq \gamma_j \hat{p} + (1 - \gamma_j)(\omega + \mathbb{E}[\omega_E | \omega_E \geq \hat{p} - \omega]) \leq \gamma_j \hat{p} + (1 - \gamma_j)(\omega + \mathbb{E}[\omega_E | \omega_E \leq \hat{p} - \omega]) < \gamma_j \hat{p} + (1 - \gamma_j)(\omega + \mathbb{E}[\omega_E | \omega_E \geq \hat{p} - \omega]) = \omega + \mathbb{E}[\omega_I],
\]

such that $\gamma_j = 0$ is a best response for any shareholder $j$ after any $p < \hat{p}$. For $E$, deviating to a price above $\hat{p}$ yields to a purchase of $\lambda$ shares with certainty but at a higher cost. Deviating to a price smaller than $\hat{p}$ yields no takeover and zero profits. Hence, $E$ does not want to deviate.

**Step 3:** Suppose $s_j < \lambda$ for all $j \in \{1, \ldots, J\}$. Then, there is an equilibrium where $p_E^*(\omega_E = 1) = \omega + 1$ and $\omega_E = 1$ is the only bidder type who secures a takeover. Further, $\Gamma^*(p_E^*(1)) \geq \lambda$.

Suppose $\gamma_j^*(p) = 0$ for all $p < \omega + 1$ and $\Gamma^*(p_E^*(1)) \geq \lambda$. Further suppose that $p_E^*(\omega_E = 0) = 0$ for all $\omega_E < 1$ and $p_E^*(\omega_E = 1) = \omega + 1$. In the conjectured equilibrium, a takeover occurs only after $p_E^* = \omega + 1$. Any $\Gamma^*(p_E^* = 1) \geq \lambda$ can be supported in equilibrium because security benefits after a successful takeover equal the tender offer. If a shareholder was pivotal at $p_E^* = \omega + 1$, i.e., she could block the takeover by not tendering, she would refrain from doing so as $\omega + \mathbb{E}[\omega_I] < \omega + 1 = p_E^*(1)$ by the full support assumption. Therefore, $T^*(p_E^*(1)) \geq \lambda$.

No bidder type $\omega_E < 1$ has an incentive to deviate to $p_E = \omega + 1$ as this would imply strictly negative profits. Independent of off-path beliefs regarding $\omega_I$ and $\omega_E$, given the other shareholders’ tendering strategy, it is optimal for any shareholder $j$ not to tender after any price $p_E < \omega + 1$ because she is not pivotal ($s_j < \lambda$ for all $j \in \{1, \ldots, J\}$). Bidder type $\omega_E = 1$ does not want to deviate downwards or upwards as this would also imply at most zero profits.

**Step 4:** In any equilibrium in which a takeover occurs with non-zero probability, there exists a unique price $\hat{p}_E \leq \omega + 1$ with $\mathbb{P}[\text{takeover} | \hat{p}_E] = 1$ played on the equi-
First note that since we consider pure strategy equilibria, $\mathbb{P}[\text{takeover}|p_E] \in \{0, 1\}$. Then, suppose, on the way to a contradiction, the statement was false, i.e., there are at least two prices $\hat{p}_E \neq p'_E$ s.t. $\mathbb{P}[\text{takeover}|\hat{p}_E] = \mathbb{P}[\text{takeover}|p'_E] = 1$ and both prices are played by some bidder type on the equilibrium path. W.l.o.g. assume $\hat{p}_E < p'_E$. Then, it must hold that $\Gamma^*(p'_E) > \Gamma^*(\hat{p}_E) \geq \lambda$ as otherwise $p'_E$ implies higher costs while leaving the benefits constant. For $\Gamma^*(p'_E) > \lambda$ to be part of an equilibrium, it has to hold that $p'_E = \omega + \mathbb{E}[\omega | p'_E]$. Otherwise, if $p'_E < \omega + \mathbb{E}[\omega | p'_E]$, any shareholder tendering a positive amount of shares had a profitable deviation to selling fewer shares while still making the takeover succeed. Because negative bidder profits cannot be part of an equilibrium, $p'_E = \omega + \mathbb{E}[\omega | p'_E]$ is only possible if type $\omega_E = p'_E - \omega$ alone posts $p'_E$. But this implies zero profits, so this type has a profitable deviation to $\hat{p}_E$, yielding the contradiction.

**Step 5:** In any equilibrium in which a takeover occurs with non-zero probability, all types $\omega_E > \hat{p}_E - \omega$ post $\hat{p}_E$.

Since there is a unique price on the equilibrium path that leads to a takeover, the only other possibility is that there is some $\omega'_E > \hat{p}_E - \omega$ that posts a price that does not realize a takeover. This, however, would imply zero profits, yielding a profitable deviation for $\omega'_E$ to $\hat{p}_E$.

**Step 6:** In any equilibrium in which a takeover occurs with non-zero probability, all $\omega_E < \hat{p}_E - \omega$ post a price that does not realize a takeover.

Posting $p_E \geq \hat{p}_E$ implies strictly negative profits. Any $p_E < \hat{p}_E$ cannot yield $\Gamma^*(p_E) \geq \lambda$ as otherwise $\hat{p}_E$ would not be the unique price after which a takeover is implemented.

**Step 7:** $\Gamma^*(\hat{p}_E) = \lambda$ for $\hat{p}_E < \omega + 1$. 
Suppose this was not true. However, we know that \( \hat{p}_E \) is unique and that all \( \omega_E > \hat{p}_E - \omega \) post-\( \hat{p}_E \) on the equilibrium path. Hence, \( \omega + \mathbb{E}[\omega_E|\hat{p}_E] > \hat{p}_E \) for all \( \hat{p}_E < \omega + 1 \). Thus, if \( \Gamma^*(\hat{p}_E) > \lambda \), any shareholder could profitably deviate and tender strictly less shares but make the takeover still succeed.

**Step 8:** By steps 1–3 we have established the existence of three different kinds of equilibria depicted in Proposition 3. Steps 4–7 establish that in any equilibrium in which a takeover occurs there is a unique tender offer \( \hat{p}_E \) on which all bidder types beyond a certain cutoff pool. The other possibility that no takeover ever occurs (step 1). Thus, we have characterized the set of equilibria. \( \square \)

**Omitted Proof from Online Appendix**

**Proof of Proposition A.**

**Step 0:**

If \( p^*_E(\omega_E) = \begin{cases} \omega + \mathbb{E}[\omega_I|\omega_I \leq \omega^{**}_I(\omega_E)] + b_I, & \text{if } \omega_E \geq b_I \\ \omega + \omega_E, & \text{otherwise,} \end{cases} \)

then, it has to hold that \( \gamma^* = \frac{1}{1-\gamma} \).

Given the conjectured equilibrium tender offer, all and only bidder types \( \omega_E > b_I \) have a positive takeover probability. Further, it holds true that \( p^*_E(\omega_E) = \omega + \mathbb{E}[\omega_I|\omega_I \leq \max(\omega_E - b_I; 0)] + b_I < \omega + \omega_E \) whenever \( \mathbb{P}[\text{takeover}|p^*_E(\omega_E)] > 0 \). Thus, if \( \gamma^* > \frac{1}{1-\gamma} \) after some \( p^*_E(\omega_E) \) and \( m_I(\omega_I \leq \omega^{**}_I) \), the shareholder would have a profitable deviation to some \( \gamma' \in [\frac{1}{1-\gamma}, \gamma^*). \)

**Step 1:** Necessary condition for a fully separating tender offer

Let \( \omega_E \) be the bidder’s true type. Since, if a takeover succeeds \( \gamma^* = \frac{1}{1-\gamma} \), the bidder’s optimal bid price \( p \), given the conjectured equilibrium, is

\[
\argmax_{p \in \mathbb{R}^+} F_I[\omega^{**}_I(p^*_E^{-1}(p))] \lambda [\omega + \omega_E - p],
\]

where \( \omega^{**}_I = \omega_E - b_I \) for \( \omega_E \geq b_I \) and zero, otherwise. Suppose \( \omega_E \geq b_I \). Replicating
the same steps as in the proof of Theorem 1 (with \( b_I = 0 \)) yields

\[
p_E'(\omega_E) = \frac{f_I(\omega_E - b_I)}{F_I(\omega_E - b_I)}(\omega + \omega_E - p_E(\omega_E)). \tag{A.11}
\]

It can be shown that the general solution to (A.11) is given by

\[
p_E^*(\omega_E) = \frac{\int_{b_I}^{\omega_E} f_I(z - b_I)(z + \omega)dz + C}{F_I(\omega_E - b_I)}, \tag{A.12}
\]

where \( C = 0 \) in equilibrium because the type \( \omega_E = b_I \) has a takeover probability of zero. Observe that we can further rewrite the price function stated in (A.12):

\[
\int_{b_I}^{\omega_E} f_I(z - b_I)(z + \omega)dz = \int_{0}^{\omega_E-b_I} f_I(z)(z + \omega + b_I)dz
\]
\[
= \frac{\omega}{F_I(\omega_E - b_I)} \int_{0}^{\omega_E-b_I} f_I(z)dz + \int_{0}^{\omega_E-b_I} f_I(z)zdz + b_I \int_{0}^{\omega_E-b_I} f_I(z)dz
\]
\[
= \omega + \mathbb{E}[^{\omega_E-b_I}f_I(z)dz] + b_I. \tag{A.13}
\]

Hence, \( p_E^*(\omega_E) = \omega + \mathbb{E}[\omega_I | \omega_I \leq \omega_E - b_I] + b_I \) for \( \omega_E \geq b_I \).

For \( \omega_E < b_I \), a takeover never occurs in the conjectured equilibrium because \( \omega_I^* = 0 \). Further, all types below \( b_I \) do not want to deviate to a price posted by some \( \omega_E \geq b_I \) since this would yield strictly negative profits. Hence, offering the true type (plus the common component) \( p_E = \omega + \omega_E < \omega + b_I \) is a best response relative to any other on-path tender offer.

Further, given the proposed \( p_E^*(\omega_E) \), all bidder types make at least zero profits.

**Step 2:** Sufficiency

This step is identical to the case with \( b_I = 0 \).

**Step 3:** The outside shareholder sells after \((p_E^*, m_I^*(\omega_I \leq \omega_I^*))\)
First note that \( m^*_I(\omega_I \leq \omega^*_I) \) only occurs with positive probability if \( \omega_E > b_I \). Then, for \( p^*_E \) and \( m^*_I(\omega_I \leq \omega^*_I) \), it has to hold that there is a \( \gamma \geq \frac{1}{1-s} \) such that

\[
\gamma p^*_E(\omega_E) + (1 - \gamma)(\omega + \mathbb{E}[\omega_E|p^*_E(\omega_E)]) \geq \omega + \mathbb{E}[\omega_I|\omega_I \leq \omega^*_I], \tag{A.14}
\]

Plugging in \( p^*_E, \omega^*_I \) and \( \omega_E > b_I \), this becomes

\[
\gamma(\omega + \mathbb{E}[\omega_I|\omega_I \leq \omega_E - b_I]) + (1 - \gamma)(\omega + \omega_E) \geq \omega + \mathbb{E}[\omega_I|\omega_I \leq \omega_E - b_I], \tag{A.15}
\]

which holds true for any \( \gamma \in [0, 1] \) since \( \mathbb{E}[\omega_I|\omega_I \leq \omega_E - b_I] < \omega_E \) by full support.

**Step 4:** The outside shareholder does not sell after \((p^*_E, m^*_I(\omega_I > \omega^*_I))\)

For \( p^*_E \) and \( m^*_I(\omega_I > \omega^*_I) \), there is no \( \gamma \geq \frac{1}{1-s} \) such that

\[
\gamma p^*_E(\omega_E) + (1 - \gamma)(\omega + \mathbb{E}[\omega_E|p^*_E(\omega_E)]) \geq \omega + \mathbb{E}[\omega_I|\omega_I > \omega^*_I(\omega_E)]. \tag{A.16}
\]

This condition needs to be checked for both \( \omega_E < b_I \) and \( \omega_E \geq b_I \). If \( \omega_E < b_I \), the condition becomes

\[
\gamma(\omega + \omega_E) + (1 - \gamma)(\omega + \omega_E) \geq \omega + \mathbb{E}[\omega_I]. \tag{A.17}
\]

Since \( \omega_E < b_I \), a sufficient condition such that the shareholder follows \( I \)'s recommendation is that \( b_I \leq \mathbb{E}[\omega_I] \). If \( \omega_E \geq b_I \), plugging in yields

\[
\gamma(\omega + \mathbb{E}[\omega_I|\omega_I \leq \omega_E - b_I] + b_I) + (1 - \gamma)(\omega + \omega_E) < \omega + \mathbb{E}[\omega_I|\omega_I > \omega_E - b_I], \tag{A.18}
\]

which is equivalent to

\[
b_I < \mathbb{E}[\omega_I|\omega_I > \omega_E - b_I] - \mathbb{E}[\omega_I|\omega_I \leq \omega_E - b_I] + \frac{1 - \gamma}{\gamma}(\mathbb{E}[\omega_I|\omega_I > \omega_E - b_I] - \omega_E). \tag{A.19}
\]

By continuity, there exists a bias \( \bar{b}_I \) sufficiently small such that the constraint is ful-
Step 4: There are no profitable deviations to $p_E \notin [p_E^*(0), p_E^*(1)]$ given that off-path beliefs satisfy the intuitive criterion and assign probability 1 to $\omega_E = 0$ for any $p_E < \omega$.

Deviations to $p_E^{\text{dev}} > p_E^*(1)$ can be profitable if either the outside shareholder also tenders if $I$ does not recommend it since this increases the takeover likelihood from $F_I(\omega_I^*(1))$ to 1, or if the outside shareholder tenders more than $\lambda$ shares, or both.

We start by showing that the outside shareholder never tenders more than $\lambda$ shares. $\gamma^*(p_E^{\text{dev}}) > \lambda$ requires that off-path beliefs induce $p_E^{\text{dev}} \geq \omega + \mathbb{E}[\omega_E|p_E^{\text{dev}}]$. Hence, off-path beliefs assign positive probability to types that make weakly negative profits by the deviation. Because $p_E^{\text{dev}} > p_E^*(1) > \omega + b_I$, any $\omega_E \leq b_I$ posting $p_E^{\text{dev}}$ makes a strict loss. Any $\omega_E > b_I$ may make zero profits by the deviation but makes strictly positive expected profits on the equilibrium path. Thus, off-path beliefs inducing $p_E^{\text{dev}} \geq \omega + \mathbb{E}[\omega_E|p_E^{\text{dev}}]$ are not consistent with the intuitive criterion.

Hence, $p_E^{\text{dev}} > p_E^*(1)$ can only be profitable if the outside shareholder also tenders if $I$ does not recommend it. $I$ does not recommend a takeover in the proposed equilibrium if $\omega_I > \omega_I^{**} = \max\{\mathbb{E}[\omega_E|p_E^{\text{dev}}] - b_I; 0\}$. Then, a profitable deviation requires that for some $\gamma \geq \frac{1}{\lambda - s} > 0$,

$$
\gamma p_E^{\text{dev}} + (1 - \gamma)(\omega + \mathbb{E}[\omega_E|p_E^{\text{dev}}]) \geq \omega + \mathbb{E}[\omega_I|\omega_I > \mathbb{E}[\omega_E|p_E^{\text{dev}}] - b_I]. \tag{A.20}
$$

The intuitive criterion of Cho and Kreps (1987) again excludes types $\omega_E \leq p_E^{\text{dev}} - \omega$ because these would make weakly negative profits. Note that if there is a $\omega_E = p_E^{\text{dev}} - \omega \leq 1$, this type makes strictly positive profits on the equilibrium path as $p_E^{\text{dev}} > p_E^*(1) > \omega + b_I$. Thus, off-path beliefs consistent with the intuitive criterion need to imply that $\mathbb{E}[\omega_E|p_E^{\text{dev}}] > p_E^{\text{dev}}$ such that the left hand side in (A.20) is strictly smaller than $\mathbb{E}[\omega_E|p_E^{\text{dev}}]$. Since $\lambda > 0$, for (A.20) to hold it is a necessary condition that

$$
\mathbb{E}[\omega_E|p_E^{\text{dev}}] > \mathbb{E}[\omega_I|\omega_I > \mathbb{E}[\omega_E|p_E^{\text{dev}}] - b_I]. \tag{A.21}
$$
But by continuity and full support, there exists a $b_1 > 0$ such that for all $b_1 \leq b_1^2$: $\mathbb{E}[\omega_E|p^{dev}] \leq \mathbb{E}[\omega_1|\omega_1 > \mathbb{E}[\omega_E|p^{dev}] - b_1]$ such that no upward deviation is profitable for $b_1 \leq b_1^2$. Further, the equilibrium can be supported by off-path beliefs assigning probability 1 to $\omega_E = 0$ for deviations to off-path prices below $\omega$. The reason is that then the outside shareholder would never tender at such a deviation because her expected payoff from tendering would be strictly below $\omega$.

**Step 5:** Take $\bar{b} := \min\{\mathbb{E}[\omega_1], b_1, b_1^2\}$ and the claim follows.

□

### References


