Heterogeneous Global Booms and Busts

Maryam Farboodi and Péter Kondor

Online Appendix

The appendix is organized as follows. Appendix A introduces the required formalization to solve for the credit market equilibrium, with endogenous and exogenous prudence.

Appendix B solves for the general form of credit market equilibrium, at $t = 1$, in a global equilibrium with exogenous prudence. Appendix C uses the credit market equilibrium in Appendix B to construct the equilibrium in real market, at $t = 0$, specialized to a simple global equilibrium. To be more precise, Appendices B and C separately address the two sub-problems that a firm solves. First, the firm chooses initial and maintained investment levels, $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_\theta$ at $t = 0$, solved in Appendix C in a simple global equilibrium. Second, the firm chooses how to raise the required liquidity on the international markets at $t = 1$, solved in Appendix B in a global equilibrium.

Finally, Appendix D provides the proofs for the results in the text, and Appendix E discusses extensions.

Appendices A and B build heavily on Kurlat (2016).

A International Credit Market Formalization

There are many markets at $t = 1$, indexed by m, open simultaneously, where firms can demand credit. M denotes the set of all markets. Each market in aggregate state θ is defined by two features. The first feature is the market interest rate, $\tilde{r}(m; \theta)$, paid by firms to international investor in exchange for bonds. If in market m only firms from a single opacity ω are serviced, we use $r_\theta(\omega) = \tilde{r}(m; \theta)$ to denote the interest rate associated with that market m.

The second is a clearing algorithm. A clearing algorithm is a rule that determines which bonds are traded first, as a function of demand and supply in a market. Since investors have different information sets, different clearing algorithms result in different allocations and we need to specify what algorithm will be used. We will expand on clearing algorithms in Section A.3.

A.1 Firm Problem

We start with two definitions.

Definition A.2 **[Maximum Market Demand]** *There is a maximum amount of credit each firm j can demand in each market m, denoted by $\bar{\sigma}_\theta$. We require $\bar{\sigma}_\theta \geq \max_\omega \ell(\omega, g; \theta)$.*

We need to impose an exogenous upper bound on how much demand for bond issuance firm can submit, in order to prevent firms from submitting excess demand $\hat{y}$ at $t = 1$ to undo the rationing. To keep the analysis as simple as possible, we set the maximum in each state such that the best good firms are not restricted and the maximum repayment promised by any bad firm is consistent with the highest repayment promised by good firms, $\bar{\sigma}_\theta = \max_\omega \ell(\omega, g; \theta)$, $\theta = H, L$. In particular, in the proof of Propositions 1, 2, and Lemma 1, we have $\bar{\sigma}_H \equiv \frac{1}{r_H}$ and $\bar{\sigma}_L \equiv \frac{1}{\underline{r}}$. For the rest of the paper, $\bar{\sigma}_\theta \equiv \xi \frac{I(0,g)}{1+r(0,g;\theta)}$.

Definition A.3 **[Rationing Function]** *A rationing function η assigns a measure $\eta(., \omega, \tau; \theta)$ on M to each bond issued by firm $j = (\omega, \tau)$.*

Let $M_0 \subset M$ denote a set of markets. Then $\eta(M_0, \omega, \tau; \theta)$ is the number of bonds firm (ω, τ) issues if he submits one unit of credit demand to each market $m \in M_0$ in aggregate state θ. The firm receives one unit per bond issued, and $r(\omega, \tau; \theta)$ denotes the average interest rate firm $j = (\omega, \tau)$ has to pay back if aggregate state is θ.

Firm Optimization in International Markets. The firm participates in the international markets in each state θ if he is hit by the liquidity shock, to raise liquidity required to maintain investment. We closely map the problem of the firm in the international market to the seller problem of Kurlat (2016). In order to do so, we introduce the following auxiliary variable, $\hat{y}$.

Definition A.4 **[Credit Capacity]** *$\hat{y}\,(\omega, \tau; \theta, r_H, r_L)$ is the maximum number of bonds that the firm $j = (\omega, \tau)$ can issue when aggregate state is θ, and the firm faces interest rate $r_{\theta'}$ in state $\theta' \in \{H, L\}$. By definition, $\hat{y}(\omega, \tau; \theta, r_H, r_L) \leq \bar{\sigma}_\theta$.*

We define the firm's problem on the international credit market as an independent problem, which takes one state variable, credit capacity $\hat{y}(\omega, \tau; \theta, r_H, r_L)$. When $\beta = 0$, the credit capacity of firm j is $\hat{y}(\omega, \tau; \theta, r_H, r_L) \equiv \frac{1}{r(\omega,\tau;\theta)}$, as explained in the text. When $\beta > 0$, we will relate $\hat{y}\,(\omega, \tau; \theta, r_H, r_L)$ to the firm's pledgeability constraint and the technological constraint $\bar{\sigma}_\theta$, in Section C. Here, we assume $\hat{y}(\omega, \tau; \theta, r_H, r_L)$ is continuous and weakly decreasing in $r_{\theta'}$, $\forall \theta'$. Later, in Section C, we verify that in equilibrium, $\hat{y}(\omega, \tau; \theta, r_H, r_L)$ is weakly decreasing in r_H and r_L.

Finally, we show that $y(\omega, \tau; \theta, r_H, r_L)$ in problem (A.1) below maps to $\ell(\omega, \tau; \theta)$ defined in Equation (5).

$$V_{\omega,\tau}\left(\hat{y}\left(.;\theta, r_H, r_L\right)\right) \equiv \max_{\{\sigma(m,\omega,\tau;\theta)\}_m} \left(1 + r(\omega, \tau; \theta)\right) \left(\frac{\rho_\tau}{\xi} - \mathbb{1}_{\tau=g}\right) y(\omega, \tau; \theta, r_H, r_L) \quad \text{(A.1)}$$

s.t.

$$\begin{aligned} & y(\omega, \tau; \theta, r_H, r_L) = \int_M \sigma(m, \omega, \tau; \theta) d\eta(m, \omega, \tau; \theta) \\ & y(\omega, \tau; \theta, r_H, r_L) \leq \hat{y}(\omega, \tau; \theta, r_H, r_L) \qquad \text{(A.2)} \\ & 0 \leq \sigma(m, \omega, \tau; \theta) \leq \bar{\sigma}_\theta \\ & r(\omega, \tau; \theta) = \frac{\int_M \tilde{r}(m; \theta)\sigma(m, \omega, \tau; \theta) d\eta(m, \omega, \tau; \theta)}{\int_M \sigma(m, \omega, \tau; \theta) d\eta(m, \omega, \tau; \theta)} \end{aligned}$$

To any unit of bonds that the firm issues to international investors, he adds $r(\omega, \tau; \theta)$ units of what he has saved using the bankers. He then injects this as the required liquidity to maintain investment. Thus by issuing $y(\omega, \tau; \theta, r_H, r_L)$ bonds, the firm continues at scale $\frac{1+r(\omega,\tau;\theta)}{\xi} y(\omega, \tau; \theta, r_H, r_L)$, which pays off ρ_τ at date $t = 2$. Good firms then have to pay back $1 + r(\omega, \tau; \theta)$ per unit bond issued, which leads to the objective (A.1).

Similar to Kurlat (2016), the choice of $\sigma(m, \omega, \tau; \theta)$ for any single market m such that $\eta(m, \omega, \tau; \theta) = 0$ has no effect on the funding obtained by the firm. Formally, this implies that program (A.1) has multiple solutions. We follow Kurlat (2016) and assume that when this is the case, the solution has to be robust to small positive perturbations of $\eta(m, \omega, \tau; \theta)$, meaning that the firm must attempt to issue bonds in all the markets where if he could he would want to, and must not attempt to issue bonds in any market where if he could he would not want to.

Definition A.5 [Robust Program] *A solution to program* (12) *is robust if for each* θ *and every* (m_0, ω_0, τ_0) *such that* $\eta(m_0, \omega_0, \tau_0; \theta) = 0$ *there exists a sequence of strictly positive real numbers* $\{z_n\}_{n=1}^{\infty}$ *and sequences of credit demand* σ^n *and total credit allocation* y^n*, such that defining*

$$\eta^n(M_0, \omega_0, \tau_0; \theta) = \eta(M_0, \omega_0, \tau_0; \theta) + z_n \mathbb{I}(m_0 \in M_0)\mathbb{I}\left(j = j_0\right),$$

(i) $\forall\, \theta$*, given* $\hat{y}(\omega, \tau; \theta, r_H, r_L)$*,* $\{\sigma^n, y^n\}$ *solve program*

$$\max_{\sigma,y} \quad (1+r(\omega,\tau;\theta))\left(\frac{\rho_\tau}{\xi}-\mathbb{1}_{\tau=g}\right)y(\omega,\tau;\theta,r_H,r_L) \tag{A.3}$$
$$s.t.$$
$$y(\omega,\tau;\theta,r_H,r_L)=\int_M \sigma(m,\omega,\tau;\theta)d\eta^n(m,\omega,\tau;\theta)$$
$$y(\omega,\tau;\theta,r_H,r_L)\leq \hat{y}(\omega,\tau;\theta,r_H,r_L)$$
$$0\leq\sigma(m,\omega,\tau;\theta)\leq\bar{\sigma}_\theta$$

(ii) $z_n \to 0$

(iii) $\sigma^n \to \sigma,\ y^n \to y;\ \forall(\omega,\tau),m$

Cross-sectional differences across $\sigma(m,\omega,\tau;\theta)$, at the same market m_0, do not reveal the identity of the firm $j=(\omega,\tau)$. This is so because wee assume that $\sigma(m,\omega,\tau;\theta)$ is divisible, and firms submit unit by unit. The investment that can potentially serve as collateral, if $\tau=g$, is verified and "marked", to avoid double promising.

A.2 International Investor Problem

$\int_0^s w(s')ds'$ denotes the mass of investors with skill not higher than s. Each investors is endowed with one unit of wealth. We refer to $w(s)$ as investor skill distribution and require it to be a non-negative function on $s\in[0,1]$ and continuous almost everywhere. International investors consume at dates $t=1,2$ and participate in international markets at $t=1$. Since they are active after realization of aggregate shock θ, we will suppress the dependence of their decisions on θ.

Definition A.6 [Acceptance Rule] *An acceptance rule is a function* $\chi:[0,1]\times\{g,b\}\times[0,1]\times\{0,1\}\to\{0,1\}$.

Definition A.7 [Feasibility] *An acceptance rule* χ *is feasible for investor* s *if it is measurable with respect to his information set, i.e. if*

$$\chi(\omega,\tau;s,\iota)=\chi(\omega',\tau';s,\iota)\quad \textit{whenever}\quad x(\omega,\tau,s,\iota)=x(\omega',\tau',s,\iota).$$

Let X denote the set of all possible acceptance rules, and X_s the set of acceptance rules that are feasible for investor s.

Definition A.8 [Allocation Function] *An allocation function* A *assigns a measure* $A(.;\chi,m,\theta)$ *on* $[0,1]$ *to each acceptance rule-market pair* $(\chi,m)\in X\times M$*.* $a(.;\chi,m,\theta)$ *denotes the density of the allocation function.*

Consider an investor demanding to buy one unit in market m and imposing acceptance rule χ. If $I_0 \subseteq [0,1] \times \{g, b\}$, then $A(I_0; \chi, m, \theta)$ represents the amount of bonds of firms $j = (\omega, \tau) \in I_0$ she will obtain, i.e. the fraction of her one unit that goes to financing firms j.

In each aggregate state θ, investor s chooses the test ι (or endowed with one), the market she participates in, m, how many bonds he intends to finance δ, and a feasible acceptance rule χ to maximize

$$\max_{\iota, m, \chi, \delta} c_1 + c_2$$

s.t.

$$\chi \in X_s \tag{A.4}$$

$$\delta \int_{(\omega,\tau)} dA(\omega, \tau; \chi, m, \theta) \leq 1 \tag{A.5}$$

$$c_1 = 1 - \delta \int_{(\omega,\tau)} dA(\omega, \tau; \chi, m; \theta) - \delta\kappa \tag{A.6}$$

$$c_2 = \big(1 + \tilde{r}(m; \theta)\big)\delta \int_{\omega} dA(\omega, g; \chi, m, \theta) \tag{A.7}$$

Constraint (A.4) restrict the investor to using feasible rules. Constraint (A.5) says that each investor can only provide credit from her own wealth. Constraint (A.6) says the investor consumes her leftover endowment at $t = 1$ net of cost of sampling, while (A.7) says that at $t = 2$ she is paid back by good firms and consumes. Substitute the consumption into investor utility function and simplify to get the objective function (8) in the text.

A.3 Clearing Algorithm

A clearing algorithm is a total order on X, which determines which acceptance rule is executed first. We will use the clearing algorithms proposed by Kurlat (2016), adjusted to our framework. In order to adopt these algorithms to our settings, let $T = \{g, b\}$.

Definition A.9 [LRF Clearing Algorithm] *ζ is a less-restrictive-first (LRF) algorithm if it orders nested acceptance rules according to $\chi(.; s, .) <_\zeta \chi(.; s', .)$ if $\chi(.; s', .)$ is nested in $\chi(.; s, .)$; i.e. the less restrictive acceptance rule first.*

Thus, acceptance rules of the form $\chi(\omega, \tau; s, \iota) = \mathbb{1}\left(\tau \in T' \subset T \mid\mid (\tau \in T - T' \; \& \; \omega \geq s)\right)$ are ordered according to $\chi(.; s, .) <_\zeta \chi(.; s', .)$ if $s < s'$, when ζ is an LRF clearing algorithm. Given the signal structure of investors when $\theta = H$, the less restrictive acceptance rule is also the less accurate.

Definition A.10 [NMR Clearing Algorithm] *ζ is a nonselective-then-more-restrictive-first (NMR) algorithm if it orders nested acceptance rules according to $\chi(.;s,.)$ first if it imposes no restriction, and among acceptance rules with restrictions, the more restrictive acceptance rule first; i.e. $\chi(.;s,.) <_\zeta \chi(.;s',.)$ if $\chi(.;s,.)$ is nested in $\chi(.;s',.)$.*

Thus acceptance rules of the form $\chi(\omega,\tau;s,\iota) = \mathbb{1}\left(\tau \in T' \subseteq T \ \&\ \omega \leq s\right)$, if ζ is an NMR clearing algorithm, are ordered according to

(i) $\chi(\omega,T;s,\iota) <_\zeta \chi(\omega,T' \subset T;s,\iota)$;

(ii) $\chi(\omega,T';s,\iota) <_\zeta \chi(\omega,T';s',\iota)$ if $s < s'$, for all $s,s' < 1$;

Given the signal structure of investors when $\theta = L$, the more restrictive acceptance rule is the less accurate.

Let $S(m,\omega,\tau;\theta)$ denote the total measure of bonds (ω,τ) offered in market m, which is the total bonds that firms of type (ω,τ) who are hit by a liquidity shock supply in market m when the aggregate state is θ.

$$S(m,\omega,\tau;\theta) = \phi\sigma(m,\omega,\tau;\theta). \qquad \forall\ \theta$$

Furthermore, let $S^s(m,\omega,\tau;\theta)$ denote the residual supply that is faced by an investor with skill s. The allocation function A and rationing function η are determined in the identical manner as Appendix B in Kurlat (2016). Note that in the proofs, we will suppress the dependence of $S(.)$ and $S^s(.)$ on θ as it is clear from the context.

Kurlat (2016) proves that in the presence of markets with different clearing algorithms, there exist an equilibrium where investors self-select into markets using LRF algorithm when the information structure is akin to ours in $\theta = H$, and markets using NMR algorithm when the information structure is that of $\theta = L$. For simplicity, we will directly assume that the clearing algorithm is LRF when $\theta = H$ and NMR when $\theta = L$. These algorithms guarantee that each investor receives a representative sample of the overall supply of bonds he is willing to accept, in the market where he participates.

B Global Equilibrium. Construction of Equilibrium in Credit Market (t = 1)

At $t = 1$, we take firm credit capacity $\hat{y}(\omega,\tau;\theta,r_H,r_L)$, i.e. the maximum level of liquidity that it can raise in the credit market, satisfying the properties of Definition A.4, as given. Given $\hat{y}\left(\omega,\tau,\theta,r_H,r_L\right)$, the equilibrium in international mar-

kets is such that firms maximize problem (A.1)**, international investors maximize problem** (8)**, and active markets clear, under certain parametric restrictions.**

We construct a more general version of the equilibrium compared to the one used in the main text.

We solve for the credit market equilibrium state-by state. We start with a lemma which simplifies the set of relevant strategies of firms in the credit market.

Lemma B.1 *Every solution to robust program* (A.3) *satisfies*

$$\begin{cases} \sigma(m,\omega,\tau;\theta) \geq \hat{y}\left(\omega,\tau;\theta,r_H(\omega),r_L(\omega)\right) & \text{if } \tilde{r}(m;\theta) < r^R(\omega,\tau;\theta) \\ \sigma(m,\omega,\tau;\theta) = 0 & \text{if } \tilde{r}(m;\theta) > r^R(\omega,\tau;\theta) \end{cases}$$

for some reservation interest rate, $r^R(\omega,\tau;\theta)$*.*

Furthermore, if $\tilde{r}(m;\theta) < r^R(\omega,\tau;\theta)$*,* $\frac{d\sigma(m,\omega,\tau;\theta)}{d\tilde{r}(m;\theta)} \leq 0$*.*

Proof. We start with the first part of the proposition. For simplicity, let j denote the firm (ω,τ), $\hat{y}(\omega,\tau) \equiv \hat{y}\left(\omega,\tau;\theta,r_H,r_L(\omega)\right)$, $\sigma(m,j) \equiv \sigma(m,\omega,\tau;\theta)$, and $\eta(m,j) \equiv \eta(m,\omega,\tau;\theta)$. Also, we suppress the dependence of interest rate on prudence shock $\theta = H, L$ and write $\tilde{r}(m)$. Each individual firm is small and takes the prices as given, and does not affect the schedule of prices either.

Assume the contrary. This implies that there are two markets, m and m' with $\tilde{r}(m') < \tilde{r}(m)$ such that, for some j, the firm chooses $\sigma(m,j) > 0$ and $\sigma(m',j) < \hat{y}(\omega,\tau)$. There are four possible cases:

(i) $\eta(m;i) > 0$ and $\eta(m',j) > 0$. Then the firm can increase his utility by choosing demand $\tilde{\sigma}$ with $\tilde{\sigma}(m',j) = \sigma(i,m') + \epsilon$ and $\tilde{\sigma}(m,j) = \sigma(m',j) - \epsilon\frac{\eta(m',j)}{\eta(m,j)})$ for some positive ϵ.

(ii) $\eta(m,j) > 0$ and $\eta(m',j) = 0$. Consider a sequence such that $\eta^n(m',j) > 0$. By the argument in part 1, for any n the solution to robust firm problem must have either $\sigma^n(m,j) = 0$ or $\sigma^n(m',j) \geq \hat{y}(\omega,\tau)$ (or both). Therefore either the condition that $\sigma^n(m,j) \to \sigma(m,j)$ or the condition that $\sigma^n(m',j) \to \sigma(j,m')$ in a robust solution is violated.

(iii) $\eta(m,j) = 0$ and $\eta(m',j) > 0$. Consider a sequence such that $\eta^n(m',j) > 0$. By the argument in part 1, for any n the solution to robust firm problem must have either $\sigma^n(m,j) = 0$ or $\sigma^n(m',j) \geq \hat{y}(\omega,\tau)$ (or both). Therefore either the condition that $\sigma^n(m,j) \to \sigma(m,j)$ or the condition that $\sigma^n(m',j) \to \sigma(m',j)$ in a robust solution is violated.

(iv) $\eta(m,j) = \eta(m',j) = 0$. Consider a sequence such that $\eta^n(m',j) > 0$ and suppose that there is a sequence of solutions to robust firm problem which satisfies $\sigma^n(m',j) \to \sigma(m',j) < \hat{y}(\omega,\tau)$. This implies that for any sequence such that $\eta^n(m,j) > 0$ and for any n, the solution to robust firm problem must have $\sigma^n(m,j) = 0$. Therefore the condition that $\sigma^n(m,j) \to \sigma(m,j)$ in a robust solution is violated.

■

Lemma B.1 implies that firms use a threshold strategy across markets with different interest rates: They submit demand to all the markets with prevailing interest rate lower than a threshold $r^R(\omega,\tau;\theta)$. The threshold interest rate depends on both the firm and the aggregate state.

To save on notation we often suppress the dependence on r_H, and r_L, and sometimes the dependence on θ, unless useful to clarify the context. Finally, we suppress the argument ι of $\chi(\omega,\tau;s,\iota)$, as it is implied by θ in each subsection.

B.1 $\theta = \mathbf{H}$: Bold International Investors

Equilibrium description. The equilibrium consists of a a single active market, m_H, pair (r_H, s_H), firm and investor optimization, an allocation function, and a rationing function. Market m_H is the market defined by interest rate r_H and an LRF algorithm. The equilibrium is described as follows.

(i) $(\mathbf{r_H}, \mathbf{s_H})$ is the solution to the pair of equations

$$r = \frac{(1-\lambda)\int_s^1 \hat{y}(\omega,b;H)d\omega}{\lambda\int_0^1 \hat{y}(\omega,g;H)d\omega} \tag{B.8}$$

$$\phi = \int_s^1 \frac{1}{(1-\lambda)\int_{s'}^1 \hat{y}(\omega,b;H)d\omega + \lambda\int_0^1 \hat{y}(\omega,g;H)d\omega} w(s')ds' \tag{B.9}$$

(ii) **Firm optimization**

- Good firm

$$\sigma(m,\omega,g;H) = \begin{cases} \min\{\bar{\sigma}_H, \hat{y}(\omega,g;H)\} = \hat{y}(\omega,g;H) & \text{if } \tilde{r}(m) = r_H \\ \bar{\sigma}_H & \text{if } \tilde{r}(m) < r_H \\ 0 & \text{otherwise} \end{cases}$$

where the first line in $\sigma(.)$ follows from Definition A.2 along with construction of $\hat{y}(\omega,g;H)$.

- Bad firm

$$\sigma(m, \omega, b; H) = \min \{\bar{\sigma}_H, \hat{y}(\omega, b; H)\} = \hat{y}(\omega, b; H) \qquad \forall\, m$$

(iii) **International investor optimization**

- $s < s_H$

$$\begin{aligned} \delta_s &= 0 \\ m_s &= m_H \\ \chi(\omega, \tau; s) &= \mathbb{I}\left(\tau = g \;||\; (\tau = b \;\&\; \omega \geq s)\right) \end{aligned}$$

- $s \geq s_H$

$$\begin{aligned} \delta_s &= 1 \\ m_s &= m_H \\ \chi(\omega, \tau; s) &= \mathbb{I}\left(\tau = g \;||\; (\tau = b \;\&\; \omega \geq s)\right) \end{aligned}$$

(iv) **Allocation function**

- For market m_H and $\chi(\omega, \tau; s) = \mathbb{1}\left(\tau = g \;||\; (\tau = b \;\&\; \omega \geq s)\right)$ for some $s \in [0, 1]$

$$a(\omega, \tau; \chi, m_H) = \frac{\left(\mathbb{I}(\tau = g) + \mathbb{I}(\tau = b \;\&\; \omega \geq s)\right) \sigma(m_H, \omega, \tau; H)}{\lambda \int_0^1 \sigma(m_H, \omega', g; H) d\omega' + (1 - \lambda) \int_s^1 \sigma(m_H, \omega', b; H) d\omega'} \tag{B.10}$$

- For market m_H and any other acceptance rule

$$a(\omega, \tau; \chi, m_H) = \begin{cases} \dfrac{\chi(\omega,\tau;s)\sigma(m_H,\omega,\tau;H)[1-\eta(m_H,\omega,\tau;H)]}{\sum_{\tau'} \int_{\omega'} \chi(\omega',b;s')\sigma(m_H,\omega',b;H)[1-\eta(m_H,\omega',b;H)]d\omega'} & \\ \qquad \text{if } \chi(\omega, \tau; s) \notin X_0 \;\&\; \sum_{\tau'} \int_{\omega'} \chi(\omega', b; s')\sigma(m_H, \omega', b)[1 - \eta(m_H, \omega', b)]d\omega' > 0 & \\ \dfrac{\chi(\omega,\tau;s)\sigma(m_H,\omega,\tau;H)[1-\eta(m_H,\omega,\tau;H)]}{\sum_{\tau'} \sum_{\omega'} \chi(\omega',\tau';s)\sigma(m_H,\omega',\tau';H)[1-\eta(m_H,\omega',\tau';H)]} & \\ \qquad \text{if } \chi(\omega, \tau; s) \notin X_0 \;\&\; \sum_{\tau'} \int_{\omega'} \chi(\omega', b; s')\sigma(m_H, b, \tau')[1 - \eta(m_H, \omega', b)]d\omega' = 0, & \\ \qquad \text{but } \sum_{\tau'} \sum_{\omega'} \chi(\omega', b'; s')\sigma(m_H, \omega', b)[1 - \eta(m_H, \omega', b)] > 0 & \\ 0 & \\ \qquad \text{otherwise} & \end{cases} \tag{B.11}$$

where $\eta(m_H, \omega, \tau; H)$ is defined below.

- For any other market

$$a(\omega,\tau;\chi,m) = \tag{B.12}$$
$$\begin{cases} \frac{\chi(\omega,\tau;s)S(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega'} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega' > 0 \\ \frac{\chi(\omega,\tau;s)S(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega' = 0, \\ & \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau') > 0 \\ 0 & \text{otherwise} \end{cases}$$

where

$$S(m,\omega,\tau) = \begin{cases} \phi\sigma(m,\omega,\tau;H) & \text{if } \begin{cases} \tau = b \\ \text{or} \\ \tau = g \,\&\, \tilde{r}(m) \in (0, r_H] \\ \text{or} \\ \tilde{r}(m) \leq 0 \end{cases} \\ 0 & \text{if } \tau = g,\ \tilde{r}(m) > r_H \end{cases}$$

(v) **Rationing function**

$$\eta(M_0,\omega,\tau;H) = \tag{B.13}$$
$$\begin{cases} 1 & \text{if } m_H \in M_0 \text{ and } \tau = g \\ \int_{s_H}^{\omega} \frac{1}{\phi(1-\lambda)\int_s^1 \hat{y}(\omega',b;H)d\omega' + \phi\lambda\int_0^1 \hat{y}(\omega',g;H)d\omega'} w(s)ds & \text{if } m_H \in M_0 \text{ and } \tau = b \text{ and } \omega \geq s_H \\ 0 & \text{otherwise} \end{cases}$$

Proof.

(i) $(\mathbf{r_H}, \mathbf{s_H})$. There is a single market m_H, with $\tilde{r}(m_H) = r_H$, where all trades take place. In this market, firms try to issue as many bonds as they can. Total supply is therefore $\lambda \int_0^1 \hat{y}(\omega', g; H)d\omega'$ good bonds and $(1-\lambda)\int_0^1 \hat{y}(\omega', b; H)d\omega'$ bad bonds. Supply decisions in markets $m \neq m_H$ have no effect on firm utility since $\eta(m,\omega,\tau;H) = 0$, so they are determined in equilibrium by the robustness requirement.

Buying from markets with interest rate other than r_H is not optimal for investors. At interest rates above r_H, the supply includes only bad firms, so investors prefer to stay away, whereas at interest rate below r_H, the supply of bonds is exactly the same as at interest rate r_H but the interest rate is lower. This does not settle the question of whether an investor chooses to buy at all. Investor optimization below then shows that investor with $s = s_H$ faces terms of trade of $\upsilon(s_H) = 1$ in market m_H, and is indifferent between buying and not buying. This results in Equation (B.8).

All investors with $s > s_H$ thus spend all of their wealth buying in market m_H and those with $s < s_H$ choose not to buy at all. The fraction of bonds by firm $j = (\omega, \tau)$ that can be issued in market m_H is given by the ratio of the total allocation of that bond across investors, to the supply of that bond. Noticing that only firms hit by liquidity shock issue bonds, and adding across investors and imposing that all good bonds are issued results in (B.9).

Proposition 7 uses an appropriate monotonic transformation of Equation (B.8) along with Lemma D.1 to show that a solution to the pair of Equations (B.8) and (B.9) such that $r_H \geq 0$ and $0 \leq s_H \leq 1$, constitutes an equilibrium.

(ii) **Firm optimization.** Taking the equilibrium market structure, rationing function and allocation function as given, $y(\omega, \tau) = \sigma(m_H, \omega, \tau)\eta(m_H, \omega, \tau)$. Since $\frac{\rho_\tau}{\xi} - \mathbb{1}_{\tau=g} > 0$, firm j's optimal choice of $\sigma(m_H, \omega, \tau)$ is determined by the corresponding constraints. For a good firm, $\eta(m_H, \omega, g) = 1$ from rationing function (B.13), which implies $y(\omega, g) = \sigma(m_H, \omega, g)$. As such, condition (A.2) is the binding constraint which in turn implies $y(\omega, g) = \sigma(m_H, \omega, g) = \hat{y}(\omega, g)$, when $\theta = H$.

For a bad firm $\eta(m_H, \omega, b) = \int_{s_H}^{\omega} \frac{1}{(1-\lambda)\int_s^1 \hat{y}(\omega', b; H)d\omega' + \lambda \int_0^1 \hat{y}(\omega', g; H)d\omega'} \frac{w(s)}{\phi} ds$ from rationing function (B.13). From equation (B.9), $\eta(m_H, 1, b) = 1$, so $\eta(m_H, \omega, b) < 1$, $\forall\ s_H \leq \omega < 1$, thus $y(\omega, b) \leq \sigma(m_H, \omega, b)$. Since $\sigma(m_H, \omega, b) = \hat{y}(\omega, b) = \bar{\sigma}_H$, constraint (A.2) is satisfied, which in turn implies $y(\omega, b) = \eta(m_H, \omega, b)\hat{y}(\omega, b)$.

Put together, the rationing function (B.13) implies that in market m^H, all good firms will be able to issue as many bonds as they demand to issue. A bad firm with opacity ω will be able to sell a fraction $\eta(m_H, \omega, b) < 1$ of bonds they demand to issue. No other bond can be issued. Thus

$$y(\omega, \tau) = \begin{cases} \hat{y}(\omega, \tau) & \text{if } \tau = g \\ \eta(m_H, \omega, \tau)\hat{y}(\omega, \tau) = \eta_H(\omega)\hat{y}(\omega, \tau) & \text{if } \tau = b \end{cases} \tag{B.14}$$

Off equilibrium, in all cheaper markets (lower interest rate), all good firms submit $\bar{\sigma}_H$. In all more expensive markets, they submit zero demand. All bad firms submit the maximum that they can submit, $\bar{\sigma}_H$, on every other market. These decisions satisfy the robust program (A.3). Note that the equilibrium $\sigma(m, \omega, \tau)$ satisfy the form of Lemma B.1.

(iii) **International investor optimization.** Choosing any feasible acceptance rule other than $\chi(\omega, \tau; s) = \mathbb{1}(\tau = g \,||\, (\tau = b \,\&\, \omega \geq s)) = 1 - \mathbb{1}[x(\omega, \tau, s, 1) = b]$ by investor s using test $\iota = 1$ in market m_H would, according to (B.10) and (B.11), result in a lower fraction of good assets, so choosing $\chi(\omega, \tau; s) = \mathbb{1}(\tau = g \,||\, (\tau = b \,\&\, \omega \geq s))$ is optimal.

Let $\upsilon(m, \chi)$ denote the terms of trade that an investor obtains in market m with acceptance rule χ,

$$\upsilon(m, \chi) = \begin{cases} \frac{(1+\tilde{r}(m)) \int_\omega \mathbb{1}[\tau=g] \sigma(m,\omega,g;\theta) dA(\omega,g;\chi,m,\theta)}{\int_{\omega,\tau} \sigma(m,\omega,\tau;\theta) dA(\omega,\tau;\chi,m,\theta)} & \text{if } A(\{g,b\},[0,1];\chi,m) > 0 \\ 0 & \text{otherwise} \end{cases}$$

which is her expected repayment per unit of bond she finances, i.e. the principal and interest rate she receives at $t = 2$. Let

$$\upsilon^{max}(s) \equiv \max_{m \in M, \chi \in X_s} \upsilon(m, \chi)$$

be the best term of trade that investor s can achieve, and let $M^{max}(s)$ be the set of markets where investor s can obtain terms of trade υ^{max} with a feasible acceptance rule.

Necessary and sufficient condition for investor optimization are that investors for whom $\upsilon^{max} < 1$ choose not to finance any bonds, investors for whom $\upsilon^{max} > 1$ spend their entire endowment in a market $m \in M^{max}(s)$, and investors for whom $\upsilon^{max} = 1$ choose a market $m \in M^{max}(s)$. Using Equation (B.10), an investor s who uses acceptance rule $\chi(\omega, \tau; s) = \mathbb{1}(\tau = g \,||\, (\tau = b \,\&\, \omega \geq s))$ in market m obtains terms of trade

$$\upsilon(m, \chi) = \begin{cases} \frac{(1+\tilde{r}(m))\lambda \int_0^1 \hat{y}(\omega,g;H) d\omega}{(1-\lambda)\int_s^1 \hat{y}(\omega,b;H) d\omega + \lambda \int_0^1 \hat{y}(\omega,g;H) d\omega} & \tilde{r}(m) \leq r_H \\ 0 & \text{otherwise} \end{cases}$$

Thus for all investors

$$\upsilon^{max}(s) = \frac{(1 + r_H)\lambda \int_0^1 \hat{y}(\omega, g; H) d\omega}{(1 - \lambda) \int_s^1 \hat{y}(\omega, b; H) d\omega + \lambda \int_0^1 \hat{y}(\omega, g; H) d\omega}$$

and the maximum is attained in any market where the interest rate is r_H, including m_H. Rewrite

$$\upsilon^{max}(s) = (1 + r_H) J(s)$$

$$J(s) = \frac{\lambda \int_0^1 \hat{y}(\omega, g; H) d\omega}{(1 - \lambda) \int_s^1 \hat{y}(\omega, b; H) d\omega + \lambda \int_0^1 \hat{y}(\omega, g; H) d\omega},$$

Note that from Equation (B.8), $J(s_H) = \frac{1}{1+r_H}$, so $\upsilon^{max}(s_H) = 1$. Moreover, $J'(s) > 0$. This implies that investors $s < s_H$ have $\upsilon^{max}(s) < 1$, so not financing any bonds is optimal for them. Investors of types $s \geq s_H$ have $\upsilon^{max}(s) \geq 1$, so financing bonds such that they spend their entire wealth in market m_H at $t = 2$ is optimal for them.

(iv) **Allocation function.** In all markets except m_H (off equilibrium path), there are no investors, so for any clearing algorithm the residual set of bonds any investor faces is just the original set of bonds demanded by firm on that market. In this case, (B.12) follows from Appendix A, Equation (39) and Appendix B, Equation (65) in Kurlat (2016).

For market m_H, the LRF algorithm implies that an investor who imposes $\chi(\omega, \tau; s) = \mathbb{1}\,(\tau = g \,||\, (\tau = b \;\&\; \omega \geq s))$ faces a residual firm credit demand of acceptable bonds that is proportional to the original firm credit demand (credit demand $\equiv$ bond supply). Therefore, the measure of assets she will obtain is the same as if she traded first. Therefore (B.10) follows from Appendix A, Equation (37) and Appendix B, Equation (65) in Kurlat (2016).

For market m_H and rules that are not of the form $\chi(\omega, \tau; s) = \mathbb{1}\,(\tau = g \,||\, (\tau = b \;\&\; \omega \geq s))$, (off equilibrium path), their trades clear after all other investors, so the bond financing demand they face only includes bonds of bad firms. Therefor (B.11) follows from Appendix A, Equation (38) and Appendix B, Equation (65) in Kurlat (2016)..

(v) **Rationing function.** Equation (B.13) follows from (B.9) using Appendix B of Kurlat (2016), Equation (67). It is the fraction of bonds that the firm is able to issue, out of the total bonds he offers (i.e. a number between zero and one).

■

B.2 $\theta =$ L: Cautious International Investors

Equilibrium description. A global equilibrium consists of an interest rate schedule $0 \leq r_L(\omega) \leq \bar{r}(r_H)$, cut-offs $\{\omega_k\}_{k=1..K} \in [0, 1]$, $K \geq 3$, firm and investor optimization, an allocation function, and a rationing function. Any global equilibrium has at least 3 thresholds, $\omega_1 < \omega_2 < \omega_3$. However, in general K can exceed 3 in a global equilibrium, with $\omega_1 < \omega_{k'} < \omega_2$ for $k' > 3$. A simple global equilibrium is a global equilibrium where there is no nonselective region, and thus $K = 3$.

For any $\omega \in [0, 1]$, let $m(\omega)$ denote the market where the price is $r_L(\omega)$, where $r_L(\omega)$ is found by the procedure described in the proof below, and the clearing algorithm is NMR.

Because of bunching, $m(\omega)$ could mean the same market for different ω. For any $\Omega_0 \subseteq [0,1]$, let the set of markets $M(\Omega_0)$ be $M(\Omega_0) = \{m(\omega) : \omega \in \Omega_0\}$. The set of active markets is $M([0,1])$. A global equilibrium is described as follows.

(i) **Premium schedule $\mathbf{0} \leq \mathbf{r_L}(\omega) \leq \bar{\mathbf{r}}(\mathbf{r_H})$** such that the interest rate falls into one of the cash-in-the-market, bunching, bunching-with-scarcity, or nonselective regions as described below.

(ii) **Firm optimization**

- Good firm

$$\sigma(m,\omega,g;L) = \begin{cases} \min\{\bar{\sigma}_L, \hat{y}(\omega,g;L)\} = \hat{y}(\omega,g;L) & \text{if } \tilde{r}(m) = r_L(\omega), \omega < \omega_2 \\ \hat{y}(\omega_2,g;L) & \text{if } \tilde{r}(m) = r_L(\omega), \omega \geq \omega_2 \\ \bar{\sigma}_L & \text{if } \tilde{r}(m) < r_L(\omega) \\ 0 & \text{otherwise} \end{cases}$$

$$y(\omega,g;L) = \int_{M([0,\omega])} \sigma(m,\omega,g;L)d\eta(m,\omega,g;L)$$

where the first line in σ follows from Definition A.2 along with construction of $\hat{y}(\omega,g;L)$.

- Bad firm

$$\sigma(m,\omega,b;L) = \begin{cases} \min\{\bar{\sigma}_L, \hat{y}(\omega,b;L)\} = \hat{y}(\omega,b;L) & m \in \text{nonselective region} \\ \bar{\sigma}_L & \text{otherwise} \end{cases}$$

$$y(\omega,b;L) = \int_{M([0,1])} \sigma(m,\omega,b;L)d\eta(m,0,b;L)$$

The rationing functions $\eta(m,\omega,\tau;L)$ are defined in Equations (B.20) and (B.21).

(iii) **International investor optimization**

Recall that $\hat{s}(\omega) = \omega$ is the lowest-skill investor who recognizes the type of a firm with opacity ω. Furthermore, let ω_1 denote the highest-ω opacity whose firm face a zero interest rate when investors are cautious, $r_L(\omega) = 0$. Define s_N by

$$\int_{s_N}^{\hat{s}(\omega_1)} w(s)ds = \phi\lambda \int_0^{\omega_1} \hat{y}(\omega,g;L)d\omega.$$

We have assumed that there is sufficient wealth by all investors to cover the aggregate credit demand by all firms, which implies that $s_N \geq 0$. The above equation implies that the aggregate wealth of investors in the interval $[s_N, \hat{s}(\omega_1)]$ is just sufficient to finance all the bonds offered by good firms with opacity $\omega < \omega_1$ at interest rate 0, and each of these investors can identify some good bond in this interval. Furthermore, any investor $s \leq s(\omega_1)$ breaks even, and is indifferent between market participation or not. We focus on an equilibrium where investors with $s \in [s_N, \hat{s}(\omega_1)]$ finance all the bonds offered by good firm $\omega < \omega_1$ at interest rate 0, while investors with $s < s_N$ either buy nonselectively or do not buy at all.

Let $\varepsilon(\omega)$ denote the unfinanced fraction of bonds that are offered in market $m(\omega)$, i.e., those whose supply is not fully absorbed in markets $M([0, \omega))$.

Define the function $\tilde{s}(\omega)$ as the solution to the following differential equation

$$\tilde{s}'(\omega) = -\frac{1}{w\left(\tilde{s}(\omega)\right)}\phi\left[\lambda \int_{\omega}^{1} \hat{y}(\omega', g; L)d\omega' + (1-\lambda)\int_{0}^{1} \hat{y}(\omega', b; L)d\omega'\right]\varepsilon'(\omega) \tag{B.15}$$

with boundary condition $\tilde{s}(1) = s_N$. Finally, let $s_0 = \tilde{s}(0)$ and define $\tilde{\omega}(s)$ for $s \in [s_0, s_N]$ by

$$\tilde{\omega}(s) = \min\left\{\omega : \tilde{s}(\omega) = s\right\}$$

(a) for $s \geq s_N$

$$\begin{aligned} \delta_s &= 1 \\ m_s &= m(s) \\ \chi(\omega, \tau; s) &= \mathbb{I}(\tau = g \;\&\; \omega \leq s) \end{aligned}$$

(b) $s \in [s_0, s_N)$

$$\begin{aligned} \delta_s &= 1 \\ m_s &= m(\tilde{\omega}(s)) \\ \chi(\omega, \tau; s) &= 1 \end{aligned}$$

(c) $s < s_0$

$$\begin{aligned} \delta_s &= 0 \\ m_s &= m(1) \\ \chi(\omega, \tau; s) &= 1 \end{aligned}$$

Investors $s \geq s_N$ spend their entire endowment financing bonds in market $m(s)$, i.e. the market for the most opaque firms for which they can observe a good signal, and they use the selective acceptance rule $\mathbb{I}(\tau = g \;\&\; \omega \leq s)$, which only accepts good assets. Some of these investor are in cash-in-the-market region, some in bunching, and some in bunching-with-scarcity. Investors $s \in [s_0, s_N)$ are nonselective. The function $\tilde{\omega}(s)$ assigns each one to a market: in market $m(\omega)$, nonselective investors bring down the fraction of unfinanced bonds by $\varepsilon'(\omega)$, which requires financing $\varepsilon'(\omega)\phi\lambda \int_\omega^1 \hat{y}(\omega', g; L)d\omega'$ good firms and $\varepsilon'(\omega)\phi\,(1-\lambda)\int_0^1 \hat{y}(\omega', b; L)d\omega'$ bad firms. If investor $\tilde{s}(\omega)$ is the nonselective investor that buys in market $m(\omega)$ then the total nonselective wealth available in that market is $-w\left(\tilde{s}(\omega)\right)\tilde{s}'(\omega)$, so market clearing implies (B.15). Inverting this function results in investor s choosing market $m\left(\tilde{\omega}(s)\right)$. Investors $s < s_0$ don't finance (buy) anything. Since they are indifferent between buying and not buying, many other patterns of demand among nonselective investors are possible.

(iv) **Allocation function**

- For markets $m(\omega) \in M([0,1])$ where ω falls in either a cash-in-the-market or a nonselective region

$$a(\omega, \tau; \chi, m) = \begin{cases} \frac{\chi(\omega,\tau;s)S(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega'} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega' > 0 \\ \frac{\chi(\omega,\tau;s)S(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega' = 0, \\ & \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega' > 0 \\ 0 & \text{otherwise} \end{cases} \tag{B.16}$$

where

$$S(m,\omega,\tau) = \begin{cases} \phi\sigma(m,\omega,\tau;L) & \text{if } \begin{cases} \tau = b \\ \text{or} \\ \tau = g \;\&\; \tilde{r}(m) \in (0, r_L(\omega)] \\ \text{or} \\ \tilde{r}(m) \leq 0 \end{cases} \\ 0 & \text{if } \tau = g,\ \tilde{r}(m) > r_L(\omega) \end{cases}$$

- For market $m(\omega)$ where ω falls in $[\omega^z, \omega^{z+1}]$ for some z which is either a bunching or bunching-with-scarcity region; and $\chi = \mathbb{I}(\tau = g \;\&\; \omega \leq s)$.

$$a(\omega, \tau; \chi, m) = \tag{B.17}$$
$$\begin{cases} \frac{\chi(\omega,\tau;s)S^s(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S^s(m,\omega',\tau')d\omega'} \\ \qquad\qquad \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S^s(m,\omega',\tau')d\omega' > 0 \\ \frac{\chi(\omega,\tau;s)S^s(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S^s(m,\omega',\tau')} \\ \qquad\qquad \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S^s(m,\omega',\tau')d\omega' = 0, \\ \qquad\qquad \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S^s(m,\omega',\tau')d\omega' > 0 \\ 0 \\ \qquad\qquad \text{otherwise} \end{cases}$$

where $S^s(m, \omega, \tau)$ is the solution to differential equation

$$\frac{dS^s(m,\omega,\tau)}{ds} = \begin{cases} -w(s)\frac{S^s(m,\omega,\tau)\mathbb{I}[\omega^z \leq \omega \leq s]}{\sum_{\tau'}\int_{\omega^z}^{s} S^s(m,\omega',\tau')d\omega'} & \text{if } \tau = g \text{ and } s \in [\omega^z, \omega^{z+1}] \\ 0 & \text{otherwise} \end{cases} \tag{B.18}$$

with boundary condition

$$S^0(m,\omega,\tau) = \begin{cases} \phi\sigma(m,\omega,\tau;L) & \text{if } \tau = b \text{ or } (\tau = g \text{ and } \omega \in [0, \omega^{z+1}]) \\ 0 & \text{otherwise} \end{cases} \tag{B.19}$$

Note that if $m(\omega)$ is in a bunching-with-scarcity region, $\sigma(m, \omega, g; L) = \sigma(m, \omega^z, g; L)$, $\forall\ \omega$. Furthermore, note that $s = 0$ is the least skilled investor, who imposes the most restrictive acceptance rule, which is cleared first via NMR algorithm.

Except for bunching and bunching-with-scarcity markets, the clearing algorithm implies that all investors draw bonds from a sample that is proportional to the original supply. This results in (B.16). In bunching markets, investor s imposes acceptance rule of the form $\chi_s(\omega, \tau; s) = \mathbb{I}(\tau = g \;\&\; \omega \leq s)$; therefore when he buys his bond portfolio, the supply of bonds from good firms in opacity ω falls in proportion to his wealth, $w(s)$, times the ratio between the supply of bonds by good firms with opacity ω and all the other bonds acceptable by investor s. This results in differential Equation (B.18) which characterizes how the supply for bonds fall as the clearing algorithm progresses.

(v) **Rationing function**

- Firm (ω, τ), $\omega \leq \omega_3$

$$\eta(M([0,l]),\omega,\tau;L)=\begin{cases}1 & \omega\leq l \text{ and } \tau=g\\ 1-\varepsilon(l) & \text{otherwise}\end{cases} \tag{B.20}$$

- Firm (ω,τ), $\omega>\omega_3$

$$\eta(M([0,l]),\omega,\tau;L)=\begin{cases}\int_{\omega}^{1}\frac{1}{R_D(\omega_3,\omega_2,\bar{r}(r_H),1)+(s-\omega_3)\phi\lambda\hat{y}(\omega_2,g;L)}w(s)ds & \omega\leq l \text{ and } \tau=g\\ 1-\varepsilon(l) & \text{otherwise}\end{cases} \tag{B.21}$$

where $R_D(.)$ and $\tilde{\omega}$ are defined in Equations (B.26) and (B.27), respectively. For good firms with opacity $\omega\leq\omega_3$ and $\omega>\omega_3$, the rationing function is separately defined. It says that if $\omega\leq\omega_3$, good firms with opacity $\omega\leq l$ are fully financed on the markets with interest rate $r(m)\in[0,r_L(l)]$. Alternatively, good firms with $\omega>\omega_3$ are rationed. They can only raise up to the maximum specified in Equation (B.21), which is the same as $\eta_L(\omega)=\eta(m(\omega),\omega,g;L)$ defined in Equation (C.9), and can be achieved at the market with (maximum) interest rate $\bar{r}(r_H)$.
Every other firm, including good firms with opacity $\omega>l$, who offers a bond at markets with interest rate $r(m)\in[0,r_L(l)]$, $(r_L(l)<\bar{r}(r_H))$ will be able to issue a fraction $1-\varepsilon(l)$, so that the unfinanced fraction of these bonds at market l is $\varepsilon(l)$. If the issuer is a good firm with $\omega<\omega_3$, the $\varepsilon(\omega)$ fraction can be issued in market $m(\omega)$.

Before moving to the proof, we present the following lemma which we will use in what follows.

Lemma B.2 *In any equilibrium $r_L(\omega)$ is non-decreasing in ω everywhere.*

Proof.

Assume the contrary. Then when $\theta=L$, there exists bonds offered by good firms with opacity ω,ω' with $\omega'<\omega$ such that $r_L(\omega')>r_L(\omega)$.For this to be consistent with firm optimization, it must be that

$$\eta(M_0,\omega',g;L)<\eta(M_0,\omega,g;L)=1,$$

where the inequality follows from firm optimization, and the equality from definition of $r_L(\omega)$ and M_0, where $M_0=\{m:\tilde{r}(m)\leq r_L(\omega)\}$. But investor optimization and the signal structure when $\theta=L$ requires that investors only use rules of the form $\chi(\omega,\tau;s)=$

$\mathbb{1}(\tau = g \;\&\; \omega \le s)$. This implies that for any $M_0 \subset M$,

$$\eta(M_0, \omega', g; L) \ge \eta(M_0, \omega, g; L),$$

a contradiction. ■

Proof. Lemma B.1 expresses the decision of each firm in terms of a reservation interest rate $r^R(\omega, \tau; L)$ when $\theta = L$. Here we show the following statements: all bad firms are identified under equilibrium acceptance rule, so $r^R(\omega, b; L) = 0$. $r^R(\omega, g; L)$ is different for good firms of different ω, unlike $\theta = H$. Finding $r^R(\omega, g; L)$ is equivalent to finding the highest interest rate at which bonds of a good firm with opacity ω trades.

Moreover, unlike $\theta = H$, a firm (ω, g) might be able to sell some bonds at interest rate below $r^R(\omega, g; L)$, so the equilibrium must characterize $r^R(\omega, g; L)$ and any other prices at which bonds of firm (ω, g) are sold.

(i) **Premium schedule $\mathbf{0} \le \mathbf{r_L}(\omega) \le \bar{\mathbf{r}}(\mathbf{r_H})$.** Let $r_L(\omega) = r^R(\omega, g; L)$ denote the highest interest rate at which bonds of a good firm with opacity ω trades. $r_L(\omega)$ falls into four possible classes: a "cash-in-the-market" interest rate, a"bunching" interest rate, a"bunching-with-scarcity" interest rate, or a "nonselective" interest rate.

Cash-in-the-market. The cash-in-the-market interest rate $r^C(\omega)$ for the bonds issued by the good firm of opacity ω is determined by equating demand and supply in the corresponding market. The total amount of liquidity demanded by firm $j = (\omega, g)$ at interest rate $r^C(\omega)$ should be equal to total wealth of investor $\hat{s}(\omega) = \omega$ which is the financier in that market.

$$\varepsilon(\omega)\phi\lambda\hat{y}(\omega, g; L, r_H, r^C(\omega)) = w(\omega) \tag{B.22}$$

As long as $r^C(\omega)$ is a strictly increasing function and in the correct range, the equilibrium would be a cash-in-the-market pricing equilibrium. Each good firm of opacity ω demands bonds in all markets where $r(m) \le r^C(\omega)$, and no market with a higher interest rate, while bad firms demand maximum bonds on every (active) market. Given the prudence shocks, each investor imposes $\chi(\omega, \tau; s) = \mathbb{I}(\tau = g \;\&\; \omega \le s)$, i.e. she finances bond in the most profitable (highest interest rate) market for which he observes $x(g; \omega, s, 0) = g$. Now consider a market with $r = r^C(\omega)$. Firms with opacity $\omega' \ge \omega$ demand credit in that market, but no firm with opacity $\omega' < \omega$ demand in this market because they have been able to issue all the bonds that they want at lower interest rate. Investor $s = \omega$ is able to recognize good assets in this markets, but investors

$s < \omega$ are not. Moreover, if $r^C(\omega)$ is strictly increasing, this is the highest interest rate where $s = \omega$ can detect good firms, so he will spend his entire wealth financing bonds demanded on this market. Then Equation (B.22) implies all the bonds demanded by firm $j = (\omega, g)$ are financed at this market, and there will be non of them for sale at interest rate higher than $r^C(\omega)$.

Bunching. If $r^C(\omega)$ turns out to be downward sloping in any range, the logic of cash-in-the-market pricing breaks down because it implies the good firm with a lower opacity is paying a higher interest rate to issue bonds, $\omega < \omega'$ and $r^C(\omega) > r^C(\omega')$. The investor who is financing the firm from higher opacity, ω', can also identify the firm from a lower opacity, ω, so he is better off financing the more transparent firm and collect a higher interest rate $r^C(\omega) > r^C(\omega')$, so there will be no financier for the more opaque firm ω'; a contradiction. In this region, there will be "bunching" of all the firms $[\omega, \omega']$ at a single price, i.e. an ironing procedure that restores a weakly monotone function. The clearing algorithm is such that the lower s investor picks the bonds that she finances first in a bunching market.

Since $w(.)$ function is decreasing in s, and $\hat{y}(.)$ function is decreasing in r^C, for low enough ω, and appropriate set of parameters, Equation (B.22) requires $r^C(\omega) < 0$. Let $\hat{\omega} = \max\ \omega$ such that $r^C(\omega) \leq 0$, then as long as $r^C(\omega)$ is increasing, or ironed as explained above, $\forall\ \omega'$ s.t. $\hat{\omega} > \omega' \geq 0$, $r^C(\omega') < 0$. Thus the requirement that there is a zero lower bound on the interest rate (no negative interest rate), implies there is a range of transparencies at the bottom, $\omega \leq \hat{\omega}$, whose good firms face zero interest rate in issuing bonds. Investors with $s \leq \hat{\omega}$ have idle wealth that is not financing any bonds, as there is not enough credit demand from good firms that they can recognize. In order for $\hat{\omega} > 0$ it must be that the least skilled investor has sufficient wealth to cover all the demand of the most transparent good firm, i.e.,

$$w(0) > \phi\lambda\hat{y}(0, g; L), \tag{B.23}$$

where we have used that $r_L(0) = 0$. Later in proof of Proposition 7 we make the appropriate parametric assumption to ensure this condition holds.

Nonselective pricing. Consider a market m with interest rate $r = \tilde{r}(m)$, where good firms with opacity ω submit credit demand in that market. That implies all the good firms with opacity $\omega' > \omega$ also submit demand in market m, as well as all the bad firms with any level of opacity. An investor can choose to impose $\chi_s(\omega, \tau; s) = 1$ in market m and buy a representative sample of the pool.

The terms of trade that he will get is

$$\begin{aligned} \upsilon^N(r) &= \frac{(1+r)\lambda[g \text{ supply at interest rate r in FN}]}{((1-\lambda)\,[b \text{ supply at interest rate r in FN}] + \lambda[g \text{ supply at interest rate r in FN}])} \\ &= \frac{(1+r)\lambda \int_\omega^1 \hat{y}(\omega, g; L) d\omega'}{(1-\lambda) \int_0^1 \hat{y}(\omega, b; L) d\omega' + \lambda \int_\omega^1 \hat{y}(\omega, g; L) d\omega'} \end{aligned}$$

As long as $\omega_1 > 0$, there are (low expertise) international investors who finance bonds issued by good firms with opacity $\omega < \omega_1$. The interest rate for these bonds is zero, so these investors make zero profits and are indifferent between financing and not financing bonds. Alternatively, if they trade nonselectively at a market at interest rate r, they can get the above terms of trade. As a result, if $\upsilon^N(r) > 1$ these investors are better off trading at interest rate r nonselectively, which in turn implies no good bond from opacity ω can be offered at a interest rate above $r^{NS}(\omega)$. With some algebra, one can show that $\upsilon_N(r) \leq 1$ is equivalent to $r \leq r^{NS}(\omega)$ where

$$r^{NS}(\omega) \equiv \frac{(1-\lambda) \int_0^1 \hat{y}(\omega, b; L) d\omega'}{\lambda \int_\omega^1 \hat{y}(\omega', g; L) d\omega'}. \tag{B.24}$$

When this upper bound interest rate is operative, bonds are finances in markets where both selective and nonselective buyers are active. In the markets where the interest rate is $r^{NS}(\omega)$, nonselective buyers will buy just enough assets (distributed pro-rate among the assets offered) such that the interest rate $r^C(\omega)$ is pushed down such that marginal investor $s = \omega$ can charge exactly interest rate $r^{NS}(\omega)$:

$$\varepsilon(\omega) = \frac{w(\omega)}{\phi\lambda\hat{y}(\omega, g; L, r^{NS}(\omega))} \tag{B.25}$$

In other words, if international investors with skill $s = \omega$ are poor, that requires a high interest rate to push the demand of firms (ω, g) down so that Equation (B.25) is satisfied. At this high interest rate, investors financing low ω good firms will enter this market and be nonselective financiers. This takes some bonds off of the market, which in turn implies a lower interest rate.

Bunching-with-scarcity. If there is a maximum interest rate $\bar{r}(r_H)$ that firms are willing to pay to get bonds from investors, and if the wealth of smart investors, in the sense precisely defined below, is in short supply, then there will be a bunching region where some good firms will be rationed.

At any interest rate $r > \bar{r}(r_H)$, good firms have zero demand for bonds, and (with linear objective function) at $r = \bar{r}(r_H)$ they are indifferent between all levels of bond issued. So if the interest rate hits $\bar{r}(r_H)$ in any market, it cannot increase any further than that.

Let $\bar{m}$ denote the market with interest rate $\bar{r}(r_H)$, $\tilde{r}(\bar{m}) = \bar{r}(r_H)$, and let $\bar{\omega}$ denote the lowest opacity level whose good firms demand credit on market $\bar{m}$. Firms $(\bar{\omega}, g)$ submit $\sigma(\bar{m}, \bar{\omega}, g; L) = \hat{y}(\bar{\omega}, g; L)$ on market $\bar{m}$ and by definition their demand is exactly fully satisfied at interest rate $\bar{r}(r_H)$. Good firms with opacity $\omega > \bar{\omega}$ also demand credit on this market. Since these firms are indifferent about how many bonds they raise on market $\bar{m}$ (given the linearity of $t = 0$ objective function), we assume that all of them submit the maximum that they can, $\hat{y}(\bar{\omega}, g; L)$: $\forall\ \omega > \bar{\omega}$, $\sigma(\bar{m}, \bar{\omega}, g; L) = \hat{y}(\bar{\omega}, g; L)$; and how many bond they raise is determined by rationing explained next.[20]

Bad firms with any opacity level also demand credit on market $\bar{m}$, but none is able to issue any bonds in this market. Thus the demand submitted on market $\bar{m}$ is given by

$$\sigma(\bar{m}, \omega, \tau; L) = \begin{cases} \hat{y}(\omega, b; L) & \text{if } \tau = b \\ \hat{y}(\bar{\omega}, g; L) & \text{if } \tau = g \text{ and } \omega > \bar{\omega} \\ 0 & \text{otherwise} \end{cases}$$

As such, if

$$(1 - \bar{\omega}) \times \phi \lambda \hat{y}(\bar{\omega}, g; L) > \int_{\bar{\omega}}^{1} w(s) ds,$$

then the wealth of investors who are able to recognize good firms with opacity $\omega \in (\bar{\omega}, 1)$ is collectively in short supply. As such, some of the good firm demand is rationed at maximum interest rate $\bar{r}(r_H)$. We next determine the subset of good firms whose credit demand is fully satisfied at interest rate $\bar{r}(r_H)$, i.e. those who are not rationed. In order to do so, introduce the following function.

[20] This is slightly stronger than what we actually need to simplify the equilibrium derivation. What we need is that when $\theta = L$, on the market where interest rate is $\bar{r}(r_H)$, no good firm submits more than the credit capacity of the lowest-opacity good firm in that market. The latter firm is $j = (\omega_2, g)$, and even absent this assumption, $\hat{y}(\omega, g; L) = \hat{y}(\omega_2, g; L)$ for $\omega_3 \geq \omega > \omega_2$. So what we need is $\hat{y}(\omega, g; L) = \hat{y}(\omega_3, g; L)$ for $\omega > \omega_3$, weaker than what specified here.

$\mathbf{R_D}(\omega', \omega, \mathbf{r}, \varepsilon)$**.** For $\omega' > \omega$, and interest rate r, let

$$R_D(\omega', \omega, r, \varepsilon) \equiv \varepsilon\phi\lambda \int_{\omega}^{\omega'} \hat{y}(z, g; L, r)dz - \int_{\omega}^{\omega'} w(s)ds. \tag{B.26}$$

where x is a parameter.

$R_D(\omega', \omega, r, \varepsilon)$ measures the excess residual bonds offered, ε, by good firms with opacity in the interval (ω, ω'), at interest rate r, which is not met by the cumulative wealth of the investors who are able to identify some good firm in this interval but no good firms with opacity $\omega'' > \omega'$, i.e. $\omega \leq s \leq \omega'$.

For $\omega = \bar{\omega}$ and $\varepsilon(\bar{\omega}) = 1$, we have

$$R_D(\omega', \bar{\omega}, \bar{r}(r_H), 1) = \phi\lambda(\omega' - \bar{\omega})\hat{y}(\bar{\omega}, g; L, \bar{r}(r_H)) - \int_{\bar{\omega}}^{\omega'} w(s)ds.$$

Recall that in markets where there is bunching, the clearing algorithm used lets lower-s investors, who impose more restrictive acceptance rules, trade before higher-s investors.

Moreover, note that $R_D(\omega', \bar{\omega}, \bar{r}(r_H), 1) > 0, \forall\, \omega' > \bar{\omega}$. The reason is the following. By the logic of cash-in-the-market pricing, $\bar{r}(r_H)$ is the interest rate at which demand of good firms of opacity $\bar{\omega}$ is exactly absorbed by wealth of the marginal investor $s = \bar{\omega}$. Consider a good firm with opacity ω' right above $\bar{\omega}$. Let $\tilde{r}'$ denote the hypothetical interest rate which clears the market for such good firm $\omega' > \bar{\omega}$, if this firm was still in a cash-in-the-market pricing. Again, using the logic of cash-in-the-market pricing, and the downward sloping skill distribution of investors, it must be that $\tilde{r}' > \bar{r}(r_H)$ as $\omega' > \bar{\omega}$. However, since $\bar{r}(r_H)$ is the maximum interest rate any good firm accept, good firm $\omega' > \bar{\omega}$ faces a lower interest rate compared to what would clear his demand using only the wealth of his marginal investors, $s = \omega'$. Let $\tilde{\omega} \in (\bar{\omega}, 1)$ be the highest opacity where the demand of good firms is fully absorbed by all the investors active in market $\bar{m}$.

$$R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1) = \int_{\tilde{\omega}}^{1} \frac{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1)}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1) + (s - \tilde{\omega})\,\phi\lambda\hat{y}(\bar{\omega}, g; L, \bar{r}(r_H))} w(s)ds$$

which implies $\tilde{\omega}$ is the solution to

$$1 = \int_{\tilde{\omega}}^{1} \frac{1}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1) + (s - \tilde{\omega})\,\phi\lambda\hat{y}(\bar{\omega}, g; L, \bar{r}(r_H))} w(s)ds \tag{B.27}$$

In Proposition 7 we argue that under our assumptions, $\tilde{\omega} < 1$.

For a good firm from any opacity $\omega > \tilde{\omega}$, none of his offered bonds can be bought by investors of expertise $s < \tilde{\omega}$, since those investors cannot identify him as good. Thus he can only sell what can be absorbed by the residual wealth of the subset of investors $s > \tilde{\omega}$ who can identify him, $s > \omega > \tilde{\omega}$.

For $s > \tilde{\omega}$, let

$$\zeta(s) = \frac{(s-\tilde{\omega})\,\phi\lambda\hat{y}(\bar{\omega}, g; L, \bar{r}(r_H))}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1) + (s-\tilde{\omega})\,\phi\lambda\hat{y}(\bar{\omega}, g; L, \bar{r}(r_H))}$$

$\zeta(s)$ captures how much of the portfolio held by investor $s > \tilde{\omega}$ are bonds issued "collectively" by good firms with opacity $\omega > \tilde{\omega}$ that s can identify. The measure of those good firms is $(s-\tilde{\omega})\,\phi\lambda$. Thus for an individual firm of opacity $\omega > \tilde{\omega}$, aggregating over holdings of his bonds, by all the investors $s > \omega$, we find how much $j = (\omega, g)$ can issue.

Let $\eta_L(\omega) = \eta(\bar{m}, \omega, g; L)$ denote the rationing function in this market. The above argument implies

$$\begin{aligned}\eta(\bar{m}, \omega, g; L) = \eta_L(\omega) &= \frac{1}{\hat{y}(\bar{\omega}, g; L; \bar{r}(r_H))}\int_{\omega}^{1} \frac{1}{(s-\tilde{\omega})\,\phi\lambda}\zeta(s)w(s)ds \\ &= \int_{\omega}^{1} \frac{1}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1) + (s-\tilde{\omega})\,\phi\lambda\hat{y}(\bar{\omega}, g; L; \bar{r}(r_H))} w(s)ds\end{aligned}$$

This is the rationing function stated in equation (B.21) in part (v) of the equilibrium, for $\bar{\omega} = \omega_2$ and $\tilde{\omega} = \omega_3$. For good firms with opacity $\tilde{\omega} \geq \omega > \bar{\omega}$, $\eta_L(\omega) = 1$.

Interest Rate Regimes. Next, we determine when $r_L(\omega)$ falls into each of the four possible classes of "cash-in-the-market", "bunching", "bunching-with-scarcity", or "nonselective" interest rate." In order to do so, introduce the following function.

$\mathbf{E}(\omega, \mathbf{r}, \varepsilon)$. Define

$$E(\omega, r, \varepsilon) \equiv \max_{\omega' \in [\omega, 1]} \int_{\omega}^{\omega'} w(s)ds - \varepsilon\phi\left((1-\lambda)\int_{\omega}^{\omega'} \hat{y}(z, b; L; r)dz + \lambda\int_{\omega}^{\omega'} \hat{y}(z, g; L; r)dz \right)$$

For a bond issued by good firm of opacity ω, interest rate r and remaining firm demand for bonds issuance ε, $E(\omega, r, \varepsilon)$ measures the maximum over $\omega' > \omega$ of the difference between the endowment of all investors with skill $s \in [\omega, \omega']$, and how much is needed to finance ε units of all the bonds in $[\omega, \omega']$ which firms offer, if they all face interest rate r. A bond interest rate can only be determined by cash-in-the-market if $E(\omega, r^C(\omega), \varepsilon(\omega)) = 0$. A strictly positive value would mean that there exists a range of investors $s \in [\omega, \omega']$ for some $\omega' > \omega$, all of whom can identify some bond in the range $[\omega, \omega']$ as a good bond (but not any bonds offered by firms with opacity higher than ω') and whose collective endowment exceeds what is necessary to finance all the bonds demanded by firms in $[\omega, \omega']$ facing a interest rate $r^C(\omega)$. Since these investors will want to spend their entire endowment financing bonds, it must be that some bond in the range $[\omega, \omega']$ must face a interest rate lower than $r^C(\omega)$. This is because firms' credit demand is downward sloping, hence a lower interest rate would push the firm credit demand up and bring firm credit demand closer to investor credit supply. But then monotonicity implies that the interest rate faced by a good firm with opacity ω must be lower than $r^C(\omega)$, a contradiction.

Next, suppose one knows that $\breve{\omega}$ is the lower limit of one type of region. In a similar manner to (Kurlat, 2016), the following procedure finds the higher end of that region, the type of region immediately above and the prices within the region.

1. For a cash-in-the-market region, the higher end is

$$\inf\{\omega > \breve{\omega} : r^{NS}(\omega) < r^C(\omega) \text{ or } E(\omega, r^C(\omega), \varepsilon(\omega); r_H) > 0 \text{ or } r^C(\omega) > \bar{r}(r_H)\} \tag{B.28}$$

   and the region to the right is a nonselective region (first condition) or a bunching region (second condition), and bunching-with-scarcity (third condition), respectively. Within the region, $r_L(\omega) = r^C(\omega)$ and $\varepsilon(\omega) = \varepsilon(\breve{\omega})$.

2. For a bunching region, the higher end is

$$\min\{\omega > \breve{\omega} : E(\omega, r^C(\bar{\omega}), \varepsilon(\bar{\omega}); r_H) = 0\} \tag{B.29}$$

   and the region to the right is a cash-in-the-market region. Within the region, $r_L(\omega) = r_L(\breve{\omega})$ and $r(\omega) = r(\breve{\omega})$.

3. For a nonselective region, the higher end is

$$\inf\left\{\omega > \breve{\omega} : \frac{w\,(\omega)}{\phi\lambda\hat{y}(\omega, g; L, r^{NS}(\omega))} > r(\omega') \text{ for some } \omega' \in (\breve{\omega}, \omega)\right.$$
$$\left.\text{or } E(\omega, r^C(\breve{\omega}), \varepsilon(\breve{\omega}); r_H) > 0 \text{ or } r^{NS}(\omega) \geq \bar{r}(r_H)\right\} \tag{B.30}$$

and the region to the right is a cash-in-the-market region (former condition), bunching region (second condition), and bunching-with-scarcity (third condition), respectively. Within the region, $r_L(\omega) = r^{NS}(\omega)$ and $\varepsilon(\omega) = \frac{w(\hat{s}(\omega))}{\phi\lambda\hat{y}(\omega,g;L,r^{NS}(\omega))}$.

4. For a bunching-with-scarcity-region, the higher end is 1. Within the region, $r_L(\omega) = \bar{r}(r_H)$ and $\varepsilon(\omega) = \varepsilon(\breve{\omega})$.

We have assume that the aggregate wealth of investors is sufficient to finance all good bonds. Thus the first region to the left is a bunching region with $\breve{\omega} = 0$, $r_L(0) = 0$, and $\varepsilon(0) = \phi\lambda\bar{\sigma}_L$. This region ends at $\omega = \omega_1$, and the region to the right is always a cash-in-the-market region. There after, if either one of the sets defined by (B.28), (B.29), (B.30) is empty, that region extends up. Since there is a maximum interest rate $\bar{r}(r_L)$ that investors are willing to accept, $r_L(\omega) \leq \bar{r}(r_L)$, firms from very opaque countries all bunch at the same interest rate. Furthermore, as we assume that wealth of skilled investors is in short supply, this is a bunching-with-scarcity region. Thus at $\omega = \omega_2$ bunching-with-scarcity region starts, with $r_L(\omega_2) = \bar{r}(r_H)$, and extends all the way to 1. Note that $\omega_2 = \bar{\omega}$ in the proof above. $\omega_3 \in (\omega_2, 1)$ is the degree of opacity at which scarcity starts, i.e. good firms that are more opaque than this level are rationed even at the maximum interest rate. Note that $\omega_3 = \tilde{\omega}$ in the proof above.

(ii) **Firm optimization.** Bond issuance decisions follow the reservation interest rate strategy. A good firm (ω, g) raises total liquidity equal to all the bonds they are able to sell on all $M([0,\omega])$ markets. A bad firm (ω, b) tries to sell in all markets $M([0,1])$. Since in equilibrium all bad assets sell at the same ratio, $\eta(m, \omega, b; L) = \eta(m, 0, b; L)$, $\forall\, m, \forall\, \omega \in [0,1]$.

Let ω_2 denote the lowest opacity firm in the bunching-with-scarcity region, i.e. the lowest ω firm who face the interest rate $\bar{r}(r_H)$

$$w(\omega_2) = \phi\lambda\hat{y}(\omega_2, g; L),$$

and let ω_3 denote the index of the lowest ω opacity whose good firms do face rationing in the bunching-with-scarcity region, defined as the solution to Equation (B.27).

For any good firm with opacity $\omega < \omega_2$, since $r(\omega) > 0$, the rationing function (B.20) implies that in order to issue all of the firm bonds, the reservation interest rate should be $r_L(\omega)$. Good firm with opacity $\omega > \omega_2$ are indifferent between raising any number of bonds, so the issuance decision is optimal. For any bad firm, the rationing function implies that reservation interest rate is $\bar{r}(r_H)$. Therefor credit issuance decisions are optimal for all firms and the total number of bonds they issue follows directly.

(iii) **International investor optimization.** For $s \in [s_N, 1]$, each investor chooses the highest interest rate market on which there is an opacity level ω such that $x(g;\omega,s,0) = g$ and $S(m,\omega,g) > 0$. Since $\tilde{r}(m) \geq 0$, this is optimal. For $s \in [s_0, s_N)$, investors only place weight on markets where nonselective pricing prevails. Equation (B.24) implies they are indifferent between financing bonds and staying out, since the highest interest rate market where there is a ω such that $x(g;\omega,s,0) = g$ and $S(m,\omega,g) > 0$ has $\tilde{r}(m) = 0$, there is no other market in which they would strictly prefer to finance. For $s < s_0$, the same logic implies that no financing is optimal.

(iv) **Allocation function.** For any market $m(\omega)$ where ω falls in either a cash-in-the-market or a nonselective range, the NMR algorithm implies that all the investors face a residual supply proportional to the original supply, so Equation (B.16) follows from Appendix B of Kurlat (2016), Equation (65).

For markets $m(\omega)$ where ω falls in a bunching or bunching-with-scarcity region as described above and χ is of the form $\chi(\omega,\tau;s) = \mathbb{I}\left(\tau = g \;\&\; \omega < s\right)$, then the differential Equations (B.18) follows from Appendix B of Kurlat (2016), Equation (66), along with Equation (D.19). Then (B.17) follows from applying the NMR algorithm.

(v) **Rationing function.** Follows from applying Equation (67) in Appendix B of Kurlat (2016).

■

C Simple Global Equilibrium. Construction of Equilibrium in Real Investment ($\mathbf{t = 0}$)

In this appendix we restrict attention to a simple global equilibria in the financial market at $t = 1$. A simple global equilibrium is a global equilibrium where there is no nonselective pricing region.

We first connect $\hat{y}(\omega, \tau; \theta, r_H, r_L)$ to date zero variables, and then show that it satisfies the required conditions in equilibrium. At $t = 0$, investors are not active. Firms anticipate the date $t = 1$ continuation value and choose the initial and maintained investment levels, $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_\theta$, to maximize their expected utility as defined in program (12).

We start by constructing $\hat{y}(\omega, \tau; \theta)$, i.e. the maximum liquidity that a firm can raise on the international markets. Maintaining $i(\omega, \tau; \theta)$ units allows a good firm to issue up to $\ell(\omega, \tau; \theta) = \frac{1}{1+r(\omega,\tau;\theta)} \xi i(\omega, \tau; \theta)$ bonds, with unit face value each, without violating the pledgeability constraint.

Bad firms value each unit of continued investment more than good firms since investors cannot seize anything from their output. Moreover, (1) they do not need any liquidity if $\theta = L$, since they cannot continue if hit by a liquidity shock, and (2) they face the same financing condition as good firms if $\theta = H$ but can only partially continue. It follows that bad firms save less liquidity, and they have enough collateral (initial scale) to issue up to $\bar{\sigma}_\theta$. See section "Firm problem given the optimal choice of issuance" in C.2 for more detail.

Putting this together we have[21]

$$\hat{y}(\omega, \tau; \theta) = \begin{cases} \ell(\omega, \tau; \theta) & \tau = g;\ \theta = H \text{ or } (\theta = L \text{ and } \omega \leq \omega_2) \\ \ell(\omega_2, \tau; \theta) & \tau = g;\ \theta = L \text{ and } \omega > \omega_2 \\ \bar{\sigma}_\theta & \tau = b;\ \forall\, \theta, \forall\, \omega \end{cases} \tag{C.1}$$

Remark. Recall that in Section B we assumed $\hat{y}(\omega, \tau; \theta, r_H, r_L)$ is decreasing in the (common) interest rate when $\theta = H$, and in the firm specific r_L. With the above mapping, we need to verify that the equilibrium $\ell(\omega, \tau; \theta)$ is in fact downward sloping in r_H, r_L, which we will do in this section.

We impose the following parameter restrictions going forward, to focus on a simple global equilibrium. They are also a sufficient condition for existence of a simple global equilibrium, as shown in the proof of Proposition 7.

Assumption C.1

(i) $\xi \geq \frac{1}{1-\phi}$

(ii) $\frac{1-\lambda}{\lambda} \leq \frac{(\rho_g - \xi)}{\rho_g \xi (1-\phi) + (\rho_g - \xi)(\phi \pi_L \xi - 1)}$

(iii) $w(0) \geq \phi\lambda\xi$ *and* $\lim_{s \to 1} w(s) = 0$.

(iv) $\min\left\{ \frac{(\rho_g - \xi)(1+(1-\lambda)\phi\xi\pi_H)}{(\rho_g(1-\phi)+\phi(\rho_g-\xi)\pi_H)\xi}, \frac{\xi\phi\lambda - w(\omega)}{\xi\phi(\lambda + w(\omega)\pi_L)} \right\} \leq \frac{1-\lambda}{1-\lambda\omega} \qquad \forall \omega$

[21]We will show that when $\theta = L$, good firms with opacity $\omega > \omega_2$ are indifferent in the scale at which they continue. Thus the above $\hat{y}$ is an equilibrium. We pick this tie-breaking rule because it simplifies the exposition. For more detail see Section B.2, Bunching-with-scarcity.

Condition (i) ensures that when there is liquidity risk, $\phi > 0$, without access to credit markets firms prefer to invest all of their initial endowment and do not use any part of it to manage liquidity risk. It also implies that without access to credit markets firms do want to invest (rather than consume right away), which requires $\rho_\tau > \frac{1}{1-\phi}$ and follows since $\forall\ \tau,\ \rho_\tau > \xi$. Condition (ii) ensures that the common interest rate is not prohibitively high when $\theta = H$, so that firms use international markets and part of their own endowment to manage liquidity risk, as opposed to investing all of their initial endowment. Condition (iii) ensures two properties of the investor skill distribution function. First, low-expertise investors have sufficient wealth so that some bonds are issued at zero interest rate. Second, expert capital is in short supply. Condition (iv) ensures that when investors are cautious, there is no equilibrium interest rate for which some investors are willing to buy up all the offered securities independent of their signal, given the investor skill distribution function $w(s)$ in the next section.

Proof of Proposition 7 provides a formal description of what each condition guarantees.

C.1 No Nonselective Region. Investor Skill Distribution

Consider $r^C(\omega)$ and $r^{NS}(\omega)$ defined in Equation (B.22) and (B.24), respectively. First, note that we have assumed the investor skill distribution function is monotonically decreasing, $w'(s) < 0$, so $r^C(\omega)$ does not become non-monotone. As such, bunching region can only emerge below some threshold, $0 \leq \omega < \underline{\omega}$, and bunching-with-scarcity only above some threshold, $\bar{\omega} < \omega \leq 1$.

In what follows, we derive a parametric assumption to ensure that nonselective region does not emerge. Nonselective interest rate schedule is an upper bound on the prevailing interest rate in each market. Thus a sufficient condition for this upper bound to never be active, i.e. for the nonselective pricing region not to emerge, is to have $r^C(\omega) \leq r^{NS}(\omega)$ for markets where $0 < \tilde{r}(m) < \bar{r}(r_H)$.

$$r^C(\omega) = \ell^{-1}\left(\frac{w\left(\omega\right)}{\phi\lambda}\right) \leq \frac{\left(1-\lambda\right)\bar{\sigma}_L}{\lambda\int_\omega^1 \ell(\omega, g; L, r_H, r^C(\omega))d\omega'},$$

where $\ell^{-1}(.)$ denotes the inverse of function $\ell(\omega, g; L; \{r_H, r^C(\omega)\})$ with respect to $r^C(\omega)$, and $\{r_H, r^C(\omega)\}$ indicates the dependence of demand function on (H, L) interest rate explicitly.[22] Note that $\hat{y}(\omega', g; L) = \bar{\sigma}_L$ ($\forall\ \omega'$) minimizes the right hand side on the above equation, which

[22]We have also used that $\hat{y}(\omega, g; L; \{r_H, r^C(\omega)\}) = \ell(\omega, g; L; \{r_H, r^C(\omega)\})$ for $r^C(\omega) < \bar{r}(r_H)$, and that no nonselective region in equilibrium implies $\varepsilon(\omega) = 1\ \forall\omega$.

yields the following sufficient condition

$$r^C(\omega) = \ell^{-1}\left(\frac{w(\omega)}{\phi\lambda}\right) \leq \frac{(1-\lambda)}{\lambda(1-\omega)}. \tag{C.2}$$

This implies that under Assumption C.1, there is no nonselective region when $\theta = L$, $\omega_1 > 0$ and $\omega_3 < 1$. The equilibrium pricing regions are thus characterized by three thresholds $\omega_1 < \omega_2 < \omega_3$ such that starting from the most transparent firms, $\omega = 0$:

(i) Good firms with opacity $0 \leq \omega \leq \omega_1$ are in bunching region and face zero interest rate.

(ii) Good firms with opacity $\omega_1 < \omega \leq \omega_2$ are in cash-in-the-market pricing region.

(iii) Good firms with opacity $\omega_2 < \omega \leq \omega_3$ are in bunching-with-scarcity market $\bar{m}$ at interest rate $\bar{r}(r_H)$, defined in (C.4), and $\eta(\bar{m}, \omega, g; L) = 1$.

(iv) Good firms with opacity $\omega_3 < \omega \leq 1$ are in bunching-with-scarcity market $\bar{m}$ at interest rate $\bar{r}(r_H)$, defined in (C.4), and $\eta(\bar{m}, \omega, g; L) < 1$.

(v) No bad firm issues any bonds in any market.

In this equilibrium

$$y(\omega, \tau; L) = \begin{cases} \hat{y}(\omega, \tau; L) & \text{if } \tau = g \text{ and } \omega \leq \omega_3 \\ \eta(m(\bar{r}), \omega, \tau; L)\hat{y}(\omega, \tau; L) = \eta_L(\omega)\hat{y}(\omega, \tau; L) & \text{if } \tau = g \text{ and } \omega > \omega_3 \\ 0 & \text{if } \tau = b \end{cases} \tag{C.3}$$

C.2 Firm Optimal Decision

Consider the firm problem (12). Each firm j takes his optimal behavior at $t = 1$ as given, which along with $t = 1$ prices in different prudence shocks, the allocation function and the rationing function fully describes firm j continuation payoff. Firm j then chooses his business plan to maximize his expected utility given this continuation payoff.

Derivation of firm optimal choice of bond issuance, Equations (2) **and** (13)**.** A firm hit by liquidity shock has three possible options, at $t = 0$, in how to manage a liquidity shock in each aggregate state at $t = 1$. First, the firm can choose not to insure against the liquidity risk and abandon investment if a liquidity shock happen. This would lead to the highest scale of operation, $I(\omega, \tau)$. Second, the firm can choose to save enough out of his own endowment, through the banker, such that he has sufficient liquidity at $t = 1$ and does not need to raise any extra financing on the international markets. This option leads to the lowest scale of operation. Third, the firm can choose to save a lower amount from his initial endowment and borrow the rest from international investors. This leads to an intermediate level of scale of operation.

From the linearity of the firm problem, the firm chooses the same option for all units of investment. Moreover, Assumption C.1.(i) implies the first option dominates the second. Assumption C.1.(iii) implies that borrowing on the international markets is sufficiently cheap that the third option dominates the first one, which in turn leads to firm's optimal liquidity choice, Equation (2).

Alternatively, a good firm who is not hit by a liquidity shock is indifferent between issuing bonds or not if $r(\omega, \tau; \theta) = 0$, and otherwise prefers not to issue. Thus these firms do not participate in the international markets. It follows that, if a bad firm not hit by a liquidity shock tries to issue bonds, his type is revealed and he does not succeed in raising funding, and he will not participate either. As such, only firms hit by liquidity shock attempt to raise funding from international investors at $t = 1$, which in turn implies the exante budget constraint (13).

Firm problem given the optimal choice of issuance. Since problem (12) is linear, Equations (4)-(11) determine the optimal firm choices, $i(\omega, \tau; \theta)\ \forall\ \theta$ whenever they are non-zero. Plugging these solutions into (13) determines $I(\omega, \tau)$.

The rest of the argument follows from a parallel logic to (Holmström and Tirole, 1998, 2011).

Good firms. Consider a good firm $j = (\omega, g)$. First, conjecture that good firms continue at full scale in high state, $i(\omega, g; H) = I(\omega, g)$. Next, let $0 \leq x \leq 1$ denote the fraction of the initial scale that firm $j = (\omega, g)$ chooses to continue when $\theta = L$. Formally $x \equiv \frac{i(\omega,g;L)}{I(\omega,g)}$.

Use the $t = 2$ interest rate along with Equation (13) to get $I(\omega, \tau)$. Substitute $I(\omega, \tau)$ and x in the objective function (12). The objective function of the good firm then boils down to

$$\Pi(x) = \frac{\phi(\rho_g - \xi)(\pi_H + \pi_L x) + (1 - \phi)\rho_g}{1 + \phi\xi(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r_L(\omega)}{1+r_L(\omega)} x)} - 1.$$

The optimal investment is the continuation scale x such that $\Pi'(x) = 0$, where

$$\Pi'(x) = \frac{\pi_L \phi \Big(\rho_g - \xi - \pi_H \phi \xi^2 \big(\frac{r_H}{1+r_H} - \frac{r_L(\omega)}{1+r_L(\omega)} \big) - \rho_g \xi \big(\frac{r_L(\omega)}{1+r_L(\omega)} \left(1 - \pi_L \phi\right) - \pi_H \frac{r_H}{1+r_H} \phi \big) \Big)}{\Big(1 + \phi \xi \big(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r_L(\omega)}{1+r_L(\omega)} x \big) \Big)^2}.$$

The numerator of $\Pi'(x)$ is independent of x. As such, if the numerator is strictly positive (negative), the firm chooses $x^* = 1 (x^* = 0)$. If the numerator is zero, good firm j is indifferent between any level of continuation when he receives a liquidity shock in $\theta = L$.

This implies that iff

$$r_L(\omega) < \bar{r}(r_H) \equiv \frac{(\rho_g - \xi) \left(1 + \phi \xi \pi \frac{r_H}{1+r_H} \right)}{\rho_g \left(1 - \phi\right) \xi + (\rho_g - \xi) \left(\frac{\phi \pi \xi}{1+r_H} - 1 \right)}, \tag{C.4}$$

a good firm $j = (\omega, g)$ hit by a liquidity shock continues at full scale when $\theta = L$. If $r_L(\omega) > \bar{r}(r_H)$ he does not maintain any investment if he receives a liquidity shock. If $r_L(\omega) = \bar{r}(r_H)$ the good firm might be rationed when issuing bonds on the international markets and continue at lower scale.

Next, we need to make sure that our conjecture for continuation at full scale in high state, $i(\omega, g; H) = I(\omega, g)$, is correct. For this conjecture to hold, it must be that $r_H < \bar{r}_H$ such that every good firm j prefers to submit liquidity demand to international markets when $\theta = H$. Using Assumption C.1.(i), the alternative is to set $i(\omega, \tau; H) = 0$, do not do any liquidity risk management and abandon production if hit by a liquidity shock in state $\theta = H$, and instead increase $I(\omega, \tau)$. Since firms with opacity $\omega = 0$ are those who face zero interest rate in $\theta = L$ such deviation is most profitable for them. Thus it is sufficient to ensure that they do not want to deviate. Thus $\bar{r}_H$ solves

$$\rho_g(1 - \phi) + (\rho_\tau - \xi)\phi \pi_L = \frac{\rho_g(1 - \phi) + (\rho_g - \xi)\phi}{1 + \phi \pi_H \xi \frac{\bar{r}_H}{1+\bar{r}_H}}.$$

It follows that if

$$r_H < \bar{r}_H \equiv \frac{(\rho_g - \xi)}{\rho_g \xi (1 - \phi) + (\rho_g - \xi)(\phi \pi_L \xi - 1)}, \tag{C.5}$$

all good firms prefer to do liquidity management using a combination of own saving and international markets.

Finally, consider the most transparent good firm, $j_{0,g} = (0, g)$. When $\theta = L$, this firm faces zero interest rate and thus does not need to hold any precautionary liquidity against this state. When $\theta = H$, every good firm, including $j_{0,g}$, prefers to do liquidity management against the liquidity shock and save $\pi_H \phi \frac{r_H}{1+r_H}$ per unit of scale, as long as $r_H < \bar{r}_H$. Putting the two states together, $j_{0,g}$ faces the lowest possible interest rate in both states of the world, and has the highest investment level among all good firms, $I(0,g)$. As explained at the end of this section, we have chosen $\bar{\sigma}_\theta \equiv \xi \frac{I(0,g)}{1+r(0,g;\theta)}$.

Bad firms. Consider any bad firm. Assumption C.1.(i) implies that firms either do liquidity management using international markets, or do not do any liquidity management. When $\theta = L$ a bad firms hit by a liquidity shock is not able to raise any international financing, so he has to fully liquidate his initial investment. Thus bad firms do not save any liquidity against $\theta = L$ aggregate state. Next, consider the most opaque bad firm, $j_{1,b} = (1,b)$. When $\theta = H$, $\eta_H(1) = 1$, thus $j_{1,b}$ is not rationed, and is treated as a good firm. Thus he needs to save $\pi_H \phi \frac{r_H}{1+r_H}$, per unit of scale, to be able to continue at full scale. It follows that $j_{1,b}$ saves the same amount of liquidity as $j_{0,g}$, and thus chooses the same level of investment to maintain.

Every other bad firm, $\omega < 1$ is rationed when $\theta = H$, thus they hold lower liquidity, compared to $j_{1,b}$, against this state of the world. This in turn implies they choose a larger scale of operation: $I(\omega, b) > I(1, b)$, $\forall\, \omega < 1$ by (13). Furthermore, bad firms face the same interest rate r_H as good firms when $\theta = H$, and moreover they do not pay back, so if good firms participate in the international markets when $\theta = H$, it is optimal for bad firms to do so as well. Because these firms are rationed when $\theta = H$ they choose maximal credit demand $\bar{\sigma}_\theta$. It follows that $\hat{y}(\omega, b; \theta, r_H, r_L)$ as defined in Equation (C.1) is optimal.

Firm investment at $\mathbf{t = 0}$. Next we characterize the scale of operation of the firm at $t = 0$. For $\tau = g$ firms, substitute the optimal continuation decisions (16)-(17), as well as the interest rates into Equation (13) to get the optimal investment decision

$$I(\omega, g) = \begin{cases} \frac{1}{1+\phi\xi\left(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r_L(\omega)}{1+r_L(\omega)}\right)} & \text{if } \omega < \omega_2 \\ \frac{1}{1+\phi\xi\left(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{\bar{r}(r_H)}{1+\bar{r}(r_H)}\right)} & \text{if } \omega_2 \leq \omega < \omega_3 \\ \frac{1+(1-\eta_L(\omega))\frac{\phi\xi\pi_L \frac{\bar{r}(r_H)}{1+\bar{r}(r_H)}}{1+\phi\xi\pi_H \frac{r_H}{1+r_H}}}{1+\phi\xi\left(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r_L(\omega)}{1+r_L(\omega)}\right)} & \text{if } \omega \geq \omega_3 \end{cases} \tag{C.6}$$

where $\eta_L(\omega) = \eta\left(m(\bar{r}(r_H)), \omega, g; L\right)$, ω_1 is defined by (D.28).

Alternatively, for $\tau = b$ firms,

$$I(\omega, b) = 1 - r_H \xi \phi \pi_H \eta_H(\omega) \bar{\sigma}_H \tag{C.7}$$

where $\eta_H(\omega) = \eta(m_H, \omega, b; H)$.

Next we verify that for good firms who do payback the international investors, the liquidity a firm raises at $t = 1$ on the international market, $y(\omega, \tau; \theta)$ in problem (A.1), is equal to its liquidity need, $\ell(\omega, \tau; \theta)$ associated with optimal investment decision (C.6). This is immediate from comparing Equations (B.14), (C.3) and (C.1).

Consider $D(\cdot)$ and $\bar{D}(\cdot)$, the individual and aggregate expenditure on maintenance, as defined by (D.18) and (D.19) in Section D, respectively. It follows that firm j's realized issuance of bonds on the international market, $\ell(\omega, \tau; \theta)$, is given by:

(i) Good firm $j = (\tau, g)$

$$\ell\left(\omega, g; \theta\right) = \frac{\xi I(\omega, \tau)}{1 + r(\omega, g; \theta)} \eta\left(m_{j,\theta}, \omega, g; \theta\right) \tag{C.8}$$

where $\eta\left(m, \omega, g; \theta\right)$ is given by

$$\eta\left(m, \omega, g; \theta\right) = \begin{cases} \int_{\omega}^{1} \frac{1+\bar{r}(r_H)}{\phi\lambda D(\frac{\bar{r}(r_H)}{1+\bar{r}(r_H)}; \frac{r_H}{1+r_H})(s-\omega_2) - \int_{\omega_2}^{\omega_3} w(s)ds} w(s)ds, & \tilde{r}(m) = \bar{r}(r_H) \ \&\ \omega > \omega_3 \ \&\ \theta = L \\ 1 & \Big(\tilde{r}(m) < \bar{r}(r_H) \text{ or } \big(\tilde{r}(m) = \bar{r}(r_H) \ \&\ \omega \leq \omega_3\big)\Big) \\ & \&\ \theta = L \\ & \text{or} \\ & \tilde{r}(m) = r_H \ \&\ \theta = H \\ 0 & \text{otherwise} \end{cases} \tag{C.9}$$

In each aggregate state, each good firm $j = (\omega, g)$ issues bonds in a single market, $m_{j,\theta}$. $m_{j,\theta}$ is given by $r(m) = r_H$ if $\theta = H$, and is defined in Section C.1 if $\theta = L$.

(ii) Bad firm $j = (\tau, b)$

$$\ell\left(\omega, \tau; \theta\right) = \eta(m_{j,\theta}, \omega, \tau; \theta)\bar{\sigma}_\theta \tag{C.10}$$

where $\eta\left(m, \omega, b; \theta\right)$ is given by

$$\eta(m, \omega, b; \theta) = \begin{cases} \int_{s_H}^{\omega} \frac{1+r_H}{(1-\lambda)(1-s)D(0; \frac{r_H}{1+r_H}) + \lambda\bar{D}(\frac{r_H}{1+r_H})} \frac{w(s)}{\phi} ds & \tilde{r}(m) = r_H \ \&\ \omega \geq s_H \ \&\ \theta = H \\ 0 & \text{otherwise} \end{cases} \tag{C.11}$$

Bad firms $j = (\omega, b)$ issue bonds on a single market when $\theta = H$ and do not issue any bonds if $\theta = L$. Thus $m_{j,\theta}$ is given by $r(m) = r_H$ if $\theta = H$, and $m_{j,\theta} = \emptyset$ if $\theta = L$.

To complete the proof we need to verify that there is a fixed point to the joint $t = 0, 1$ problem, i.e. date $t = 0$ optimal outcomes do constitute an equilibrium in the international markets at $t = 1$. We do this in Proposition 7.

D Proofs

Proof of Propositions 1, 2, and Lemma 1 . We prove the results in Section 3 backward. However, in the main text the results are stated forward. As such, we will present the 3 proofs jointly and point to the corresponding result accordingly. Furthermore, the credit market is as described in Appendix A. Furthermore, with the same argument as in Appendix B, a bold investor extends loans to all firms who do not produce conclusive evidence of being bad, while a cautious investor only extends loans to firms who produce conclusive evidence for being good.

We proceed in two steps. First we conjecture investors' choice of test (information choice) and acceptance rule in each state, describe the equilibrium for the conjectured test and acceptance rule, and prove that it is an equilibrium. We then step back and prove that investors' optimal choice of test is consistent with the conjectured choice and the proposed equilibrium in each state.

High Aggregate Shock ($\theta = \lambda_{\mathbf{H}}$.) We conjecture that all investors are bold in this state and accept applications with $x(\omega, \tau) = g$ only. In equilibrium, a fraction of unskilled and all investors with $s > s_0$ advertise r_H. All good firms demand $\bar{\sigma}_H \equiv \frac{1}{r_H}$ credit at all interest rates not higher than r_H and all bad firms demand $\bar{\sigma}_H$ credit at all interest rates. Good firms' demand is fulfilled at r_H while bad firms are allocated

$$\ell(\omega, b; \lambda_H) = \begin{cases} 0 & \text{if } \omega \in [0, s_0] \\ \frac{1}{r_H} - \frac{w(s_1)}{\lambda_H + (1-\lambda_H)(1-s_1)} - \frac{1}{\lambda_H} w(1) & \text{if } \omega \in [s_0, s_1] \\ \frac{1}{r_H} - \frac{1}{\lambda_H} w(1) & \text{if } \omega \in [s_1, 1] \end{cases} \tag{D.1}$$

at r_H. r_H is defined using the condition that unskilled investors break even. It is the solution to

$$\frac{\lambda_H}{\lambda_H + (1 - \lambda_H)(1 - s_0)} (1 + r_H) - \kappa \left(\frac{1}{\lambda_H + (1 - \lambda_H)(1 - s_0)} \right) - 1 = 0, \tag{D.2}$$

i.e. the zero profit condition of an unskilled bold investor when all firms submit the same credit demand at r_H, and Assumption A.9 implies that she is the first investor who samples the pool. $\lambda_H + (1-\lambda_H)(1-s_0)$ is the fraction of applications which do not provide conclusive evidence that they are bad in the bold test, when used by an investor with skill s_0, and the investor lends to them. Therefore, the investor has to test $\frac{1}{\lambda_H+(1-\lambda_H)(1-s_0)}$ applications to be able to lend out her 1 unit. Out of these projects, λ_H fraction are actually good and pay back, generating the total revenue from lending. The second term is the cost of these tests. Therefore, the unskilled group is indifferent whether to lend at r_H or stay out.

$$\frac{\lambda_H w(s_0)}{\lambda_H + (1-\lambda_H)(1-s_0)} > \frac{\lambda_H}{r_H} - w_1 - \frac{\lambda_H w(s_1)}{\lambda_H + (1-\lambda_H)(1-s_1)} > 0 \tag{D.3}$$

is sufficient to ensure that there exists an $s_H < s_0$ such that

$$\left(1 - \frac{s_H}{s_0}\right) \frac{\lambda_H w(s_0)}{\lambda_H + (1-\lambda_H)(1-s_0)} + \frac{\lambda_H w(s_1)}{\lambda_H + (1-\lambda_H)(1-s_1)} + w(1) = \frac{\lambda_H}{r_H} \tag{D.4}$$

In turn, (D.4) ensures that at interest rate r_H, the total credit supplied by a fraction $\left(1 - \frac{s_H}{s_0}\right)$ of unskilled investors along with every other investors is exactly sufficient to satisfy the credit demand of all good firms.

The allocation of credit to bad firms is given by market clearing conditions. In particular, we have

$$\left(1 - \frac{s_H}{s_0}\right) \frac{(1-\lambda_H)(1-s_1) w(s_0)}{\lambda_H + (1-\lambda_H)(1-s_0)} + \frac{(1-\lambda_H)(1-s_1) w(s_1)}{\lambda_H + (1-\lambda_H)(1-s_1)} = (1-s_1)(1-\lambda_H)\,\ell(\omega, b; \lambda_H)$$

$$\left(1 - \frac{s_H}{s_0}\right) \frac{(1-\lambda_H)(s_1-s_0) w(s_0)}{\lambda_H + (1-\lambda_H)(1-s_0)} = (s_1-s_0)(1-\lambda_H)\,\ell(\omega, b; \lambda_H)$$

for $\omega \in [s_1, 1]$ and $\omega \in [s_0, s_1]$ respectively, which, together with Equation (D.4), implies Equation (D.1).

Any investor with $s > s_0$ strictly prefers to enter at r_H as they make strictly fewer bad loans. These investors do not advertise a higher rate as good firms do not demand credit at higher rates. All entrants lend out all of their capital, thus none of them advertise a lower rate either. All good firms can borrow up to their credit capacity, therefore they do not demand credit at a higher rate. As all bad investors are rationed, by our robustness requirement and the fact that they do not intend to pay back, they demand maximum credit at higher interest rates as well, but do not raise any credit in those markets.

Low Aggregate Shock ($\theta = \lambda_{\mathbf{L}}$.) We conjecture that all investors are cautious in this state and accept applications with $x(\omega, \tau) = g$ only. In equilibrium, unskilled advertise the rate $\underline{r}$, moderately skilled advertise $\hat{r}$, and the most skilled advertise $\bar{r}$. Bad firms are not allocated any credit, hence by our robustness requirement, all bad firms demand maximum credit $\bar{\sigma}_L \equiv \frac{1}{\underline{r}}$ at every interest rate. Let $r_L(\omega)$ denote the interest rate at which good firms from country ω can raise financing.

Assume the following two conditions hold:

$$w(0) \geq s_0 \lambda_L \frac{1}{\underline{r}}, \tag{D.5}$$

$$w(1) < (1 - s_1) \lambda_L \frac{1}{\bar{r}}, \tag{D.6}$$

Then, $r_L(\omega)$ is given by

$$r_L(\omega) = \begin{cases} \underline{r} & \text{if } \omega \in [0, s_0] \\ \hat{r} = \max\{\min\{\frac{\lambda_L(s_1 - s_0)}{w(s_1)}, \bar{r}\}, \underline{r}\} & \text{if } \omega \in [s_0, s_1] \\ \bar{r} & \text{if } \omega \in [s_1, 1], \end{cases}$$

If in addition, we have

$$\frac{1}{\bar{r}} \leq \frac{w(s_1)}{(s_1 - s_0)\lambda_L} \leq \frac{1}{\underline{r}} \tag{D.7}$$

then the interest rate schedule corresponds to the following credit allocation for good firms on the international market:

$$\ell(\omega, g; \lambda_L) = \begin{cases} \frac{1}{\underline{r}} & \text{if } \omega \in [0, s_0] \\ \frac{1}{\hat{r}} & \text{if } \omega \in [s_0, s_1] \\ \frac{w(1)}{(1-s_1)\lambda_L} & \text{if } \omega \in [s_1, 1]\ . \end{cases}$$

Demand of the good firms from least opaque countries, $\omega \in [0, s_1]$, is fully satisfied at the market with minimum interest rate, $r_L(\omega) = \underline{r}$, i.e. $\ell(\omega, g; \lambda_L) = \sigma(m(r_L(\omega)), \omega, g; \theta) = \bar{\sigma}_L$. $\underline{r}$ is the solution to the zero profit condition of unskilled cautious investors when all type of firms submit the same credit demand at the given interest rate,

$$(1 + \underline{r}) - \kappa \left(\frac{1}{\lambda_L s_0}\right) - 1 = 0. \tag{D.8}$$

For these investors, only fraction $\lambda_L s_0$ of the sampled firms provide conclusive evidence that they are good, which implies the second term is the cost. However, all the passed applications are from good firms, hence the revenue is the first term. Therefore, unskilled investors are indifferent whether to advertise interest rate $\underline{r}$ or stay inactive. The fraction of unskilled entrants is consistent with market clearing if condition (D.5) is satisfied. The acceptance rule is rationalized by the same argument as in state $\theta = \lambda_H$.

Moderately skilled investors offer a higher rate $\hat{r}$, hence they make a positive rent. $\hat{r}$ is determined by the market clearing condition of these investors, through cash-in-the-market pricing,

$$w\left(s_1\right) = \left(s_1 - s_0\right)\lambda_L \frac{1}{\hat{r}}.$$

Thus, if condition (D.7) is satisfied, then $\hat{r} = \frac{(s_1 - s_0)\lambda_L}{w(s_1)}$, and otherwise $\hat{r}$ takes the boundary values of $\underline{r}$ and $\bar{r}$, correspondingly. The completes proof of Lemma 1.

Note that if an unskilled or moderately skilled investor were to advertise a higher rate, she would not receive any applications she could pass her their skill level.

As long as skilled capital is in short supply, i.e. condition (D.6) is satisfied, skilled investors advertise the maximal interest rate any good firm is willing to accept $\bar{r}$.

Firms from the most opaque countries $\omega \in [s_1, 1]$ who cannot raise capital at any other advertised interest rate demand the maximum $\bar{\sigma}_L$ both at and under $r_L(\omega) = \bar{r}$. They can only raise financing in the market with highest interest rate $\bar{r}$ tho since only in that market they are recognized as good firms. Furthermore since skilled capital is in short supply, good firms are rationed at this rate. Thus, $\ell(\omega, g; \lambda_L)$ for this group is given by the market clearing condition

$$w(1) = (1 - s_1)\lambda_L \ell(\omega, g; \lambda_L).$$

Assumption 2.(ii) ensures that Equations (D.3), and (D.5)-(D.6) hold. This completes the proof of Proposition 2.

Optimal Choice of Test. Now we show that in each aggregate state, each investor prefers to choose the conjectured test and acceptance rule. Note that the skilled group's choice is immaterial as they observe τ of all firms regardless of their choice of test.

Acceptance rule. For the unskilled investors, recall that the acceptance rule has to be measurable with respect to the signal. That is, for any test, investors have three choices.(1) they can accept applications generating $x(\omega, \tau) = g$ only as conjectured, (2) they can reject all applications regardless of the signal, (3) they can accept all applications regardless of the signal. (2) is dominated by choosing to be inactive, while (3) is dominated by choosing the bold test and following (1). The latter is so, because a bold test rejects only bad firms, which is surely better than accepting all for any given pool. Therefore, our conjectured acceptance rule has to be optimal for any choice of test.

Test. First we show that the optimal test is either $\iota = 0$ or $\iota = 1$. Consider an investor with skill s, running an ι-test and advertising an interest rate r understanding that the the corresponding market m, $\gamma_\tau(m,s)$ fraction of τ the applications are submitted by $\omega > s$ firms. Thus, lending out her one unit of capital at market m leads to the profit

$$\frac{\lambda(\gamma_g + \iota(1-\gamma_g))(1+r) - \kappa}{\lambda\gamma_g + \iota((1-\gamma_g)\lambda + (1-\lambda)(1-\gamma_b))} - 1$$

where we have omitted arguments of $\gamma_\tau(m,s)$ for brevity. This expression is consistent with the zero profit conditions (D.2) and (D.8).

The first order condition with respect to the optimal test is

$$\frac{\partial\left(\frac{\lambda(\gamma_g+\iota(1-\gamma_g))(1+r)-\kappa}{\lambda\gamma_g+\iota((1-\gamma_g)\lambda+(1-\lambda)(1-\gamma_b))} - 1\right)}{\partial\iota} = \frac{\kappa((1-\gamma_b)(1-\lambda) + \lambda(1-\gamma_g)) - \lambda\gamma_g(1-\lambda)(1-\gamma_b)(1+r)}{(\lambda\gamma_g + \iota((1-\gamma_g)\lambda + (1-\lambda)(1-\gamma_b)))^2}. \tag{D.9}$$

Observe that the sign is independent of ι, implying a corner solution. Therefore, when (D.9) is positive the investor chooses the bold test, and when it is negative she chooses the cautious test.

Now we show the optimal choice when $\theta = \lambda_H$ is the bold test for both the moderately skilled and unskilled investor groups. For that (D.9) must be positive for r_H. As all firms submit demand at that rate, $\gamma_g = \gamma_b = s$. Therefore, we need

$$\kappa((1-s)(1-\lambda) + \lambda(1-s)) - \lambda s(1-\lambda)(1-s)\left(1 + \frac{(\kappa + (1-\lambda)(1-s))}{\lambda}\right) > 0$$

or:

$$(1-s)(1-s(1-\lambda))(-s+\kappa+s\lambda) > 0.$$

Thus

$$\frac{\kappa}{s_0} > \frac{\kappa}{s_1} > 1 - \lambda_H \tag{D.10}$$

ensures that both unskilled and moderately skilled strictly prefers to enter as bold.

Finally, we show that $\theta = \lambda_L$ implies that the optimal choice is a cautious test for the moderately skilled and unskilled investors. An unskilled investor, in the market with $\underline{r}$ where all firms submit and so $\gamma_g = \gamma_b = s_0$, we need

$$\begin{aligned}&\kappa((1-s_0)(1-\lambda_L) + \lambda_L(1-s_0)) - \lambda_L s_0(1-\lambda_L)(1-s_0)\left(1 + \frac{\kappa}{\lambda_L s_0}\right)\\ &= \lambda_L(1-s_0)(\kappa - s_0 + \lambda_L s_0) < 0\end{aligned}$$

or

$$\frac{\kappa}{s_0} < 1 - \lambda_L. \tag{D.11}$$

For moderately skilled investors, (D.9) is decreasing in r. Therefore, it is sufficient to show that they prefer to enter as cautious at interest rate $\min \hat{r} = \underline{r}$. Importantly, in the candidate equilibrium $\gamma_g = (s_1 - s_0), \gamma_b = s_1$ as only $\omega \in [s_0, s_1]$ good firms and all bad firms participate in the market with interest rate $\hat{r}$. Therefore, it is sufficient if

$$\begin{aligned}&\kappa\left((1-s_1)(1-\lambda_L)+\lambda_L(1-(s_1-s_0))\right)\\&-\lambda_L(s_1-s_0)(1-\lambda_L)(1-s_1)\left(1+\frac{(\kappa+(1-\lambda_L)(1-s_1))}{\lambda_L}\right)<0\end{aligned}$$

or

$$\frac{\kappa(1-(s_1-s_0))}{(1-s_1)(s_1-s_0)} < 1 - \lambda_L. \tag{D.12}$$

Equations (D.10)-(D.12) follow from Assumptions 2.(i) and 2.(iv). This completes the proof of Proposition 1. ■

Proof of Proposition 3.

The firm problem at date $t = 1$ is defined in (A.1).

(i) The general form of equilibrium for $\theta = H$ is characterized in Section B.1. (r_H, s_H) are given by Equations (B.8) and (B.9), respectively, using $\hat{y}$ defined in (C.1).

(ii) The general form of equilibrium for $\theta = L$ is characterized in Section B.2. The form in (14) is then derived by specializing the investor skill distribution function in Section C.1, which also uses $\hat{y}$ defined in (C.1) as well as Assumption C.1.

■

Proof of Proposition 4.

The firm problem at date $t = 1$ is defined in (A.1).

(i) The general form of equilibrium for $\theta = H$ is characterized in Section B.1. Section C.2 shows that the optimal continuation decision is determined by the constraint. It follows that the equilibrium amount that the firm raises is given by $y(\omega, \tau; H, r_H, r_L)$ in program (A.1), using $\hat{y}$ defined in (C.1) and Equations (2) and (13) with the optimal $i(\omega, \tau; H)$.

(ii) The general form of equilibrium for $\theta = L$ is characterized in Section B.2, and specialized in Section C.1 by specializing the investor skill distribution function under Assumption C.1. Section C.2 shows that the optimal continuation decision is determined by the constraint. It follows that the equilibrium amount that the firm raises is given by $y(\omega, \tau; L, r_H, r_L)$ in program (A.1), using $\hat{y}$ defined in (C.1) and Equations (2) and (13) with the optimal $i(\omega, \tau; L)$ and $\alpha = \xi$.

■

Proof of Proposition 5.

The derivation of optimal firm scale of operation, as well as the optimal continuation decision, is provided in Section C.2. (15) follows from (C.6), where the rationing function is defined in (C.9). ■

Proof of Proposition 6.

The derivation of optimal firm exante investment, as well as the optimal continuation decision, is provided in Section C.2. (18) follows from (C.7), where the rationing function is defined in (C.11). ■

Lemma D.1 *Assume $G(x)$ and $H(x, z)$ are continuous in x. Equation* (D.13) *has a fixed point $x \in [0, 1]$,*

$$F(x) = \frac{(1-\lambda)\int_{s_H(x)}^{1} H(x,z)dz}{(1-\lambda)\int_{s_H(x)}^{1} H(x,z)dz + \lambda G(x)}; \tag{D.13}$$

where $s_H(x)$ is the solution to

$$\int_{s_H(x)}^{1} \frac{1}{(1-\lambda)\int_{s}^{1} H(x,z)dz + \lambda G(x)} w(s)ds = \phi(1-x), \tag{D.14}$$

if Equation (D.14) *has a solution, and $s_H(x) = 0$ otherwise.*

Proof of Lemma D.1.

First note that if Equation (D.14) has a solution in $s_H(x)$, it will be $s_H(x) \in [0, 1]$. The reason is that $w(s) = 0$ for $s > 1$ and $s < 0$, so moving s_H outside the $[0, 1]$ interval does not change the left hand side of Equation (D.14).

Case 1 [Equation (D.14) **holds with equality, $s_H \in [0, 1]$].** Consider the case where s_H is interior. Consider the self-map on $F : [0, 1] \mapsto [0, 1]$. We use Brouwer's fixed-point theorem to prove existence of a fixed point. $[0,1]$ is a compact convex set. We need to show is that $F(x)$ is a continuous function, and maps $[0,1]$ to itself, which is immediate since the ratio in $F(x)$ is positive and (weakly) smaller than one.

Next we move to proving continuity. $G(x)$ is continuous in x. $H(x,z)$ is also continuous in x, and so is $\int H(x,z)dz$. Thus if a solution $s_H(x)$ to Equation (D.14) exists, it is also continuous.

This implies that if a solution to Equation (D.14) exists, then everything on the right hand side of Equation (D.13) is continuous, so $F(x)$ is a continuous map from $[0,1]$ to $[0,1]$, which implies by Brouwer's theorem a fix point exists.

Case 2 [Equation (D.14) **only holds with inequality, thus** $s_H = 0$**].** Then Equation (D.13) becomes one equation in one unknown in x, which with the same argument as the previous case has a fixed point. ■

Proof of Proposition 7. Let

$$\underline{\lambda} \equiv \frac{\rho_g \xi(1-\phi) + (\rho_g - \xi)(\phi \pi_L \xi - 1)}{\rho_g \xi(1-\phi) + (\rho_g - \xi)\phi \pi_L \xi} \text{ and}$$

$$\Lambda(\omega) \equiv \min\left\{ \frac{(\rho_g - \xi)\left(1 + (1-\lambda)\,\phi \xi \pi_H\right)}{\left(\rho_g\,(1-\phi) + \phi\,(\rho_g - \xi)\,\pi_H\right)\xi}, \frac{\xi \phi \lambda - w(\omega)}{\xi \phi(\lambda + w(\omega)\pi_L)} \right\}.$$

Then, the sufficient conditions of Proposition 7 are equivalent with Assumption C.1 (i)-(iv). Here, we proceed in steps. First, we show that the existence of a simple global equilibrium can be mapped to a fixed point problem. After describing the problem we explain the mapping between Equations (D.15)-(D.30) to the solution developed in Sections C and B. Then, we use Lemma D.1 to prove the existence of equilibrium. Second, we explain the role of Assumption C.1 (i)-(iv).

Equilibrium Existence as a Fixed Point Problem

Equations (D.15)-(D.30) spell out how the equilibrium objects $r_H, r_L(\omega), s_H, \omega_1, \omega_2, \omega_3, \eta_L(\omega)$ and $\eta_H(\omega)$ are constructed.

$$F\left(x\right) = \frac{\left(1-\lambda\right)\left(1-s_H(x)\right) D\left(0;x\right)}{\left(1-\lambda\right)\left(1-s_H(x)\right) D(0;x) + \lambda \bar{D}(x)} \tag{D.15}$$

where $s_H(x)$ solves

$$\int_{s_H(x)}^{1} \frac{1}{\left(1-\lambda\right)(1-s)D(0;x) + \lambda \bar{D}(x)} w(s)ds = (1-x)\phi, \tag{D.16}$$

if Equation (D.16) has a positive solution, and $s_H(x) = 0$ otherwise.

Moreover

$$\bar{y}(x) = \frac{(\rho_g - \xi)(1 + \phi\xi\pi_H x)}{(\rho_g(1-\phi) + \phi(\rho_g - \xi)\pi_H)\xi} \tag{D.17}$$

$$D(y;x) = \frac{\xi}{1 + \phi\xi(\pi_H x + \pi_L y)} \tag{D.18}$$

$$\begin{aligned}\bar{D}(x) =& \omega_1(x) D(0;x) + \int_{\omega_1(x)}^{\omega_2(x)} D(y^C(\omega);x) d\omega \\ &+ \left(1 - \omega_2(x) + \frac{\phi\xi\pi_L\bar{y}(x)}{1+\phi\xi\pi_H x}\int_{\omega_3(x)}^{1}(1-\eta_L(\omega))d\omega\right) D(\bar{y}(x);x).\end{aligned} \tag{D.19}$$

where

$$y^C(\omega;x) \equiv \frac{\xi\phi\lambda - w(\omega)(1+\phi\xi\pi_H x)}{\xi\phi(\lambda + w(\omega)\pi_L)} \qquad \omega \in [\omega_1(x), \omega_2(x)]\,. \tag{D.20}$$

The rationing functions are given as follows

$$\eta_L(\omega) = \min\left(1, \int_{\omega}^{1} \frac{1}{\phi\lambda(1-\bar{y}(x))D(\bar{y}(x);x)(s-\omega_2(x)) - \int_{\omega_2(x)}^{\omega_3(x)} w(s)ds} w(s)ds\right) \tag{D.21}$$

$$\eta_H(\omega) = \min\left(1, \int_{s_H(x)}^{\omega} \frac{1}{(1-\lambda)(1-s)D(0;x) + \lambda\bar{D}(x)} \frac{w(s)}{\phi(1-x)} ds\right) \tag{D.22}$$

and $\omega_1(x), \omega_2(x), \omega_3(x)$ are defined as follows.

Let $\hat{\omega}_1(x)$ and $\hat{\omega}_2(x)$ be the solution to the following two equations, respectively:

$$w(\omega_2) - \phi\lambda(1-\bar{y}(x))D(\bar{y}(x);x) = 0, \tag{D.23}$$

$$w(\omega_1) - \phi\lambda D(0;x) = 0. \tag{D.24}$$

Then

$$\omega_2(x) = \min\{\max\{\hat{\omega}_2(x), 0\}, 1\}, \tag{D.25}$$

$$\omega_1(x) = \min\{\max\{\hat{\omega}_3(x), 0\}, 1\}. \tag{D.26}$$

Moreover, let $\hat{\omega}_3$ be the solution to

$$1 = \int_{\omega_3}^{1} \frac{1}{\phi\lambda(1-\bar{y}(x))D(\bar{y}(x);x)(s-\omega_2(x))) - \int_{\omega_2(x)}^{\omega_3(x)} w(s)ds} w(s)ds \tag{D.27}$$

$$\omega_3(x) = \min\{\max\{\hat{\omega}_3(x), 0\}, 1\}. \tag{D.28}$$

Finally, given the fixed point x^*, $\bar{r}(x^*) = \frac{\bar{y}(x^*)}{1-\bar{y}(x^*)}$, and interest rates r_H and $r_L(\omega)$ are given by

$$r_H = \frac{x^*}{1-x^*} \tag{D.29}$$

$$\hat{r}(\omega) = \frac{y^C(\omega; x^*)}{1-y^C(\omega; x^*)}. \tag{D.30}$$

and (14).

To simplify the formulas, the proposition is stated in terms of premia rather than interest rates, using the monotone transformation

$$q = \frac{r}{1+r}. \tag{D.31}$$

Using this transformation, Equation (B.8), which defines the interest rate when $\theta = H$, can be written as

$$q = \frac{(1-\lambda)\int_s^1 \hat{y}(\omega, b; H)d\omega}{(1-\lambda)\int_s^1 \hat{y}(\omega, b; H)d\omega + \lambda\int_0^1 \hat{y}(\omega, g; H)d\omega} \tag{D.32}$$

Let $H(r_H) = \hat{y}(\omega, b; H, r_H)$ and $G(r_H) = \int_0^1 \hat{y}(\omega, g; H, r_H)d\omega$, noting that $H(r_H) = \hat{y}(\omega, g; H)$ depends on q_H through r_H, using Equation (D.31). We have shown in Section C that $\hat{y}$ is continuous in r_H, and r_H is by construction continuous in q_H. As such, existence of a solution to pair of Equations (D.32) and (B.9) such that $q_H \geq 0$ and $0 \leq s_H \leq 1$, implies that there exists a solution to pair of Equations (B.8) and (B.9) such that $r_H \geq 0$ and $0 \leq s_H \leq 1$.
We proceed in two steps. We first explain the mapping between Equations (D.15)-(D.30) to the solution developed in Sections C and B. We will then use Lemma D.1 to prove the existence of equilibrium.

Equation (D.18) writes the general form of expected maintenance cost of a good firm who faces premia x when $\theta = H$, and y when $\theta = L$, or interest rates $\frac{x}{1-x}$ and $\frac{y}{1-y}$, respectively. It uses the equilibrium firm scale of operation, defined by Equation (C.6), and optimal continuation scale. Substitute in Equation (2) (with $\alpha = \xi$) to get firm liquidity demand in international markets: $\ell(\omega, g; \theta) = \frac{D(\frac{r_L(\omega)}{1+r_L(\omega)}; \frac{r_H}{1+r_H})}{1+r(\omega,g;\theta)}$. Using this demand functions, $\hat{y}$ at $t = 1$ is defined in (C.1). It is straight forward to verify that using (C.1) to solve for the firm problem at $t = 1$ (Section B), $y(\omega, \tau; \theta) = \ell(\omega, \tau; \theta)$.

Under the appropriate sufficient conditions on the parameters (see the end of this proposition), firms choose to participate in international markets when $\theta = H$, with the equilibrium described in B.1, and when $\theta = L$ with the equilibrium described in sections B.2 and C.1,. Under this equilibrium structure, Equation (D.19) aggregates the total required maintenance across the pricing regions when $\theta = L$.

Equation (D.17) rewrites the maximum premium $\bar{q}$ in $\theta = L$, defined in Equation (C.4), when the common premium in high state is x. The threshold transparencies ω_1, ω_2 and ω_3 are defined in Equations (D.26), (D.25), and (D.28), respectively, and $\omega_3 = \tilde{\omega}$ and $\omega_2 = \bar{\omega}$ in the $\theta = L$ equilibrium in Section B.2. This leads to the rationing function in Equation (C.9). Equation (D.24) determines the threshold where bunching region ends, at zero interest rate, given liquidity demand function (D.18). Equation (D.23) determines the threshold where bunching-with-scarcity region starts, at premium $\bar{q}$ (interest rate $\bar{r}(r_H)$). Equation (D.27) determines the threshold where rationing starts in bunching-with-scarcity region, given the liquidity demand

Finally, Equations (D.15) and (D.16) jointly determine the pooling premium and marginal investor when $\theta = H$, at the above liquidity demand levels.

The last equilibrium object to determine is demand function for credit $\{\sigma(m, \omega, \tau; \theta)\}_{\theta=H,L}$. It is implied from Lemma B.1 and Equation (C.1), Proposition 4, and Equations (C.6) and (C.7) to relate each firm demand for credit in each state to its the equilibrium scale of operation.

Lastly, we will use Lemma D.1 to prove existence of equilibrium. Let $G(x) = \bar{D}(x)$ and $H(x) = D(0; x)$. As such we need to show both functions are continuous.

$\bar{y}(x)$ is continuous. $D(y; x)$ is continuous in x for any $x, y > 0$ since $1+\phi\xi(\pi_H x+\pi_L y) > 0$. Thus $D(0; x)$ and $D(\bar{y}(x); x)$ are also continuous.

Now turn to $\omega_1(x)$, $\omega_2(x)$ and $\omega_3(x)$. $w(.)$ and $D(y; x)$ are continuous in x. $D(y, x)$ is constant in ω and $w(.)$ is increasing in ω, so Equations (D.23) and (D.24) have a unique solution in ω, so $\hat{\omega}_1(x)$ and $\hat{\omega}_2(x)$ exist, are unique, and continuous.

Next, $D(0, x)$ is decreasing in x. Moreover

$$D(\bar{y}(x); x) - (1 - \bar{y}(x))D(\bar{y}(x); x) = \bar{y}(x)D(\bar{y}(x); x) = \frac{\rho_H - \xi}{\rho_g - \phi\xi},$$

$$\frac{d\Big((1 - \bar{y}(x))D(\bar{y}(x); x)\Big)}{dx} = \frac{dD(\bar{y}(x); x)}{dx} = \frac{\xi^2\pi_H\phi(\rho_g(1 - \phi) + \phi\pi_H(\rho_g - \xi))}{(\rho_g - \phi\xi)(1 + \phi\xi\pi_H x)^2} < 0.$$

Thus both $\hat{\omega}_2(x)$ and $\hat{\omega}_1(x)$ are monotonically decreasing in x. Since $\hat{\omega}_2(x)$ and $\hat{\omega}_1(x)$ are continuous, (D.25) and (D.26) imply that $\omega_2(x)$ and $\omega_1(x)$ are also continuous and weakly decreasing in x.

Next consider the right hand side of (D.27). $\omega_2(x)$ is continuous. Moreover, (D.27) is the simplified version of (B.27). We have already shown that $R_D(\omega_1, \omega_2(x), \bar{y}(x), 1) > 0$, and $\hat{y}(\omega_2(x), g; L) = D(\bar{y}(x); x)(1-x) > 0$, thus the denominator is positive. Each term is also continuous in x, which in turn implies the right hand side is continuous in x. Thus $\hat{\omega}_3(x)$ is continuous as well, and using Equation (D.28), $\omega_3(x)$ is also continuous.

Finally, continuity of $\omega_i(x)$ $i = 1, 2, 3$, along with continuity of $w(.)$, $\eta(.)$ and $D(y; x)$ (in x) implies $\bar{D}(x)$ is continuous. So by Lemma D.1, the fixed point exists.

Explanation of Parameter Restrictions

Optimal Firm Decision without Access to International Market [$\xi \geq \frac{1}{1-\phi}$.] Assume the firm does not have access to international investors. So the firm can do one of the two things. The first option is to invest all of his initial endowment. Then the firm continues with a high scale, $I_I = 1$, if not hit by a liquidity shock, and terminate the project if hit. Thus the payoff is $\Pi_I = \rho_\tau(1-\phi)I_1 = \rho_\tau(1-\phi)$. Alternatively, the firm can save enough of his own endowment using bankers to insure against the liquidity shock in either or both aggregate states. Since the aggregate state is only relevant in the interaction with the international investors, if the firm choose to insure against liquidity shock from own endowment, it will be for both aggregate states. The firm investment scale is given by $I_S = \frac{1}{1+\phi\xi}$, and his expected payoff is $\Pi_I = \rho_\tau I_2$. Thus for $\Pi_I > \Pi_S$ we need

$$1 - \phi > \frac{1}{1+\phi\xi} \Rightarrow \phi < \frac{\phi\xi}{1+\phi\xi} \Rightarrow \xi > \frac{1}{1-\phi},$$

which is Assumption C.1.(i). Under this assumptions when firms can access the international credit market, we only need to compare borrowing on the international markets with investing all of their endowment. This is the next parametric restriction that we consider.

Sufficient Condition for Inequality (C.5) [$\underline{\lambda} < \lambda$] Equations (D.30), (D.15), and the monotonicity of (D.15) in s_H implies

$$\frac{r_H}{1+r_H} = \frac{(1-\lambda)\left(1-s_H(r_H)\right)}{(1-\lambda)\left(1-s_H(r_H)\right) + (\lambda)\frac{\bar{D}(\frac{r_H}{1+r_H})}{D(0;\frac{r_H}{1+r_H})}} \leq \frac{(1-\lambda)}{(1-\lambda) + \lambda\frac{\bar{D}(\frac{r_H}{1+r_H})}{D(0;\frac{r_H}{1+r_H})}},$$

which in turn implies

$$r_H \leq \frac{1-\lambda}{\lambda} \frac{D(0; \frac{r_H}{1+r_H})}{\bar{D}(\frac{r_H}{1+r_H})}.$$

So to find an upper bound on r_H, it is sufficient to find an upper bound on $\frac{D(0; \frac{r_H}{1+r_H})}{\bar{D}(\frac{r_H}{1+r_H})}$. Note that $D(0; \frac{r_H}{1+r_H})$ is what the most transparent good firms spending on maintenance. As these firms face the most favourable credit market conditions, $\bar{D}(\frac{r_H}{1+r_H}) \leq D(0; \frac{r_H}{1+r_H})$ holds which in turn implies

$$r_H \leq \frac{(1-\lambda)}{\lambda} \Rightarrow q_H \leq (1-\lambda)$$

In Assumption C.1.(ii) we assume $\frac{(1-\lambda)}{\lambda} \leq \bar{r}_H$, where $\bar{r}_H$ is defined in Equation (C.5). This in turn insures that $r_H \leq \bar{r}_H$. Moreover, one can substitute $(1-\lambda)$ for x in (D.17) to get an upper bound on $\bar{q}$.

Sufficient Condition for Inequality (B.23) $[w(0) \geq \phi\lambda\xi]$. Using (C.1), we can write condition (B.23) as

$$w(0) > \phi\lambda\ell(0, g; L) = \phi\lambda D(0; r_H)$$

Note that in $\hat{\omega} = \omega_3(r_H)$. A sufficient condition for the above inequality to hold is

$$w(0) \geq \phi\lambda\xi, \tag{D.33}$$

which ensure that $\omega_1 > 0$, and constitutes the first part of Assumption C.1.(iii).

Sufficient Condition for a smaller than one solution to Equation (B.27). $[\lim_{s\to 1} w(s) = 0]$ This condition directly ensures insures that $\tilde{\omega}$ that solves Equation (B.27) is smaller than 1, i.e. $1 > \omega_3$. This is the second part of Assumption C.1.(iii).

Sufficient Condition for Inequality (C.2). $[\min\left\{\frac{(\rho_g-\xi)(1+(1-\lambda)\phi\xi\pi_H)}{(\rho_g(1-\phi)+\phi(\rho_g-\xi)\pi_H)\xi}, \frac{\xi\phi\lambda-w(\omega)}{\xi\phi(\lambda+w(\omega)\pi_L)}\right\} \leq \frac{1-\lambda}{1-\lambda\omega}\ \forall\omega]$. The only set of markets we need to consider are those with cash-in-the-market pricing. Let $q^C(\omega) = \frac{r^C(\omega)}{1+r^C(\omega)}$ and $\bar{q}(r_H) = \frac{\bar{r}(r_H)}{1+\bar{r}(r_H)}$. From (C.2)

$$q^C(\omega) \leq \frac{(1-\lambda)}{(1-\lambda)+\lambda(1-\omega)}$$

Start by noting that $\bar{q}(r_H)$ is the maximum $q^C(\omega)$ can achieve, so a sufficient condition for inequality (C.2) is

$$\min\{\bar{q}(r_H), q^C(\omega)\} \leq \frac{(1-\lambda)}{(1-\lambda)+\lambda(1-\omega)}.$$

Next from (D.17)

$$\bar{q}(r_H) = \frac{(\rho_g - \xi)(1+\phi\xi\pi_H q_H)}{(\rho_g(1-\phi)+\phi(\rho_g-\xi)\pi_H)\,\xi} \leq \frac{(\rho_g-\xi)\,(1+(1-\lambda)\,\phi\xi\pi_H)}{(\rho_g\,(1-\phi)+\phi\,(\rho_g-\xi)\,\pi_H)\,\xi}$$

where the inequality used part (ii) to replace q_H with it's maximum, $(1-\lambda)$. Next, from (D.20)

$$q^C(\omega) = \frac{\xi\phi\lambda - w(\omega)(1+\phi\xi\pi_H x)}{\xi\phi(\lambda + w(\omega)\pi_L)} \leq \frac{\xi\phi\lambda - w(\omega)}{\xi\phi(\lambda+w(\omega)\pi_L)}$$

where the inequality just uses $q_H \geq 0$. Substitute both back to get a sufficient condition

$$\min\left\{\frac{(\rho_g-\xi)\,(1+(1-\lambda)\,\phi\xi\pi_H)}{(\rho_g\,(1-\phi)+\phi\,(\rho_g-\xi)\,\pi_H)\,\xi}, \frac{\xi\phi\lambda - w(\omega)}{\xi\phi(\lambda+w(\omega)\pi_L)}\right\} \leq \frac{1-\lambda}{1-\lambda\omega}.$$

which is Assumption C.1.(iv).

■

E Further Results and Extensions

E.1 Determination of Exposure Groups: Analytical Results

As we note in the main text, we can decompose the total effect of our parameters to the relative size of exposure groups into two parts. First, keeping the interest rate in the high state (r_H) fixed, changes in parameters have a direct effect on maintained investment $i(\omega, \tau, L)$. Second, there is an indirect effect through the spill-over across aggregate states. A change in credit demand in the low state affects scale $I(\tau, g)$ and, through the budget constraint (13), affects credit demand in the high state as well. This in turn changes the equilibrium interest rate in the high state, r_H, which then feeds back into the initial and maintained investment in both high and low states.

In the following proposition, we characterize the direct effect. In particular, we show that an increase in the probability or the size of the liquidity shock, ϕ and ξ, in the fraction of good firms, λ, and in the probability of the low aggregate state, π_L, all increase the total credit demand of good firms at zero interest rate, and, consequently, shrink the set of core countries. Similarly, an increase in the size of the liquidity shock, in the fraction of good firms, and in the productivity of good firms increases the total credit demand of good firms at $\bar{r}(r_H)$ interest rate, and, consequently, increases the set of peripheral countries. While we do not have analytical results on the indirect effect, the direct effect dominates in all our numerical simulations.

Proposition E.1 *In a simple global equilibrium, keeping r_H fixed,*

(i) the set of low exposure countries shrinks if there is an increase in ξ, ϕ, λ, or π_L,

$$\frac{\partial \omega_1}{\partial \xi}, \frac{\partial \omega_1}{\partial \phi}, \frac{\partial \omega_1}{\partial \lambda}, \frac{\partial \omega_1}{\partial \pi_L}|_{r_H \text{ fixed}} < 0.$$

(ii) The set of high exposure countries grows if there is an increase in ξ, λ or ρ_g,

$$\frac{\partial \omega_3}{\partial \xi}, \frac{\partial \omega_3}{\partial \lambda}, \frac{\partial \omega_3}{\partial \rho_g}|_{r_H \text{ fixed}} < 0$$

Proof of Proposition E.1. Using Equation (D.24), the size of the low exposure group is determined by

$$w(\omega_1) = \phi \lambda \xi i\left(\omega, g, L\right)|_{\omega \in [0, \omega_1]}. \tag{E.34}$$

The direct effects come from simple differentiation using Equations (15) and (17) and noting that $\eta_H(\omega) = \eta_L(\omega) = 1$ in the low exposure region. The size of the group of high exposure countries is defined implicitly in Equation (D.27). Let $Z_1 = \phi\left(\lambda\right) \frac{\xi}{1+\bar{r}(r_H)} i\left(\tau_j = \omega, g, L\right)|_{\omega \in [\omega_1, \omega_2]}$, the amount an unrationed representative good firm borrow facing the maximum interest rate $\bar{r}$. In the left panel of Figure 3, we plot the supply of capital of a $k \geq \omega_3$ firm, $\eta_L(\omega) Z_1$ as the dashed curve, which, using the definition of ω_2 in (D.23), we can rewrite as

$$Z_1 \int_{\omega}^{1} \frac{1}{Z_1\left(s - w^{-1}\left(Z_1\right)\right) - \int_{w^{-1}(Z_1)}^{\omega_3} w(s) ds} w(s) ds. \tag{E.35}$$

By definition, ω_1 is determined by the point where this curve is equal to the demand Z_1, the dashed line, as this is the least transparent country where firms demand for credit is fully met. While a change in Z_1 moves both curves, using the implicit function theorem, we can verify that $\frac{\partial \omega_3}{\partial (Z_1)} < 0$. The direct effects then come from simple differentiation using equations (15) and (17) and noting that $\eta_H(\omega) = \eta_L(\omega) = 1$ in the region $\omega \in [\omega_1, \omega_2]$. ■

E.2 Partitioned Opacity Groups

In the baseline model, we assume that investors have an uninformative prior about ω, the average opacity of firms in a given country. That is, if an investor does not find conclusive evidence on a firm, the country of origin does not help her do any further inference.

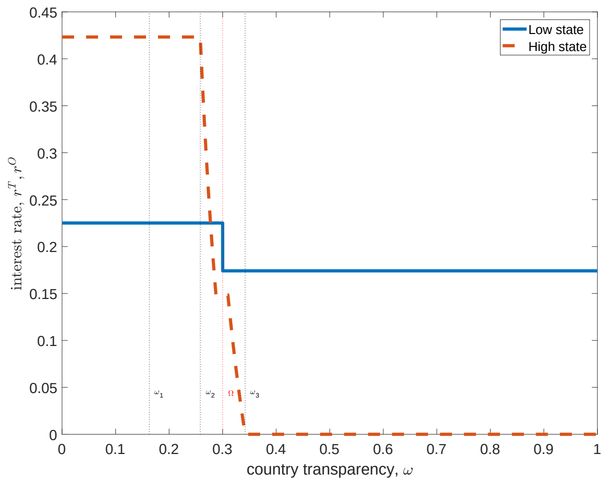

Figure 5: Interest rates for transparent and opaque countries in the high (solid) and low (dashed) aggregate state.

In this section, we weaken this assumption. In particular, suppose that a public signal partitions countries into a transparent and an opaque group. That is, observing the public signal, each investor knows that the opacity, ω of the given country is $\omega > \Omega$ or $\omega < \Omega$, where Ω is an arbitrary cut-off. Intuitively, investors understand that a firm from a southern country in Europe tends to be more opaque than a northern country firm, but they have no information on how firms in different south European countries compare to each other.

Figure 5 illustrates the effect of this treatment on the equilibrium interest rate schedules. Compared to the corresponding figure for the baseline case, the left panel of Figure 1, it is clear that the qualitative difference is small. The main effect of the extra signal is the partial separation in the high aggregate state. With the public signal, investors have an additional choice. They can choose to accept only firms from the transparent group to lend to. For less skilled investors, this implies a portfolio with less bad firms, as their mistakes are concentrated in opaque countries. Therefore, in equilibrium, less skilled investors lend to firms from the transparent group only, albeit at a lower interest rate. On the other hand, more skilled investors lend to firms from the opaque group but for higher interest rate. The marginal investor who is just indifferent between these two choices is determined in equilibrium.[23]

While it is an intuitive assumption that investors have some prior knowledge on the average opacity of firms in different countries, we assume this away in the baseline model

[23]The public signal also introduces a small bunching region around Ω in the low aggregate state interest rate schedule. As we explain in Appendix B, this comes from the requirement that the interest rate schedule has to be weakly monotonically decreasing in ω, and is obtained by an ironing procedure.

because of two main reasons. First, we believe the additional analytical complexity does not justify the additional insight. Second, one of the main focuses of our analysis is how investors endogenously classify countries into low and high exposure groups in equilibrium. As this extension illustrates, a public signal on ω classify countries exogenously, and obscures our analysis.