# Online Appendix for: Monetary Policy with Opinionated Markets

Ricardo J. Caballero    Alp Simsek

## A. Model and the log-linearized equilibrium

In this appendix, we describe the details of the model and derive the log-linearized equilibrium conditions that we use in our analysis. The model and the analysis closely follows the textbook treatment in Galí (2015), except that the central bank sets the interest rate *before* observing the aggregate demand shock within the period (to capture monetary policy transmission lags in a simple way).

**Representative household (the market).** The economy is set in discrete time with periods $t \in \{0, 1, ...\}$. In each period, a representative household ("the market") makes consumption-savings and labor supply decisions. Formally, the market solves,

$$\max_{\{C_{t+h}, N_{t+h}\}_{h=0}^{\infty}} \overline{E}_t^M \left[ \sum_{h=0}^{\infty} \beta^h \left( \log C_{t+h} - \frac{N_{t+h}^{1+\eta}}{1+\eta} \right) \right] \tag{A.1}$$

$$\text{s.t. } P_t C_t + \frac{B_{t+1}}{R_t^f} = B_t + W_t N_t + \int_0^1 \Pi_t(\nu)\, d\nu.$$

$C_t$ denotes consumption, $N_t$ denotes the labor supply, and $\eta$ is the inverse labor supply elasticity. The market has log utility—we describe the role of this assumption subsequently. The expectations operator $E_t^M[\cdot]$ corresponds to the market's belief after the realization of the demand shock in period $t$ (see Figure 3).

In the budget constraint, $R_t^f$ denotes the gross risk-free nominal interest rate between periods $t$ and $t+1$. The term $B_t$ denotes the one-period risk-free bond holdings. In equilibrium, the risk-free asset is in zero net supply, $B_t = 0$. The term $\Pi_t(\nu)$ denotes the profits from intermediate good firms (that we describe subsequently). For simplicity, we do not allow households to trade the firms (in equilibrium, there would be no trade since this is a representative household).

The optimality conditions for problem $(A.1)$ are standard and given by,

$$\frac{W_t}{P_t} = \frac{N_t^{\eta}}{C_t^{-1}} \tag{A.2}$$

$$C_t^{-1} = \beta R_t^f \overline{E}_t^M \left[ \frac{P_t}{P_{t+1}} C_{t+1}^{-1} \right]. \tag{A.3}$$

**Final good firms.** There is a competitive final good sector that combines intermediate inputs from a continuum of monopolistically competitive firms indexed by $\nu \in [0,1]$. The final good sector produces according to the technology,

$$Y_t = \left( \int_0^1 Y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} \, d\nu \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A.4}$$

This firm's optimality conditions imply a demand function for the intermediate good firms,

$$Y_t(\nu) = \left( \frac{P_t(\nu)}{P_t} \right)^{-\varepsilon} Y_t \tag{A.5}$$

$$\text{where } P_t = \left( \int_0^1 P_t(\nu)^{1-\varepsilon} \, d\nu \right)^{1/(1-\varepsilon)}. \tag{A.6}$$

$P_t$ denotes the ideal price index.

**Intermediate good firms.** Each intermediate good firm produces according to the technology

$$Y_t(\nu) = A_t N_t(\nu)^{1-\alpha}. \tag{A.7}$$

Firms take the demand for their goods as given and set price to $P_t(\nu)$ to maximize the current market value of their profits, as we describe subsequently.

**Market clearing conditions.** The aggregate goods and labor market clearing conditions are given by,

$$Y_t = C_t \tag{A.8}$$

$$N_t = \int_0^1 N_t(\nu) \, d\nu. \tag{A.9}$$

**Potential (flexible-price) outcomes.** We start by characterizing a potential (flexible-price) benchmark around which we log-linearize the equilibrium conditions. In this benchmark, firms are symmetric and set the same price $P_t^*$. The optimal price solves [cf. $(A.5)$ and $(A.7)$]:

$$\max_{P_t^*, Y_t^*, N_t^*} P_t^* Y_t^* - W_t N_t^* \tag{A.10}$$
$$\text{s.t. } Y_t^* = A_t \left(N_t^*\right)^{1-\alpha} = \left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} Y_t.$$

$Y_t$ denotes the aggregate output that firms take as given. The solution is

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{(1-\alpha) A_t \left(N_t^*\right)^{-\alpha}}. \tag{A.11}$$

Hence, firms operate with a constant markup over their marginal cost. By symmetry, aggregate output is given by $Y_t = Y_t^* = A_t \left(N_t^*\right)^{1-\alpha}$. Combining these observations with Eqs. $(A.2)$ and $(A.8)$, we solve for the potential labor supply

$$N^* = \left(\frac{\varepsilon - 1}{\varepsilon} (1-\alpha)\right)^{1/(1+\eta)}. \tag{A.12}$$

Likewise, potential output is

$$Y_t^* = A_t \left(N^*\right)^{1-\alpha}. \tag{A.13}$$

Note that potential output is determined by current productivity and is independent of expectations about the future.

**Nominal rigidities.** We next describe the nominal rigidities. In each period, a randomly selected fraction, $1-\theta$, of firms reset their nominal prices. The firms that do not adjust their price in period $t$, set their labor input to meet the demand for their goods (since firms operate with a markup and we focus on small shocks). Consider the firms that adjust their price in period $t$. These firms' optimal price, $P_t^{adj}$, solves

$$\max_{P_t^{adj}} \sum_{h=0}^{\infty} \theta^h \overline{E}_t^M \left\{ M_{t,t+h} \left( Y_{t+h|t} P_t^{adj} - W_{t+h} N_{t+h|t} \right) \right\} \tag{A.14}$$
$$\text{where } Y_{t+h|t} = A_{t+h} N_{t+h|t}^{1-\alpha} = \left(\frac{P_t^{adj}}{P_{t+h}}\right)^{-\varepsilon} Y_{t+h}$$
$$\text{and } M_{t,t+h} = \beta^h \frac{1/C_{t+h}}{1/C_t} \frac{P_t}{P_{t+h}}.$$

The terms, $N_{t+h|t}, Y_{t+h|t}$, denote the input and the output of the firm (that resets its price in period $t$) in a future period $t+h$. The term, $M_{t,t+h}$, is the stochastic discount factor between periods $t$ and $t+h$ (determined by the firm owners' preferences). Note that firms share the same belief as the representative household. The optimality condition is

$$\sum_{h=0}^{\infty}\theta^h\overline{E}_t^M\left\{M_{t,t+h}P_{t+h}^{\varepsilon}Y_{t+h}\left(P_t^{adj}-\frac{\varepsilon}{\varepsilon-1}\frac{W_{t+h}}{(1-\alpha)\,A_{t+h}N_{t+h|t}^{-\alpha}}\right)\right\}=0 \qquad \text{(A.15)}$$

$$\text{where } N_{t+h|t}=\left(\frac{P_t^{adj}}{P_{t+h}}\right)^{\frac{-\varepsilon}{1-\alpha}}\left(\frac{Y_{t+h}}{A_{t+h}}\right)^{\frac{1}{1-\alpha}}.$$

**The New-Keynesian Phillips curve.** We next combine Eq. $(A.15)$ with the remaining equilibrium conditions to derive the New-Keynesian Phillips curve. Specifically, we log-linearize the equilibrium around the allocation that features real potential outcomes and zero inflation, that is, $N_t = N^*, Y_t = Y_t^*$ and $P_t = P^*$ for each $t$. Throughout, we use the notation $\tilde{x}_t = \log(X_t/X_t^*)$ to denote the log-linearized version of the corresponding variable $X_t$. We also let $Z_t = \frac{W_t}{A_t P_t}$ denote the normalized (productivity-adjusted) real wage.

We first log-linearize Eq. $(A.2)$ (and use $Y_t = C_t$) to obtain

$$\tilde{z}_t = \eta\tilde{n}_t + \tilde{y}_t. \qquad \text{(A.16)}$$

Log-linearizing Eqs. $(A.4 - A.7)$ and $(A.9)$, we also obtain

$$\tilde{y}_t = (1-\alpha)\,\tilde{n}_t. \qquad \text{(A.17)}$$

Finally, we log-linearize Eq. $(A.15)$ to obtain

$$\sum_{h=0}^{\infty}(\theta\beta)^h\,\overline{E}_t^M\left\{\tilde{p}_t^{adj}-\left(\tilde{z}_{t+h}+\alpha\tilde{n}_{t+h|t}+\tilde{p}_{t+h}\right)\right\}=0, \qquad \text{(A.18)}$$

$$\text{where } \tilde{n}_{t|t+h}=\frac{-\varepsilon\left(\tilde{p}_t^{adj}-\tilde{p}_{t+h}\right)}{1-\alpha}+\tilde{n}_{t+h}.$$

The second line uses $\tilde{y}_t = (1-\alpha)\,\tilde{n}_t$.

We next combine Eqs. $(A.16 - A.18)$ and rearrange terms to obtain a closed-form solution for the price set by adjusting firms

$$\begin{aligned}\tilde{p}_t^{adj} &= (1-\theta\beta)\sum_{h=0}^{\infty}(\theta\beta)^h\,\overline{E}_t^M\left[\Theta\tilde{y}_{t+h}+\tilde{p}_{t+h}\right],\\ \text{where } \Theta &= \frac{1+\eta}{1-\alpha+\alpha\varepsilon}\end{aligned}$$

Since the expression is recursive, we can also write it as a difference equation

$$\tilde{p}_t^{adj}=(1-\theta\beta)\left(\Theta\tilde{y}_t+\tilde{p}_t\right)+\theta\beta\overline{E}_t^M\left[\tilde{p}_{t+1}^{adj}\right]. \qquad \text{(A.19)}$$

Here, we have used the law of iterated expectations, $\overline{E}_t^M[\cdot] = \overline{E}_t^M\left[\overline{E}_{t+1}^M[\cdot]\right]$.

Next, we consider the aggregate price index $(A.6)$

$$\begin{aligned} P_t &= \left((1-\theta)\left(P_t^{adj}\right)^{1-\varepsilon} + \int_{S_t}(P_{t-1}(\nu))^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)} \\ &= \left((1-\theta)\left(P_t^{adj}\right)^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon}\right)^{1/(1-\varepsilon)}, \end{aligned}$$

where we have used the observation that a fraction $\theta$ of prices are the same as in the last period. The term, $S_t$, denotes the set of sticky firms in period $t$, and the second line follows from the assumption that adjusting terms are randomly selected. Log-linearizing the equation, we further obtain $\tilde{p}_t = (1-\theta)\tilde{p}_t^{adj} + \theta\tilde{p}_{t-1}$. After substituting inflation, $\pi_t = \tilde{p}_t - \tilde{p}_{t-1}$, this implies

$$\pi_t = (1-\theta)\left(\tilde{p}_t^{adj} - \tilde{p}_{t-1}\right). \tag{A.20}$$

Hence, inflation is proportional to the price change by adjusting firms.

Finally, note that Eq. $(A.19)$ can be written in terms of the price change of adjusting firms as

$$\tilde{p}_t^{adj} - \tilde{p}_{t-1} = (1-\theta\beta)\Theta\tilde{y}_t + \tilde{p}_t - \tilde{p}_{t-1} + \theta\beta\overline{E}_t^M\left[\tilde{p}_{t+1}^{adj} - \tilde{p}_t\right].$$

Substituting $\pi_t = \tilde{p}_t - \tilde{p}_{t-1}$ and combining with Eq. $(A.20)$, we obtain the New-Keynesian Phillips curve (1) that we use in the main text

$$\begin{aligned} \pi_t &= \kappa\tilde{y}_t + \beta\overline{E}_t^M[\pi_{t+1}] \\ \text{where } \kappa &= \frac{1-\theta}{\theta}(1-\theta\beta)\frac{1+\eta}{1-\alpha+\alpha\varepsilon}. \end{aligned} \tag{A.21}$$

**Aggregate demand shocks.** We focus on aggregate demand shocks, which we capture by assuming log productivity, $a_{t+1}$, follows the process

$$a_{t+1} = a_t + g_t. \tag{A.22}$$

$g_t$ denotes the growth rate of productivity between periods $t$ and $t+1$, which is realized in period $t$.

**The IS curve.** Finally, we log-linearize the Euler equation $(A.3)$ to obtain Eq. (2) in the main text,

$$\tilde{y}_t = -\left(i_t - \overline{E}_t^M[\pi_{t+1}] - \rho\right) + g_t + \overline{E}_t^M[\tilde{y}_{t+1}].$$

$i_t = \log R_t^f$ denotes the nominal risk-free interest rate and $\rho = -\log\beta$ is the discount rate. We have used the market clearing condition, $Y_t = C_t$ [cf. Eq. $(A.8)$], the definition of the potential output, $Y_t^* = A_t(N^*)^{1-\alpha}$ [cf. $(A.13)$], and the evolution of productivity, $A_{t+1} = A_t e^{g_t}$ [cf.

$(A.22)$]. The equation illustrates $g_t$ has a one-to-one effect on aggregate spending and output in period $t$. Hence, we refer to $g_t$ as the *aggregate demand shock* in period $t$.

**Monetary policy and equilibrium.** To capture transmission lags, the Fed sets the interest rate at the beginning of the period, *before* observing the aggregate demand shock for the current period. Following much of the literature, we assume the Fed's objective function is given by $E_t^F \left[\sum_{h=0}^{\infty} \beta^h \left(\gamma \tilde{y}_{t+h}^2 + \pi_{t+h}^2\right)\right]$. Here, $\gamma$ denotes the relative weight on the output gap. Recall also that we also assume the Fed sets policy *without commitment*. Thus, the Fed solves the problem in (3)

$$\min_{i_t} E_t^F \left[\gamma \tilde{y}_t^2 + \pi_t^2\right] + E_t^F \left[V_{t+1}^F\right] \text{ where } V_{t+1}^F = \sum_{h=1}^{\infty} \beta^h \left(\gamma \tilde{y}_{t+h}^2 + \pi_{t+h}^2\right),$$
$$\text{s.t. } (1) \text{ and } (2).$$

As long as $E_t^F \left[V_{t+1}^F\right]$ is exogenous to the Fed's current policy decision, which will be the case for the equilibria we will analyze, the Fed effectively solves a sequence of static problems. This completes the equilibrium conditions.

**Price of the market portfolio.** For future reference, we also derive the equilibrium price of "the market portfolio." Specifically, in every period $t$, agents can also invest in a security in zero net supply whose payoff is proportional to output in subsequent periods, $\{Y_{t+h}\}_{h\geq 1}$. We let $\omega_t$ denote the market's holding of this security and modify the budget constraint as follows

$$P_t C_t + \frac{B_{t+1}}{R_t^f} + \omega_t P_t Q_t = B_t + \omega_{t-1} P_t \left(Y_t + Q_t\right) + W_t N_t + \int_0^1 \Pi_t \left(\nu\right) d\nu.$$

$Q_t$ denotes the *ex-dividend* and *real* price of this security (excluding the current dividends and adjusted for the nominal price level). Using the optimality condition for $\omega_t$, we obtain

$$Q_t = \overline{E}_t^M \left[\beta \frac{C_{t+1}^{-1}}{C_t^{-1}} \left(Q_{t+1} + Y_{t+1}\right)\right].$$

Solving the equation forward, and using the transversality condition, we further obtain

$$Q_t = \overline{E}_t^M \left[\sum_{h\geq 1} \beta^h \frac{\left(C_{t+h}\right)^{-1}}{\left(C_t\right)^{-1}} Y_{t+h}\right].$$

After substituting $Y_t = C_t$ and simplifying, we find

$$Q_t = \frac{\beta}{1-\beta} Y_t. \tag{A.23}$$

Hence, in view of log utility, the equilibrium price of the market portfolio is proportional to output. Substituting $Y_t = \exp(\tilde{y}_t) Y_t^*$ and $Y_t^* = A_t (N^*)^{1-\alpha}$ and taking logs, we obtain Eq. (8) in the main text

$$q_t = q^* + a_t + \tilde{y}_t, \text{ where } q^* = \log\left(\frac{\beta}{1-\beta}(N^*)^{1-\alpha}\right).$$

In equilibrium, asset prices are proportional to output. Therefore, asset prices change either when productivity ($a_t$) changes or when the output gap ($\tilde{y}_t$) changes.

# B. Omitted derivations

This appendix presents the derivations omitted from the main text.

## B.1. Omitted derivations in Section 3.3

**Proof of Lemma 1.** We characterize agents' beliefs more generally, even when they might not have yet reached a learning steady state. We obtain the beliefs along the learning steady state by taking the limit of the variance of agents' beliefs as $t \to \infty$.

Fix a period $t-1$ and suppose that at the end of this period the agent has the prior belief $\mathbf{g}_{t-1} \sim N\left(\overline{\mathbf{g}}_{t-1}^j, \sigma^2_{\overline{\mathbf{g}},t-1}\right)$. Using $\mathbf{g}_t = \mathbf{g}_{t-1} + \varepsilon_t$, the agent's prior about $\mathbf{g}_{t-1}$ implies a prior about $\mathbf{g}_t$ given by $N\left(\overline{\mathbf{g}}_{t-1}^j, \sigma^2_{\overline{\mathbf{g}},t-1} + \sigma^2_\varepsilon\right)$. Note also that the agent has the following signals about the permanent component in period $t$:

$$s_t + \mu_t^j =^j \mathbf{g}_t + e_t$$

$$g_t = \mathbf{g}_t + v_t,$$

where recall that $=^j$ implies equality under agent $j$'s belief. Combining these observations, the agent's pre-shock and post-shock beliefs in the next period are given by $\mathbf{g}_t \sim N\left(\mathbf{g}_t^j, \sigma^2_{\mathbf{g},t}\right)$ and $\mathbf{g}_t \sim N\left(\overline{\mathbf{g}}_t^j, \sigma^2_{\overline{\mathbf{g}},t}\right)$, where the variances satisfy

$$\begin{aligned} \frac{1}{\sigma^2_{\mathbf{g},t}} &= \frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1} + \sigma^2_\varepsilon} + \frac{1}{\sigma^2_e} \\ \frac{1}{\sigma^2_{\overline{\mathbf{g}},t}} &= \frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1} + \sigma^2_\varepsilon} + \frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_v}, \end{aligned} \tag{B.1}$$

and the conditional means satisfy

$$\mathbf{g}_t^j = \frac{\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}\overline{\mathbf{g}}^j_{t-1}}{\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}} + \frac{\frac{1}{\sigma^2_e}\left(s_t+\mu^j_t\right)}{\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}}, \tag{B.2}$$

$$\overline{\mathbf{g}}_t^j = \frac{\left(\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}\right)\mathbf{g}^j_t}{\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}} + \frac{\frac{1}{\sigma^2_v}g_t}{\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}}.$$

Next note that the second equation in $(B.1)$ implies

$$\frac{1}{\sigma^2_{\overline{\mathbf{g}},t}} = f\left(\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}}\right) \text{ where } f(x) = \frac{1}{\frac{1}{x}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}.$$

We have $\frac{d(f(x)-x)}{dx} = \frac{1}{(1+x\sigma^2_\varepsilon)^2} - 1 < 0$ for each $x > 0$. We also have $\lim_{x\to 0} f(x) - x > 0$ and $\lim_{x\to\infty} f(x) - x < 0$. These observations imply that $f(x)$ has a unique fixed point over the range $x > 0$ denoted by $x^* > 0$. Moreover, starting with any $x_0 > 0$, the sequence, $x_t = f(x_{t-1})$, converges to the unique fixed point, $\lim_{t\to\infty} x_t = x^*$. It follows that $\lim_{t\to\infty} \sigma^2_{\overline{\mathbf{g}},t} = \sigma^2_{\overline{\mathbf{g}}} > 0$ where $\sigma^2_{\overline{\mathbf{g}}}$ is the unique positive solution to

$$\frac{1}{\sigma^2_{\overline{\mathbf{g}}}} = f\left(\frac{1}{\sigma^2_{\overline{\mathbf{g}}}}\right) = \frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}. \tag{B.3}$$

Taking the limit of the first equation in $(B.1)$, we also obtain $\lim_{t\to\infty} \sigma^2_{\mathbf{g},t} = \sigma^2_{\mathbf{g}}$ where $\frac{1}{\sigma^2_{\mathbf{g}}} = \frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}$.

Next suppose sufficient time has passed and agents have already reached a learning steady state in which the variances of their beliefs are constant, $\sigma_{\overline{\mathbf{g}},t} = \sigma_{\overline{\mathbf{g}}}$ and $\sigma_{\mathbf{g},t} = \sigma_{\mathbf{g}}$. Substituting these expressions into $(B.2)$, we establish the two equations in (13). Combining these equations, we also obtain Eq. (14) with coefficients

$$\varphi = \frac{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}}{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}} = \frac{\sigma^2_{\overline{\mathbf{g}}}}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon},$$

$$\omega^s = \frac{\frac{1}{\sigma^2_e}}{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}}$$

$$\text{and } \omega^g = \frac{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}}{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}}\frac{\frac{1}{\sigma^2_v}}{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}}.$$

It remains to establish the comparative statics of the persistence of beliefs, $\varphi = \frac{\sigma^2_{\overline{\mathbf{g}}}}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}$.

Rewriting this expression, we obtain $\frac{\sigma_\varepsilon^2}{\sigma_{\overline{\mathbf{g}}}^2} = \frac{1-\varphi}{\varphi}$. Note also that Eq. $(B.3)$ implies,

$$\frac{\sigma_\varepsilon^2}{\sigma_{\overline{\mathbf{g}}}^2} = \frac{1}{\frac{\sigma_{\overline{\mathbf{g}}}^2}{\sigma_\varepsilon^2}+1} + \sigma_\varepsilon^2\left(\frac{1}{\sigma_e^2}+\frac{1}{\sigma_v^2}\right).$$

After substituting $\frac{\sigma_\varepsilon^2}{\sigma_{\overline{\mathbf{g}}}^2} = \frac{1-\varphi}{\varphi}$ and rearranging terms, we characterize $\varphi$ as the unique solution (in the range $(0,1)$) to:

$$\frac{(1-\varphi)^2}{\varphi} = \sigma_\varepsilon^2\left(\frac{1}{\sigma_e^2}+\frac{1}{\sigma_v^2}\right).$$

Since the left side is a decreasing function of $\varphi$, the solution is decreasing in $\sigma_\varepsilon^2$ and increasing in $\sigma_v^2$ and $\sigma_e^2$. This completes the proof. □

**Proof of Lemma 2.** Follows from Eq. (14). □

**Proof of Lemma 3.** Note that the identities trivially hold when $h = 0$. Suppose $h > 0$. First consider an agent's belief about their own belief in period $t+h$. For each agent $j$, we have

$$E_t^j\left[\mathbf{g}_{t+h}^j\right] = E_t^j\left[E_{t+h}^j\left[\mathbf{g}_{t+h}\right]\right] = E_t^j\left[\mathbf{g}_{t+h}\right] = E_t^j\left[\mathbf{g}_t + \sum_{\tilde{h}=1}^{h}\varepsilon_{t+\tilde{h}}\right] = E_t^j\left[\mathbf{g}_t\right] = \mathbf{g}_t^j.$$

The first and the last equalities substitute the definition of $\mathbf{g}_{t+h}^j$ and $\mathbf{g}_t^j$. The second equality applies the law of iterated expectations. The third equality substitutes the dynamics for the permanent component of demand from (9). The fourth equality uses the fact that $\varepsilon_{t+\tilde{h}}$ has zero mean for each period $t+\tilde{h}$. This proves Eq. (16).

Next consider an agent's belief about the other agent's belief in period $t+h$. For each agent $j$ and $j' \neq j$, we have

$$E_t^j\left[\mathbf{g}_{t+h}^{j'}\right] = E_t^j\left[\mathbf{g}_{t+h}^j + \mathbf{g}_{t+h}^{j'} - \mathbf{g}_{t+h}^j\right] = \mathbf{g}_t^j + E_t^j\left[\mathbf{g}_{t+h}^{j'} - \mathbf{g}_{t+h}^j\right] = \mathbf{g}_t^j + \varphi^h\left(\mathbf{g}_t^{j'} - \mathbf{g}_t^j\right).$$

The first equality rewrites $\mathbf{g}_{t+h}^{j'}$, the second equality applies Eq. (16), and the third equality uses Lemma 2 (which implies that disagreements follow an AR(1) process according to each agent $j$). This implies Eq. (17) and completes the proof. □

## B.2. Omitted derivations in Section 4

**Proof of Proposition 1, part (i).** Most of the proof is provided in the main text. It remains to verify the conjectures we have made for the Fed's expected continuation value, $E_t^F\left[V_{t+1}^F\right]$, and the market's expected output gap for the next period, $\overline{E}_t^M\left[\tilde{y}_{t+1}\right]$.

The Fed's expected continuation value is given by $E_t^F\left[V_{t+1}^F\right] = E_t^F\left[\sum_{h=1}^{\infty}\beta^h\left(\gamma\tilde{y}_{t+h}^2\right)\right]$. In view of Eq. (22), future output gaps, $y_{t+h} = g_{t+h} - \mathbf{g}_{t+h}^F$, are exogenous to the current policy rate. This verifies our conjecture that $\frac{dE_t^F\left[V_{t+1}^F\right]}{di_t}=0$. In particular, the Fed's problem (3) is effectively static and the optimality condition is given by (4).

Using (26), the market's expected output gap for the next period is given by $\overline{E}_t^M\left[\tilde{y}_{t+1}\right]=\varphi\left(\mathbf{g}_t^M-\mathbf{g}_t^F\right)$. This verifies our conjectures that $\frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}\right]}{di_t}=0$ and that agents know $\overline{E}_t^M\left[\tilde{y}_{t+1}\right]$ before the realization of the demand shock for the period, completing the proof. □

**Proof of Proposition 1, part (ii).** The derivation of the market's expected interest rates is presented in the main text. Here, we derive the Fed's expected interest rates. Taking the expectation of Eq. (21) according to the Fed's belief, we obtain,

$$\begin{aligned}E_t^F\left[i_{t+h}\right] &= \rho+(1-\varphi)E_t^F\left[\mathbf{g}_{t+h}^F\right]+\varphi E_t^F\left[\mathbf{g}_{t+h}^M\right]\\ &= \rho+(1-\varphi)\mathbf{g}_t^F+\varphi\left(\varphi^h\mathbf{g}_t^M+\left(1-\varphi^h\right)\mathbf{g}_t^F\right)\\ &= \rho+\mathbf{g}_t^F+\varphi^{h+1}\left(\mathbf{g}_t^M-\mathbf{g}_t^F\right).\end{aligned}$$

The second line uses Lemma 3. This proves Eq. (24). Taking the limit as $h\to\infty$, we also obtain $\lim_{h\to\infty}E_t^M\left[i_{t+h}\right]=\rho+\mathbf{g}_t^M$, completing the proof. □

## B.3. Omitted derivations in Section 5

**Proof of Proposition 2.** We verify that it is optimal for the Fed to follow the interest rate rule in (21),

$$i_t=\rho+(1-\varphi)\mathbf{g}_t^F+\varphi\mathbf{g}_t^M.$$

Recall that, after seeing this interest rate, the market thinks the Fed's belief is

$$\mathbf{G}^F\left(i_t\right)\equiv\frac{i_t-\rho-\varphi\mathbf{g}_t^M}{1-\varphi}.$$

Along the equilibrium path, the market's inference is correct, $\mathbf{G}^F\left(i_t\right)=\mathbf{g}_t^F$.

Consider a continuation path in which the Fed sets an arbitrary policy rate $i_t$ in period $t$ and follows the interest rate rule in (21) starting period $t+1$ onward. Since the Fed reveals its belief in period $t+1$, the equilibrium in future periods is the same as in Section 4. In particular, as in the proof of Proposition 1, future output gaps are exogenous to the current policy rate. This verifies that $\frac{dE_t^F\left[V_{t+1}^F\right]}{di_t}=0$ and ensures the Fed's optimality condition is still given by (4).

After seeing the policy rate $i_t$, the market *thinks* the equilibrium in future periods will be the same as in Section 4 given the Fed's period-$t$ belief, $\mathbf{G}^F\left(i_t\right)$. Therefore, the market's expected output gap in period $t+1$ depends on the current policy rate. In particular, using Eq. (26), we have

$$\overline{E}_t^M\left[\tilde{y}_{t+1}|i_t\right]=\varphi\left(\mathbf{g}_t^M-\mathbf{G}^F\left(i_t\right)\right)\text{ where }\mathbf{G}^F\left(i_t\right)=\frac{i_t-\rho-\varphi\mathbf{g}_t^M}{1-\varphi}.$$

This implies $\frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}|i_t\right]}{di_t}=-\frac{\varphi}{1-\varphi}$. Substituting this into the Fed's optimality condition (4), we obtain $E_t^F\left[\left(1+\frac{\varphi}{1-\varphi}\right)\tilde{y}_t\right]=0$. The optimality condition simplifies to Eq. (5) as before, $E_t^F[\tilde{y}_t]=0$. Thus, the Fed's optimal interest rate is still given by Eq. (6)

$$\begin{aligned} i_t &= \rho + E_t^F[g_t] + E_t^F\left[\overline{E}_t^M[\tilde{y}_{t+1}|i_t]\right] \\ &= \rho + \mathbf{g}_t^F + \varphi\left(\mathbf{g}_t^M - \mathbf{G}^F(i_t)\right). \end{aligned}$$

The second line substitutes $\overline{E}_t^M[\tilde{y}_{t+1}|i_t]$ as well as the Fed's belief, $E_t^F[g_t]=\mathbf{g}_t^F$. Substituting the equilibrium condition, $\mathbf{G}^F(i_t)=\mathbf{g}_t^F$, we obtain the interest rate rule in (21). This verifies that it is optimal for the Fed to follow the interest rate rule.

Finally, note that Eq. (30) follows from considering Eq. (27) before and after the Fed's interest rate decision. To establish Eq. (31), first note that Eq. (8) implies

$$\Delta E_t^M[\tilde{q}_{t+h}] = \Delta E_t^M[q^* + a_{t+h} + \tilde{y}_{t+h}] = \Delta E_t^M[\tilde{y}_{t+h}].$$

Here, the last line follows because $\Delta E_t^M[a_{t+h}]=0$ (the Fed belief surprise does not change the market's expectation for future productivity). Eq. (31) then follows from considering Eq. (28) before and after the interest rate decision. This completes the proof. □

**Proof of Proposition 3.** Note that Lemma 2 also applies after replacing the market's belief with the DGP. Using this observation, we characterize the expected future output gap under the DGP

$$\begin{aligned} E_t^{DGP}[\tilde{y}_{t+h}] = E_t^{DGP}\left[g_{t+h} - \mathbf{g}_{t+h}^F\right] &= E_t^{DGP}\left[\mathbf{g}_{t+h}^{DGP} - \mathbf{g}_{t+h}^F\right] \\ &= \varphi^h\left(\mathbf{g}_t^{DGP} - \mathbf{g}_t^F\right) \\ &= \varphi^h\left[\varphi\left(\mathbf{g}_{t-1}^{DGP} - \mathbf{g}_{t-1}^F\right) + \omega^s\left(\mu_t^{DGP} - \mu_t^F\right)\right]. \end{aligned}$$

The first equality uses Eq. (22), the second equality uses the law of iterated expectations, and the last two lines use Lemma 2. This implies the future output gap follows

$$\tilde{y}_{t+h} = \varphi^h\left[\varphi\left(\mathbf{g}_{t-1}^{DGP} - \mathbf{g}_{t-1}^F\right) + \omega^s\left(\mu_t^{DGP} - \mu_t^F\right)\right] + \tilde{\varepsilon}_{t+h}.$$

Here, $\tilde{\varepsilon}_{t+h}$ is a random variable that has zero mean and is uncorrelated with all information available before the demand shock in period $t$, including $\mu_t^{DGP}, \mu_t^F, \mu_t^M$. On average, future output gaps depend on the *past* belief differences between the DGP and the Fed, $\varphi\left(\mathbf{g}_{t-1}^{DGP} - \mathbf{g}_{t-1}^F\right)$, and on the *current* interpretation differences between the DGP and the Fed, $\omega^s\left(\mu_t^{DGP} - \mu_t^F\right)$.

We next use Eqs. (29−30) to characterize the interest rate shock

$$\Delta i_t = (1-\varphi)\,\Delta\mathbf{g}_t^F = (1-\varphi)\,\omega^s\tilde{\mu}_t^F \text{ where } \tilde{\mu}_t^F = \mu_t^F - \rho_\mu\mu_t^M.$$

The interest rate shock depends on the Fed's residual interpretation after controlling for the market's interpretation.

Next, we combine the expressions for the future output gap and the interest rate shock to obtain:

$$\begin{aligned}\beta^{DGP}\left(\tilde{y}_{t+h},\Delta i_t\right) &= \frac{cov^{DGP}\left(\varphi^h\omega^s\left(\mu_t^{DGP}-\mu_t^F\right),(1-\varphi)\,\omega^s\tilde{\mu}_t^F\right)}{var^{DGP}\left((1-\varphi)\,\omega^s\tilde{\mu}_t^F\right)}\\ &= \frac{\varphi^h}{1-\varphi}\frac{cov\left(\mu_t^{DGP}-\mu_t^F,\tilde{\mu}_t^F\right)}{var\left(\tilde{\mu}_t^F\right)}. \qquad \text{(B.4)}\end{aligned}$$

The regression coefficient depends on the covariance between the interpretation differences between the DGP and the Fed, $\mu_t^{DGP}-\mu_t^F$, and the Fed's residual interpretation after controlling for the market's interpretation, $\tilde{\mu}_t^F$. To calculate this covariance, we rewrite Eq. (33) to obtain

$$\begin{aligned}\mu_t^{DGP}-\mu_t^F &= \left(\beta^F-1\right)\mu_t^F+\beta^M\mu_t^M+\varepsilon_t^{DGP}\\ &= \left(\beta^F-1\right)\left(\tilde{\mu}_t^F+\rho^M\mu_t^M\right)+\beta^M\mu_t^M+\varepsilon_t^{DGP}\\ &= \left(\beta^F-1\right)\tilde{\mu}_t^F+\left(\beta^M+\left(\beta^F-1\right)\rho^M\right)\mu_t^M+\varepsilon_t^{DGP} \qquad \text{(B.5)}\end{aligned}$$

Here, the second line substitutes the Fed's residual interpretation from Eq. (29) and the last line rearranges terms. By construction, $\mu_t^M$ and $\varepsilon_t^{DGP}$ are both uncorrelated with $\tilde{\mu}_t^F=\mu_t^F-\rho_\mu\mu_t^M$. Therefore, substituting $(B.5)$ into $(B.4)$, we obtain

$$\beta^{DGP}\left(\tilde{y}_{t+h},\Delta i_t\right)=\frac{\varphi^h}{1-\varphi}\frac{cov\left(\left(\beta^F-1\right)\tilde{\mu}_t^F,\tilde{\mu}_t^F\right)}{var\left(\tilde{\mu}_t^F\right)}=\frac{\varphi^h}{1-\varphi}\left(\beta^F-1\right).$$

This completes the proof. □

## B.4. Omitted derivations in Section 7

**Proof of Proposition 4.** Most of the proof is provided in the main text. Here we complete the remaining steps. We first describe the processes for the output gap and inflation associated with the equilibrium characterized in the proposition.

Consider the IS equation (2)

$$\tilde{y}_t=-\left(i_t-\overline{E}_t^M\left[\pi_{t+1}\right]-\rho\right)+g_t+\overline{E}_t^M\left[\tilde{y}_{t+1}\right].$$

Recall our conjecture that the agents know $\overline{E}_t^M\left[\pi_{t+1}\right],\overline{E}_t^M\left[\tilde{y}_{t+1}\right]$ before the realization of the demand shock in period $t$. Then, the IS equation implies

$$\begin{aligned}\tilde{y}_t &= E_t^M\left[\tilde{y}_t\right]+g_t-\mathbf{g}_t^M\\ &= g_t-\mathbf{g}_t^M+\Gamma^M\left(\mathbf{g}_t^M-\mathbf{g}_t^F\right)\ \text{ where }\Gamma^M=\frac{\left(\gamma+\kappa^2\right)(1-\beta\varphi)}{\gamma+\kappa^2-\gamma\beta\varphi}. \qquad \text{(B.6)}\end{aligned}$$

Here, the second line substitutes (49). This characterizes the process for the output gap. The first line also implies $E_t^F[\tilde{y}_t] = E_t^M[\tilde{y}_t] + \mathbf{g}_t^F - \mathbf{g}_t^M$, which verifies our conjecture that Eq. (25) still applies.

Next consider the NKPC (1)

$$\begin{aligned}
\pi_t &= \kappa\tilde{y}_t + \kappa\overline{E}_t^M[\pi_{t+1}] \\
&= \kappa\tilde{y}_t + \kappa\beta\Pi^M\varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right) \\
&= \kappa\left(g_t - \mathbf{g}_t^M\right) + \left(\kappa\Gamma^M + \beta\varphi\Pi^M\right)\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right) \text{ where } \Pi^M = \frac{\gamma+\kappa^2}{\gamma+\kappa^2-\gamma\beta\varphi}\kappa. \qquad \text{(B.7)}
\end{aligned}$$

Here, the second line substitutes (47) and the last line substitutes ($B$.6). This characterizes the process for inflation.

We next verify that the equilibrium satisfies the conjectures we have made for the Fed's expected continuation value, $E_t^F\left[V_{t+1}^F\right]$, and the market's expected inflation and output gap for the next period, $\overline{E}_t^M[\tilde{y}_{t+1}], \overline{E}_t^M[\pi_{t+1}]$.

The Fed's expected continuation value is given by $E_t^F\left[V_{t+1}^F\right] = E_t^F\left[\sum_{h=1}^{\infty}\beta^h\left(\gamma\tilde{y}_{t+h}^2 + \pi_{t+h}^2\right)\right]$. In view of Eqs. ($B.6 - B.7$), future output gaps and inflation, $y_{t+h}, \pi_{t+h}$, do not depend on the Fed's current policy rate. This verifies that $\frac{dE_t^F\left[V_{t+1}^F\right]}{di_t} = 0$.

Using Eq. (47), the market's expected inflation in the next period is given by

$$\overline{E}_t^M[\pi_{t+1}] = \Pi^M\varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right).$$

Likewise, using Eq. (49), the market's expected output in the next period is given by,

$$\overline{E}_t^M[\tilde{y}_{t+1}] = \overline{E}_t^M\left[E_{t+1}^M[\tilde{y}_{t+1}]\right] = \overline{E}_t^M\left[\Gamma^M\left(\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F\right)\right] = \Gamma^M\varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right). \qquad \text{(B.8)}$$

The last equality uses Lemma 2. These expressions verify our conjectures that $\frac{d\overline{E}_t^M[\tilde{y}_{t+1}]}{di_t} = \frac{d\overline{E}_t^M[\pi_{t+1}]}{di_t} = 0$ and that agents know $\overline{E}_t^M[\pi_{t+1}], \overline{E}_t^M[\tilde{y}_{t+1}]$ before the realization of the demand shock in period $t$.

Finally, we derive the Fed's optimality conditions (43) and (44). Using (3), the Fed's problem is

$$\begin{aligned}
&\min_{i_t} \gamma E_t^F\left[\tilde{y}_t^2\right] + E_t^F\left[\pi_t^2\right] + E_t^F\left[V_{t+1}^F\right] \\
\text{s.t.} \quad &\tilde{y}_t = -\left(i_t - \overline{E}_t^M[\pi_{t+1}] - \rho\right) + g_t + \overline{E}_t^M[\tilde{y}_{t+1}] \\
&\pi_t = \kappa\tilde{y}_t + \beta\overline{E}_t^M[\pi_{t+1}].
\end{aligned}$$

Using $\frac{dE_t^F\left[V_{t+1}^F\right]}{di_t} = \frac{d\overline{E}_t^M[\tilde{y}_{t+1}]}{di_t} = \frac{d\overline{E}_t^M[\pi_{t+1}]}{di_t} = 0$, the Fed's problem is effectively static and the optimality condition is given by

$$\gamma \frac{d\tilde{y}_t}{di_t} E_t^F [\tilde{y}_t] + \frac{d\pi_t}{di_t} E_t^F [\pi_t] = -\gamma E_t^F [\tilde{y}_t] - \kappa E_t^F [\pi_t] = 0.$$

Rearranging this expression, we obtain $E_t^F [\tilde{y}_t] = -\frac{\kappa}{\gamma} E_t^F [\pi_t]$. Substituting this into the NKPC under the Fed's belief (1), $E_t^F [\pi_t] = \kappa E_t^F [\tilde{y}_t] + \beta \overline{E}_t^M [\pi_{t+1}]$, we obtain Eqs. (43) and (44). This completes the characterization of equilibrium. □

**Proof of Corollary 2**. Substituting Eq. $(B.8)$ and (48) into Eq. (50), we obtain $r_t = \rho + (1 - \tilde{\varphi}) \mathbf{g}_t^F + \tilde{\varphi} \mathbf{g}_t^M$ where

$$\tilde{\varphi} = \Gamma^M \varphi - \Gamma^F = \left( \frac{(\gamma + \kappa^2)(1 - \beta\varphi) + \kappa^2 \beta}{\gamma + \kappa^2 - \gamma\beta\varphi} \right) \varphi = \left( 1 + \frac{\kappa^2 \beta (1 - \varphi)}{\gamma + \kappa^2 - \gamma\beta\varphi} \right) \varphi.$$

Note that $\tilde{\varphi} > \varphi$. We also have $\tilde{\varphi} < 1$ since $\gamma + \kappa^2 - \gamma\beta\varphi > \kappa^2 \beta\varphi$. This establishes $\tilde{\varphi} \in (\varphi, 1)$ and completes the proof. □

**Proof of Corollary 3.** The expressions for $E_t^F [\tilde{y}_t]$ and $E_t^F [\pi_t]$ follow from Eqs. (48) and (45) after substituting $u_t = \beta\kappa\varphi \left( \mathbf{g}_t^M - \mathbf{g}_t^F \right)$. These expressions are the same as Eqs. (3.4) and (3.5) in Clarida, Gali and Gertler (1999); Galí (2015) after appropriately adjusting the notation: specifically, by setting $E_t^F [\tilde{y}_t] = x_t, E_t^F [\pi_t] = \pi_t, \kappa = \lambda, \gamma = \alpha$, and $\varphi = \rho$. □

# C. Tantrum shocks, gradualism, communication

Section 6 extends the baseline model to analyze tantrum shocks and gradualism. In the main text we present only the key equations and the intuitions. In this appendix, we present the formal results omitted form the main text along with the proofs.

Recall that, to capture tantrums, we allow the market and the Fed to disagree about the short-term component of demand. Fix a period $t$ and suppose in (only) this period the Fed and the market can disagree about the transitory demand shock, $v_t$. The Fed believes $v_t \sim N \left( \Delta \mathbf{v}_t^F, \sigma_v^2 \right)$, whereas the market still believes $v_t \sim N \left( 0, \sigma_v^2 \right)$ [see (9)]. Here, $\Delta \mathbf{v}_t^F$ captures the Fed's belief change for the short-term component in period $t$. The market does not observe the Fed's belief change and thinks it has mean zero and is drawn independently of all other variables, $\Delta \mathbf{v}_t^F \sim N \left( 0, \sigma_{\mathbf{v}^F}^2 \right)$.

As before, the Fed and the market can also disagree about the long-term component due to different interpretations of the public signal. As in Section 5, the market does not observe the Fed's interpretation. Recall that at the beginning of period $t$ the market thinks the Fed's long-term belief change, $\Delta \mathbf{g}_t^F = \mathbf{g}_t^F - E_t^M \left[ \mathbf{g}_t^F \right]$, has mean zero and variance $(\omega^s)^2 \left( 1 - \rho_\mu^2 \right) \sigma_\mu^2$ [see (29)].

These assumptions create a signal extraction problem for the market. Unlike in Section 5, the interest rate does not fully reveal the Fed's belief, because the market is uncertain about both dimensions of the Fed's belief. Instead, we will establish that the market interprets a surprise interest rate change according to the parameter

$$\tau = \frac{(\omega^s)^2 \left(1-\rho_\mu^2\right)\sigma_\mu^2}{(\omega^s)^2 \left(1-\rho_\mu^2\right)\sigma_\mu^2 + \sigma_{\mathbf{v}^F}^2}. \tag{C.1}$$

We refer to $\tau \in [0,1]$ as *the market's reaction type.*

The key assumption of this appendix is that *the Fed does not know the market's reaction type* $\tau$. We start with a benchmark case in which the Fed knows $\tau$. We then analyze an extreme case in which the Fed underestimates $\tau$, which is useful to illustrate the mechanics of tantrum shocks. We then consider a more common case in which the Fed is uncertain about $\tau$, which is useful to analyze the policy implications of tantrum shocks. We show that the fear of tantrums induces the Fed to act more gradually than in the baseline model. Finally, we discuss how communication policies between the Fed and the market can help mitigate tantrums.

## C.1. Benchmark when the Fed knows $\tau$

Our next result characterizes the equilibrium for the benchmark case in which the Fed knows the market's reaction type.

**Proposition 5.** *Consider the setup in which in (only) period $t$ the Fed believes the short-term component is distributed according to, $v_t \sim N\left(\Delta\mathbf{v}_t^F, \sigma_v^2\right)$. Suppose the market believes the Fed's short-term belief change is drawn from the distribution, $\Delta\mathbf{v}_t^F \sim N\left(0, \sigma_{\mathbf{v}^F}^2\right)$, and is independent of other random variables. Let $\tau \in (0,1)$ denote the market's reaction type given by (C.1). Suppose the Fed knows $\tau$.*

*In period $t$, the equilibrium interest rate is given by* (34)

$$\begin{aligned} i_t &= E_t^M[i_t] + (1-\varphi\tau)\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right), \\ \textit{where } E_t^M[i_t] &= \rho + (1-\varphi) E_t^M\left[\mathbf{g}_t^F\right] + \varphi \mathbf{g}_t^M. \end{aligned} \tag{C.2}$$

*The market's posterior belief after observing the interest rate is given by*

$$E_t^M\left[\mathbf{g}_t^F | i_t\right] - E_t^M\left[\mathbf{g}_t^F\right] = \tau \frac{i_t - E_t^M[i_t]}{1-\varphi\tau}, \tag{C.3}$$

*which is equal to $\tau\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)$ along the equilibrium path. A surprise increase in Fed optimism in period $t$ increases the current and the forward rates according to the market's reaction type:*

$$\frac{\Delta i_t}{\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F} = 1 - \varphi\tau \textit{ and } \frac{\Delta E_t^M[i_{t+h}]}{\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F} = \tau\varphi^h(1-\varphi) \textit{ for } h \geq 1. \tag{C.4}$$

*The surprise reduces the market's expectation for the current and future gaps as follows:*

$$\frac{\Delta E_t^M\left[\tilde{y}_t\right]}{\Delta \mathbf{g}_t^F+\Delta \mathbf{v}_t^F}=-1 \text{ and } \frac{\Delta E_t^M\left[\tilde{y}_{t+h}\right]}{\Delta \mathbf{g}_t^F+\Delta \mathbf{v}_t^F}=-\tau \varphi^h \text{ for } h \geq 1. \tag{C.5}$$

*From period $t+1$ onward, the equilibrium is the same as in Proposition 2.*

Eq. $(C.2)$ describes the Fed's *optimal* interest rate policy. The market's expected interest rate is determined as in Proposition 1. If the Fed is more optimistic than what the market expected, then it adjusts the interest rate according to the market's reaction type, $\tau$. Notice that an optimistic Fed hikes the interest rate by the same amount, $(1-\varphi\tau)\left(\Delta \mathbf{g}_t^F+\Delta \mathbf{v}_t^F\right)$, regardless of whether that optimism concerns the long-term or the short-term belief.

Eq. $(C.3)$ describes the market's posterior belief given the interest rate it observes. Along the equilibrium path, the interest rate is given by Eq. $(C.2)$. Thus, the market's posterior belief is *Bayesian* and given by $E_t^M\left[\mathbf{g}_t^F|i_t\right]-E_t^M\left[\mathbf{g}_t^F\right]=\tau\left(\Delta \mathbf{g}_t^F+\Delta \mathbf{v}_t^F\right)$. Higher-than-expected interest rates reveal a bundled signal of Fed optimism, but not whether the optimism is short term or long term. The market interprets the signal according to its reaction type, $\tau=\frac{(\omega^s)^2\left(1-\rho_\mu^2\right)\sigma_\mu^2}{(\omega^s)^2\left(1-\rho_\mu^2\right)\sigma_\mu^2+\sigma_{\mathbf{v}^F}^2}$. When $\tau$ is high, the market is more uncertain about the long-term belief and attributes high interest rates to long-term optimism.

Eq. $(C.4)$ describes how the current and the forward interest rates respond to a surprise increase in Fed optimism. The interest rate responses are determined by the market's reaction type—rather than by the Fed's actual belief type. When the market is reactive, $\tau=1$, the responses are the same as in the baseline model with long-term optimism (see Proposition 2). When the market is unreactive, $\tau=0$, the current rate increases substantially (by the amount of the Fed's optimism) but the forward rates remain unchanged.

Why does the market's reaction type drive the current and forward interest rates? Forward rates are naturally determined by the market's reaction, as these rates reflect the market's belief. The current rate is also determined by the market's reaction, because the Fed *optimally* responds to the market's reaction. As before, the Fed targets an overall increase in the current and forward interest rates that counteracts its initial optimism. When the market is more reactive, the forward rates increase by more and the Fed hikes the current interest rate by less—closer to the baseline case in which it has long-term optimism [cf. Figure 4].

Finally, Eq. $(C.5)$ describes how the market's expected output gaps respond to a surprise increase in Fed optimism. For the current period ($h=0$), the market's expected output gap decreases one-to-one with the Fed's optimism, as in the baseline model (see Proposition 2). The reason is that the Fed targets an overall change in the current and forward rates that exactly counteracts its optimism. Therefore, a surprise increase in Fed optimism reduces the market's expectation of the current output gap one-to-one (and it leaves the Fed's expectation of the current output gap unchanged). For future periods ($h>1$), the market's expected output gap responds according to its reaction type. When the market is reactive, $\tau=1$, it expects a decline also in future output gaps. When the market is unreactive, $\tau = 0$, it does not expect a decline in future output gaps.

**Proof of Proposition 5.** First consider the equilibrium from period $t+1$ onward. Since the Fed does not have a short-term belief change in these periods, the equilibrium is the same as in Proposition 2. In particular, in subsequent periods the Fed's interest rate decision fully reveals its belief. This also verifies that $\frac{dE_t^F\left[V_{t+1}^F\right]}{di_t} = 0$. As before, the Fed's problem (3) in period $t$ is effectively static and the optimality condition is given by (4).

Next consider the equilibrium in period $t$. We conjecture that the interest rate rule in Eq. $(C.2)$ is optimal for the Fed and the belief updating rule in Eq. $(C.3)$ is Bayesian for the market along the equilibrium path.

Consider the Fed's optimal interest rate decision in period $t$. As before, this depends on the market's expected output gap for the next period $t+1$. The output gap in period $t+1$ is still given by Eq. (22), $\tilde{y}_{t+1} = g_{t+1} - \mathbf{g}_{t+1}^F$. Combining this with Lemma 2, the market's expected output gap in period $t$ *after* the interest rate decision is given by

$$\begin{aligned} \overline{E}_t^M\left[\tilde{y}_{t+1}\right] &= \overline{E}_t^M\left[\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F\right] = \varphi\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F | i_t\right]\right), \\ \text{where } & E_t^M\left[\mathbf{g}_t^F | i_t\right] - E_t^M\left[\mathbf{g}_t^F\right] = \tau \frac{i_t - E_t^M\left[i_t\right]}{1-\varphi\tau}. \end{aligned} \tag{C.6}$$

The second line substitutes the market's belief updating rule from $(C.3)$.

Eq. $(C.6)$ implies $\frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}\right]}{di_t} = -\frac{\varphi\tau}{1-\varphi\tau}$. Substituting this into the Fed's optimality condition (4), we obtain $E_t^F\left[\left(1+\frac{\varphi\tau}{1-\varphi\tau}\right)\tilde{y}_t\right] = 0$. Since $\frac{\varphi\tau}{1-\varphi\tau}$ is constant, this simplifies to Eq. (5) as before, $E_t^F\left[\tilde{y}_t\right] = 0$. Thus, the Fed's optimal interest rate is still characterized by Eq. (6)

$$\begin{aligned} i_t &= \rho + E_t^F\left[g_t\right] + E_t^F\left[\overline{E}_t^M\left[\tilde{y}_{t+1}\right]\right] \\ &= \rho + E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F + \varphi\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F | i_t\right]\right) \\ &= \rho + E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F + \varphi\left(\mathbf{g}_t^M - \left(E_t^M\left[\mathbf{g}_t^F\right] + \tau\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)\right)\right) \\ &= E_t^M\left[i_t\right] + (1-\varphi\tau)\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right). \end{aligned}$$

The second line substitutes Eq. $(C.6)$ along with the Fed's belief for period $t$, $E_t^F\left[g_t\right] = E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F$. The third line substitutes $E_t^M\left[\mathbf{g}_t^F | i_t\right] = E_t^M\left[\mathbf{g}_t^F\right] + \tau\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)$, which holds along the equilibrium path. The last line simplifies the expression. This proves that the interest rate rule in Eq. $(C.2)$ is optimal for the Fed.

Next consider the market's belief updating rule in period $t$. Along the equilibrium path, the interest rate policy in $(C.2)$ provides the market with an imperfect signal of the Fed's long-term belief change,

$$\frac{i_t - E^M\left[i_t\right]}{1-\varphi\tau} = \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F \sim N\left(\Delta\mathbf{g}_t^F, \sigma_{\mathbf{v}^F}^2\right).$$

The market combines the signal with its prior belief, $\Delta \mathbf{g}_t^F \sim N\left(0, (\omega^s)^2 \left(1-\rho_\mu^2\right) \sigma_\mu^2\right)$ [see (29)]. The Bayesian posterior is then given by,

$$E_t^M\left[\Delta \mathbf{g}_t^F | i_t\right] = \frac{\frac{1}{\sigma_{\mathbf{v}^F}^2}}{\frac{1}{(\omega^s)^2\left(1-\rho_\mu^2\right)\sigma_\mu^2} + \frac{1}{\sigma_{\mathbf{v}^F}^2}} \frac{i_t - E^M\left[i_t\right]}{1-\varphi\tau} = \tau \frac{i_t - E^M\left[i_t\right]}{1-\varphi\tau},$$

where $\tau$ is given by Eq. $(C.1)$. In particular, the belief updating rule in Eq. $(C.3)$ is Bayesian for the market along the equilibrium path. This verifies the conjectured equilibrium.

Next consider Eq. $(C.4)$ that describes the impact of a Fed belief surprise on the current and forward interest rates. The impact on the current rate follows from Eq. $(C.2)$. Consider the impact on the forward rates for horizons $h \geq 1$. From period $t+1$ onward, the equilibrium is the same as before. Taking expectations of Eq. (21) in period $t+1$ conditional on $\mathbf{g}_{t+1}^F$ and using Lemma 3, we obtain

$$\begin{aligned} E_{t+1}^M\left[i_{t+h} | \mathbf{g}_{t+1}^F\right] &= \rho + (1-\varphi) E_{t+1}^M\left[\mathbf{g}_{t+h}^F | \mathbf{g}_{t+1}^F\right] + \varphi\left(E_{t+1}^M\left[\mathbf{g}_{t+h}^M\right]\right) \\ &= \rho + (1-\varphi)\left(\varphi^{h-1}\mathbf{g}_{t+1}^F + \left(1-\varphi^{h-1}\right)\mathbf{g}_{t+1}^M\right) + \varphi \mathbf{g}_{t+1}^M. \end{aligned}$$

Taking the expectation of this expression in period $t$ *after the interest rate decision*, and using Lemma 3 once more, we obtain

$$\begin{aligned} E_t^M\left[i_{t+h} | i_t\right] &= \rho + (1-\varphi)\varphi^{h-1} E_t^M\left[\mathbf{g}_{t+1}^F | i_t\right] + \left(1-(1-\varphi)\varphi^{h-1}\right) E_t^M\left[\mathbf{g}_{t+1}^M\right] \\ &= \rho + (1-\varphi)\varphi^{h-1}\left(\varphi E_t^M\left[\mathbf{g}_t^F | i_t\right] + (1-\varphi)\mathbf{g}_t^M\right) + \left(1-(1-\varphi)\varphi^{h-1}\right)\mathbf{g}_t^M \\ &= \rho + (1-\varphi)\varphi^h E_t^M\left[\mathbf{g}_t^F | i_t\right] + \left(1-(1-\varphi)\varphi^h\right)\mathbf{g}_t^M. \end{aligned}$$

Taking the expectation in period $t$ *before the interest rate decision*, we have the same expression with $E_t^M\left[\mathbf{g}_t^F | i_t\right]$ replaced by $E_t^M\left[\mathbf{g}_t^F\right]$. Combining these observations, and using $E_t^M\left[\mathbf{g}_t^F | i_t\right] - E_t^M\left[\mathbf{g}_t^F\right] = \tau\left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right)$, we obtain

$$\frac{\Delta E_t^M\left[i_{t+h}\right]}{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F} = \tau(1-\varphi)\varphi^h \text{ for } h \geq 1.$$

This establishes Eq. $(C.4)$.

We finally establish Eqs. $(C.5)$ that describe the impact of a Fed belief surprise on the market's expectation for the current and future output gaps.

First consider the market's expected output gaps in future periods ($h \geq 1$). From period $t+1$ onward, the equilibrium is the same as before. Therefore, taking the expectation of Eq. (22) conditional on $\mathbf{g}_{t+1}^F$ and using Lemma 2, we obtain

$$E_{t+1}^M\left[\tilde{y}_{t+h} | \mathbf{g}_{t+1}^F\right] = E_{t+1}^M\left[\mathbf{g}_{t+h}^M - \mathbf{g}_{t+h}^F\right] = \varphi^{h-1}\left(\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F\right).$$

Taking the expectation of this expression in period $t$ *after* the interest rate decision, and using Lemma 2 once more, we obtain

$$E_t^M\left[\tilde{y}_{t+h}|i_t\right] = \varphi^h\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F|i_t\right]\right) \text{ for } h \geq 1.$$

Taking the same expectation *before* the interest rate decision, we obtain the same expression with $E_t^M\left[\mathbf{g}_t^F|i_t\right]$ replaced by $E_t^M\left[\mathbf{g}_t^F\right]$. Taking the difference between these expressions and substituting $E_t^M\left[\mathbf{g}_t^F|i_t\right] = E_t^M\left[\mathbf{g}_t^F\right] + \tau\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)$, we prove Eq. $(C.5)$ for future output gaps, $\Delta E_t^M\left[\tilde{y}_{t+h}\right] = -\tau\varphi^h$ for $h \geq 1$.

Next consider the market's expected output gap in the current period ($h = 0$). Recall that Eq. (7) still applies,

$$\tilde{y}_t = g_t - E_t^F\left[g_t\right] + \overline{E}_t^M\left[\tilde{y}_{t+1}\right] - E_t^F\left[\overline{E}_t^M\left[\tilde{y}_{t+1}\right]\right].$$

From the previous analysis, we have $\overline{E}_t^M\left[\tilde{y}_{t+1}\right] = \varphi\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F|i_t\right]\right)$. This in turn implies $\overline{E}_t^M\left[\tilde{y}_{t+1}\right] = E_t^F\left[\overline{E}_t^M\left[\tilde{y}_{t+1}\right]\right]$, because the Fed knows $E_t^M\left[\mathbf{g}_t^F|i_t\right] = E_t^M\left[\mathbf{g}_t^F\right] + \tau\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)$. Combining these observations and substituting $E_t^F\left[g_t\right] = \mathbf{g}_t^F + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F$, the current output gap satisfies

$$\tilde{y}_t = g_t - \left(\mathbf{g}_t^F + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right).$$

Taking the expectation of the output gap under the market's belief *before* and *after* the interest rate decision, we obtain

$$E_t^M\left[\tilde{y}_t|i_t\right] = \mathbf{g}_t^M - \left(\mathbf{g}_t^F + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right) \text{ and } E_t^M\left[\tilde{y}_t\right] = \mathbf{g}_t^M - \mathbf{g}_t^F.$$

Taking the difference between these expressions, we establish Eq. $(C.5)$ for the current period, $\frac{\Delta E_t^M\left[\tilde{y}_t\right]}{\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F} = -1$. This completes the proof of the proposition. □

## C.2. Mechanics of tantrum shocks

We next analyze an extreme case in which the Fed thinks the market is unreactive ($\tau = 0$), whereas the market is actually reactive ($\tau = 1$). This case helps illustrate the mechanics of tantrum shocks. The analysis of this case is mostly presented in the main text. Here, we derive Eqs. $(36 - 38)$.

As we explain in the main text, the Fed sets the policy rate in (35) because it believes the market is unreactive, $\tau = 0$. However, the market is actually reactive, $\tau = 1$, and thinks the Fed knows this. Therefore, after seeing the policy rate in (35), the market's posterior belief for the Fed's long-term belief becomes

$$E_t^M\left[\mathbf{g}_t^F|i_t\right] - E_t^M\left[\mathbf{g}_t^F\right] = \frac{\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F}{1 - \varphi}. \tag{C.7}$$

Proposition 5 then applies for the market reaction type, $\tau = 1$, and the "as-if" Fed belief change in $(C.7)$. In particular, Eq. $(C.4)$ implies

$$\Delta E_t^M \left[ i_{t+h} \right] = \frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi} \varphi^h \left(1-\varphi\right) \text{ for } h \geq 1.$$

This proves Eq. (36). Likewise, Eqs. $(C.5)$ imply

$$\Delta E_t^M \left[ \tilde{y}_t \right] = -\frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi} \text{ and } \Delta E_t^M \left[ \tilde{y}_{t+h} \right] = -\frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi} \varphi^h \text{ for } h \geq 1.$$

This proves Eq. (37).

Finally, we establish Eq. (38) that describes the Fed's expected output gap in the current period. The output gap is still given by (7),

$$\tilde{y}_t = g_t - E_t^F \left[ g_t \right] + \overline{E}_t^M \left[ \tilde{y}_{t+1} | \tau = 1 \right] - E_t^F \left[ \overline{E}_t^M \left[ \tilde{y}_{t+1} | \tau = 0 \right] \right]. \tag{C.8}$$

Here, we have written conditional expectations to incorporate the fact that the Fed sets the output gap thinking the market is unreactive, $\tau = 0$, but the market actually is reactive, $\tau = 1$. Note also that that Eq. $(C.6)$ applies conditional on the market's reaction type, $\tau$, and its posterior belief, $E_t^M \left[ \mathbf{g}_t^F | i_t \right]$. Applying the equation for $\tau = 1$, we obtain

$$\overline{E}_t^M \left[ \tilde{y}_{t+1} | \tau = 1 \right] = \varphi \left( \mathbf{g}_t^M - E_t^M \left[ \mathbf{g}_t^F | i_t \right] \right) = \varphi \left( \mathbf{g}_t^M - E_t^M \left[ \mathbf{g}_t^F \right] \right) - \varphi \frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi}.$$

Applying the equation for $\tau = 0$, we obtain

$$\overline{E}_t^M \left[ \tilde{y}_{t+1} | \tau = 0 \right] = \varphi \left( \mathbf{g}_t^M - E_t^M \left[ \mathbf{g}_t^F \right] \right).$$

Substituting these expressions into Eq. $(C.8)$, we obtain

$$\tilde{y}_t = g_t - E_t^F \left[ g_t \right] - \varphi \frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi}.$$

Taking the expectation under the Fed's belief, we prove Eq. (38), $E_t^F \left[ \tilde{y}_t | \tau = 1 \right] = -\frac{\varphi \left( \Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F \right)}{1-\varphi}$. This completes the derivations omitted from the main text.

In this extreme case, the Fed operates under the assumption that the market is unreactive and will interpret its interest rate change as temporary. Thus, the Fed is ex-post surprised when the market is revealed to be reactive and misses its output gap target even under its own belief.

### C.3. Policy implication of tantrums: Gradualism

We next turn to the policy implications of tantrums. To analyze policy, we analyze a less extreme case in which the Fed is uncertain about the market's reaction type. Our next result characterizes the equilibrium for this case and shows that the Fed acts *even more gradually* than in our baseline model.

**Proposition 6.** *Consider the setup in Proposition 5 with the difference that in period $t$ the market can have one of two types, $\tau \in \{0,1\}$. The Fed believes $\tau = 1$ with probability $\delta \in (0,1)$. The market knows $\delta$.*

*In period $t$, the equilibrium interest rate is given by Eq.* (39)

$$i_t = E_t^M [i_t] + \left(1 - \varphi\tilde{\delta}\right) \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right), \tag{C.9}$$

*where $\tilde{\delta}$ is the unique root of the following quadratic over the range $x \in (\delta, 1)$:*

$$P(x) = x^2 \varphi - x(1 + 2\delta\varphi) + \delta(\varphi + 1). \tag{C.10}$$

*The market's posterior belief after observing the interest rate is given by*

$$E_t^M \left[\mathbf{g}_t^F | i_t\right] - E_t^M \left[\mathbf{g}_t^F\right] = \tau \frac{i_t - E_t^M [i_t]}{1 - \varphi\tilde{\delta}}, \tag{C.11}$$

*which is equal to $\tau\left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right)$ along the equilibrium path. The Fed's ex-ante expected output gap in period $t$ is given by Eq.* (40)

$$E_t^F [\tilde{y}_t] = \left(\tilde{\delta} - \delta\right) \varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right). \tag{C.12}$$

*The Fed's output gap conditional on the market's type is given by*

$$E_t^F [\tilde{y}_t | \tau = 1] = -\left(1 - \tilde{\delta}\right) \varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right) \text{ and } E_t^F [\tilde{y}_t | \tau = 0] = \tilde{\delta}\varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right). \tag{C.13}$$

*From period $t+1$ onward, the equilibrium is the same as in Proposition 2.*

Eq. ($C$.11) is the same as in Section C.1 in which the Fed knows the market's reaction type (see ($C$.3)). Along the equilibrium path, the market extracts a bundled optimism signal and forms a posterior belief that depends on its reaction type. Eq. (39) is different and says that the Fed acts as if the market is more reactive than implied by its ex-ante mean, $\tilde{\delta} > \delta = E_t^F [\tau]$ (cf. ($C$.2)).

Eq. ($C$.12) characterizes the Fed's expected output gap. The Fed misses its *ex-ante* output gap target *even under its own belief.* For instance, when the Fed is more optimistic than what the market expected, $\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F > 0$, it leaves a positive output gap on average. Eqs. ($C$.13) characterize the Fed's expected output gap *conditional on the market's type.* An optimistic Fed expects a negative output gap when the market is revealed to be reactive—a milder version of the tantrum shocks from the earlier Section C.2. Conversely, the Fed expects a positive output gap when the market is unreactive.

Why does the Fed hike the interest rate more cautiously and leave a positive output gap on average? Unlike in the previous versions of the model, the Fed is uncertain about how a *change* in its policy interest rate $i_t$ will affect the output gap. If the market is reactive, an interest rate hike increases the market's perception for the Fed's long-term optimism. Consequently, the interest rate hike also reduces the market's expected future output gap, $\frac{d\overline{E}_t^M[\tilde{y}_{t+1}|i_t,\tau=1]}{di_t} < 0$. In view of the IS curve (2), this creates a large impact on the current output gap, $\frac{d\tilde{y}_t[\tau=1]}{di_t} < -1$. In contrast, if the market is unreactive, an interest rate hike does not change the market's expected future output gap, $\frac{d\overline{E}_t^M[\tilde{y}_t|i_t,\tau=0]}{di_t} = 0$, and it has a smaller impact on the current output gap, $\frac{d\tilde{y}_t[\tau=0]}{di_t} = -1$. Since the economy is more sensitive to the Fed's interest rate decision when the market is reactive, the Fed overweights that case in its decision, $\tilde{\delta} > \delta$ [cf. Eq. (4)]. Therefore, the Fed acts as if the market is more reactive than implied by its prior mean belief, and adjusts the interest rate by a small amount. By acting conservatively, the Fed misses its output gap on average but it mitigates the tantrum shock that exacerbates its miss when the market is revealed to be reactive.

**Proof of Proposition 6.** We first check that the quadratic in $(C.10)$ has a unique root over the range, $x \in (\delta, 1)$. Note that $P(\delta) = \delta(1-\delta)\varphi > 0$ and $P(1) = (1-\delta)(\varphi - 1) < 0$. Since $P(\cdot)$ is an upward sloping parabola, these conditions imply that $P(x)$ has exactly one root that falls in the interval $(\delta, 1)$.

Next consider the equilibrium from period $t+1$ onward. Once the Fed sets the interest rate and observes the forward interest rate's reaction to it, the Fed learns the market's reaction type $\tau$ (see Proposition 5). Therefore, the equilibrium in subsequent periods is the same as in Proposition 2. This also verifies that $\frac{dE_t^F[V_{t+1}^F]}{di_t} = 0$. As before, the Fed's problem (3) in period $t$ is effectively static and the optimality condition is given by (4).

Consider the equilibrium in period $t$. We conjecture that the interest rate rule in Eq. $(C.9)$ is optimal for the Fed and the belief updating rule in Eq. $(C.11)$ is Bayesian for the market along the equilibrium path.

Consider the Fed's optimal interest rate decision in period $t$. Once the Fed learns $\tau$, the analysis is the same as in Section C.1. Following the steps in the proof of Proposition 5, we obtain the following analogue of $(C.6)$:

$$\overline{E}_t^M[\tilde{y}_{t+1}|\tau] = \varphi\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F|i_t,\tau\right]\right) \quad \text{(C.14)}$$
$$\text{where } E_t^M\left[\mathbf{g}_t^F|i_t,\tau\right] - E_t^M\left[\mathbf{g}_t^F\right] = \tau\frac{i_t - E_t^M[i_t]}{1-\varphi\tilde{\delta}}.$$

This expression implies $\frac{d\overline{E}_t^M[\tilde{y}_{t+1}|\tau]}{di_t} = -\frac{\varphi\tau}{1-\varphi\tilde{\delta}}$. Substituting this into the Fed's optimality condition (4), we obtain,

$$E_t^F\left[\frac{d\tilde{y}_t}{di_t}\tilde{y}_t\right] = 0 \text{ where } \frac{d\tilde{y}_t}{di_t} = -\left(1 + \frac{\varphi\tau}{1-\varphi\tilde{\delta}}\right). \quad \text{(C.15)}$$

The marginal policy impact, $\frac{d\tilde{y}_t}{di_t}$, depends on the market's reaction type, $\tau$. Since the Fed is uncertain about the market's type, this term does *not* drop out of the expectation. Therefore, unlike the equilibria we analyzed so far, the Fed's expected output gap, $E_t^F[\tilde{y}_t]$, is not necessarily zero.

To characterize the optimal policy further, we rewrite Eq. $(C.15)$ in terms of conditional expectations

$$0 = -E_t^F\left[\frac{d\tilde{y}_t}{di_t}\tilde{y}_t\right] = \delta\left(1+\frac{\varphi}{1-\varphi\tilde{\delta}}\right)E_t^F[\tilde{y}_t|\tau=1] + (1-\delta)E_t^F[\tilde{y}_t|\tau=0].$$

Note that the root of the quadratic in Eq. $(C.10)$ satisfies

$$\tilde{\delta} = \frac{\delta\left(1+\frac{\varphi}{1-\varphi\tilde{\delta}}\right)}{\delta\left(1+\frac{\varphi}{1-\varphi\tilde{\delta}}\right)+1-\delta}.$$

Therefore, the Fed's optimality condition can be equivalently written as

$$0 = \tilde{\delta}E_t^F[\tilde{y}_t|\tau=1] + \left(1-\tilde{\delta}\right)E_t^F[\tilde{y}_t|\tau=0]. \tag{C.16}$$

Hence, the Fed targets a *weighted average* of the output gap over the cases in which the market is reactive and unreactive. The weight for the reactive case is given by the endogenous parameter, $\tilde{\delta}$, which exceeds the prior probability of this case, $\tilde{\delta} > \delta$.

We next solve for the optimal interest rate. Substituting the IS curve (2) into $(C.16)$, we obtain

$$i_t = \rho + E_t^F[g_t] + \tilde{\delta}\overline{E}_t^M[\tilde{y}_{t+1}|\tau=1] + \left(1-\tilde{\delta}\right)\overline{E}_t^M[\tilde{y}_{t+1}|\tau=0]. \tag{C.17}$$

This in turn implies

$$\begin{aligned}
i_t &= \rho + E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F + \varphi\begin{pmatrix} \tilde{\delta}\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F|i_t,\tau=1\right]\right) \\ +\left(1-\tilde{\delta}\right)\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F|i_t,,\tau=0\right]\right)\end{pmatrix} \\
&= \rho + E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F + \varphi\begin{pmatrix} \tilde{\delta}\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F\right] - \left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)\right) \\ +\left(1-\tilde{\delta}\right)\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F\right]\right)\end{pmatrix} \\
&= E_t^M[i_t] + \left(1-\varphi\tilde{\delta}\right)\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right).
\end{aligned}$$

Here, the first line substitutes Eq. $(C.14)$ along with the Fed's belief for period $t$, $E_t^F[g_t] = E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F$. The second line substitutes $E_t^M\left[\mathbf{g}_t^F|i_t,\tau\right] = E_t^M\left[\mathbf{g}_t^F\right] + \tau\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)$ for each $\tau \in \{0,1\}$, which holds along the equilibrium path. The last line simplifies the expression. This proves that the interest rate rule in $(C.9)$ is optimal for the Fed.

Next consider the market's belief updating rule in period $t$. Along the equilibrium path, the interest rate policy in $(C.9)$ provides the market with an imperfect signal of the Fed's long-term belief change, $\frac{i_t - E^M[i_t]}{1-\varphi\tilde{\delta}} = \Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F$. Following the same steps as in the proof of Proposition 5, we establish that the belief updating rule in $(C.11)$ is Bayesian for the market along the equilibrium path. This verifies the conjectured equilibrium path.

We next establish Eqs. $(C.12 - C.13)$. To this end, we substitute the interest rate from $(C.17)$ into the IS curve (2) to obtain

$$\tilde{y}_t = g_t - E_t^F[g_t] + \overline{E}_t^M[\tilde{y}_{t+1}|\tau] - \left\{\tilde{\delta}\overline{E}_t^M[\tilde{y}_{t+1}|\tau=1] + \left(1-\tilde{\delta}\right)\overline{E}_t^M[\tilde{y}_{t+1}|\tau=0]\right\}.$$

Taking the Fed's expectation conditional on $\tau = 1$, we obtain

$$\begin{aligned} E_t^F[\tilde{y}_t|\tau=1] &= \left(1-\tilde{\delta}\right)\left(\overline{E}_t^M[\tilde{y}_{t+1}|\tau=1] - \overline{E}_t^M[\tilde{y}_{t+1}|\tau=0]\right) \\ &= -\left(1-\tilde{\delta}\right)\varphi\begin{pmatrix} \left(\overline{E}_t^M\left[\mathbf{g}_t^F|i_t,\tau=1\right] - E_t^M\left[\mathbf{g}_t^F\right]\right) \\ -\left(\overline{E}_t^M\left[\mathbf{g}_t^F|i_t,\tau=0\right] - E_t^M\left[\mathbf{g}_t^F\right]\right) \end{pmatrix} \\ &= -\left(1-\tilde{\delta}\right)\varphi\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right). \end{aligned}$$

The second line substitutes Eqs. $(C.14)$ and the third line substitutes $E_t^M\left[\mathbf{g}_t^F|i_t,\tau\right] = E_t^M\left[\mathbf{g}_t^F\right] + \tau\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)$ for each $\tau \in \{0,1\}$. Likewise, taking the Fed's expectation conditional on $\tau = 0$, we obtain

$$\begin{aligned} E_t^F[\tilde{y}_t|\tau=0] &= \tilde{\delta}\left(\overline{E}_t^M[\tilde{y}_{t+1}|\tau=0] - \overline{E}_t^M[\tilde{y}_{t+1}|\tau=1]\right) \\ &= -\tilde{\delta}\varphi\begin{pmatrix} \left(\overline{E}_t^M\left[\mathbf{g}_t^F|i_t,\tau=0\right] - E_t^M\left[\mathbf{g}_t^F\right]\right) \\ -\left(\overline{E}_t^M\left[\mathbf{g}_t^F|i_t,\tau=1\right] - E_t^M\left[\mathbf{g}_t^F\right]\right) \end{pmatrix} \\ &= \tilde{\delta}\varphi\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right). \end{aligned}$$

This proves Eq. $(C.13)$. Finally, note that the unconditional expectation is given by

$$\begin{aligned} E_t^F[\tilde{y}_t] &= \delta E_t^F[\tilde{y}_t|\tau=1] + (1-\delta)E_t^F[\tilde{y}_t|\tau=0] \\ &= \left(-\delta\left(1-\tilde{\delta}\right) + (1-\delta)\tilde{\delta}\right)\varphi\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right) \\ &= \left(\tilde{\delta} - \delta\right)\varphi\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right). \end{aligned}$$

This establishes Eq. $(C.12)$ and completes the proof of the proposition. □

## C.4. Policy implication of tantrums: Communication

Proposition 6 shows that, despite acting conservatively, the Fed misses its output gap conditional on the market's type. Therefore, the possibility of tantrum shocks increases the Fed's ex-ante expected gaps in (3). When the market is uncertain about the Fed's belief, its reaction type $\tau$ becomes a key parameter for policy. If the Fed is confused about $\tau$, there can be extreme tantrum shocks as in Section C.2. If the Fed is uncertain about $\tau$, there are still (milder) tantrum shocks that make the Fed miss its output target more often than without these shocks.

The welfare losses induced by tantrum shocks create a natural role for communication between the Fed and the market. First, the Fed can try to figure out the market's reaction type $\tau$. Second, the Fed can try to reveal its own belief to the market—making the market's reaction type irrelevant to the equilibrium and therefore mitigating the tantrum shocks. The following result formalizes the second point. In our model with two belief types, the Fed can reveal its belief by announcing the average interest rate it plans to set in the next period in addition to the current rate.

**Proposition 7.** *Consider the setup in Proposition 6 in which the Fed can have a short-term belief change in period t as well as a long-term belief change, and the Fed is uncertain about the market's reaction type $\tau$. Suppose in period t the Fed announces both the current interest rate, $i_t$, and the interest rate it expects to set in the next period, $i^F_{t+1} \equiv E^F_t [i_{t+1}]$. In equilibrium, the Fed's announcements are truthful and given by*

$$\begin{aligned} i_t &= \rho + (1-\varphi)\,\mathbf{g}^F_t + \varphi \mathbf{g}^M_t + \Delta \mathbf{v}^F_t \\ i^F_{t+1} = E^F_t [i_{t+1}] &= \rho + \left(1-\varphi^2\right)\mathbf{g}^F_t + \varphi^2 \mathbf{g}^M_t . \end{aligned} \tag{C.18}$$

*These announcements fully reveal both dimensions of the Fed's belief change, $\Delta \mathbf{g}^F_t = \mathbf{g}^F_t - E^M_t \left[\mathbf{g}^F_t\right]$ and $\Delta \mathbf{v}^F_t$. The Fed achieves a zero expected output gap under its belief regardless of the market's reaction type, $E^F_t [\tilde{y}_t] = E^F_t [\tilde{y}_t|\tau] = 0$.*

Intuitively, by announcing two interest rates, the Fed can fully reveal both dimensions of its belief change. The expected rate in the next period reveals the Fed's long-term belief change, $\Delta \mathbf{g}^F_t$. The current rate then reveals the Fed's short-term belief change, $\Delta \mathbf{v}^F_t$. Once the market learns the Fed's belief, the equilibrium is similar to the baseline setting in which the market's reaction type does not play a role (cf. Proposition 1). We only need to adapt the analysis to incorporate the fact that the Fed's short-term belief, $\Delta \mathbf{v}^F_t$, is not necessarily zero. The current interest rate increases one-to-one with $\Delta \mathbf{v}^F_t$, since this belief change does not persist into future periods and the market knows this.

Proposition 7 provides a rationale for the enhanced Fed communication that we have seen in recent years, e.g., "the forward guidance" or "the dot curve". In our model, the role of these policies is *not* to persuade the market—the market is opinionated. Rather, communication is useful because it helps reveal the Fed's belief to the market, reducing the chance of tantrum shocks in which the market misinterprets the Fed's belief.

**Proof of Proposition 7.** We verify that it is optimal for the Fed to announce the interest rates in ($C$.18) in period $t$. After seeing the announcements, $\left(i_t, i^F_{t+1}\right)$, the market infers the

Fed's long-term and short-term beliefs as

$$\mathbf{G}^F\left(i_t, i_{t+1}^F\right) \equiv \frac{i_{t+1}^F - \rho - \varphi^2 \mathbf{g}_t^M}{1-\varphi^2}, \tag{C.19}$$
$$\boldsymbol{\Delta}\mathbf{V}^F\left(i_t, i_{t+1}^F\right) \equiv i_t - \rho - (1-\varphi)\,\mathbf{G}^F\left(i_t, i_{t+1}^F\right) - \varphi \mathbf{g}_t^M.$$

Along the equilibrium path, the market's inferences are correct, $\mathbf{G}^F\left(i_t, i_{t+1}^F\right) = \mathbf{g}_t^F$ and $\boldsymbol{\Delta}\mathbf{V}^F\left(i_t, i_{t+1}^F\right) = \Delta \mathbf{v}_t^F$.

As before, the equilibrium in subsequent periods is the same as in Proposition 2. This implies the Fed's expected continuation value $E_t^F\left[V_{t+1}^F\right]$ does not depend on its policy choice in period $t$. Thus, the Fed's problem (3) is effectively static and its optimality conditions are given by the following analogues of Eq. (4),

$$E_t^F\left[\left(-1 + \frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}\right]}{di_t}\right)\tilde{y}_t\right] = E_t^F\left[\left(-1 + \frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}\right]}{di_{t+1}^F}\right)\tilde{y}_t\right] = 0. \tag{C.20}$$

After seeing the Fed's policy announcements, the market *thinks* the continuation equilibrium will be the same as in Proposition 2 given the Fed's beliefs in ($C$.19). In particular, Eq. (26) implies

$$\overline{E}_t^M\left[\tilde{y}_{t+1}\right] = \varphi\left(\mathbf{g}_t^M - \mathbf{G}^F\left(i_t, i_{t+1}^F\right)\right).$$

Using Eq. ($C$.19), we obtain $\frac{d\overline{E}_t^M[\tilde{y}_{t+1}]}{di_t} = 0$ and $\frac{d\overline{E}_t^M[\tilde{y}_{t+1}]}{di_{t+1}^F} = -\frac{\varphi}{1-\varphi^2}$. Substituting these expressions into ($C$.20), the Fed's optimality conditions simplify to $E_t^F\left[\tilde{y}_t\right] = 0$ as in the baseline model (see (5)).

We next show that the policy announcements in ($C$.18) achieve a zero output gap under the Fed's belief, $E_t^F\left[\tilde{y}_t\right] = 0$, and therefore are optimal. Along the equilibrium path, the market infers the Fed's belief correctly and thinks, $\overline{E}_t^M\left[\tilde{y}_{t+1}\right] = \varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right)$. Substituting this into the IS curve (2) along with the expression for the current interest rate we obtain

$$\begin{aligned}\tilde{y}_t &= -\left((1-\varphi)\,\mathbf{g}_t^F + \varphi \mathbf{g}_t^M + \Delta \mathbf{v}_t^F\right) + g_t + \varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right) \\ &= g_t - \mathbf{g}_t^F - \Delta \mathbf{v}_t^F.\end{aligned}$$

Taking the expectation under the Fed's belief establishes $E_t^F\left[\tilde{y}_t\right] = 0$. Thus, it is optimal for the Fed to follow the announcements in ($C$.18). This analysis also implies $E_t^F\left[\tilde{y}_t\right] = E_t^F\left[\tilde{y}_t | \tau\right] = 0$, completing the proof. □

# D. Data details and omitted empirical results

This appendix contains the details of our data sources and variable construction, and the empirical results omitted from the main text.

## D.1. Data sources

**Federal funds rate (FFR).** In Figure 1, we plot the Federal funds rate (FFR). These data are public and obtained from the Fed Board, retrieved through FRED. The corresponding FRED ticker is "FEDFUNDS". We use the monthly version of the series.

**Dates of FOMC meetings:** These data are public and obtained from Nakamura and Steinsson (2018*a*,*b*).

**The Fed's Greenbook/Tealbook forecasts.** In the left panel of Figure 1 as well as in Figure 2, we use Greenbook/Tealbook forecasts for the FFR and for the GDP price index. These forecasts are produced by the Fed research staff before each FOMC meeting. The data come from two sources:

- **Digital Greenbook/Tealbook data from the Philadelphia Fed.** These data are public and obtained from the Philadelphia Fed.
  - The predictions for the GDP price index inflation come from the main Greenbook data set (available at `https://www.philadelphiafed.org/-/media/frbp/assets/surveys-and-data/greenbook-data/documentation/gbweb_row_format.xlsx`). This data set is at the FOMC-meeting frequency and quarterly forecasting horizon. For most of our sample period, the data report the Greenbook projections for the annualized quarterly growth rate in the price index for GDP.[1] The data are available until the end of 2013.
  - The predictions for the FFR come from the supplement data set (available at `https://www.philadelphiafed.org/-/media/frbp/assets/surveys-and-data/greenbook-data/greenbook_financial_assumptions_interestrates_web.xls`). This data set contains the Fed staff's assumptions for the future values of financial variables, including the FFR, other interest rates, and equity prices. We focus on the assumptions for the FFR. These assumptions are available at a quarterly forecasting horizon. Each assumption corresponds to the quarterly average effective federal funds rate. The data are available until the sixth meeting of 2008.[2]
- **Hand-collected Greenbook/Tealbook data:** We collected these data from public sources (Caballero and Simsek (2022)). Specifically, since the digital data for the FFR

[1] Before December 11, 1991, the data report the projections for the price index growth for the GNP implicit deflator. Between December 11, 1991 and March 21, 1996, the data report the projections for the price index growth for the GDP implicit deflator.

[2] In some cases, the assumption is appended with a "+" or "-", suggesting that the value was likely to be a bit higher (in the case of a "+") or a bit lower (in the case of a "–") than the value to which they were appended. We have ignored these additional suffixes. For further information on this data set, see the notes on the Philadelphia Fed's website: `https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/gap-and-financial-data-set`

predictions are available for a shorter time period than the digital data for inflation predictions, we hand-collected the FFR predictions for the missing meetings to obtain a longer time series. We obtained the predictions from either the corresponding Greenbook PDF file (from the sixth meeting of 2008 until the third meeting of 2010) or the corresponding Tealbook PDF file (from the fourth meeting of 2010 until the last meeting of 2015).[3] The hand-collected data are at the FOMC meeting-frequency and reflect *yearly* forecasts. To match the forecasting horizon of the digital data set, we linearly interpolate these yearly forecasts to obtain quarterly forecasts. For the nearby quarters, we use the FFR in the last quarter as the interpolation anchor.

It might be useful to give one example. Consider the FOMC meeting on August 10, 2010. The data for this meeting come from the Tealbook PDF file dated August 4, 2010. On page 32, there is a table that contains the forecasts for several macroeconomic variables:

**The Long-Term Outlook**

(Percent change, Q4 to Q4, except as noted)

| Item | 2010 | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|---|
| Real GDP | 2.7 | 3.6 | 4.8 | 5.0 | 4.6 |
| Civilian unemployment rate[1] | 9.7 | 8.9 | 7.6 | 6.2 | 5.3 |
| PCE prices, total | 1.3 | 1.1 | 1.0 | 1.2 | 1.4 |
| Core PCE prices | 1.1 | .9 | 1.0 | 1.1 | 1.4 |
| Federal funds rate[1] | .1 | .1 | .4 | 2.1 | 3.3 |

1. Percent, average for the final quarter of the period.

We hand-collect all of these data but focus on the predictions for the FFR. For the last two quarters of 2010, we interpolate the average FFR in Q2 (which was 0.19) with the prediction for the last quarter of 2010. For the subsequent quarters, we interpolate the predictions for the last quarters of the neighboring years. This results in the following forecasts at quarterly frequency (q0 corresponds to the current quarter):

| fomc_date | ffrFq0 | ffrFq1 | ffrFq2 | ffrFq3 | ffrFq4 | ffrFq5 | ffrFq6 | ffrFq7 | ffrFq8 | ffrFq9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 10aug2010 | 0.15 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.18 | 0.25 | 0.33 | 0.40 |

We combine the hand-collected data with the digital data to obtain a time series for the FFR and inflation predictions that runs until the end of 2013. The combined data set is at the FOMC-meeting frequency and quarterly forecasting horizon. In the left panel of Figure 1, we plot the FFR predictions for select FOMC meetings up to four-quarters ahead (for q0-q4). Since the date of the Greenbook/Tealbook is slightly earlier than that of the FOMC meeting,

[3] We collected the predictions for several additional macroeconomic variables even though we use the hand-collected data only for the Fed funds rate. We include all of our hand-collected data in our replication package.

the predictions in Figure 1 are matched to the date of the corresponding FOMC meeting. For Figure 2 as well as for the robustness analyses in Online Appendix D.2, we convert the data into a quarterly time series by averaging the predictions made in each FOMC meeting within the quarter.

**The Fed's SEP and the dot curve.** Beginning with the October 2007 FOMC meeting, FOMC meeting participants submit individual forecasts of various economic variables in conjunction with four FOMC meetings a year. The Summary of Economic Projections (SEP) provides a summary of these forecasts such as the range, the mean, and the median. These summary forecasts are released to the public shortly after the corresponding FOMC meeting (beginning in April 2011, it is released with the Chairman's post-meeting press conference). Individual forecasts are made available to the public after five years. Beginning in 2012, the SEP began to include the forecasts for the FFR—also known as "the dot curve." Each dot corresponds to an FOMC member's forecast for the FFR.

In the right panel of Figure 1, we plot the median SEP prediction for the FFR for select FOMC meetings. We collected this data from public sources (Caballero and Simsek (2022)). Specifically, we collect the individual participants' forecasts for the available years (between 2012-2015) from the Fed's SEPs published on `https://www.federalreserve.gov/monetarypolicy/fomc_historical.htm`. We collect the median participant's forecast for the more recent years (between 2016-2021Q2) from the advance releases of the SEPs published on `https://www.federalreserve.gov/monetarypolicy/fomccalendars.htm`.[4] These data have a *yearly* forecasting horizon: specifically, we have the prediction for the FFR for the end of the current year, for the end of the next year, and so on. In the right panel of Figure 1, we plot the median prediction for the end of the current and the next year.

**The forward rates extracted from the FFR futures.** In Figure 1, we also plot the forward interest rates corresponding to select FOMC meetings. These data are proprietary and obtained from Bloomberg. We use the Bloomberg Terminal to download the daily FFR futures prices for up to 35 months ahead. The corresponding Bloomberg tickers are "FF1 Comdty-FF36 Comdty." We obtain these data from March 1, 2002 until February 28, 2020.[5]

We then convert the futures prices to implied forward rates. Each futures contract settles at the end of the month at 100 minus the average FFR observed in the corresponding month. We extract the implied forward interest rate for the corresponding month using the conversion,

$$\text{(Forward interest rate)}_{t,h} = 100 - \text{(FFR futures price)}_{t,h}.$$

Here, $t$ is a trading day and $h \in \{0, .., 35\}$ is the monthly horizon ($h = 0$ corresponds to the

[4] As with the Greenbook/Tealbook data, we hand-collected more data than we use. We include all of our hand-collected data in our replication package.

[5] Upon inspection, two prices (93.425, 93.245) seem to be filler for misising data. We change all instances of these prices to missing.

current month). We use end-of-day futures prices: that is, the forward interest rates reflect the market's predictions *after* the FOMC meeting.

We adjust the forward interest rates to match the format of the Fed's predictions in each panel of Figure 1. For the right panel, where we plot the SEP predictions for the end of the current and the next year, we plot the forward interest rate for the last month of the current year and the last month of the next year. For the left panel, where we plot the Greenbook/Tealbook predictions at a quarterly forecasting horizon (up to four quarters ahead), we plot the forward interest rates for the corresponding quarter. Specifically, for the current quarter (q0), we average over the forward rates for the remaining months in the current quarter. For subsequent quarters (q1-q4), we average over the forward rates for all of the months in the corresponding quarter.

**Blue Chip Financial Forecasts.** In Figure 2, we also use data from Blue Chip Financial Forecasts. This is a proprietary database that contains the forecasts made by member financial institutions for future interest rates and economic activity. The source data set is at a monthly frequency and quarterly forecasting horizon. It contains individual as well as the consensus (average) forecasts up to five quarters ahead.

We have access to the digital Blue Chip data from January 2001 until February 2020. We have access to PDF files for years before 2001. Blue Chip starts in 1983 but it has the forecasts for the GDP price index starting in 1986. Therefore, we hand-collect the consensus predictions for the FFR and the GDP price index from 1986 to 2000. We combine the hand-collected and the digital data to obtain a time series for consensus predictions that runs from 1986 to 2020. For Figure 1 as well as for the robustness analyses in Online Appendix D.2, we convert the data into a quarterly time series by averaging the predictions made in each month within the quarter, and we use the subset of the data from 1990 to 2013.

**Inflation breakevens from the TIPS market.** In Figure D.2 of Online Appendix D.2, we show that the correlations illustrated in Figure 2 are robust to measuring the market's beliefs from asset price data as opposed to survey data. To this end, we need asset-price-based measures of interest rate and inflation predictions.

For the interest rate predictions, we use the forward rates data that we described earlier. We use the version of the data at a trading-day frequency and a quarterly-forecasting horizon. We convert this data to quarterly frequency by averaging over all trading days within the quarter.

For the inflation predictions, we use the inflation breakevens implied by the TIPS market. To this end, we obtain data for both the nominal yield curve and the TIPS yield curve. These data are public and come from the Fed Board, estimated based on the approach by Gürkaynak, Sack and Wright (2007, 2010). We use these data to calculate the nominal and real (inflation-adjusted) rates at the appropriate forecasting horizon. We then obtain the inflation breakevens by taking the difference between the nominal rate and the real rate.

More specifically, Gürkaynak, Sack and Wright (2007, 2010) estimate yield curve by properly tuning the parameters of the Nelson-Siegel-Svensson yield curve. This approach assumes the instantaneous forward rates $n$ years ahead are characterized by the formula:

$$f_t(n) = \beta_{0,t} + \beta_{1,t}\exp(-\frac{n}{\tau_{1,t}}) + \beta_{2,t}\frac{n}{\tau_{1,t}}\exp\left(-\frac{n}{\tau_{2,t}}\right). \tag{D.1}$$

The approach then estimates the parameters $\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_{1,t}, \tau_{2,t}$ for each trading day $t$ to fit the actual market data on that day. We obtain these daily parameters for both the nominal yield curve (available from `https://www.federalreserve.gov/data/nominal-yield-curve.htm`) and for the TIPS yield curve (available from `https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm`). This enables us to construct the instantaneous nominal and TIPS forwards rates at arbitrary horizons.

Figure D.2 requires data at a quarterly forecasting horizon. We therefore use the formula in $(D.1)$ to construct implied predictions at a quarterly forecasting horizon. We associate the predictions for a particular quarter with the instantaneous forward rate for the last day of the quarter. In particular, for each trading day $t$ and forecasting horizon $h \in \{0, 1, ..\}$, we first calculate the (yearly) distance between the current day and the last day of the forecasted quarter: that is, $n_{t,h} = (t(q_h) - t + 1)/365$, where $t(q_h)$ denotes the last day of $h$ quarters ahead. We then apply Eq. $(D.1)$ with the parameters for the trading day $t$ along with $n = n_{t,h}$. We use this approach to calculate both the nominal forward rate and the TIPS forward rate. We calculate the breakeven inflation by taking the difference between the two forward rates, $\pi_{t,h}^{bkeven} = f_t^{nom}(n_{t,h}) - f_t^{tips}(n_{t,h})$. The resulting data has a trading-day frequency and a quarterly forecasting horizon. We convert this data to quarterly frequency by averaging over all trading days within the quarter. Our data set for inflation breakevens runs from 2004-Q1 until 2021-Q2 (because the TIPS yield curve parameters are available starting in 2004).

## D.2. Robustness of Fed-market disagreement patterns

In Section 2, we show that the disagreements between the Fed and the market about future interest rates are correlated with the disagreements about future aggregate demand (proxied by inflation). We also show that these disagreements about demand are persistent over time. In this appendix, we show that these patterns are robust to using a regression analysis, focusing on different prediction horizons, and measuring the market's belief from asset price data as opposed to survey data.

**Regression analysis.** In the main text, we focus on a graphical analysis. Table D.1 shows that the results illustrated by Figure 2 also hold in a regression analysis. Column 1 shows that the disagreement between the Fed and market on the future interest rate is correlated with the disagreement on future inflation. Column 2 shows that the disagreement on future inflation is correlated with its lagged value. The coefficient on the lag term is large, indicating that disagreements are quite persistent.

Table D.1: Fed-market disagreements on interest rates vs. inflation

| | (1) FFR disagreement b/se | (2) Inflation disagreement b/se |
|---|---|---|
| Inflation disagreement | 0.87** | |
| | (0.16) | |
| Inflation disagreement last quarter | | 0.70** |
| | | (0.06) |
| $R^2$ (adjusted) | 0.37 | 0.48 |
| Observations | 96 | 96 |

Note: The sample is a quarterly time series of Greenbook and Blue Chip forecasts between 1990-2013. Disagreement is the difference between the Greenbook and the Blue Chip forecast for 4 quarters ahead. FFR is the quarterly average (percent) and inflation is the annualized quarterly growth rate of the GDP price index (percent). Estimation is via OLS. Newey-West standard errors with a bandwidth of 4 quarters are in parentheses. +, *, and ** indicate significance at 0.1, 0.05, and 0.01 levels, respectively.

**Alternative forecast horizons.** In the main text, we focus on forecasts for the fourth quarter (beyond the current quarter). Figure D.1 shows that the patterns also hold when we use alternative forecast horizons. However, the correlation between disagreements on interest rates and inflation becomes weaker for shorter horizons. This might be because disagreements about aggregate demand are smaller over shorter time horizons, since macroeconomic uncertainty tends to grow with time.[6]

**Measuring the market's belief using asset price data.** In the main text, we measure the market's belief using the Blue Chip survey data. One concern is that the survey data might not be fully representative of financial markets. For instance, the dominant belief that determines asset prices might be different than the consensus (average) belief that we use (in models with disagreements, the dominant belief is typically a *wealth-weighted* average belief). To address this concern, we next redo the analysis by measuring the market's belief from asset price data. We measure the market's interest rate predictions from forward interest rates (as in Figure 1) and the inflation predictions from inflation breakevens in the TIPS market. As before, we measure the Fed's beliefs from Greenbook/Tealbook. The combined data set is at a quarterly frequency and quarterly forecasting horizon, and it runs from 2004-Q1 to 2013-Q4. Online Appendix D.1 contains details about data sources and construction.

Figure D.2 shows that the correlations illustrated in Figure 2 apply also when we measure the market's belief from asset price data. The direction of the disagreements implied by asset prices is generally similar to the direction of the disagreements implied by the Blue Chip survey data. The exception is the Global Financial Crisis (GFC) period 2008-2009, during which the inflation disagreement implied by asset prices is very large and has the opposite sign of the inflation disagreement implied by survey data. However, during the GFC inflation breakevens experienced

[6] Figure D.1 shows that there are sizeable disagreements about inflation also for the first horizon. This is arguably driven by unmodeled factors, e.g., disagreement about relative prices in the GDP price index.

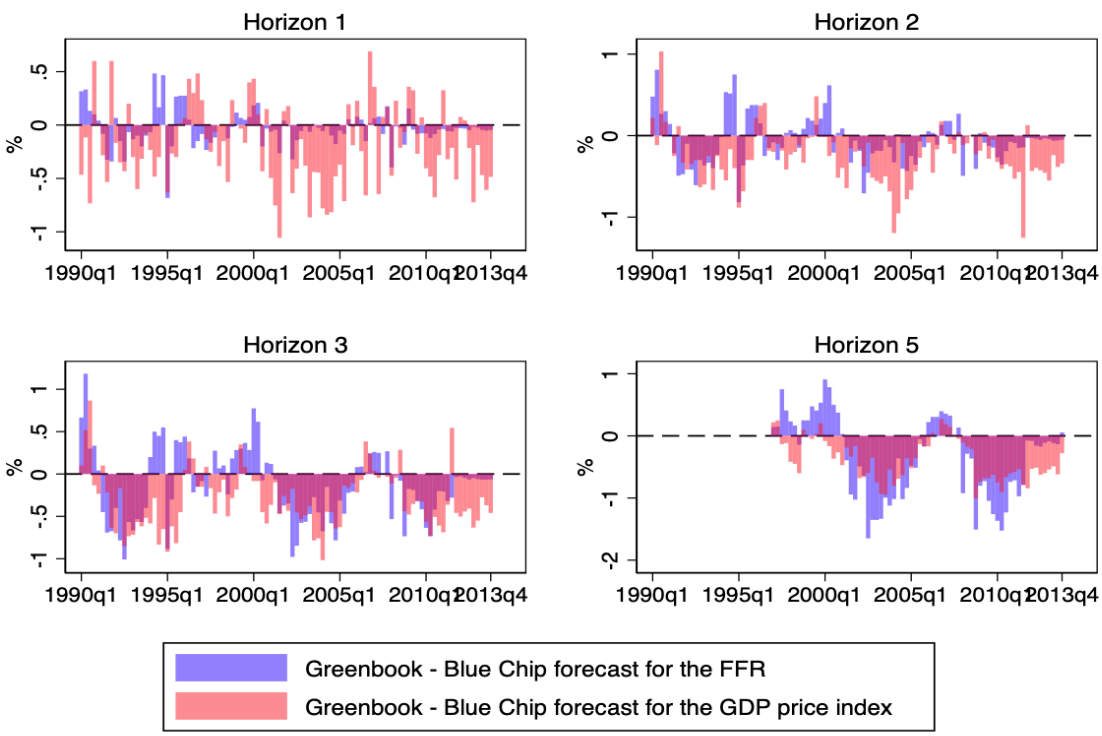


Figure D.1: The bars denote the disagreement between the Fed's Greenbook forecast and the consensus BlueChip forecasts. Each panel corresponds to forecasts for a different horizon. The blue (resp. red) bars correspond to disagreements on the FFR (resp. GDP price index).

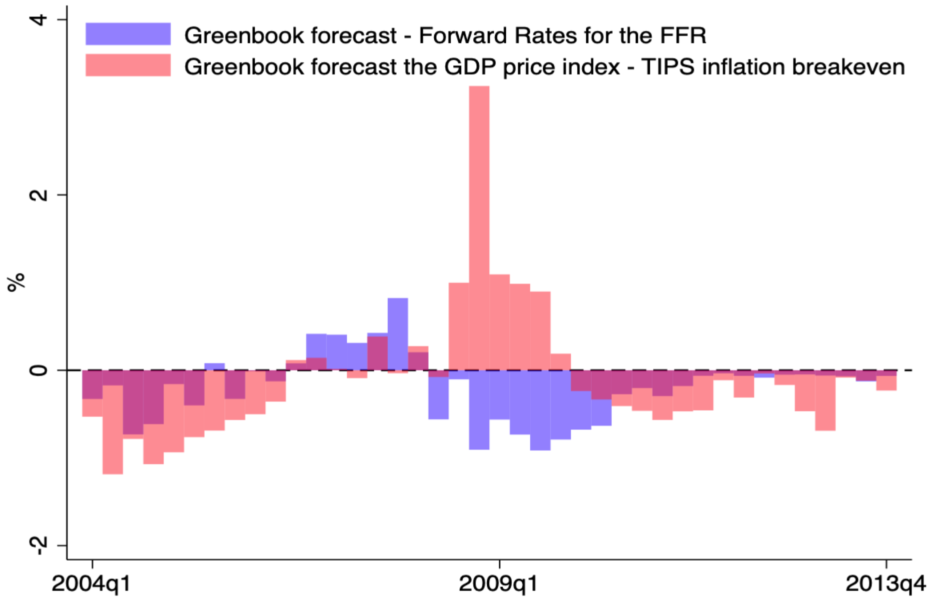


Figure D.2: The bars denote the disagreement between the Fed's Greenbook forecast and the market's forecast inferred from asset prices. The forecasts are for 4 quarters ahead. The blue (resp. red) bars correspond to disagreements on the FFR (resp. inflation). The market's FFR forecast is inferred from the FFR futures prices. The market's inflation forecast is obtained as the breakeven inflation rate in the TIPS market (see Online Appendix D.1 for details).

a large liquidity premium spike. The TIPS rates increased relative to the corresponding nominal treasury rates, not because of a change in inflation expectations, but because the market became very illiquid. This imported a downward bias to the market's expected inflation and an upward bias to the Fed-market inflation disagreements.

The breakdown of the correlation patterns during the GFC highlights the shortcomings of using asset prices to measure the market's predictions. While the asset price data might more accurately reflect the dominant belief, it can be confounded by liquidity premia or risk premia. These confounding premia tend to be especially large when financial markets are in distress. Therefore, we adopt the analysis with the survey data as our baseline approach and present the analysis with asset price data as a robustness check.