Online Appendix for "Reputation, Bailouts, and Interest Rate Spread Dynamics"

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A Omitted Proofs

A.1 Proof of Proposition 1

We prove that a monotone continuous equilibrium exists by considering the limit of a sequence of finite horizon equilibria. We show that this limit exists and that the equilibrium objects associated with it are continuous and increasing in the government's reputation.

Define $Q(\sigma,\pi) = qP_H + q(1-\pi)\sum_s p_s h(\theta_L|s)\,\sigma(\pi,s)$,

$$\begin{aligned}
\omega^*(Q,s) &= \lambda h(\theta_H|s)\left[\theta_H k(Q)^\alpha - b(Q)\right] + (1-\lambda)\left[h(\theta_H|s)\,b(Q) - k(Q)\right] \\
\omega(Q,s) &= \lambda h(\theta_H|s)\left[\theta_H k(Q)^\alpha - b(Q)\right] + (1-\lambda)\left[h(\theta_H|s)\,b(Q) - \psi h(\theta_L|s)\,b(Q) - k(Q)\right] \\
\Delta\omega(Q,s) &= \psi h(\theta_L|s)\,b(Q)
\end{aligned}$$

with $k(Q) = (\alpha\theta_H Q)^{1/(1-\alpha)}$, $b(Q) = (\alpha\theta_H)^{1/(1-\alpha)}\,Q^{\alpha/(1-\alpha)}$. Let

$$X = \{\Delta W : [0,1] \to \mathbb{R} \text{ and } \Delta W(\cdot) \text{ is increasing and continuous and } \Delta W(p_{nc}) = 0\}$$

and $f : X \to \Sigma$ as $f(\Delta W) = \sigma$ to be the *smallest*[1] solution to the functional equation

$$\sigma(\pi,s) = \begin{cases} 0 & \text{if } \beta\Delta W(p_{nc} + \pi\Delta p) \geqslant \Delta\omega(Q(\sigma,\pi),s) \\ \tilde{\sigma} & : \beta\Delta W\left(p_{nc} + \frac{\pi\Delta p}{\pi + (1-\pi)(1-\tilde{\sigma})}\right) = \Delta\omega(Q(\sigma,\pi),s) \\ 1 & \text{if } \beta\Delta W(p_c) < \Delta\omega(Q(\sigma,\pi),s) \end{cases} \tag{1}$$

Define the operator $\mathbb{T}_\Delta : X \to X$ as

$$\Delta W'(\pi) = \mathbb{T}_\Delta(\Delta W)(\pi) = W'(\pi) - W'(p_{nc})$$

where

$$\begin{aligned}
W'(\pi) = &\sum_s p_s \sigma(\pi,s)\,\omega^*(Q(\sigma,\pi),s) \\
&+ \sum_s p_s\left[1-\sigma(\pi,s)\right]\left[\omega(Q(\sigma,\pi),s) + \beta\Delta W\left(p_{nc} + \frac{\pi\Delta p}{\pi + (1-\pi)(1-\sigma(\pi,s))}\right)\right].
\end{aligned}$$

and $\sigma = f(\Delta W)$. Note that if $\Delta W = \mathbb{T}_\Delta \Delta W$ then $\sigma = f(\Delta W)$ is an equilibrium for the

[1] By doing so we are selecting the best equilibrium. We could alternatively look for the worst by choosing the largest solution.

model. In fact, if we have a fixed point then we can compute

$$W\left(p_{nc}\right)=\sum_{s} p_{s}\sigma\left(p_{nc},s\right)\frac{\omega^{*}\left(Q\left(\sigma,p_{nc}\right),s\right)}{\left(1-\beta\right)}$$
$$+\sum_{s} p_{s}\left(1-\sigma\left(p_{nc},s\right)\right)\frac{\left[\omega\left(Q\left(\sigma,p_{nc}\right),s\right)+\beta\Delta W\left(p_{nc}+\frac{\pi\Delta p}{p_{nc}+\left(1-p_{nc}\right)\left(1-\sigma\left(p_{nc},s\right)\right)}\right)\right]}{1-\beta}$$

and

$$W\left(\pi\right)=W\left(p_{nc}\right)+\Delta W\left(\pi\right)$$

and so all equilibrium conditions are satisfied.

We first establish a preliminary result that characterizes $f\left(\Delta W\right)$:

Claim. Given $\Delta W\in X$, $\sigma\left(\pi,s\right)=f\left(\Delta W\right)$ is decreasing in π. Moreover, if $\Delta W_{H}\geqslant\Delta W_{L}$ then $f\left(\Delta W_{H}\right)\leqslant f\left(\Delta W_{L}\right)$.

Proof. Note that the smallest solution of (1) is the smallest fixed point of the following operator: $\mathbb{T}_{\sigma}:\Sigma\to\Sigma$ defined as

$$\mathbb{T}_{\sigma}\sigma\left(\pi,s\right)=\begin{cases}0 & \text{if } \beta\Delta W\left(p_{nc}+\pi\Delta p\right)\geqslant\Delta\omega\left(Q\left(\sigma,\pi\right),s\right)\\ \tilde{\sigma} & :\beta\Delta W\left(p_{nc}+\frac{\pi\Delta p}{\pi+\left(1-\pi\right)\left(1-\tilde{\sigma}\right)}\right)=\Delta\omega\left(Q\left(\sigma,\pi\right),s\right)\\ 1 & \text{if } \beta\Delta W\left(p_{c}\right)<\Delta\omega\left(Q\left(\sigma,\pi\right),s\right)\end{cases}$$

where $\Sigma=\{\sigma:[0,1]\times S\to[0,1]\}$. First we show that for any $\sigma\in\Sigma$ and $\Delta W\in X$, $\sigma'\left(\pi,s\right)=\left(\mathbb{T}_{\sigma}\sigma\right)\left(\pi,s\right)$ is decreasing in π. Suppose by way of contradiction that there exists $\pi_{L}<\pi_{H}$ and $\sigma'\left(\pi_{L},s\right)<\sigma'\left(\pi_{H},s\right)$ for some s so that the bailout probability is larger if we start from a higher prior.

Suppose first that $\sigma'\left(\pi_{L},s\right)=0$ then

$$\Delta\omega\left(Q\left(\sigma,\pi_{L}\right),s\right)\leqslant\beta\left[W\left(p_{nc}+\pi_{L}\Delta p\right)-W_{1}\left(p_{nc}\right)\right].$$

We also have

$$\Delta\omega\left(Q\left(\sigma,\pi_{H}\right),s\right)<\Delta\omega\left(Q\left(\sigma,\pi_{L}\right),s\right)$$

and

$$\beta\left[W\left(p_{nc}+\pi_{L}\Delta p\right)-W\left(p_{nc}\right)\right]\leqslant\beta\left[W\left(p_{nc}+\pi_{H}\Delta p\right)-W\left(p_{nc}\right)\right]$$

where the first inequality follows from $Q\left(\sigma,\pi_{H}\right)<Q\left(\sigma,\pi_{L}\right)$ and the fact that $B\left(Q\right)$ is

increasing, and the second inequality from $W(\pi)$ being an increasing function. Therefore,

$$\Delta\omega\left(Q\left(\sigma,\pi_H\right),s\right) \leqslant \beta\left[W\left(p_{nc}+\pi_H\Delta p\right)-W\left(p_{nc}\right)\right]$$

and $\sigma'\left(\pi_H,s\right)=0$, yielding a contradiction.

Next, suppose that $0<\sigma'\left(\pi_L,s\right)<\sigma'\left(\pi_H,s\right)<1$. Then,

$$\beta\left[W\left(p_{nc}+\frac{\pi\Delta p}{\pi+(1-\pi)\left(1-\sigma'\left(\pi_H,s\right)\right)}\right)-W\left(p_{nc}\right)\right]=\Delta\omega\left(Q\left(\sigma,\pi_H\right),s\right),$$
$$\beta\left[W\left(p_{nc}+\frac{\pi\Delta p}{\pi+(1-\pi)\left(1-\sigma'\left(\pi_L,s\right)\right)}\right)-W\left(p_{nc}\right)\right]=\Delta\omega\left(Q\left(\sigma,\pi_L\right),s\right).$$

Therefore, since $\Delta\omega\left(Q\left(\sigma,\pi_H\right),s\right)\leqslant\Delta\omega\left(Q\left(\sigma,\pi_L\right),s\right)$ and W is increasing, it must be that

$$\begin{aligned}
p_{nc}+\frac{\pi_L\Delta p}{\pi_L+\left(1-\pi_L\right)\left(1-\sigma'\left(\pi_L,s\right)\right)} &\geqslant p_{nc}+\frac{\pi_H\Delta p}{\pi_H+\left(1-\pi_H\right)\left(1-\sigma'\left(\pi_H,s\right)\right)}\\
\iff \frac{\left(1-\pi_H\right)}{\pi_H}\left(1-\sigma'\left(\pi_H,s\right)\right) &\geqslant \frac{\left(1-\pi_L\right)}{\pi_L}\left(1-\sigma'\left(\pi_L,s\right)\right)\\
\iff 1-\sigma'\left(\pi_H,s\right) &\geqslant \frac{\left(1-\pi_L\right)}{\pi_L}\Big/\frac{\left(1-\pi_H\right)}{\pi_H}\left(1-\sigma'\left(\pi_L,s\right)\right)>1-\sigma'\left(\pi_L,s\right)\\
\iff \sigma'\left(\pi_L,s\right) &> \sigma'\left(\pi_H,s\right)
\end{aligned}$$

obtaining a contradiction.

Finally, if $0<\sigma'\left(\pi_L,s\right)<\sigma'\left(\pi_H,s\right)=1$ then

$$\beta\left[W\left(p_c\right)-W\left(p_{nc}\right)\right]<\Delta\omega\left(Q\left(\sigma,\pi_H\right),s\right)<\Delta\omega\left(Q\left(\sigma,\pi_L\right),s\right)$$

implying $\sigma'\left(\pi_L,s\right)=1$, which is also a contradiction. Thus $\sigma'(\pi,s)=\left(\mathbb{T}_\sigma\sigma\right)(\pi,s)$ is decreasing in π.

Next, we show that $\mathbb{T}_\sigma$ is monotone in σ. That is, if $\sigma_H\geqslant\sigma_L$ then $\sigma'_H\geqslant\sigma'_L$. Suppose by way of contradiction that for some π and s, $\sigma'_H(\pi,s)<\sigma'_L(\pi,s)$. Then, it must be that $\sigma'_L(\pi,s)>0$ so

$$\beta\Delta W\left(p_{nc}+\frac{\pi\Delta p}{\pi+(1-\pi)\left(1-\sigma'_L(\pi,s)\right)}\right)\geqslant\Delta\omega\left(Q\left(\sigma_L,\pi\right),s\right).$$

Since $\sigma_H\geqslant\sigma_L$ then $Q\left(\sigma_H,\pi\right)\geqslant Q\left(\sigma_L,\pi\right)$ and

$$\Delta\omega\left(Q\left(\sigma_H,\pi\right),s\right)\geqslant\Delta\omega\left(Q\left(\sigma_L,\pi\right),s\right).$$

Thus,

$$\beta \Delta W\left(p_{nc}+\frac{\pi \Delta p}{\pi+(1-\pi)\left(1-\sigma_H'(\pi,s)\right)}\right)>\beta \Delta W\left(p_{nc}+\frac{\pi \Delta p}{\pi+(1-\pi)\left(1-\sigma_L'(\pi,s)\right)}\right)$$
$$\geqslant \Delta\omega\left(Q\left(\sigma_L,\pi\right),s\right)$$
$$\geqslant \Delta\omega\left(Q\left(\sigma_H,\pi\right),s\right)$$

which implies that $\sigma_H'(\pi,s)=1$, a contradiction. Therefore the operator $\mathbb{T}_\sigma$ is monotone and we can find f as

$$f(\Delta W)=\lim_{n\to\infty}\mathbb{T}_\sigma^n\mathbf{0} \quad \mathbf{0}(\pi,s)=0 \quad \forall(\pi,s)$$

where $\mathbf{0}$ is some initial feasible value.

Finally, we show that if $\Delta W_H \geqslant \Delta W_L$ then $f(\Delta W_H)\leqslant f(\Delta W_L)$. We know that

$$f(\Delta W_i)=\lim_{n\to\infty}\mathbb{T}_\sigma^n(\mathbf{0};\Delta W_i)$$

so it suffices to show that for all n

$$\sigma_H^n(\pi,s)=\mathbb{T}_\sigma^n(\mathbf{0};\Delta W_H)\leqslant\mathbb{T}_\sigma^n(\mathbf{0};\Delta W_L)=\sigma_H^n(\pi,s)$$

which is true since $\mathbb{T}_\sigma$ is monotone increasing in σ and monotone decreasing in ΔW. This must also be true in the limit so that $f(\Delta W_H)\leqslant f(\Delta W_L)$. □

We now show that $\mathbb{T}_\Delta \Delta W \in X$. Notice that the definition of $\mathbb{T}_\Delta$ implies that $\mathbb{T}_\Delta(\Delta W)(p_{nc})=0$. Thus, what is left is to show that the operator $\mathbb{T}_\Delta$ maps increasing functions into increasing functions.

Claim. Suppose that $\Delta W(\pi)$ is an increasing function. Then $\Delta W'(\pi)=\mathbb{T}_\Delta(\Delta W)(\pi)$ is also an increasing function.

Proof. Let $\sigma=f(\Delta W)$. Consider $\pi_L<\pi_H$ and define $\mathcal{S}_1=\{s:\sigma(\pi_L,s)=\sigma(\pi_H,s)=0\}$, $\mathcal{S}_2=\{s:\sigma(\pi_L,s)>\sigma(\pi_H,s)=0\}$, and $\mathcal{S}_3=\{s:\sigma(\pi_L,s)\geqslant\sigma'(\pi_H,s)>0\}$ so that in $\mathcal{S}_1$ there are no bailouts under both π_L and π_H, in $\mathcal{S}_2$ there is a positive probability of bailouts under π_L but not under π_H, and in $\mathcal{S}_3$ bailouts happen with positive probability under both π_L and π_H. Then

$$W'(\pi_L) = \sum_{s\in\mathcal{S}_1} p_s \left[\omega\left(Q\left(\pi_L,\sigma\right),s\right) + \beta\Delta W\left(p_{nc}+\pi_L\Delta p\right)\right] + \sum_{s\in\mathcal{S}_2} p_s \left[\omega^*\left(Q\left(\pi_L,\sigma\right),s\right) + \beta\Delta W\left(p_{nc}\right)\right] + \sum_{s\in\mathcal{S}_3} p_s \left[q\omega^*\left(Q\left(\pi_L,\sigma\right),s\right) + \beta\Delta W\left(p_{nc}\right)\right]$$

and

$$W'(\pi_H) = \sum_{s\in\mathcal{S}_1} p_s \left[\omega\left(Q\left(\pi_H,\sigma\right),s\right) + \beta\Delta W\left(p_{nc}+\pi_H\Delta p\right)\right] + \sum_{s\in\mathcal{S}_2} p_s \left[\omega\left(Q\left(\pi_H,\sigma\right),s\right) + \beta\Delta W\left(p_{nc}+\pi_H\Delta p\right)\right] + \sum_{s\in\mathcal{S}_3} p_s \left[q\omega^*\left(Q\left(\pi_H,\sigma\right),s\right) + \beta\Delta W\left(p_{nc}\right)\right].$$

Since $Q(\pi_H,\sigma) < Q(\pi_H,\sigma)$ and $\omega(Q,s)$ is decreasing in Q we have that for all s

$$\omega\left(Q\left(\pi_H,\sigma\right),s\right) - \omega\left(Q\left(\pi_L,\sigma\right),s\right) > 0,$$
$$\omega^*\left(Q\left(\pi_H,\sigma\right),s\right) - \omega^*\left(Q\left(\pi_L,\sigma\right),s\right) > 0.$$

Moreover, since ΔW is increasing we have

$$W\left(p_{nc}+\pi_H\Delta p\right) \geqslant W\left(p_{nc}+\pi_L\Delta p\right).$$

Thus, for all $s \in \mathcal{S}_1 \cup \mathcal{S}_3$ we have that the value at π_H is higher than at π_L. Finally, for $s \in \mathcal{S}_2$ we have that

$$\omega\left(Q\left(\pi_H,\sigma\right),s\right) + \beta\Delta W\left(p_{nc}+\pi_H\Delta p\right) \geqslant \omega^*\left(Q\left(\pi_H,\sigma\right),s\right) + \beta\Delta W\left(p_{nc}\right) \geqslant \omega^*\left(Q\left(\pi_L,\sigma\right),s\right) + \beta\Delta W\left(p_{nc}\right)$$

and so it follows that

$$W'(\pi_H) - W'(\pi_L) > 0 \Rightarrow \Delta W'(\pi_H) - \Delta W'(\pi_L) > 0.$$

□

Finally, consider the sequence $\{\Delta W_n\}_{n=0}^{\infty}$, where $\Delta W_0(\pi) = 0$ and $\Delta W_{n+1} = \mathbb{T}_\Delta \Delta W_n$ for all $n > 0$. We show that this sequence is monotone increasing.

Claim. If p_{nc} is sufficiently close to zero then $\Delta W_{n+1}(\pi) \geqslant \Delta W_n(\pi)$ for all π and n.

Proof. We use an induction argument to prove the result. Consider first the initial iteration. Since $\Delta W_0(\pi) = 0$ for all π, $\sigma_1 = f(\Delta W_0) = 1$ for all π and $s \neq s_H$. Thus, $Q_1(\pi) = qP_H + q(1-\pi)P_L$ and

$$W_1(\pi) = \sum_{s \neq s_H} p_s \omega^*(Q_1(\pi), s) + p_H \omega(Q_1(\pi), s_H)$$

which is strictly increasing in π. Thus, for all $\pi > p_{nc}$

$$\Delta W_1(\pi) = W_1(\pi) - W_1(p_{nc}) > 0 = \Delta W_0(\pi).$$

Moreover, we have that for all $\pi > p_{nc}$:

$$\Delta W_1(p_{nc} + \pi\Delta p) - \Delta W_1(p_{nc}(1+\Delta p)) \geqslant \Delta W_0(p_{nc} + \pi\Delta p) - \Delta W_0(p_{nc}(1+\Delta p)).$$

Consider now an arbitrary interation n:

$$\begin{aligned}W_{n+1}(\pi) = p_H &[\omega(Q_{n+1}(\pi), s_H) + \beta\Delta W_n(p_{nc} + \pi\Delta p)] \\ &+ \sum_{s \neq s_H} p_s \sigma_{n+1}(\pi)\, \omega^*(Q_{k+1}(\pi), s) \\ &+ \sum_{s \neq s_H} p_s (1 - \sigma_{k+1}(\pi)) \left[\omega(Q_{k+1}(\pi), s) + \beta\Delta W_k\left(p_{nc} + \frac{\pi\Delta p}{\pi + (1-\pi)(1-\sigma_{k+1}(\pi, s))}\right)\right].\end{aligned}$$

Assume that the sequence up to n satisfies $\Delta W_n \geqslant \Delta W_{n-1}$ and

$$\begin{aligned}&\Delta W_n(p_{nc} + \pi\Delta p) - \Delta W_n(p_{nc}(1+\Delta p)) \\ \geqslant &\Delta W_{n-1}(p_{nc} + \pi\Delta p) - \Delta W_{n-1}(p_{nc}(1+\Delta p)).\end{aligned} \tag{2}$$

For $\pi > p_{nc}$, we have that

$$W_{n+1}(\pi) - W_n(\pi) > p_H\beta\left[\Delta W_n(p_{nc} + \pi\Delta p) - \Delta W_{n-1}(p_{nc} + \pi\Delta p)\right] \tag{3}$$

because $Q_{n+1} \leqslant Q_n$, ω and ω^* are decreasing in Q, $\sigma_{n+1} \leqslant \sigma_n$, and $\Delta W_n(\pi) > \Delta W_{n-1}(\pi)$ for all $\pi > p_{nc}$ by the induction hypothesis.

If p_{nc} is sufficiently close to 0, then $\sigma_{n+1} = f(\Delta W_n)$ is such that $\sigma_{n+1}(p_{nc}, s) = 1$ for all for $s \neq s_H$ and so $Q_{n+1}(p_{nc}) = \bar{Q}(p_{nc}) = P_H + (1-\pi)P_L$ and thus does not depend on n. Therefore, for p_{nc} close to zero

$$W_{n+1}(p_{nc}) = \sum_{s\in S} p_s \omega^* \left(\bar{Q}(p_{nc}), s\right) + \beta p_H \Delta W_n (p_{nc}(1+\Delta p)).$$

and so,

$$\begin{aligned}\Delta W_{n+1}(\pi) - \Delta W_n(\pi) &= [W_{n+1}(\pi) - W_n(\pi)] - [W_{n+1}(p_{nc}) - W_n(p_{nc})] \\ &> \beta p_H [\Delta W_n (p_{nc} + \pi\Delta p) - \Delta W_{n-1}(p_{nc} + \pi \Delta p)] \\ &- \beta p_H [\Delta W_n (p_{nc}(1+\Delta p)) - \Delta W_{n-1}(p_{nc}(1+\Delta p))] \\ &\geqslant 0\end{aligned}$$

where the first inequality follows from (3) and p_{nc} being close to zero, and the second inequality follows from (2). Finally, since p_{nc} is close to zero, $\Delta W_{n+1}(p_{nc}(1+\Delta))$ is close to zero and so condition (2) holds for interation $n+1$, completing the induction argument. □

Thus, the sequence $\{\Delta W_n\}_{n=0}^{\infty} \subset X$ is increasing. Moreover, it is bounded because $\Delta W_n \leqslant \left[W^R(0) - W^R(1)\right]$. Thus, the constructed sequence converges, $\Delta W_n \to \Delta W \in X$, and the limit is an equilibrium as $\Delta W = \mathbb{T}_\Delta W$. Since $\Delta W \in X$ we have that ΔW is increasing in π. Since we showed that $\sigma = f(\Delta W)$ is decreasing in π for all $\Delta W \in X$, the equilibrium bailout probability on path, σ, is decreasing in π. Finally, $\sigma(\pi, s_L) \geqslant \sigma(\pi, s_M) \geqslant \sigma(\pi, s_H)$ follows from the fact that the static costs of not bailing out are are increasing in s. Q.E.D.

A.2 Proof of Proposition 2

We first show that under condition (21) in Assumption 1 we have $\sigma(\pi, s_L) = 1$ for all π. To this end, note that in any equilibrium $B(\pi) = \mathbf{b}(\bar{\gamma}(\pi)) \geqslant \mathbf{b}(0)$. Moreover, note that the dynamic gains from bailing out, $W(p_c) - W(p_{nc})$, are bounded by $W^R(0) - W^R(1)$, i.e.,

$$W(p_c) - W(p_{nc}) \leqslant W^R(0) - W^R(1).$$

This is because $W^R(0) = W^R \geqslant W(p_c)$ since the value of the Markov equilibrium is lower than the value of the Ramsey plan, and $W(p_{nc}) \geqslant W^R(1)$ because along the equilibrium path private agents believe that with some probability they are facing the commitment type. Hence we have that

$$\psi B(\pi) \geqslant \psi \mathbf{b}(0) > \beta \left[W^R(0) - W^R(1)\right] \geqslant \beta [W(p_c) - W(p_{nc})]$$

and so it is optimal to bail out with probability one if $s = s_L$.

Next, we show that for some π it is optimal to mix in a mild crisis under assumption (22) in the text. Since we know that $\sigma(\pi, s_L) = 1$ for all π then

$$\bar{\gamma}(\pi) = \frac{p_L(1-\pi)\sigma(\pi, s_L) + p_M\mu(1-\pi)\sigma(\pi, s_M)}{P_L} \in \left[(1-\pi)\frac{p_L}{P_L}, (1-\pi)\right]$$

so

$$B(\pi) \in \left[\mathbf{b}(1-\pi), \mathbf{b}\left((1-\pi)\frac{p_L}{P_L}\right)\right].$$

First, suppose by way of contradiction that $\sigma(\pi, s_M) = 0$ for all π. Then it must be that

$$0 < \psi\mu B(\pi) \leqslant \beta\left[W(p_{nc} + \pi\Delta p) - W(p_{nc})\right]$$

but as $p_{nc} \to 0$, for $\pi = p_{nc} = 0$ we have

$$0 < \psi\mu B(p_{nc}) \leqslant \beta\left[W(p_{nc}) - W(p_{nc})\right] = 0$$

which is a contradiction. Thus for π low enough we have $\sigma(\pi, s_M) > 0$.

We now show that it is not optimal to bail out for sure in state s_M. Suppose by way of contradiction that $\sigma(\pi, s_M) = 1$ for all π. Thus, we have that $\bar{\gamma} = 1$ so it must be that for all π

$$\psi\mu\mathbf{b}(1-\pi) \leqslant \beta\left[W(p_c) - W(p_{nc})\right].$$

Under the contradiction hypothesis, $\sigma(\pi, s_i) = 1$ for all π and $s_i = s_M, s_L$ so the continuation value for the optimizing type is

$$\begin{aligned} W(\pi) = {} & (1-2\lambda)\left[qP_H\mathbf{b}(1-\pi) - \mathbf{k}(1-\pi)\right] + \lambda\left[qP_H\mathbf{k}(1-\pi)^\alpha - \mathbf{k}(1-\pi)\right] \\ & + \beta P_H W(p_{nc} + \pi\Delta p) + \beta P_L W(p_{nc}) \end{aligned}$$

which evaluated at $\pi = p_c$ and p_{nc} reduces to

$$\begin{aligned} W(p_{nc}) &= (1-2\lambda)\left[qP_H\mathbf{b}(1-p_{nc}) - \mathbf{k}(1-p_{nc})\right] + \lambda\left[qP_H\mathbf{k}(1-p_{nc})^\alpha - \mathbf{k}(1-p_{nc})\right] \\ &\quad + \beta P_H W(p_{nc} + p_{nc}\Delta p) + \beta P_L W(p_{nc}) \\ W(p_c) &= (1-2\lambda)\left[qP_H\mathbf{b}(1-p_c) - \mathbf{k}(1-p_c)\right] + \lambda\left[qP_H\mathbf{k}(1-p_c)^\alpha - \mathbf{k}(1-p_c)\right] \\ &\quad + \beta P_H W(p_{nc} + p_c\Delta p) + \beta P_L W(p_{nc}) \end{aligned}$$

and as $p_{nc} \to 0$ and $p_c \to 1$ we have

$$\begin{aligned}
W(p_{nc}) &= (1-2\lambda)\left[qP_H\mathbf{b}(1) - \mathbf{k}(1)\right] + \lambda\left[qP_H\mathbf{k}(1)^{\alpha} - \mathbf{k}(1)\right] \\
&\quad + \beta P_H W(p_{nc}) + \beta P_L W(p_{nc}) \\
W(p_c) &= (1-2\lambda)\left[qP_H\mathbf{b}(0) - \mathbf{k}(0)\right] + \lambda\left[qP_H\mathbf{k}(0)^{\alpha} - \mathbf{k}(0)\right] \\
&\quad + \beta P_H W(p_c) + \beta P_L W(p_{nc}).
\end{aligned}$$

Thus, using $[qP_H\mathbf{b}(0) - \mathbf{k}(0)] = 0$ and subtracting the two expressions above we obtain

$$\begin{aligned}
\Delta W(p_c) &= \frac{(1-2\lambda)}{1-\beta P_H}\left\{\mathbf{k}(1) - qP_H\mathbf{b}(1)\right\} \\
&\quad + \frac{\lambda}{1-\beta P_H}\left\{\left[qP_H\mathbf{k}(0)^{\alpha} - \mathbf{k}(0)\right] - \left[qP_H\mathbf{k}(1)^{\alpha} - \mathbf{k}(1)\right]\right\}.
\end{aligned}$$

Since for all $\lambda \in [0, 1/2]$ we have that $(1-2\lambda) + \lambda = 1 - \lambda \leqslant 1$, condition (22) in the text ensures that

$$\begin{aligned}
\beta\frac{\mathbf{k}(1) - qP_H\mathbf{b}(1)}{1-\beta P_H} &> \psi\mu\mathbf{b}(1), \\
\beta\frac{\left\{\left[qP_H\mathbf{k}(0)^{\alpha} - \mathbf{k}(0)\right] - \left[qP_H\mathbf{k}(1)^{\alpha} - \mathbf{k}(1)\right]\right\}}{1-\beta P_H} &> \psi\mu\mathbf{b}(1),
\end{aligned}$$

which in turn imply that

$$\beta\Delta W(p_c) > \psi\mu(1)$$

so that it is not optimal to have $\sigma(\pi, s_M) = 1$ because the static costs of not bailing out are smaller than the dynamic benefits. This is a contradiction. Q.E.D.

A.3 Proof of Proposition 4

To prove that the conjectured equilibrium with $\sigma_1 = 1$ exists, we need to show that (26), (27), and (28) in the text hold at $\pi = p_c = 1$ and $\sigma_1 = 1$. From (25) in the text, a simple computation gives us the transitory type's continuation value:

$$U_{2T}(\theta_1, \pi) = P_H\left(q\left(P_H + (1-\pi)P_L\right)\right)^{\alpha/(1-\alpha)}\theta_H^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}(1-\alpha). \tag{4}$$

Recall that under the conjectured equilibrium, the transitory types do not receive a bailout but repay in both states.Their problem in period 1 is

$$\max_k P_H\theta_H k^{\alpha} - k/Q$$

with $Q = q$. Thus,

$$k_{1T} = (\alpha P_H \theta_H q)^{1/(1-\alpha)}$$

and

$$B_{1T} = (\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} < B_{1P}. \tag{5}$$

Using (4) and (5), conditions (26), (27), and (28) in the text can be written as (recall that $\pi = 1$)

$$- (\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} + \beta P_H q^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} (1-\alpha) > 0,$$

$$- (\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} + \beta P_H (q P_H)^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} (1-\alpha) < 0,$$

$$\beta [W_2(1) - W_2(0)] < \psi [\mu B_{1P} + (1-\mu) B_{1T}].$$

Using the expression for $W_2(\pi)$ we have

$$\begin{aligned} W_2(1) - W_2(0) &= 0 - (\alpha \theta_H)^{\frac{1}{1-\alpha}} Q(0)^{\frac{\alpha}{1-\alpha}} [q P_H - Q(0)] \\ &= (\alpha \theta_H q)^{\frac{1}{1-\alpha}} P_L. \end{aligned}$$

Moreover, since $B_{1P} > B_{1T}$, to show that (26), (27), and (28) in the text are satisfied it is sufficient to show that

$$- (\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} + \beta P_H q^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} (1-\alpha) > 0,$$

$$- (\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} + \beta P_H (q P_H)^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} (1-\alpha) < 0,$$

$$\beta [W(1) - W(0)] < \psi (\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)}.$$

It is straightforward to verify that these three inequalities are satisfied under Assumption 2. Q.E.D.

A.4 Proof of Proposition 5

We now want to show that

$$-b_1 + \beta U_2(p_{nc}) \geqslant 0.$$

Note that

$$b_1 \leqslant b_2(0) = (\alpha \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)}.$$

Thus it is sufficient to show that

$$- (\alpha \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} + \beta P_H q^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \left[\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)} \right] \geqslant 0$$

or

$$\theta_H^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} \left\{ \beta P_H \left[\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)} \right] - \alpha^{1/(1-\alpha)} \right\} \geqslant 0$$

or

$$\beta P_H - (1 + \beta P_H)\,\alpha \geqslant 0.$$

But this follows directly from (29) in the text. Q.E.D.

B Extensions

B.1 Model with N borrowers

Consider an environment with N borrowers. Let $\mathbf{B} = (b_1, ..., b_N)$, $\mathbf{K} = (k_1, ..., k_N)$ and $\Theta = (\theta_1, ..., \theta_N)$. We show that as $N \to \infty$, the equilibrium outcome is the one in the main text.

Consider first the incentives for the government in sub-period 2. If the government bails out, its static value is

$$\omega^* (\mathbf{B}, \mathbf{K}, \Theta) = \max_{T_i} \lambda \left[\sum_i \frac{1}{N} \left[\max \{ \theta_i k_i^\alpha + T_i - b_i ; 0 \} \right] \right] + \\ (1-\lambda) \left[\sum_i \frac{1}{N} b_i \mathbb{I}_{\{\theta_i k_i^\alpha + T_i - b_i \geqslant 0\}} - \sum_i \frac{1}{N} T_i \right].$$

The value with no transfers is

$$\omega (\mathbf{B}, \mathbf{K}, \Theta) = \lambda \left[\sum_i \frac{1}{N} \left[\max \{ \theta_i k_i^\alpha - b_i ; 0 \} \right] \right] + (1-\lambda) \sum_{i:\theta_i = \theta_H} \frac{1}{N} b_i - \psi \sum_{i:\theta_i = 0} \frac{1}{N} b_i.$$

Notice that since N is finite and $\lambda \leqslant 1/2$, the optimal transfers will satisfy $T \in \{\mathbf{0}, T^*\}$ since choosing any other level only imposes costs on the government. Thus, we can summarize the government's decision is sub-period 2 by the bailout policy $\sigma (\pi, \mathbf{B}, \mathbf{K}, \Theta)$. The static benefits of bailing out are

$$\Delta\omega (\mathbf{B}, \mathbf{K}, \Theta) = \psi \sum_{i:\theta_i = 0} \frac{1}{N} b_i. \tag{6}$$

Consider now the problem for a borrower. This problem is identical to the one in the main text, except that now they internalize the effect of their choices on the government's equilibrium bailout policy $\sigma (\pi, \mathbf{B}, \mathbf{K}, \Theta)$. The first order condition that characterizes debt issuance is

$$\alpha\theta_H (Q_i b_i)^{\alpha - 1} \left(Q_i + Q_{b_i} b_i \right) - 1 = 0 \tag{7}$$

where $Q_i = Q_i(\mathbf{B}, \mathbf{K}, \pi, \sigma)$ is the pricing schedule for borrower i which can be written as

$$Q_i(\mathbf{B}, \mathbf{K}, \pi, \sigma) = qP_H + q(1-\pi)\sum_s p_s \sum_\Theta \Pr(\Theta|s)\,\sigma(\pi, \mathbf{B}, \mathbf{K}, \Theta)$$

where

$$\Pr(\Theta|s) = \prod_{i=1}^{N} h(\theta_i|s)$$

and

$$Q_{bi} = \frac{\partial Q_i}{\partial b_i} = q(1-\pi)\sum_s p_s \sum_\Theta \Pr(\Theta|s)\,\frac{\partial \sigma(\pi, \mathbf{B}, \mathbf{K}, \Theta)}{\partial b_i}.$$

This is identical to the characterization with a continuum of borrowers if $Q_{bi} = 0$. We next show this is the case in the limit as $N \to \infty$.

First note that the dynamic benefits are independent of b_i and so borrowers can only affect the bailout decision by affecting the static benefits of bailing out. Differentiating (6) we obtain

$$\frac{\partial}{\partial b_i}\Delta\omega(\mathbf{B}, \mathbf{K}, \Theta) = \begin{cases} \frac{\psi}{N} & \text{if } \theta_i = 0 \\ 0 & \text{if } \theta_i = \theta_H \end{cases}$$

which converges to 0 as $N \to \infty$. Therefore Q_{bi} converges to zero and in the limit the first order condition for the borrower is $\alpha\theta_H (Q_i b_i)^{\alpha-1} Q_i - 1 = 0$ so

$$b_i(\pi) = (\alpha\theta_H)^{\frac{1}{1-\alpha}} Q_i(\pi)^{\frac{\alpha}{1-\alpha}}, \quad k_i(\pi) = (\alpha\theta_H)^{\frac{1}{1-\alpha}} Q_i(\pi)^{\frac{1}{1-\alpha}}.$$

Using the law of large numbers we have,

$$Q_i(\pi) = qP_H + q(1-\pi)\sum_s p_s \sigma(\pi, \mathbf{B}, \mathbf{K}, \Theta(s))$$

where $\Theta(s)$ is a sequence $\Theta = \{\theta_n\}_{n=1}^{\infty}$ with a share $h(\theta_L|s)$ of borrowers with realizations equal to θ_L and a share $h(\theta_H|s)$ of borrowers with realizations equal to θ_H. The expression above is the same as the one in Lemma 2 for the case with a continuum of borrowers. Since the static benefits in (6) are the same as the ones in (9) in the text then the limits of W and σ are also equal to that in the continuum case. To see why, notice that we can just apply the same argument as in Proposition 1.

B.2 Ramsey Problem

Here we show that if α is sufficiently high and ψ is sufficiently low, then a government with commitment chooses not to bail out. Thus, bailouts are not optimal ex-ante but are only optimal ex-post.

The objective of the government with commitment is to maximize

$$W^c \equiv (1-\lambda)\left[e-K+qP_HB\right]+\lambda\left[qP_H\left(\theta_HK^\alpha-B\right)\right]-\psi P_L\left(1-\bar{\gamma}\right)B \tag{8}$$

where the first term is the consumption of taxpayers and lenders, the second term is the borrowers' consumption, and the last term is the social default cost. Note that we can rewrite the first two terms of (8) as

$$\begin{aligned}
&(1-\lambda)\left[e-K+qP_HB\right]+\lambda\left[qP_H\left(\theta_HK^\alpha-B\right)\right]-\lambda K+\lambda K\\
&=(1-\lambda)\,e+(1-\lambda)\left(-K+qP_HB\right)+\lambda\left[qP_H\theta_HK^\alpha-K\right]+\lambda\left(K-qP_HB\right)\\
&=(1-\lambda)\,e+(1-2\lambda)\left(-K+qP_HB\right)+\lambda\left[qP_H\theta_HK^\alpha-K\right].
\end{aligned}$$

Thus, the Ramsey problem is

$$\max_{B,K,\bar{\gamma}}(1-\lambda)\,e-(1-2\lambda)\left(K-qP_HB\right)+\lambda\left[qP_H\theta_HK^\alpha-K\right]-\psi P_L\left(1-\bar{\gamma}\right)B$$

subject to

$$\begin{aligned}
B&=\mathbf{b}\left(\bar{\gamma}\right)=\left(\alpha\theta_H\right)^{1/(1-\alpha)}\left[q\left(P_H+P_L\bar{\gamma}\right)\right]^{\alpha/(1-\alpha)}\\
K&=\mathbf{k}\left(\bar{\gamma}\right)=\left(\alpha\theta_Hq\left(P_H+P_L\bar{\gamma}\right)\right)^{1/(1-\alpha)}.
\end{aligned}$$

Differentiating with respect to $\bar{\gamma}$ we have:

$$\begin{aligned}
\frac{\partial W^c}{\partial\bar{\gamma}}=&\,\mathbf{k}'\left(\bar{\gamma}\right)\left[-(1-\lambda)+\lambda\alpha qP_H\theta_H\mathbf{k}\left(\bar{\gamma}\right)^{\alpha-1}\right]\\
&+\mathbf{b}'\left(\bar{\gamma}\right)\left[(1-\lambda)\,qP_H-\lambda-\psi P_L\left(1-\bar{\gamma}\right)\right]+\psi P_L\mathbf{b}\left(\bar{\gamma}\right).
\end{aligned}$$

Note that

$$\begin{aligned}
\frac{\partial W^c}{\partial\bar{\gamma}}&\leqslant\lambda\left[\alpha qP_H\theta_H\mathbf{k}\left(\bar{\gamma}\right)^{\alpha-1}-1\right]\mathbf{k}'\left(\bar{\gamma}\right)-\mathbf{b}'\left(\bar{\gamma}\right)\psi P_L\left(1-\bar{\gamma}\right)+\psi P_L\mathbf{b}\left(\bar{\gamma}\right)\\
&\leqslant-\psi P_L\left[\mathbf{b}'\left(\bar{\gamma}\right)\left(1-\bar{\gamma}\right)-\mathbf{b}\left(\bar{\gamma}\right)\right]
\end{aligned}$$

where the first inequality follows from the fact that the term $(1-2\lambda)\left(K-qP_HB\right)$ is increasing in $\bar{\gamma}$, the second from the fact that $\alpha qP_H\theta_H\mathbf{k}\left(\bar{\gamma}\right)^{\alpha-1}-1<0$ and $\mathbf{k}'\left(\bar{\gamma}\right)>0$. Thus, it suffices to show that $\left[\mathbf{b}'\left(\bar{\gamma}\right)\left(1-\bar{\gamma}\right)-\mathbf{b}\left(\bar{\gamma}\right)\right]>0$ or

$$\frac{\alpha qP_L}{(1-\alpha)}\left(\alpha\theta_H\right)^{1/(1-\alpha)}\left[q\left(P_H+P_L\bar{\gamma}\right)\right]^{\alpha/(1-\alpha)-1}\left(1-\bar{\gamma}\right)-\left(\alpha\theta_H\right)^{1/(1-\alpha)}\left[q\left(P_H+P_L\bar{\gamma}\right)\right]^{\alpha/(1-\alpha)}>0$$

or

$$\frac{\alpha q P_L}{(1-\alpha)} \frac{(1-\bar{\gamma})}{[q(P_H + P_L\bar{\gamma})]} - 1 > 0$$

which is true if α is sufficiently close to 1 and $\bar{\gamma} < 1$. For $\bar{\gamma}$ close to one, we have that

$$\lim_{\bar{\gamma}\to 1} \frac{\partial W^c}{\partial \bar{\gamma}} = \mathbf{k}'(1)\left[-(1-\lambda) + \lambda\alpha q P_H \theta_H \mathbf{k}(1)^{\alpha-1}\right] + \mathbf{b}'(1)\left[(1-\lambda) q P_H - \lambda\right] + \psi P_L \mathbf{b}(1)$$

which is negative if ψ is small enough. Therefore if α is sufficiently close to one and ψ is sufficiently small then the Ramsey problem has $\bar{\gamma} = 0$.

B.3 A Consumption Smoothing Model

In this section we describe a model in which the desire to borrow arises from consumption smoothing motives. All of our results go through in this economy.

All borrowers are one-period lived and symmetric ex-ante. In sub-period 1, borrower i has income $Y_{i1} = Y_1$ and can borrow b_i from risk neutral lenders to finance consumption c_{i1}. In sub-period 2, the aggregate state s is realized according to a distribution P. As in our baseline model, assume that the state can take three values: $s \in \{s_L, s_M, s_H\}$ with probabilities p_L, p_M, and p_H respectively. Each borrower receives stochastic income θ drawn from a distribution $H(\cdot|s)$. We assume that θ can take on two values: θ_H and θ_L. In state s_H, all the borrowers receive the high endowment so $h(\theta_H|s_H) = 1$ and $h(\theta_L|s_H) = 0$. In state s_M instead, $h(\theta_H|s_M) = 1-\mu$ and $h(\theta_L|s_M) = \mu$. Finally, in state s_L all borrowers receive the low endowment, $h(\theta_L|s_L) = 1$ and $h(\theta_H|s_L) = 0$. After the realization of θ, each borrower can default on its debt. The rest of the model is unchanged.

Let $c_{i2}(s,\theta)$ denote the consumption of borrower i in sub-period 2 given (s,θ). The preferences of borrower i are given by

$$u(c_{i1}) + \delta \sum_s p_s \sum_\theta h(\theta \mid s)\, u(c_{i2}(s,\theta)) \tag{9}$$

where $u(\cdot)$ is increasing, concave, and differentiable, and δ is the borrower's discount factor across sub-periods. The budget constraint of the borrower in sub-period 1 is

$$c_{i1} = Y_1 + Q b_i$$

where b_i is the debt issued by the borrower and Q is the price of the debt. In sub-period

2, if the borrower does not default, its budget constraint is

$$c_{i2}(s,\theta) = \theta - b_i + T_i$$

where T_i are transfers from the government. We assume that the private cost of default to the borrower, $\underline{u}(s,\theta)$ is given by

$$\underline{u}(s,\theta) = \begin{cases} u(0) & \text{if } \theta = \theta_H \\ u(\theta_L) & \text{if } \theta = \theta_L \end{cases}$$

which implies that these costs exhibit a high degree of convexity. Thus, it is always optimal for borrowers to repay debt if $\theta = \theta_H$ while if $\theta = \theta_L$ there is repayment only if debt issued is zero or there is a transfer equal to at least b. Therefore, the fraction of borrowers defaulting is given by $\Delta = \sum_\theta h(\theta \mid s) \mathbb{I}_{\{u(\theta - b + T(B,s)(b,\theta)) < \underline{u}(s,\theta)\}}$ and the optimal transfer is $T \in \{\mathbf{0}, T^*\}$ where

$$T^*(B,s)(b,\theta_L) = b.$$

Given this setup, it is easy to see that all the results in the baseline model apply here as well.

B.4 Payoff Types

Here we show that the equilibrium outcome in the main text is also the equilibrium outcome of a policy game with payoff types if the cost of default for the low cost type is sufficiently small.

Suppose there are two types of governments: low and high cost types which are indexed by subscripts L and H respectively. In particular, the social default cost for the high cost type is $\psi_H = \psi$ as in the baseline model (i.e. the one in the main text) and the default cost for the low cost type is ψ_L. Here, π is the probability that the government is the low default cost type L. Let $\sigma_i(\pi, s)$ for $i = L, H$ be the probability of a bailout given prior π in state s and $\sigma(\pi, s)$ denote the equilibrium strategy for the optimizing type in the baseline model.

Proposition. *Under the assumptions in Proposition 1 and 2, there exists $\bar{\psi}_L > 0$ such that for all $\psi_L \leqslant \bar{\psi}_L$*

$$\sigma_H(\pi,s) = \sigma(\pi,s) \text{ and } \sigma_L(\pi,s) = 0, \ \forall \pi, s$$

is an equilibrium.

Proof. Given the conjectured bailout strategies, the levels of debt and capital $B(\pi)$ and $K(\pi)$ are the same as in the case with the commitment type considered in the text. Given

$B(\pi)$, $K(\pi)$, and $\sigma_L(\pi, s) = 0$, the problem of the high cost type is identical to the problem of the optimizing type in the baseline model and thus $\sigma_H(\pi, s) = \sigma(\pi, s)$ is optimal. We are left to check that never bailing out is optimal for the low default cost type. To this end, define the value for the low cost government type of following the conjectured strategy, $W_L(\pi; \psi_L)$, as the unique solution to the following functional equation:

$$W_L(\pi; \psi_L) = w(\pi) + \beta \sum_s p_s W_L\left(p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma(\pi, s))}; \psi_L\right)$$

where the static value given by

$$w(\pi; \psi_L) = (1-\lambda)e + (1-2\lambda)[qP_H B(\pi) - K(\pi)] + \lambda[qP_H\theta_H K(\pi)^\alpha - K(\pi)] - \psi_L q P_L B(\pi).$$

It is easy to see that since $w(\pi; \psi_L)$ is strictly increasing in reputation π, this property is inherited by $W_L(\pi; \psi_L)$. For $\sigma_L(\pi, s) = 0$ to be an equilibrium, it must be that for all $\pi \in [p_{nc}, p_c]$ and $s \in \{s_M, s_L\}$

$$\beta\left[W_L\left(p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma(\pi, s))}; \psi_L\right) - W_L(p_{nc}; \psi_L)\right] \geqslant \psi_L h(\theta_L | s) B(\pi). \quad (10)$$

We now show that the posterior after no bailout, $p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma(\pi, s))}$, is bounded away from p_{nc}. Note that for π close to p_{nc}, we know from the proof of Proposition 2 that $\sigma(\pi, s) > 0$. Let $0 < \underline{\sigma}(s) = \min_{\pi \in [p_{nc}, p_{nc}+\varepsilon]} \sigma(\pi, s)$ for some $\varepsilon > 0$. Thus, we have

$$p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma(\pi, s))} \geqslant \begin{cases} p_{nc} + \frac{p_{nc}(p_c - p_{nc})}{p_{nc} + (1-p_{nc})(1-\underline{\sigma}(s))} > p_{nc} & \text{for } \pi \in [p_{nc}, p_{nc} + \varepsilon] \\ p_{nc} + \varepsilon(p_c - p_{nc}) > p_{nc} & \text{for } \pi > p_{nc} + \varepsilon \end{cases}$$

so we can define

$$\eta \equiv \min_{s \in \{s_M, s_L\}} \min_{\pi \in [p_{nc}, p_c]} \left\{p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma(\pi, s))}\right\} > p_{nc}.$$

Then the dynamic benefits of not bailing out are at least

$$\beta[W_L(\eta; \psi_L) - W_L(p_{nc}; \psi_L)] > 0$$

where the strict inequality follows from $W_L(\pi; \psi_L)$ being strictly increasing in π and $\eta > p_{nc}$. In particular, for $\psi_L = 0$ we have that

$$\beta[W_L(\eta; 0) - W_L(p_{nc}; 0)] > 0$$

so condition (10) holds for $\psi_L = 0$. Thus, by continuity, there exists $\bar{\psi}_L > 0$ such that for all $\psi_L \leqslant \bar{\psi}_L$ condition (10) holds and so it is optimal for the low default cost government to follow a strategy of never bailing out. Q.E.D.

The proposition implies that any equilibrium outcome in the model with a behavioral commitment type is an equilibrium outcome in this model with payoff types.

B.5 Arbitrary Distributions and Recovery Rates

We now argue that Proposition 1 hold in more general environments with more general distributions and recovery rates. We allow both s, θ to be drawn from continuous distributions $P(s)$ and $H(\theta \mid s)$ respectively. We generalize the social cost function to allow for any increasing function $C(\cdot)$. Finally, in the event of default, we assume that lenders and borrowers can renegotiate the contract so that borrowers make a partial repayment to the lenders and avoid the default cost. Let

$$\Delta(B, K, s) = \int \mathbb{I}_{\{B > \theta K^{\alpha}\}} dH(\theta|s),$$

$$\tilde{\Delta}(B, K, s) = \int \theta K^{\alpha} \mathbb{I}_{\{B > \theta K^{\alpha}\}} dH(\theta|s)$$

where $\tilde{\Delta}(B, s)$ denotes the maximal transfer that can be extracted from the borrower such that it is indifferent between defaulting and not.

The static value of bailing out is

$$\omega^{\text{bailout}}(B, K, s) = \lambda \int \max\{\theta K^{\alpha} - B, 0\} dH(\theta|s) + (1-\lambda)\left[(1 - \Delta(B, K, s)) B + \tilde{\Delta}(B, s)\right]$$

and the static value of not bailing out (and allowing default) is

$$\begin{aligned}\omega^{\text{no-bailout}}(B, K, s) = \lambda \int \max\{\theta K^{\alpha} - B, 0\} dH(\theta|s) + (1-\lambda)\left[(1 - \Delta(B, K, s)) B + \tilde{\Delta}(B, K, s)\right] \\ - C(\Delta(B, K, s) B).\end{aligned}$$

Note that even absent a bailout, since private agents can re-negotiate contracts, lenders will extract $\tilde{\Delta}(B, K, s)$ from borrowers who default. Given this the pricing schedule for debt is

$$\begin{aligned}Q(b, k|\pi, B, K) = q\left\{\int (1 - \Delta(b, k, s)) dP(s) + \int \tilde{\Delta}(b, k, s) dP(s)\right\} \\ + q\left\{(1-\pi)\int \sigma(\pi, B, K, s)\left[\Delta(B, K, s) B - \tilde{\Delta}(B, K, s)\right] dP(s)\right\}.\end{aligned} \quad (11)$$

Given that private contracts can be renegotiated, lenders receive at least $\tilde{\Delta}(B, K, s)$ in the event of default. The expression on the second line denotes the additional transfer received in the event of a bailout. From Lemma 1, the problem for the borrower in period 1 is

$$\max_{b,k} \int \int \max\{\theta k^{\alpha} - b, 0\} \, dH(\theta|s) \, dP(s) \tag{12}$$

subject to

$$K \leqslant Q(b, k|\pi, B, K) \, b.$$

Define $\Theta^{s}_{+}(B, K) \equiv \{\theta : \theta K^{\alpha} - B \geqslant 0\}$. The following Lemma characterizes the private outcome in the stage game given the bailout policy σ if the distribution for θ is continuous:

Lemma 1. *Given π and a bailout policy σ, (B, K, Q) is a symmetric equilibrium outcome if*

$$K = (QB)^{\alpha},$$

$$\int_{s} \int_{\theta \in \Theta^{s}_{+}(B,K)} \alpha\theta (QB)^{\alpha-1} \left(Q + Q'B\right) dH(\theta \mid s) \, dP(s) - 1 = 0,$$

and $Q = Q(B, K|\pi, B, K)$ *where*

$$Q' = \left. \frac{dQ(b, k|\pi, B, K)}{db} \right|_{(b,k)=(B,K)}.$$

We show that both the existence and characterization results hold in this environment if the private equilibrium of the stage game satisfies the following condition:

Assumption 1. *For any bailout policy $\sigma(\pi, B, K, s)$ which decreasing in π for all (B, K, s), the private equilibrium outcome is such that $B(\pi)$ is a decreasing function.*

The assumption requires the debt issued to be decreasing in π which implies that the equilibrium default probabilities in each state s, to be decreasing in π. In general, as π decreases there are two effects on the equilibrium price of debt Q. First, since the probability of a bail out is higher, Q increases. However, the resulting increase in borrowing increases the probability of default which might lower Q. The assumption requires the first force to dominate so that in equilibrium the price of issuing debt decreases and thus the debt issued increases. It is easy to see that the example described in the previous section satisfies this assumption. Under this Assumption, the steps in Proposition 1 go through unchanged.

B.6 Persistent Shocks: Contagion and Shock Sensitivity

In the model with iid shocks, there is no heterogeneity among borrowers at the beginning of any period. As a result the model cannot generate the contagion effects described in the introduction. By the contagion effect, we mean the increase in the price of debt for a country not directly affected by an adverse fundamental shock. Moreover, since the price of debt depends only on π and not the state, it is not possible to generate the differential effect of reputation on the sensitivity of prices to fundamentals unless we introduce multiple types of borrowers. To show that our framework can generate such features we extend the baseline model to allow for persistent of aggregate and idiosyncratic states. As a result, the distribution functions of idiosyncratic and aggregate shocks are now $h(\theta'|s', s, \theta)$ and $P(s'|s)$.

Let's consider our simple example. To simplify the algebra we assume that the government cares only about the borrowers ($\lambda = 0$). The aggregate state s follows a Markov chain

$$P\left(s'|s\right) = \begin{bmatrix} p_{HH} & p_{HM} & p_{HL} \\ p_{MH} & p_{MM} & p_{ML} \\ p_{LH} & p_{LM} & p_{LL} \end{bmatrix}.$$

As before, in state s_H, all borrowers draw θ_H and in state s_L all borrowers draw θ_L, i.e. $h(\theta_H|s_H, \theta) = 1$ and $h(\theta_L|s_L, \theta) = 1$ for all θ. We assume that in the medium state, a fraction μ of borrowers have the low output θ_L and

$$h(\theta_L|s = s_M, s_- = s_M, \theta_- = \theta_L) = \rho_L$$
$$h(\theta_L|s = s_M, s_- = s_M, \theta_- = \theta_H) = \rho_H$$

with $\rho_L\mu + \rho_H(1-\mu) = \mu$. Thus, the productivity shock is persistent in the medium state.

Let $\mathbf{z}_- = (s_-, \theta_-)$, $\mathbf{z} = (s, \theta)$ and $\nu(\mathbf{z}_-)$ denote the fraction of type $\mathbf{z}_-$. Next, let $P_H(\mathbf{z}_-)$ and $P_L(\mathbf{z}_-)$ be probabilities of a high and low idiosyncratic endowment respectively, conditional on history $\mathbf{z}_-$. Therefore,

$$P_H(\mathbf{z}_-) = p_{s_-H} + p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\left(1-\rho_{\theta_-}\right) + \left(1-\mathbb{I}_{s_-=s_M}\right)(1-\mu)\right],$$

$$P_L(\mathbf{z}_-) = p_{s_-L} + p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\rho_{\theta_-} + \left(1-\mathbb{I}_{s_-=s_M}\right)\mu\right].$$

Next, define

$$\bar{\gamma}(\mathbf{z}_-) \equiv \frac{p_{s_-L}(1-\pi)\,\sigma(\pi, s_-, s_L) + p_{s_-M}p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\rho_{\theta_-} + \left(1-\mathbb{I}_{s_-=s_M}\right)\mu\right](1-\pi)\,\sigma(\pi, s_-, s_M)}{P_L(\mathbf{z}_-)}$$

to be the probability that an individual borrower with history $\mathbf{z}_-$ will be bailed out conditional on drawing θ_L. As in the i.i.d case this serves as a useful sufficient statistic to characterize private decisions. The price of debt in this environment is

$$\mathbb{Q}\left(\mathbf{z}_-,\bar{\gamma}\right)=qP_H\left(\mathbf{z}_-\right)+qP_L\left(\mathbf{z}_-\right)\bar{\gamma}\left(\mathbf{z}_-\right)$$

and the optimal debt level $\mathbb{B}\left(\mathbf{z}_-,\bar{\gamma}\right)$ is given by

$$\mathbb{B}\left(\mathbf{z}_-,\bar{\gamma}\right)=\left(\alpha\theta_H\right)^{\frac{1}{1-\alpha}}\mathbb{Q}\left(\mathbf{z}_-,\bar{\gamma}\right)^{\frac{\alpha}{1-\alpha}}. \tag{13}$$

Define the $\bar{\mathbb{B}}\left(s_-,\bar{\gamma}\right)$ to be aggregate level of debt where

$$\begin{aligned}
\bar{\mathbb{B}}\left(s_L,\bar{\gamma}\right) &\equiv \mathbb{B}\left(\left(s_L,\theta_H\right),\bar{\gamma}\right)=\mathbb{B}\left(\left(s_L,\theta_L\right),\bar{\gamma}\right),\\
\bar{\mathbb{B}}\left(s_M,\bar{\gamma}\right) &\equiv \mu\mathbb{B}\left(\left(s_M,\theta_L\right),\bar{\gamma}\right)+(1-\mu)\,\mathbb{B}\left(\left(s_M,\theta_H\right),\bar{\gamma}\right),\\
\bar{\mathbb{B}}\left(s_H,\bar{\gamma}\right) &\equiv \mathbb{B}\left(\left(s_H,\theta_H\right),\bar{\gamma}\right)=\mathbb{B}\left(\left(s_H,\theta_L\right),\bar{\gamma}\right).
\end{aligned}$$

Next, we characterize a set of continuous monotone equilibria for the economy for an arbitrary transition matrix P and provide sufficient conditions so that the characterization results for the iid case extend to this more general environment. Assumption 2 is the analog for Assumption 1 in the text for the case with persistent endowments.

Assumption 2. *Let* $C(x)=\psi x$. *Define* $W^R\left(s,\bar{\gamma}\right)$ as be the solution to

$$\begin{aligned}
W^R\left(s_-,\bar{\gamma}\right)=e-&\sum_{\theta}\nu\left(s_-,\theta\right)\left[\bar{\gamma}\left(s_-,\theta\right)+\psi\left(1-\bar{\gamma}\left(s_-,\theta\right)\right)\right]qP_L\left(s_-,\theta\right)\mathbb{B}\left(\left(s_-,\theta\right),\bar{\gamma}\left(s_-,\theta\right)\right)\\
&+\beta\sum_{s}p_{s_-s}W^R\left(s,\bar{\gamma}\right)
\end{aligned}$$

Assume that

$$\psi\bar{\mathbb{B}}\left(s_-,0\right)>\beta\left[W^R\left(s,0\right)-W^R\left(s,1\right)\right]\ \textit{for all } s \tag{14}$$

and

$$\mathbf{A}^{-1}\cdot\mathbf{x}>\mathbf{G} \tag{15}$$

where

$$\mathbf{A}=\begin{bmatrix}1-p_{LL} & p_{LM} & p_{LH}\\ p_{ML} & 1-p_{MM} & p_{MH}\\ p_{HL} & p_{HM} & 1-p_{HH}\end{bmatrix},$$

$$\mathbf{x}=\begin{bmatrix}q\left(p_{LM}\mu+p_{LL}\right)\bar{\mathbb{B}}\left(s_L,1\right)\\ q\mu\left(p_{MM}\rho_L+p_{ML}\right)\mathbb{B}\left(\left(s_M,\theta_L\right),1\right)+q\mu\left(p_{MM}\rho_H+p_{ML}\right)\mathbb{B}\left(\left(s_M,\theta_H\right),1\right)\\ q\left(p_{HM}\mu+p_{HL}\right)\bar{\mathbb{B}}\left(s_H,1\right)\end{bmatrix}$$

and

$$\mathbf{G} = \begin{bmatrix} \psi\mu\bar{\mathbb{B}}\left(s_L, 0\right) \\ \psi\left[\mu\rho_L\mathbb{B}\left(\left(s_M, \theta_L\right), 1\right) + \left(1-\mu\right)\rho_H\mathbb{B}\left(\left(s_M, \theta_H\right), 1\right)\right] \\ \psi\mu\bar{\mathbb{B}}\left(s_H, 0\right) \end{bmatrix}.$$

Proposition 1. *For an arbitrary transition matrix* P*, if* p_{nc} *is sufficiently small, there exists a continuous monotone equilibrium in which* $B\left(s_{-}, \pi\right) : S \times [0,1] \to \mathbb{R}$ *is decreasing in* π*,* $\sigma\left(s_{-}, \pi, s\right) : S \times [0,1] \times S \to [0,1]$ *is decreasing in* π*,* $W\left(s_{-}, \pi\right) : S \times [0,1] \to \mathbb{R}$ *is increasing in* π*,* $Q\left(s_{-}, \pi\right) : S \times [0,1] \to \mathbb{R}$ *is decreasing in* π *for all* s_{-}*, and*

$$W\left(s_L, \pi\right) < W\left(s_M, \pi\right) < W\left(s_H, \pi\right).$$

Furthermore, under Assumption 2, if $p_c \to 1$ *and* $p_{nc} \to 0$ *then it must be that:*

- *It is optimal to bailout with probability one in a severe recession,* $\sigma\left(s_{-}, \pi, s_L\right) = 1$ *for all* π *and* s_{-}*.*
- *It is optimal to mix in a mild recession for some values of* π *for all* s_{-}*.*

Proof. The proof of the first part is identical to the i.i.d case. To see the second, we first show that under condition (14) in Assumption 2 we have $\sigma\left(\pi, s_{-}, s_L\right) = 1$ for all $\left(\pi, s_{-}\right)$. To this end, note that in any equilibrium $\mathbb{B}\left(\mathbf{z}_{-}, \bar{\gamma}\right) \geqslant \mathbb{B}\left(\mathbf{z}_{-}, 0\right)$. Moreover, note that the dynamic gains from bailing out, $W\left(s, p_c\right) - W\left(s, p_{nc}\right)$, are bounded by $W^R\left(s, 0\right) - W^R\left(s, 1\right)$ in that

$$W\left(s, p_c\right) - W\left(s, p_{nc}\right) \leqslant W^R\left(s, 0\right) - W^R\left(s, 1\right)$$

because $W^R\left(s, 0\right) \geqslant W\left(s, p_c\right)$, and $W\left(s, p_{nc}\right) \geqslant W^R\left(s, 1\right)$. Hence we have that

$$\psi B\left(s_{-}, \pi\right) \geqslant \psi\bar{\mathbb{B}}\left(s_{-}, 0\right) > \beta\left[W^R\left(s, 0\right) - W^R\left(s, 1\right)\right] \geqslant \beta\left[W\left(s, p_c\right) - W\left(s, p_{nc}\right)\right]$$

and so it is optimal to bail out with probability one if $s = s_L$.

Next we show that it is optimal to mix in a mild recession under assumption (15). Suppose by way of contradiction that $\sigma\left(\pi, s_{-}, s_M\right) = 1$ for all π. Under the assumption that the government type is absorbing, the value for the optimizing type in state s for $\pi = 1$ is

$$W\left(s, 1\right) = qp_{sH}\left[0 + \beta W\left(s_H, 1\right)\right] + qp_{sM}\left[0 + \beta W\left(s_M, 0\right)\right] + qp_L\left[0 + \beta W\left(s_L, 0\right)\right].$$

For $\pi = 0$, since $\bar{\gamma}\left(0\right) = 1$ we have for $s = \{s_H, s_L\}$

$$\begin{aligned} W\left(s, 0\right) = &-q\left(p_{sM}\mu + p_{sL}\right)\bar{\mathbb{B}}\left(s, 1\right) + qp_{sH}\beta W\left(s_H, 0\right) \\ &+ qp_{sM}\beta W\left(s_M, 0\right) + qp_{sL}\beta W\left(s_L, 0\right) \end{aligned}$$

and for $s = s_M$

$$W(s_M, 0) = -q\mu(p_{MM}\rho_L + p_{ML})\mathbb{B}((s_M, \theta_L), 1) - q(1-\mu)(p_{MM}\rho_H + p_{ML})\mathbb{B}((s_M, \theta_H), 1)$$
$$+ qp_{MH}\beta W(s_H, 0) + qp_{MM}\beta W(s_M, 0) + qp_{ML}\beta W(s_L, 0)$$

and so $W(p_c) - W(p_{nc}) = W(1) - W(0)$ equals

$$W(s, 1) - W(s, 0) = x_s + q\beta \sum_{s'} p_{ss'}\left[W(s', 1) - W(s', 0)\right]$$

for some constant x_s. Hence we can write

$$\mathbf{A} \cdot \mathbf{W} = \mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 - p_{LL} & p_{LM} & p_{LH} \\ p_{ML} & 1 - p_{MM} & p_{MH} \\ p_{HL} & p_{HM} & 1 - p_{HH} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} W(s_L, 1) - W(s_L, 0) \\ W(s_M, 1) - W(s_M, 0) \\ W(s_H, 1) - W(s_H, 0) \end{bmatrix}$$

and

$$\mathbf{x} = \begin{bmatrix} q(p_{LM}\mu + p_{LL})\bar{\mathbb{B}}(s_L, 1) \\ q\mu(p_{MM}\rho_L + p_{ML})\mathbb{B}((s_M, \theta_L), 1) + q\mu(p_{MM}\rho_H + p_{ML})\mathbb{B}((s_M, \theta_H), 1) \\ q(p_{HM}\mu + p_{HL})\bar{\mathbb{B}}(s_H, 1) \end{bmatrix}$$

and so

$$\mathbf{W} = \mathbf{A}^{-1} \cdot \mathbf{x}.$$

The static gains of bailing out in a mild recession if $\pi = 1$ is given by

$$\mathbf{G} = \begin{bmatrix} \psi\mu\bar{\mathbb{B}}(s_L, 0) \\ \psi\left[\mu\rho_L\mathbb{B}((s_M, \theta_L), 1) + (1-\mu)\rho_H\mathbb{B}((s_M, \theta_H), 1)\right] \\ \psi\mu\bar{\mathbb{B}}(s_H, 0) \end{bmatrix}.$$

For the contradiction hypothesis to be valid, it must then be that even for $\pi = 1$ the government prefers not to incur the default costs, or

$$\mathbf{G} \geqslant \mathbf{A}^{-1} \cdot \mathbf{x}$$

which contradicts in Assumption 2. Hence it must be that $\sigma(\pi, s_{-}, s_M) = 1 < 1$ for some π.

We are now left to show that we cannot have that $\sigma(\pi, s_{-}, s_M) = 0$ for all π. Suppose by way of contradiction this is indeed the case. In particular, we have that $\sigma(0, s_{-}, s_M) = 0$. Hence, it must be that

$$\bar{\gamma}(\mathbf{z}_{-}) = \frac{p_L(1-\pi)\sigma(\pi, s_L) + p_M\mu(1-\pi)\sigma(\pi, s_M)}{P_L(\mathbf{z}_{-})} = \frac{p_{s_{-}L}(1-\pi)}{P_L(\mathbf{z}_{-})}.$$

The posterior after no-bailout (if $\pi = 0$), is

$$\pi' = p_{nc} + \pi(p_c - p_{nc}) = p_{nc}$$

since a no-bailout is expected under the contradiction hypothesis, and for $s_{-} \in \{s_H, s_L\}$

$$\mu\bar{\mathbb{B}}(s_{-}, \bar{\gamma}) \leqslant \beta[W(s, p_{nc}) - W(s, p_{nc})].$$

This is a contradiction since

$$0 < \mu\bar{\mathbb{B}}(s_{-}, \bar{\gamma}) \leqslant \beta[W(s, p_{nc}) - W(s, p_{nc})] = 0.$$

Hence, we cannot have that $\sigma(\pi, s_{-}, s_M) = 0$ for all π. Therefore, there is mixing for some interval of π. A similar argument holds for $s_{-} = s_M$. Q.E.D.

Thus, a continuous monotone equilibrium exists when shocks are persistent and the economy displays similar dynamics to the i.i.d case. We now show that the introduction of persistence can generate the contagion effects described previously. In the i.i.d case, there is only a single type of borrower in each period. However, with persistent shocks, if $s_{-} = s_M$, then in the following period there are two types of borrowers: (s_M, θ_L) and (s_M, θ_H). If there is no bail out and a subsequent rise in reputation, the interest rates faced by both types rise due to the presence of a common government. This provides an explanation as to why the CDS spreads for Italy rose after the perceived recovery rates for Greek bonds declined. The announcement that private creditors were expected to receive haircuts on Greek bonds signaled that EU countries were less likely to receive the benefit of a full bail out in case of default in the future. As a result, the cost of borrowing for other countries that might have been considered at risk of default rose as well.

Proposition 2. *(Contagion) If the reputation of the government increases after observing no bail out in state s_M, then the price of debt for types (s_M, θ_H) decreases.*

The proofs follows from the observation that the pricing function Q depends positively on π.

We next show that this model is capable of generating higher sensitivity to fundamentals when reputation is high.

Proposition 3. *(Sensitivity) For any s_-, the difference in the price of debt for a $\theta_- = \theta_H$ borrower and a $\theta_- = \theta_L$ borrower is increasing in the reputation of the government. That is, $Q((s_-, \theta_{H-}), \pi) - Q((s_-, \theta_{L-}), \pi)$ is increasing in π. Similarly, for any θ_-, the differences $Q((s_H, \theta_-), \pi) - Q((s_M, \theta_-), \pi)$ and $Q((s_M, \theta_-), \pi) - Q((s_L, \theta_-), \pi)$ are increasing in π for π large enough.*

Debt prices (and debt issuances) are less responsive to the state s_- when the prior is low. That is, if the probability of facing the optimizing type is low then lenders are less worried about the state of the world since they expect to get bailed out with high probability and therefore, debt prices are not sensitive to the state. These effects are illustrated in Figure 1. As the fourth plot illustrates, the difference between the price of debt across the different states is increasing in π. At $\pi = 0$, the prices are identical and equal to the risk-free rate since lenders expect to be bailed out with probability one. At $\pi = 1$, prices are driven exclusively by the probability of default and since the states are persistent, the difference in prices is large.

B.7 Learning Model

We can simplify the analysis by noting that since $\sigma_2 = 1$, the price in the secondary market simplifies to

$$q_2 = Q_2(\pi, B, s, \varepsilon|\sigma) = \begin{cases} (1-\mu) + \mu(1-\pi)\sigma_1(\pi, B, q_2) + \varepsilon & s = s_H \\ (1-\pi)[(1-\mu) + \mu\sigma_1(\pi, B, q_2)] + \varepsilon & s = s_L \end{cases}.$$

It follows that if $Q_2(\pi, s_H, \varepsilon_H) = Q_2(\pi, s_L, \varepsilon_L)$ then

$$\varepsilon_L = \varepsilon_H + (1-\mu)\pi.$$

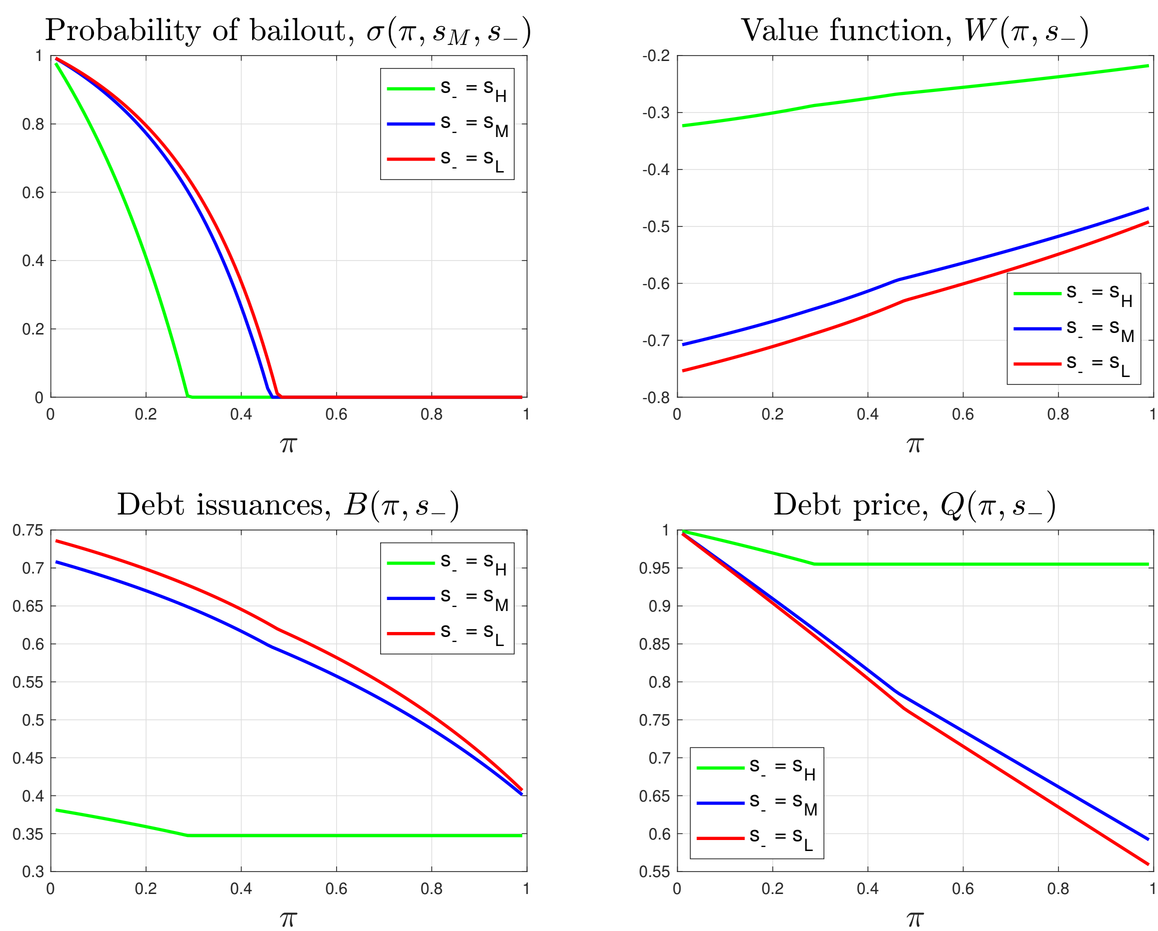
If we assume that $\text{supp}(g) = (-\infty, +\infty)$ we can then make a change of variable and express all the equilibrium objects as a function of the realization of ε in state s_H. Define

$$F(\pi) \equiv \int \sigma_1(\pi, \varepsilon)[p(s_H) g(\varepsilon) + p(s_L) g(\varepsilon + (1-\mu)\pi)] d\varepsilon \tag{16}$$

to be ex-ante probability of a bailout in the first stage of the sub-period two given prior π. Then, the price of issuing debt in the first sub-period is

$$Q(\pi) = q[p(s_H)(1-\mu) + p(s_L)(1-\mu)(1-\pi) + \mu(1-\pi)F(\pi)] \tag{17}$$

Figure 1: Equilibrium objects for computed discrete example with persistent shocks



and so the optimal choice of debt satisfies

$$B(\pi) = (\alpha\theta_H)^{\frac{1}{1-\alpha}} Q(\pi)^{\frac{\alpha}{1-\alpha}}. \tag{18}$$

The value for the government, assuming $\lambda = 0$, is given by

$$\begin{aligned} W(\pi) = &-Q(\pi) B(\pi) + \\ &+ qp(s_H)\{(1-\mu) B(\pi) \\ &+ \int \sigma_1(\pi,\varepsilon)\beta W(p_{nc}) g(\varepsilon) d\varepsilon \\ &+ \int [1-\sigma_1(\pi,\varepsilon)] \left[-c\mu B(\pi) + \beta W\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1(\pi,\varepsilon))}\Delta p\right)\right] g(\varepsilon) d\varepsilon \Big\} \\ &+ qp(s_L)\left\{\int [1-\sigma_1(\pi,\varepsilon+(1-\mu)\pi)][-c\mu B(\pi)] g(\varepsilon+(1-\mu)\pi) d\varepsilon + \beta W(p_{nc})\right\}. \end{aligned} \tag{19}$$

Finally, the probability of a bailout in the first stage $\sigma_1(\pi,\varepsilon)$ is given by

$$\sigma_1(\pi,\varepsilon) = \begin{cases} 0, & \text{if } c\mu B(\pi) \leqslant \beta\hat{p}_H(\pi,\varepsilon)[W(p_{nc}+\pi\Delta p) - W(p_{nc})] \\ \tilde{\sigma}, & \text{if } c\mu B(\pi) = \beta\hat{p}_H(\pi,\varepsilon)\left[W\left(p_{nc} + \frac{\pi\Delta p}{\pi+(1-\pi)(1-\tilde{\sigma})}\right) - W(p_{nc})\right] \\ 1, & \text{if } c\mu B(\pi) \geqslant \beta\hat{p}_H(\pi,\varepsilon)[W(p_{nc}+\Delta p) - W(p_{nc})] \end{cases} \tag{20}$$

where

$$\hat{p}_H(\pi,\varepsilon) = \frac{p(s_H)}{p(s_H) g(\varepsilon) + p(s_L) g(\varepsilon+(1-\mu)\pi)}.$$

Thus, (16)–(20) define a set of functional equations that can be solved for the equilibrium objects $F(\pi)$, $Q(\pi)$, $B(\pi)$, $W(\pi)$, and $\sigma_1(\pi,\varepsilon)$.