Online Appendix: Public Education Inequality and Intergenerational Mobility

Angela Zheng James Graham

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Model Calibration Details

Define $\lambda_{2,i} \equiv \int \mathbb{1}_i d\lambda_2$, which denotes the distribution over age 2 households that choose to live in neighborhood i. Note that $\sum_i \int d\lambda_{2,i} = 1$. As discussed in the Calibration Section, we solve for an equilibrium in which the mass of households located in neighborhoods A, B and C is 0.5, 0.3, and 0.2, respectively. Recall that housing market equilibrium requires

$$\int d\lambda_{2,A} = \bar{s}_A p_A^{\psi}, \quad \int d\lambda_{2,B} = \bar{s}_B p_B^{\psi}, \quad \int d\lambda_{2,C} = \bar{s}_C p_C^{\psi}$$

In combination with the population constraint, the land availability parameters $\bar{s}_i$ can be expressed as:

$$\bar{s}_A = \bar{s}_C \frac{\int d\lambda_{2,A}}{\int d\lambda_{2,C}} \left(\frac{p_C}{p_A}\right)^{\psi}$$
$$\bar{s}_B = \bar{s}_A \frac{\int d\lambda_{2,B}}{\int d\lambda_{2,A}} \left(\frac{p_A}{p_B}\right)^{\psi}$$
$$\bar{s}_C = \left(1 + p_C^{\psi} \left[\frac{\int d\lambda_{2,A} + \int d\lambda_{2,B}}{\int d\lambda_{2,C}}\right]\right)^{-1}$$

In the model calibration, we target the populations for neighborhoods A and B and the ratio of prices for those neighborhoods, p_B/p_A. This leaves the unknowns p_A and $\frac{p_C}{p_A}$. The land availability parameters $\bar{s}_i$ are calibrated by imposing the equilibrium populations and the target price ratio p_B/p_A taken from the data, and by searching for p_A and p_C/p_A to clear housing markets.

In this way, our equilibrium algorithm simultaneously clears housing markets and achieves the neighborhood population and relative price targets.

Model Robustness

Table 1: Calibrated Model Parameters for Each State

	Parameter	CA	FL	GA	IL	NC	NY	MI	OH	PA	TX
β	Discount factor	0.931	0.921	0.946	0.901	0.900	0.910	0.904	0.918	0.918	0.947
α	Altruism	0.503	0.940	0.809	0.695	0.943	0.946	0.700	0.502	0.502	0.909
σ_ε	Taste shock	0.393	0.102	0.073	0.274	0.140	0.309	0.109	0.140	0.140	0.273
μ_a	Mean ability	20.236	16.013	20.878	30.906	7.003	16.294	20.261	25.223	25.223	11.373
σ_a	Ability shock	0.418	0.069	0.315	0.490	0.288	0.153	0.650	0.362	0.362	0.432
ρ_a	Ability persistence	0.625	0.599	0.716	0.644	0.771	0.816	0.513	0.518	0.518	0.660
σ_y	Income shock	0.548	0.977	0.506	0.535	0.489	0.573	0.487	0.508	0.508	0.851
γ	Education elasticity	0.110	0.404	0.187	0.102	0.312	0.198	0.163	0.475	0.475	0.259
$\bar{Q}$	Non-local school funds	0.193	0.112	0.321	0.268	1.335	0.117	1.334	0.568	0.568	0.068
η_A	Available land, A	0.911	4.757	1.409	0.468	5.204	2.517	1.514	0.287	0.719	60.212
η_B	Available land, B	0.341	2.225	0.283	0.119	1.723	0.824	0.343	0.073	0.173	12.952
η_C	Available land, C	0.160	1.096	0.036	0.038	0.662	0.359	0.102	0.021	0.055	2.549

Notes:

Table 2: Model Fit for Each State

	California		Florida		Georgia		Illinois		North Carolina	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
Median networth/income	1.711	1.738	1.711	1.729	1.711	1.685	1.711	1.719	1.711	1.619
Bequests share of networth	0.003	0.001	0.003	0.028	0.003	0.007	0.003	0.004	0.003	0.002
Median rent/income	0.209	0.217	0.206	0.123	0.171	0.133	0.166	0.143	0.166	0.142
Median income, C/A	1.446	1.456	1.221	1.207	1.786	1.855	1.908	1.932	1.451	1.454
Gini coefficent, $t = 2$	0.512	0.439	0.531	0.546	0.499	0.421	0.446	0.499	0.458	0.409
$\mathbb{P}(q_1\|q_1)$	0.302	0.393	0.307	0.240	0.336	0.435	0.344	0.458	0.388	0.451
$\mathbb{P}(q_5\|q_1)$	0.102	0.063	0.061	0.162	0.043	0.046	0.070	0.037	0.046	0.033
Education share	0.019	0.015	0.019	0.072	0.019	0.062	0.019	0.009	0.019	0.041
Mean local funding ratio	0.276	0.375	0.357	0.234	0.296	0.350	0.483	0.656	0.000	0.052
Population, A	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.501
Population, B	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.299
Price ratio, B/A	1.690	1.690	1.284	1.284	1.505	1.505	1.768	1.768	1.300	1.300

	New York		Michigan		Ohio		Pennsylvania		Texas	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
Median networth/income	1.711	1.752	1.711	1.643	1.711	1.660	1.711	1.661	1.711	1.637
Bequests share of networth	0.003	0.002	0.003	0.004	0.003	0.000	0.003	0.000	0.003	0.060
Median rent/income	0.198	0.197	0.172	0.065	0.164	0.114	0.171	0.086	0.174	0.332
Median income, C/A	1.245	1.206	1.976	1.821	1.836	1.722	1.902	1.762	1.772	1.761
Gini coefficent, $t = 2$	0.471	0.382	0.391	0.466	0.407	0.475	0.401	0.477	0.512	0.579
$\mathbb{P}(q_1\|q_1)$	0.322	0.320	0.356	0.435	0.393	0.465	0.345	0.474	0.288	0.329
$\mathbb{P}(q_5\|q_1)$	0.094	0.105	0.061	0.050	0.056	0.030	0.086	0.029	0.088	0.089
Education share	0.019	0.030	0.019	0.019	0.019	0.048	0.019	0.049	0.019	0.079
Mean local funding ratio	0.471	0.477	0.182	0.140	0.441	0.416	0.430	0.408	0.434	0.526
Population, A	0.500	0.501	0.500	0.500	0.500	0.499	0.500	0.500	0.500	0.499
Population, B	0.300	0.299	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.301
Price ratio, B/A	1.677	1.677	1.735	1.735	1.434	1.434	1.745	1.745	1.475	1.475

Notes:

Details of the Model Policy Experiments

Public School Revenue Equalization

We want to find an equilibrium when school funding is equalized across neighborhoods. In that case the local revenue is shared equally, so that all neighborhoods receive equal funding per pupil. Then school quality, given by total per pupil funding, is:

$$Q_i = \sum_i \tau p_i \int \mathbb{1}_i d\lambda_2 + \bar{Q},$$

where $\mathbb{1}_i$ is an indicator function denoting a household's choice of neighborhood.

With equalized school funding, neighborhoods differ only by house prices. Absent neighborhood taste shocks, households would always choose to live in the lowest priced neighborhood and thus equilibrium would consist of equal house prices across neighborhoods. In the presence of neighborhood taste shocks, households have may have an idiosyncratic preference for one neighborhood over another even though that neighborhood is more expensive. An equilibrium in which households experience neighborhood taste shocks can support a non-degenerate distribution of neighborhood prices.

As in the benchmark model, we search for an equilibrium in the first neighborhood price and the ratio of prices between neighborhood 2 and 1: $\left(p_1, \frac{p2}{p1}\right)$. Recall that the housing supply function for neighborhood i is $\bar{s}_i p_i^{\psi}$. The population of age 2 households is normalized to one, so that total housing supply is given by $\sum_i \bar{s}_i p_i^{\psi} = 1$, where $\bar{s_1}, \bar{s_2}, \bar{s_3}$ and ψ are given by the calibrated values in Tables **??** and 2. Thus, the price of neighborhood 3 is determined residually as

$$p_3 = \left(\frac{1 - \bar{s}_1 p_1^{\psi} - \bar{s}_2 p_2^{\psi}}{\bar{s}_3}\right)^{\frac{1}{\psi}}.$$

Because land availability $\bar{s}_i$ is fixed in each neighborhood, the changes in equilibrium house prices are determined by the elasticity of housing supply, ψ.

Welfare Calculations

Consider the welfare gain for a household of moving to a new policy. Let $V_1, V_{2,A}, V_{2,B}, V_{2,C}, V_3$ represent the value functions for a household prior to the policy change, and let

$V_1^*, V_{2,A}^*, V_{2,B}^*, V_{2,C}^*, V_3^*$ represent value functions after the policy change. Denote ω as the percentage increase in consumption in every period required to leave a household indifferent between the pre- and post-policy environments. Then define the following value functions as taking the optimal household decisions prior to the policy change, but inflating consumption in each period by the welfare gain ω:

$$\tilde{V}_1 = \log(c_1(1+\omega)) + \beta \mathbb{E}_{a^c, y_2} \tilde{V}_2(m_2, h, y_2, a^c) \tag{1}$$

$$\tilde{V}_{2,i} = \log(c_{2,i}(1+\omega)) + \beta \tilde{V}_3(m_3, h^c, a^c) \tag{2}$$

$$\tilde{V}_3 = \log(c_3(1+\omega)) + \alpha \mathbb{E}_{y^c} \tilde{V}_1(m_c, h^c, y^c, a^c) \tag{3}$$

Note that we assume that consumption change applies to the household's children (and grandchildren, etc...). Now consider the utility flows cumulated across one household's lifetime:

$$\tilde{V}_1 = \log(c_1(1+\omega)) + \beta \mathbb{E}_{a^c, y_2} \max_i \{\log(c_{2,i}(1+\omega)) + \sigma_\varepsilon \varepsilon_i\} + \beta^2 \log(c_3(1+\omega)) + \beta^2 \alpha \mathbb{E}_{y^c} \tilde{V}_1 \tag{4}$$

Exploiting the log-additivity of the utility function, Letting g denote one generation, and iterating forward across future generations yields:

$$\begin{aligned}
\tilde{V}_1 &= V_1 + \log(1+\omega)\left(1 + \beta + \beta^2 + \beta^2\alpha + \beta^2\alpha\beta + \beta^2\alpha\beta^2 + \beta^2\alpha\beta^2\alpha + ...\right) && (5)\\
&= V_1 + \log(1+\omega)\left((1+\beta+\beta^2) + (\beta^2\alpha + \beta^3\alpha) + (\beta^4\alpha + \beta^4\alpha^2) + ...\right) && (6)\\
&= V_1 + \log(1+\omega)((1+\beta+\beta^2) + (1+\beta+\beta^2)\alpha\beta^2 + (1+\beta+\beta^2)\alpha^2\beta^4 + ...) && (7)\\
&= V_1 + \log(1+\omega)(1+\beta+\beta^2)\sum_{g=0}^{\infty}(\alpha\beta^2)^g && (8)\\
&= V_1 + \log(1+\omega)\frac{1+\beta+\beta^2}{1-\alpha\beta^2} && (9)\\
& && (10)
\end{aligned}$$

We set ω such that $V_1^* = \tilde{V}_1$, which means that we can compute ω as:

$$\omega = \exp\left(\frac{(V_1^* - V_1)(1-\alpha\beta^2)}{1+\beta+\beta^2}\right) - 1 \tag{11}$$

Note that if we were to compute the welfare gain with households beginning at ages 2 or 3 we get:

$$\omega_2 = \exp\left(\frac{(V_2^* - V_2)(1-\alpha\beta^2)}{1+\beta+\beta\alpha}\right) - 1 \tag{12}$$

$$\omega_3 = \exp\left(\frac{(V_3^* - V_3)(1-\alpha\beta^2)}{1+\alpha+\beta\alpha}\right) - 1 \tag{13}$$

Notice that older households discount welfare gains more heavily since, typically, $1 + \beta + \beta^2 < 1 + \beta + \beta\alpha < 1 + \alpha + \beta\alpha$.