Online Appendix to "The Long Run Evolution of Absolute Intergenerational Mobility"

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A Proof of Proposition 1

For a bivariate log-normal distribution with parameters μ_p, σ_p (for the parents' marginal distribution), μ_c, σ_c (for the children's marginal distribution) and correlation ρ, the absolute mobility is

$$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 - 2\rho\sigma_p\sigma_c + \sigma_c^2}}\right), \tag{A.1}$$

where Φ is the cumulative distribution function of the standard normal distribution. Provided $\mu_c > \mu_p$ and $\sigma_c > \sigma_p$ it follows that

$$\frac{\partial A}{\partial \rho} > 0; \tag{A.2}$$

$$\frac{\partial A}{\partial\left(\frac{\sigma_c}{\sigma_p}\right)} < 0. \tag{A.3}$$

Proof: First, by definition, the correlation ρ, between X_p and X_c equals to their covariance, divided by $\sigma_p\sigma_c$

$$\rho = \frac{\mathrm{Cov}\left[X_p, X_c\right]}{\sigma_p\sigma_c}. \tag{A.4}$$

β can be directly calculated as follows, by the linear regression slope definition:

$$\beta = \frac{\sum_{i=1}^{N}\left(X_p^i - \bar{X}_p\right)\left(X_c^i - \bar{X}_c\right)}{\sum_{i=1}^{N}\left(X_p^i - \bar{X}_p\right)}, \tag{A.5}$$

where $\bar{X}_p$ and $\bar{X}_c$ are the average parent and child log-incomes, respectively.

It follows that

$$\beta = \frac{\mathrm{Cov}\left[X_p, X_c\right]}{\sigma_p^2}. \tag{A.6}$$

We immediately obtain

$$\beta = \frac{\sigma_c}{\sigma_p}\rho. \tag{A.7}$$

Now we define a new random variable $Z = X_c - X_p$. It follows that calculating A is equivalent to calculating the probability $P\left(Z > 0\right)$.

Subtracting two dependent normal distributions yields

$$Z \sim \mathcal{N}\left(\mu_c - \mu_p, \sigma_p^2 + \sigma_c^2 - 2\mathrm{Cov}\left[X_p, X_c\right]\right) \,, \tag{A.8}$$

and it follows, due to Eq. (A.7), that

$$Z \sim \mathcal{N}\left(\mu_c - \mu_p, \sigma_p^2 \left(1 - 2\beta\right) + \sigma_c^2\right) \,. \tag{A.9}$$

If now follows that

$$\frac{Z - \left(\mu_c - \mu_p\right)}{\sqrt{\sigma_p^2 \left(1 - 2\beta\right) + \sigma_c^2}} \sim \mathcal{N}\left(0, 1\right) \,, \tag{A.10}$$

so we can now write

$$\begin{aligned} & A = P\left(Z > 0\right) = \\ & P\left(\frac{Z - \left(\mu_c - \mu_p\right)}{\sqrt{\sigma_p^2 \left(1 - 2\beta\right) + \sigma_c^2}} > -\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 \left(1 - 2\beta\right) + \sigma_c^2}}\right) = \\ & \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 \left(1 - 2\beta\right) + \sigma_c^2}}\right) = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 - 2\rho\sigma_p\sigma_c + \sigma_c^2}}\right) \,, \end{aligned} \tag{A.11}$$

where Φ is the cumulative distribution function of the standard normal distribution. Taking the partial derivative of this expression with respect to the correlation ρ we get

$$\frac{\partial A}{\partial \rho} = \frac{\left(\mu_c - \mu_p\right) \sigma_p \sigma_c e^{-\frac{\left(\mu_c - \mu_p\right)^2}{2\left(\sigma_p^2 - 2\rho\sigma_p\sigma_c + \sigma_c^2\right)}}}{\sqrt{2\pi}\left(\sigma_p^2 - 2\rho\sigma_p\sigma_c + \sigma_c^2\right)^{3/2}} \,. \tag{A.12}$$

Assuming $\mu_c > \mu_p$, it follows that

$$\frac{\partial A}{\partial \rho} > 0 \,. \tag{A.13}$$

Similarly, we can rewrite A as

$$\Phi\left(\frac{\mu_c - \mu_p}{\sigma_p \sqrt{1 - 2\rho\kappa + \kappa^2}}\right) \,, \tag{A.14}$$

where $\kappa = \sigma_c / \sigma_p$. Assuming $\kappa > 1$, *i.e.* if inequality increases between the generations, $\sqrt{1 - 2\rho\kappa + \kappa^2}$ is an increasing function of κ (because $0 < \rho < 1$). Thus, A is decreasing with κ, or

$$\frac{\partial A}{\partial \left(\frac{\sigma_c}{\sigma_p}\right)} < 0 \,. \tag{A.15}$$

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B A Comparison between Absolute Mobility and Katz-Krueger Measure of Mobility

The results of Chetty et al. (2017) show that the share of children earning more than the median parent declined from 92% in the 1940 birth cohort to 45% in the 1984 cohort (Katz and Krueger, 2017). This alternative measure of absolute mobility – the share of children earning more than the median parent – moves almost identically to A across cohorts in the United States (Katz and Krueger, 2017). Denoting it as $\tilde{A}$, it follows that in the bivariate log-normal model

$$\tilde{A} = \Phi\left(\frac{\mu_c - \mu_p}{\sigma_c}\right), \tag{B.1}$$

where Φ is the cumulative distribution function of the standard normal distribution.

Using $\tilde{A}$ has clear advantages over A. In particular, they can be "directly computed from standard public-use cross-sectional household survey data and do not require data that longitudinally link children to parents." (Katz and Krueger, 2017, p. 382) However, $\tilde{A}$ would be close to A only if the IGE is close to 1/2:

Proposition 2 *For a bivariate log-normal distribution with parameters μ_p, σ_p (for the parents' marginal distribution), μ_c, σ_c (for the children's marginal distribution) and assuming IGE of β, then*

$$A = \tilde{A} \iff \beta = \frac{1}{2}. \tag{B.2}$$

Proof Following Eq. (A.1)

$$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}}\right). \tag{B.3}$$

Following Eq. (B.1)

$$\tilde{A} = \Phi\left(\frac{\mu_c - \mu_p}{\sigma_c}\right), \tag{B.4}$$

and therefore

$$\tilde{A} = A \iff \frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}} = \pm\frac{\mu_c - \mu_p}{\sigma_c}. \tag{B.5}$$

We then obtain

$$\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}} = \pm\frac{\mu_c - \mu_p}{\sigma_c} \iff \sigma_c = \pm\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2} \iff \beta = \frac{1}{2}. \tag{B.6}$$

■

It follows that $\tilde{A}$ cannot be used as a proxy for A unless the IGE is close to 0.5. As discussed in Section I in the main text, it is therefore no surprise that for the United States A and $\tilde{A}$ are similar – Aaronson and Mazumder (2008) estimate the IGE for the 1950–1970 birth cohorts at 0.46–0.58.

C Robustness Checks I: Copulas

C.1 The Validity of the Copula Decomposition

To estimate absolute intergenerational mobility we use repeated cross-sections and combine them using a copula, the joint distribution of parent and child income ranks. This methodology is discussed in Section II in the main text. It follows that A, the measure of absolute mobility, is

$$A = \int \mathbf{1}_{\{Q^c(r^c) \geq Q^p(r^p)\}} C\left(r^c, r^p\right) dr^c dr^p \,, \tag{C.1}$$

where r^c and r^p are the child and parent income ranks, respectively; Q^c and Q^p are the respective quantile functions; C is the copula. The decomposition in Eq. (C.1) looks different from the definition in Eq. (1) (in the main text), but it is exact when the copula is given in full. This follows from Sklar's theorem (Sklar, 1959), which states that any two-dimensional distribution can be expressed as the composition of a copula and two marginal one-dimensional distributions.

In practice, the accuracy of the decomposition is somewhat limited by the resolution in which the copula is measured and reported, as copulas are typically estimated in discrete form. For example, using 10 or 100 fractiles to represent a copula could make a difference to the estimated absolute mobility. In Figure C.1 we demonstrate that the effect is small, typically of less than one percentage point when compared to an analytic result. To show that, we test the sensitivity to the resolution of the copula, with parameters fitted for France in different years (cohorts of 1950, 1960, 1970, and 1980), tested against a theoretically tractable result in the bivariate log-normal model (see Section I in the main text). This way Eq. (C.1) can be verified.

In each case we:

- Fit the parameters μ_p, σ_p, μ_c, σ_c and ρ to the data for France
- Create a random simulated bivariate log-normal sample (with 10^6 pairs of values) with these parameters
- Construct the copula for the simulated sample in a given resolution (either 10, 20, 40, 50, 80, or 100 fractiles)
- Compute the absolute mobility using Eq. (C.1) given the simulated sample for the marginal distributions and the discrete copula

The last three steps are repeated 100 times in each case to avoid an effect created by mere chance. The results are then compared to the analytic result (Eq. (3) in the main text). The comparison is presented in Figure C.1. It shows that as the copula resolution increases, Eq. (C.1) becomes closer to the analytic result, as expected. Yet, even if the resolution is as coarse as 10 fractiles only, the difference between Eq. (C.1) and the analytic result is up to 1 percentage point.

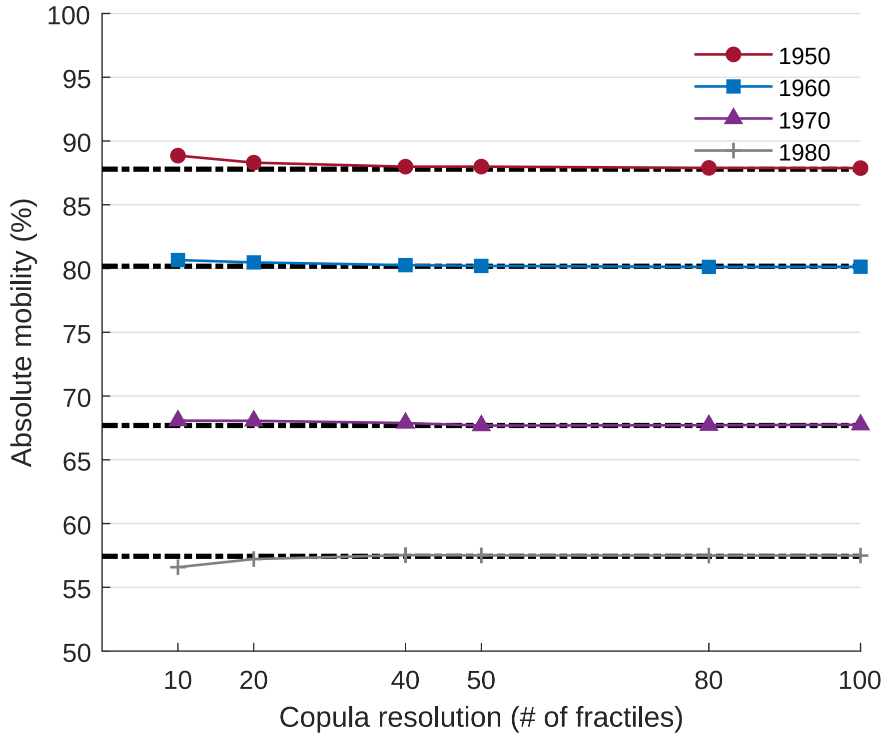

Figure C.1: The effect of the copula resolution on absolute mobility. Using fitted parameters for France in 1950, 1960, 1970, and 1980 (The World Inequality Database, 2017) we compare an analytic result when using the bivariate log-normal model to numerical simulations, where the copulas are estimated in different resolutions (10, 20, 40, 50, 80, and 100) and absolute mobility is computed using Eq. (C.1). The black dash-dotted lines are the analytic results. To eliminate the impact of randomness in the simulations, they are repeated 100 times in each case and the results averaged over the simulations (the standard deviation of the results in each case among the 100 simulated samples was less than 0.7 percentage points).

C.2 Sensitivity of Absolute Mobility to Copula Structure

In our estimates we use Spearman's rank correlation (or rank-rank slope) to describe the copula between the income distributions of parents and children. We then continue to estimate the absolute intergenerational mobility. We assume that given the marginal distributions, the rank correlation determines absolute mobility. Conceptually, the same rank correlation could result in very different absolute mobility estimates. Yet, as we describe below, this requires the copulas to be unrealistic. Realistic copulas with a different structure, but the same rank correlation, would deliver almost identical estimates of absolute mobility.

We describe two tests for the sensitivity of absolute mobility to the copula structure. First, we discuss a general way to tweak copulas so that their structure changes, but their rank correlation is preserved. This allows taking realistic copulas to the extreme (while preserving their rank correlation) and test how that affects absolute mobility. In a second test we compare several copula models from the literature that are parametrized by their rank correlation. We show that for a given rank correlation, they all result in almost identical absolute mobility estimates.

C.2.1 Impact of rank-correlation preserving moves

We consider copulas as transition (doubly stochastic) matrices $\mathbf{P} \in \mathscr{P}(N)$, where p_{ij} represents the probability of transferring to quantile j (child) for those starting in quantile i (parent) and N is the number of income quantiles. Evidence shows that the diagonal elements are generally higher and the transition probabilities decrease with the transition distance. The probability to move between two ranks i and j within two generations is a decreasing function of $|i-j|$ (see, for example Jäntti et al. (2006); Chetty et al. (2017)). Preserving the rank correlation, while creating a large effect on absolute mobility requires breaking this regularity.

The rank correlation of a transition matrix is

$$\rho_S(\mathbf{P}) = \frac{12\sum_{i=1}^{N}\sum_{j=1}^{N} ijp_{ij} - 3N(N+1)^2}{N(N^2-1)}, \tag{C.2}$$

thus, only the sum $\sum_{i=1}^{N}\sum_{j=1}^{N} ijp_{ij}$ depends on the matrix elements.

We now define a Δ-*local rank-correlation preserving move* as a change to 8 elements in the matrix – p_{i_1,j_1}, p_{i_1+1,j_1}, p_{i_1,j_1+1}, p_{i_1+1,j_1+1} and p_{i_2,j_2}, p_{i_2+1,j_2}, p_{i_2,j_2+1}, p_{i_2+1,j_2+1} in the following way:

$$\begin{aligned}
&p_{i_1,j_1} \to p_{i_1,j_1} + \Delta \\
&p_{i_1+1,j_1+1} \to p_{i_1+1,j_1+1} + \Delta \\
&p_{i_1+1,j_1} \to p_{i_1+1,j_1} - \Delta \\
&p_{i_1,j_1+1} \to p_{i_1,j_1+1} - \Delta \\
&p_{i_2,j_2} \to p_{i_2,j_2} - \Delta \\
&p_{i_2+1,j_2+1} \to p_{i_2+1,j_2+1} - \Delta \\
&p_{i_2+1,j_2} \to p_{i_2+1,j_2} + \Delta \\
&p_{i_2,j_2+1} \to p_{i_2,j_2+1} + \Delta
\end{aligned} \tag{C.3}$$

where Δ can be either positive or negative (as long as all the elements remain non-negative and not greater than 1) and i_1, j_1, i_2 and j_2 can be any quantiles between 1 and $N-1$.

Such a change trivially preserves the sum $\sum_{i=1}^{N}\sum_{j=1}^{N} ijp_{ij}$ and therefore the rank correlation. By composing many Δ-local rank-correlation preserving moves it is possible to change a given copula while preserving the rank correlation.

In general, rank-correlation preserving moves have the effect of increasing the trace while also increasing the extreme ends of the transition matrix, or vice versa. This is demonstrated in the three copulas in Figure C.2. They all share the same rank correlation (0.3), but are very different from one another. The copulas were constructed by composing several Δ-local rank-correlation preserving moves on copula **A**, which is the copula used for producing the baseline estimates of absolute mobility in France. Only copula **A** is realistic and has the typical form of the empirical copulas (compare with Jäntti et al. (2006); Eberharter (2014)). Copula **C** is far from being plausible, attaching very high probabilities to the diagonal, zeros to some off diagonal elements, but non-zero probability to make the largest possible moves between generations.

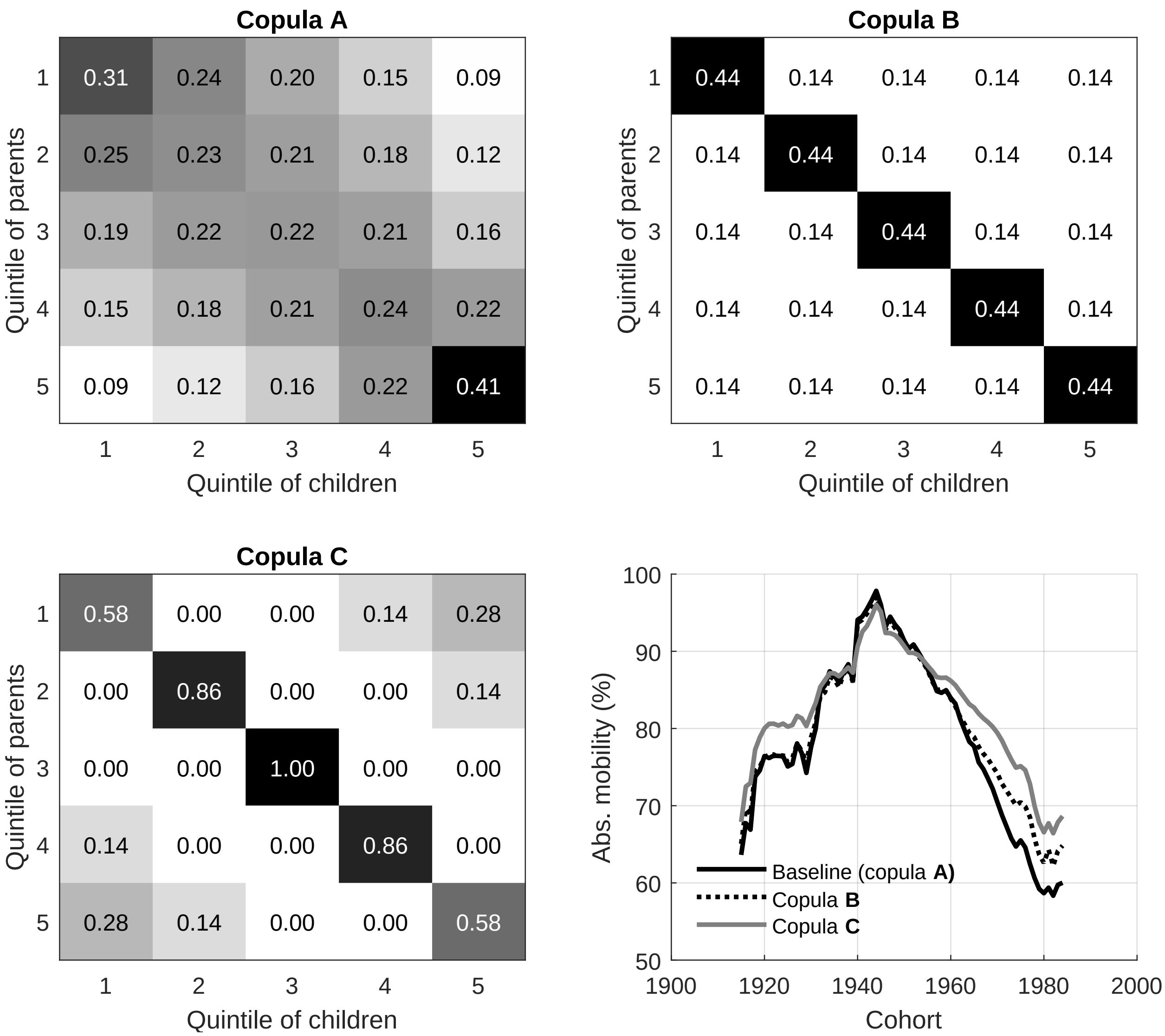

Figure C.2: Three copulas (transition matrices) constructed by composing many Δ-local rank-correlation preserving moves. Copula **A** is similar to the copula used for producing the baseline estimates of absolute mobility in France. The bottom right panel shows the absolute mobility estimates for France when using the different copulas with the same rank correlation but very different structure.

Nevertheless, copulas **A** and **B** produce almost the same absolute mobility. This was tested for France, using the same marginal distributions used for the baseline estimates. For each of the three copulas we produced a series of estimated absolute mobility values. The results are also presented in Figure C.2. As expected, copula **C** leads to results that are different from those obtained with copula **A**. Yet, the trend remains similar and the results are 3.6

percentage points higher than the baseline estimate on average. Copula **B** leads to results that are almost identical to the baseline. This is regardless of it being rather unrealistic.

C.2.2 Sensitivity to copula model

Copulas are many times modeled, *i.e.* described by parametric functions. The model choice, for a given rank correlation, means that the copula structure is slightly different, for example, more or less weight along the diagonal or possible asymmetries. The choice of copula model may affect the estimated absolute mobility. We demonstrate that as long as the rank correlation is the same, the copula model effect on estimated absolute mobility is, in practice, insignificant.

We compare four copula models – Gaussian, which is the copula in the bivariate log-normal model, as well as the Clayton, the Gumbel and the Plackett copula families (Plackett, 1965; Trivedi and Zimmer, 2007). In their study of relative intragenerational labor income mobility in France, Bonhomme and Robin (2009, p. 67) argue that the Gaussian copula "tends to underestimate the dependence in the middle of the distribution, that is, the probabilities of remaining in the second, third, and fourth quintiles" and show that the empirical copula is best estimated by the Plackett copula. Gumbel copulas, however, unlike the other models, are asymmetric along the main diagonal, *e.g.* the probability of a child whose parents are in the highest percentile to stay there are higher than those of a child whose parents are in the lowest percentile to stay there. This is a realistic property (see, for example, copula **A** in the previous section). Thus, while all the models are used in the literature and resemble real copulas, they are somewhat different from one another.

To compare between the different copula models we consider the marginal distributions in France and the United States, the same way it was done in our baseline estimates (see Section III in the main text). For each copula model we randomly create a bivariate distribution with uniformly distributed marginals (between 0 and 1) and with the appropriate copula structure (*i.e.* the chosen copula model with the right parameter that correspond to the rank correlation in France or in the US). This was done using the MATLAB procedure *copularnd*. Then we compose from the marginals and the bivariate uniform distribution the joint intergenerational income distribution of interest in each case, from which the measure of absolute mobility is immediately computed.

Figure C.3 demonstrates that the differences between the absolute mobility estimates when using different copula models, while assuming the same rank correlation, are negligible. There is not sizable impact on neither level nor trend.

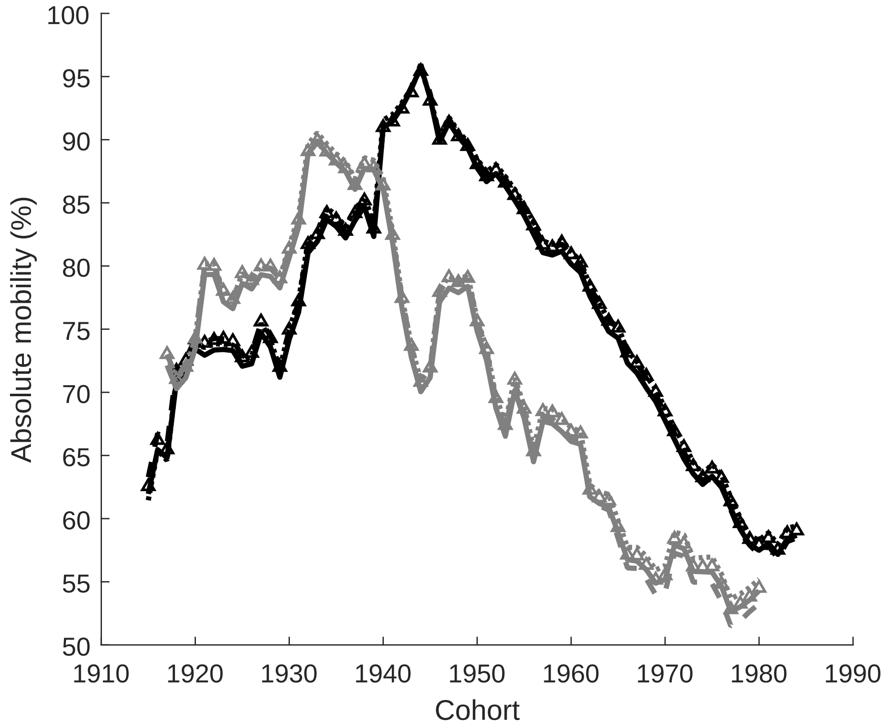

Figure C.3: The copula model effect on the absolute mobility in France (black) and the United States (gray). The copula models used were Gaussian (solid lines), Clayton (dashed), Gumbel (dotted) and Plackett (triangles).

C.3 Robustness of Trends

To assess the robustness of the results in terms of the copula's role in determining long run absolute intergenerational mobility trends we conducted two tests. First, we check how the baseline estimates for each country studied would change if the rank correlation is assumed to be either 0.1 or 0.5, in addition to the nominal value used for each country (based on the same methodology used to produce the results in Figure 4 in the main text). The results of this test are presented in Figure C.4. We conclude that in none of the cases letting the rank correlation vary within a wide range of values changes the long run trend of the absolute mobility estimates.

In a second test, we ask how the overall decrease in absolute mobility changes if instead of using the nominal rank correlation in each country, we let its value change from 0 to 1. For every rank correlation value we recalculate the resulting evolution of absolute mobility and determine the decrease in absolute mobility over a long time period (in most countries between the 1950 cohort to the latest cohort available; in some countries we begin in an earlier cohort if available). Figure C.5 shows that in practice, no realistic copula would overturn the observed decreasing absolute mobility trends. In none of the countries a rank correlation value between 0 and 0.6 led to a change of more than 7 percentage points from the baseline decline (over the entire period). In almost all countries the difference was limited to 2–3 percentage points. Bigger changes, to the extent the trend is completely overturned, require the rank correlation to be 0.8 and above, much higher than any realistic value. In some countries even such high levels of immobility would not have a large effect on the absolute mobility trend.

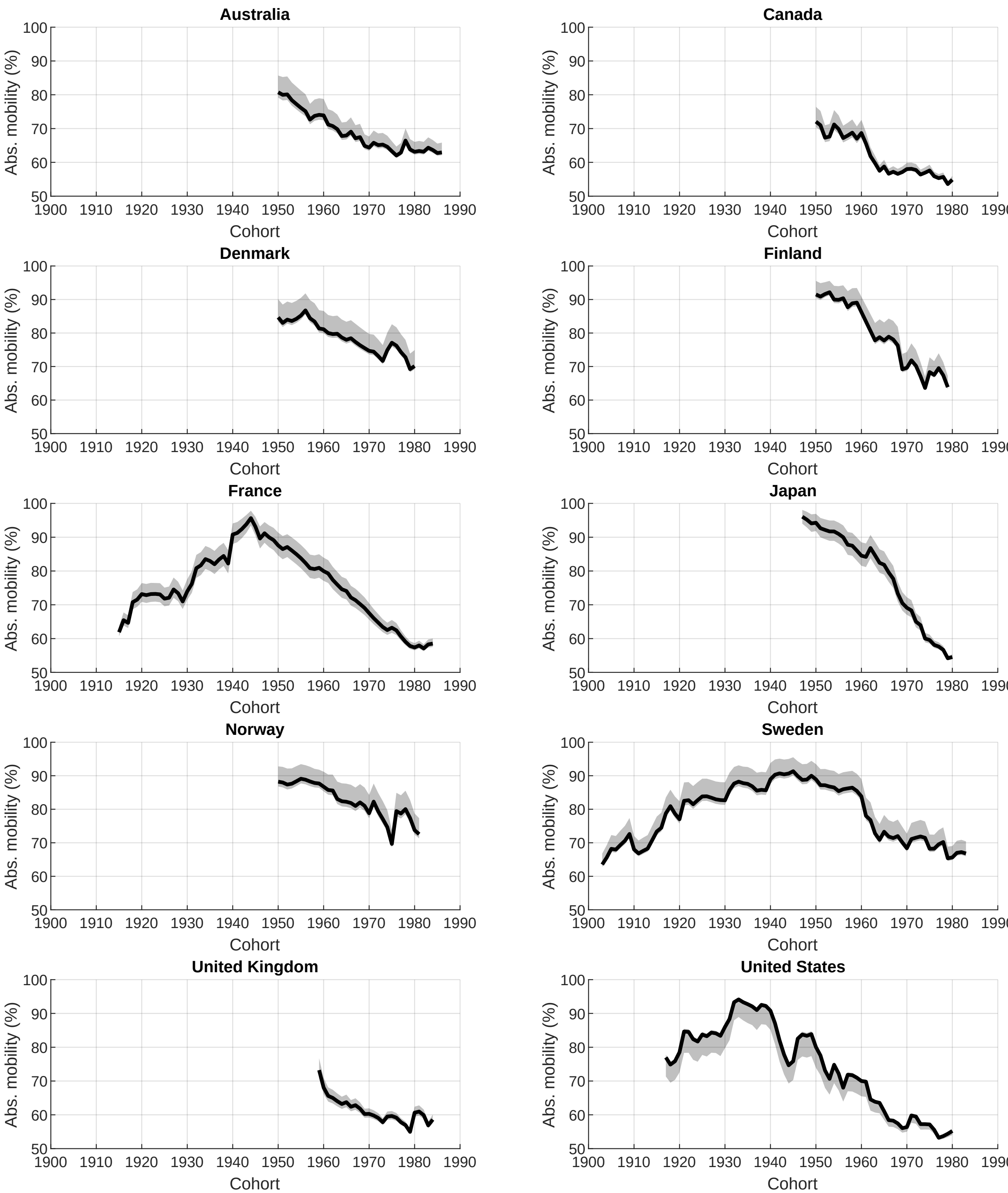

Figure C.4: The evolution of absolute intergenerational mobility in advanced economies. The absolute mobility based on nominal rank correlations is in black. The shaded gray areas are the ranges covered by the lower and upper bound of the estimates assuming the rank correlation is within the range [0.1, 0.5] for each country. In some cases the shaded areas are too narrow to be visible.

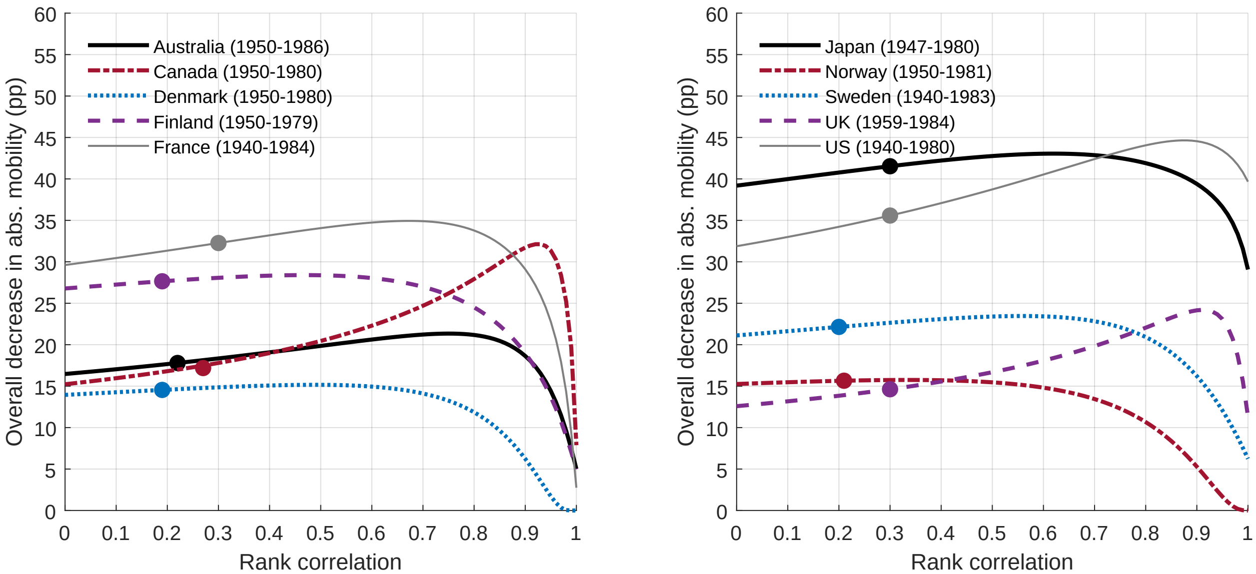

Figure C.5: The decrease in absolute mobility with changing rank correlation. In each country the magnitude of the decrease in absolute mobility since 1950 (slightly earlier or later in some countries, depending on the data availability) was calculated, while letting the rank correlation change from 0 to 1. The circles show the baseline decrease in absolute mobility for each country (using the nominal rank correlation value).

C.4 Sensitivity to absolute mobility threshold

A possible concern with the measure of absolute mobility is that it is a "crude" one. Children earning income that is just above their parents' would count the same as children earning twice their parents. One possibility is that the role of the copula is more important if we define absolute mobility as

$$A_X = \frac{\sum_{i=1}^{N} \mathbf{1}_{\{Y_{ci}/Y_{pi}>1+X/100\}}}{N}\,, \tag{C.4}$$

i.e. children are counted if their income is at least $X\%$ higher than their parents'. In particular, we can define X as the average growth rate in income (which would then neutralize the effect of income growth on the estimated absolute mobility).

The analytic result for the bivariate log-normal approximation and the simulation results shown in Figure 3 in the main text demonstrate that the sensitivity of absolute mobility to the copula generally becomes smaller with smaller growth. In other words, when setting X to be the average growth rate, the results are expected to be less sensitive to the copula than when X is zero.

Figure C.6 shows that in the four countries with the largest observed decline in absolute mobility – Australia, France, Japan and the United States – setting X to be 10% has no real impact on the trends observed. When X is the average income growth rate, the trend only reflects the impact of the changing inequality. Taking the rank correlation in this case to be either 0 or 0.6 has no real impact on the A_X trend and only a small effect on levels (slightly higher in the United States than in other countries).

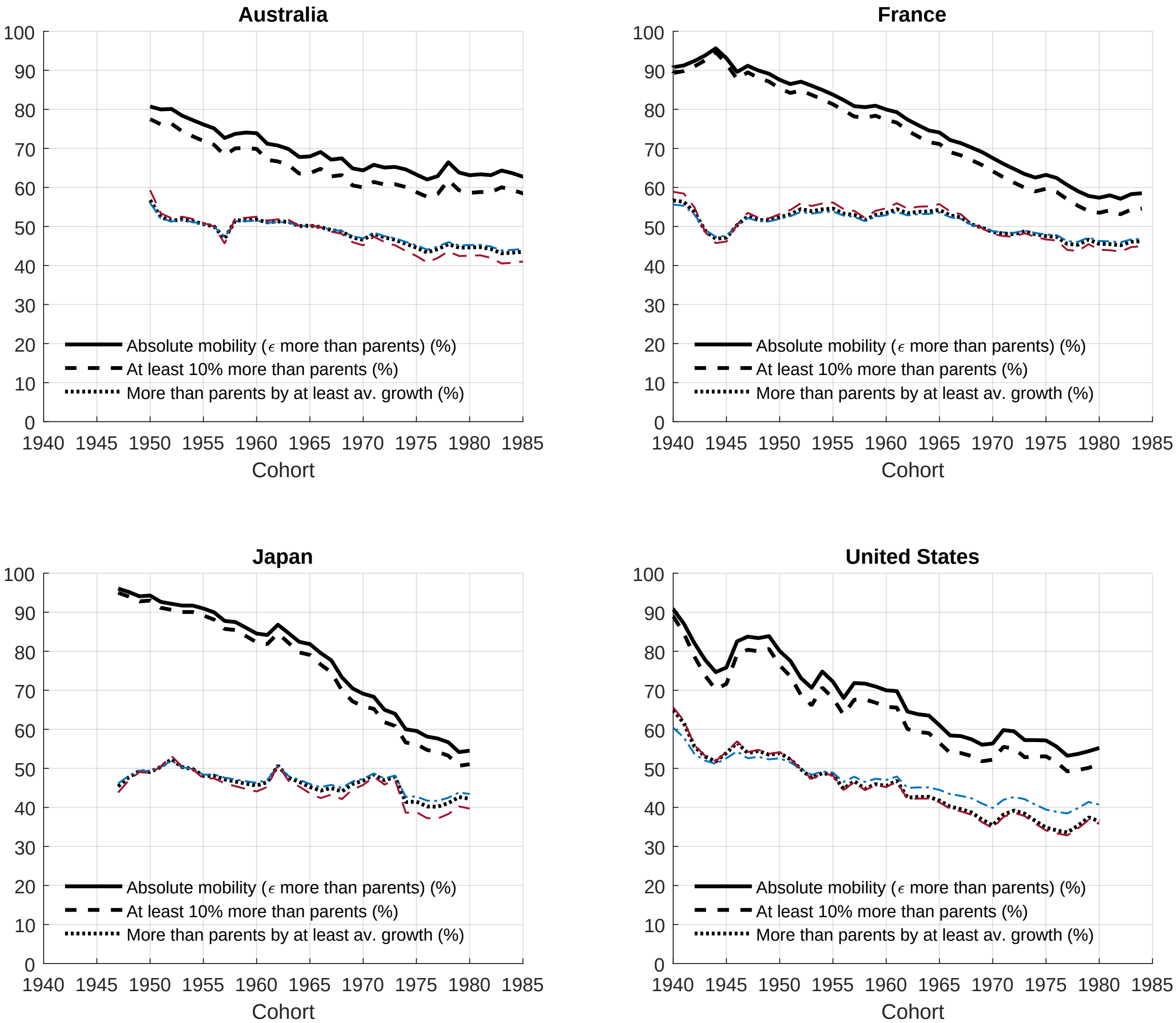

Figure C.6: The decrease in absolute mobility in Australia, France, Japan and the United States with different thresholds. In each country we compare the baseline decreasing trend of absolute mobility (with Eq. (C.4) for $X = 0$) to two cases: $X = 10\%$ and when X is the intergenerational income growth rate for every cohort. In the latter case we also compare the outcome for rank correlation of either 0 (blue) and 0.6 (red) to see whether the sensitivity of the measure of absolute mobility to the copula becomes substantial in this case.

D Empirical Copulas and Measures of Relative Mobility

In Figure 2 in the main text we use copulas measured for different birth cohorts, different countries and for both pre-tax and post-tax incomes and compare them in terms of different measures of relative mobility. Our aim was to demonstrate that although relative mobility is measured by theoretically distinct measures, in practice, differences in one measure translate into proportional changes in other measures. These measures of relative mobility are effectively interchangeable, supporting the claim that intergenerational copulas share a typical form.

Here we present the different measures used for the comparison. We consider copulas as transition (doubly stochastic) matrices $\mathbf{P} \in \mathscr{P}(N)$, where p_{ij} represents the probability of transferring to quantile j (child) for those starting in quantile i (parent) and N is the number of income quantiles. We use four standard measures of relative mobility:

- Spearman's rank correlation (Spearman, 1904) (or rank-rank slope, RRS), defined as

$$\rho_S(\mathbf{P}) = \frac{12 \sum_{i=1}^{N} \sum_{j=1}^{N} ijp_{ij} - 3N(N+1)^2}{N(N^2-1)} \tag{D.1}$$

- Bartholomew's index (Bartholomew, 1967) (average absolute jump), defined as

$$B(\mathbf{P}) = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} |i-j| \, p_{ij} \tag{D.2}$$

- Average absolute non-zero jump, defined as the average absolute jump while excluding the trace of $\mathbf{P}$, or

$$NZ(\mathbf{P}) = \frac{N \cdot B(\mathbf{P})}{N - tr(\mathbf{P})} \tag{D.3}$$

- Shorrocks' trace index (Shorrocks, 1978), defined as

$$S(\mathbf{P}) = \frac{N - tr(\mathbf{P})}{N-1} \tag{D.4}$$

The different measures are mathematically related, however they are not linearly dependent. Specifically, it is possible to construct matrices which have the same trace index, but very different rank correlation, average absolute non-zero jump measure or Bartholomew's index and vice versa. Bartholomew (1967); Shorrocks (1978); Atkinson and Bourguignon (1982); Atkinson (1983) provide several constructive examples demonstrating the differences between such measures. They describe mathematical constructions of copulas such that one measure is preserved while others may change. An example of three copulas with the same rank correlation that differ in the other measures is presented in Appendix C.2.

In addition, we would like to demonstrate that the set of 28 copulas used in Figure 2 in the main text spans indeed over a wide range. Figure D.1 displays two 5×5 matrices that represent these copulas for Denmark and the United States (the copula for the US displays lower mobility than the one used in the baseline estimates, and was taken from Eberharter (2014) and not from Chetty et al. (2017)). These copulas are the furthest away from one another in the entire set of copulas (taking the difference in the rank correlation as the measure of distance: 0.1 for Denmark and 0.44 for the US).

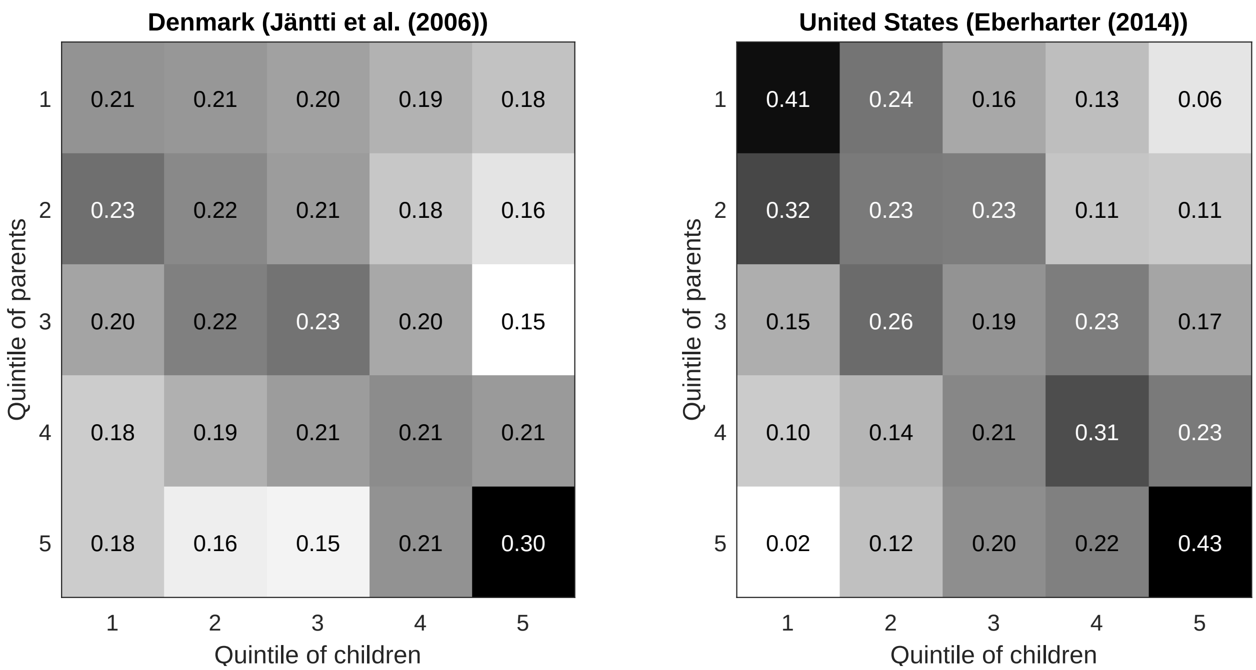

Figure D.1: 5×5 representations of intergenerational copulas for Denmark (Jäntti et al., 2006) the United States (Eberharter, 2014). These copulas are chosen to demonstrate the two copulas that are the furthest away from one another in the set of 28 copulas used to produce Figure 2 in the main text.

E Robustness Checks II: Marginal Distributions

The estimation of absolute mobility using Eq. (6) in the main text requires a copula and the marginal distributions of parents and children. In previous appendices we studied mainly the sensitivity of absolute mobility estimates to the copula, showing that this sensitivity is very low in all realistic scenarios, in terms of both levels and trends. This appendix provides a series of tests for the sensitivity of absolute mobility to changes in the marginal distributions. The tests address multiple aspects that are crucial for understanding where the assumptions made in our main analysis (Section III in the main text) might break down. More importantly, these tests would help verifying that the main conclusions of the analysis are indeed robust. In particular, we would like to address to following aspects:

- Unit of observation (individual, family, tax unit or equal-split adult)
- Using the entire adult population vs. certain age groups
- Income concept (total vs. labor income; pre-tax vs. post-tax)
- Under- and over-estimation of income growth and income inequality

E.1 Sensitivity to the Unit of Observation

In order to make our analysis results as comparable as possible to those in Chetty et al. (2017) we use equal-split adults as the unit of observation of our income data when possible: individuals in tax units that are composed of more than one income-contributing individuals are assumed to contribute each an equal part to the total income. In several countries the income data are based, however, on individual or tax unit incomes. Tax units may be either individuals or families, depending on the country and the year. In some countries taxes are declared on an individual basis today, but not in the past.

The baseline estimates do not take into account those differences. In Figure E.1 we show that under very conservative assumptions, ignoring the differences between individual and family income may lead to a downward bias of 5–6.5 percentage points in absolute intergenerational mobility.

The calculation assumes that the samples of individual incomes are randomly divided into two sub-samples of equal size – $\{A_i\}$ and $\{B_i\}$. These sub-samples are then matched assuming a Gaussian copula with rank correlation of zero (meaning perfectly random matching) so that for a specific index j, A_j corresponds to B_j. These represent spouses in a family and we assume that each family is composed of two spouses exactly. The matched incomes are then summed to create a new sample $\{C_i\}$, for $C_i = A_i + B_i$. This is done for every year and then absolute mobility is estimated the same way as the baseline estimate but assuming the $\{C_i\}$ samples rather than the original samples, based on individual incomes.

This is a conservative estimate since it ignores assortative mating. Assortative mating effectively increases the rank correlation between spouses' incomes, which was assumed 0 in the previous calculation. For a rank correlation of 1, the absolute mobility estimates will be

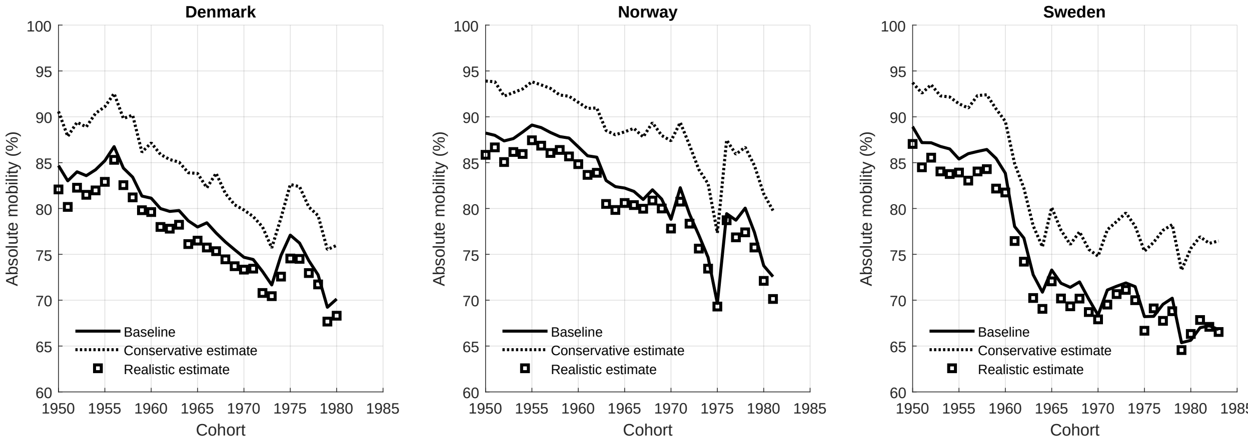

Figure E.1: The absolute intergenerational mobility in Denmark, Norway and Sweden implementing assortative mating on individual income data.

similar to those obtained without the sample splitting. The effect of sample splitting would increase with decreasing rank correlation. In addition, we assume that all families have two spouses and we ignore single-person families, for which individual data reflect family data. For these reasons, the difference between individual, equal-split and family income absolute mobility estimates is practically smaller than that created by the conservative estimate.

In Denmark, Norway and Sweden, in which the data we use for absolute mobility estimates are individualized, 40%–50% of households are single-person families (Eurostat, 2018). We consider, in addition to the zero rank correlation conservative estimate, a more realistic estimate for these countries – we assume that for each of the individual income samples half remains unchanged and the other half is divided and matched in the way explained, assuming a rank correlation of 0.3 with a Gaussian copula (the income rank correlation between spouses in the United States is 0.3 and was very stable from 1964 onward (United States Bureau of Labor Statistics, 2017)). Figure E.1 presents these estimates, which are very close to the baseline estimates, both in level and in trend.

We note that despite the results in Figure E.1 for Denmark, Norway and Sweden it is still possible that the unit of observation matters for the absolute mobility trend. This might be because the share of families with two spouses has been declining in most advanced economies. Chetty et al. (2017) show that in the United States, adjusting for family size substantially reduces the overall decline in absolute mobility from 1940 to 1984. This happens as less income is needed to maintain the same standard of living within a family when family size is declining. A similar result was found in Berman (2018) for absolute intragenerational mobility.

Using the detailed data from Garbinti, Goupille-Lebret and Piketty (2018) we can also test the sensitivity of the results for France to different units of observation. In particular, these data allow comparing absolute mobility for individual adults and equal-split adults in France after 1970. The individualized series assign zero labor income to nonworking spouses, and is more unequal than the equal-split series by design. Income growth is similar in both. Therefore, the individualized-based estimates should be lower than the baseline estimates.

Figure E.2 presents the results, showing that the individualized-based estimates are indeed lower. Yet, the differences between the estimates are small – the average difference is 1 percentage point and the overall trend is similar.

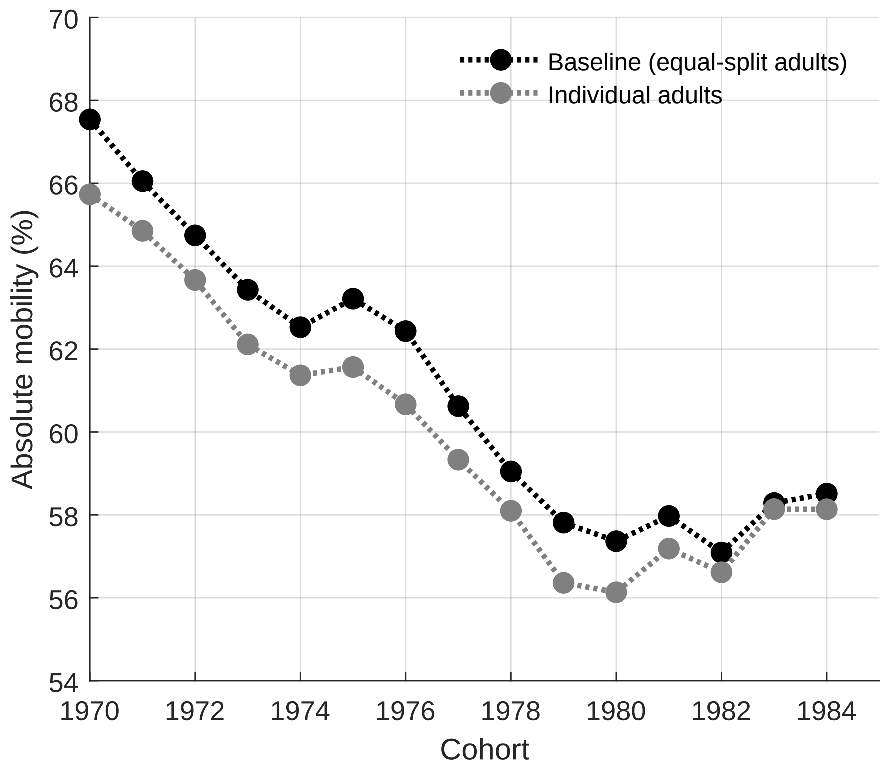

Figure E.2: Absolute intergenerational mobility in France using equal-split and individualized income data.

Further support to the small sensitivity of absolute mobility estimates to the unit of observation is given in the next section, where we consider various age groups in the United States.

E.2 Absolute Intergenerational Mobility for the Entire Population and for Age Groups

Garbinti, Goupille-Lebret and Piketty (2018, p. 64) "combine national accounts, tax and survey data in a comprehensive and consistent manner to build homogeneous annual series on the distribution of national income by percentiles over the 1900–2014 period, with detailed breakdown by age, gender and income categories over the 1970–2014 period." Using the tabulated age-grouped data, it is possible to estimate the absolute intergenerational mobility in France for 1970, 1975 and 1979 cohorts by considering the income distribution of adults aged 20–39 in 1970, 1975 and 1979 as parents and in 2000, 2005 and 2009 as children. Assuming rank correlation of 0.3, the same as in the baseline estimates, we find that the estimates assuming 20–39 year-old adults, are lower than the baseline estimates by 2–5 percentage points. This difference is not statistically significant, due to the large statistical error associated with the tabulated data. In terms of trend, these results are inline with the baseline.

In addition, we can compare our baseline estimates for the United States to those reported in Chetty et al. (2017). The latter estimates are only based on samples of 30-year-old children, rather than pooling all ages together as done for the baseline estimates.

The results are presented in Figure E.3. They illustrate the high similarity between the different estimates. Although pooling all ages together does change the results in terms of levels, this has only a small effect on the long run trend of absolute mobility.

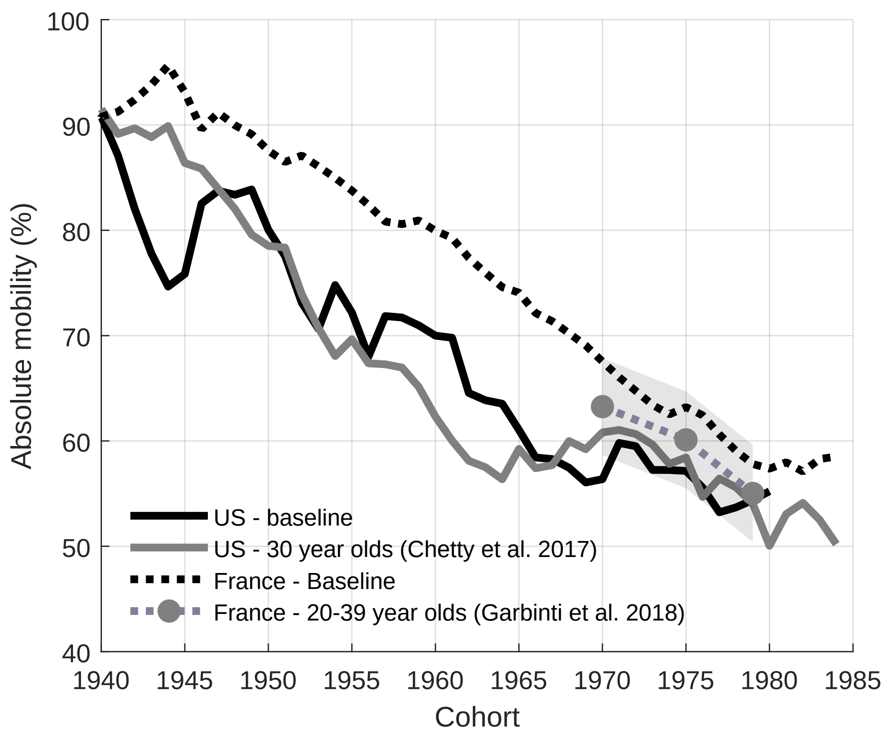

Figure E.3: The absolute intergenerational mobility in the United States and in France for all adults and for different age groups. The shaded gray area represents a 95% confidence interval for the estimates produced by bootstrapping for France, based on the tabulations in Garbinti, Goupille-Lebret and Piketty (2018).

We can also compare our baseline estimates for the United States to estimates that use the Current Population Survey (CPS) for the marginal income distributions (United States Bureau of Labor Statistics, 2017). The CPS is a detailed household survey, so it is possible to restrict the marginal distributions to certain age groups only. It also enables considering individual income and family income separately (see also previous section). These data are available from 1962 onward.

Figure E.4 presents this comparison. It shows that all specifications, taking into account all adults, only 30 year olds, only 40 year olds, 40 year old 'children' compared to 30 year old 'parents' (*i.e.* considering the populations of 30 year olds at year X and of 40 year olds at year $X + 30$), and family incomes for adults aged 35–45 (so that the sample is not restricted to families with spouses that are both at the same age), result in almost identical trends of absolute mobility. They produce a rather narrow band of values, with the 30 year olds vs. 40 year olds specification being the only case in which the deviations from the baseline were consistently and considerably larger. This occurs despite substantial differences in terms of the average income growth in several specifications. This is because in these cohorts the dominant factor in determining absolute mobility is the increase in income inequality, which is rather similar in the various specifications. It is possible that in earlier cohorts a similar test would have resulted in a slightly higher sensitivity. Yet, it is very unlikely that the long run trend will substantially change. This is demonstrated in Appendix E.4 below, discussing the impact of systematic over- and under-estimation of income growth on the long run trend of absolute mobility.

For all CPS-based specifications, absolute mobility was also calculated while considering rank correlations of 0.1 and 0.5, to reflect the potential impact of changing copulas over time. The entire range of values resulting in these calculations is presented in Figure E.4. These calculations show that when such a wide variety of adjustments to income measurement was made, including changes in the copula, absolute mobility declined by 9 (the same as in the baseline estimates) to 13 percentage points between 1962 and 1980 in the United States.

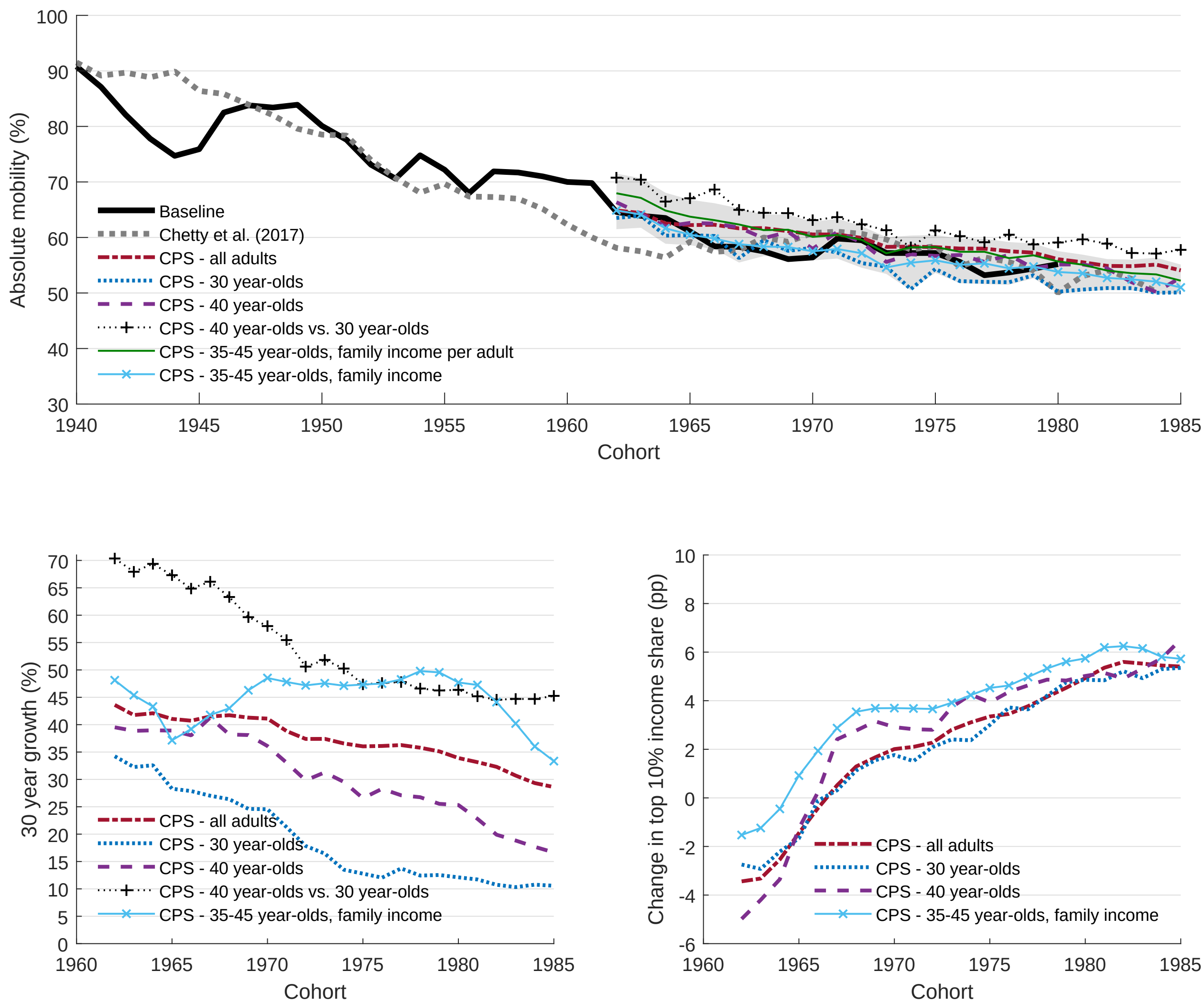

Figure E.4: A comparison of absolute intergenerational mobility estimates in the United States. The top panel shows the baseline estimates (Figure 4 in the main text) as well as the baseline estimates from Chetty et al. (2017) and other estimates based on CPS data: taking into account all adults, only 30 year olds, only 40 year olds, 40 year old 'children' compared to 30 year old 'parents', and family incomes for adults aged 35–45 (either divided by the number of adults in the family or not). In all cases we use the same copula. The shaded area presents the range of absolute mobility values in the CPS-based estimates while taking into account that the rank correlation is between 0.1 and 0.5 in all the specifications. The bottom panels show the 30 year growth rate (left) and overall change in the top 10% income share over 30 years (right) for each cohort in the different CPS specifications (the two bottom panels were smoothed using a moving average for clarity).

E.3 Robustness of Absolute Mobility to Changes in Unit of Observation and Income Concept

The data in The World Inequality Database (2017) allow considering absolute mobility in France after 1970 for labor income only rather than total income. The changes in labor income inequality are milder than for total income. This is due to the rising share of capital income in national income after the 1970s (Piketty and Zucman, 2014; Garbinti, Goupille-Lebret and Piketty, 2018). Yet, labor income growth is also slower than for total income. Thus, the differences between the absolute mobility estimates for the two income concepts are expected to be small. This was also found for the United States by Chetty et al. (2017). Detailed labor income data and its distribution are available for the United States from 1962 onward (Piketty, Saez and Zucman, 2018).

Figure E.5 presents a comparison between the baseline absolute mobility estimates and labor income-based estimates for France and the United States. We find that absolute mobility for labor income is lower than for total income. However, the differences between the estimates are small and the average difference is less than 1.5 percentage points in both countries. The trends are nearly identical.

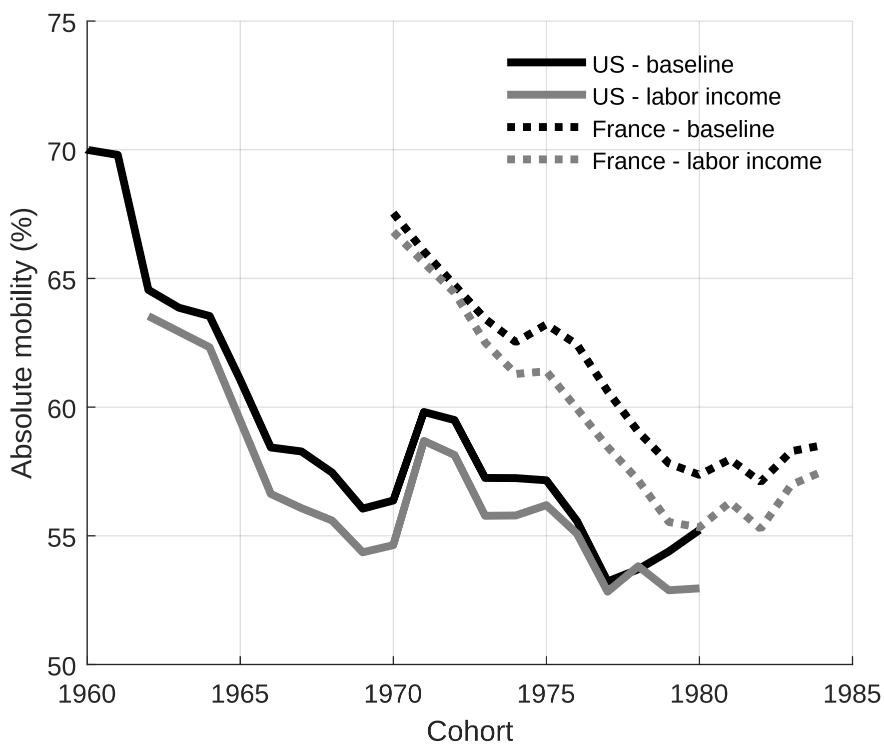

Figure E.5: Absolute intergenerational mobility in France and the United States for total income (black) and labor income (gray).

We can also compare absolute mobility for pre-tax and post-tax (after taxes and transfers) incomes. The baseline estimates use pre-tax incomes, but it is possible that redistribution changes the picture of absolute mobility, and makes decreasing absolute mobility trends milder. Using data from The World Inequality Database (2017), Figure E.6 shows that the impact of redistribution in terms of levels can be quite substantial, reaching about 10 percentage points in the later cohorts in both countries. Yet, despite having a sizable impact on absolute mobility levels, redistribution is still quite far from overturning the declining absolute mobility trend.

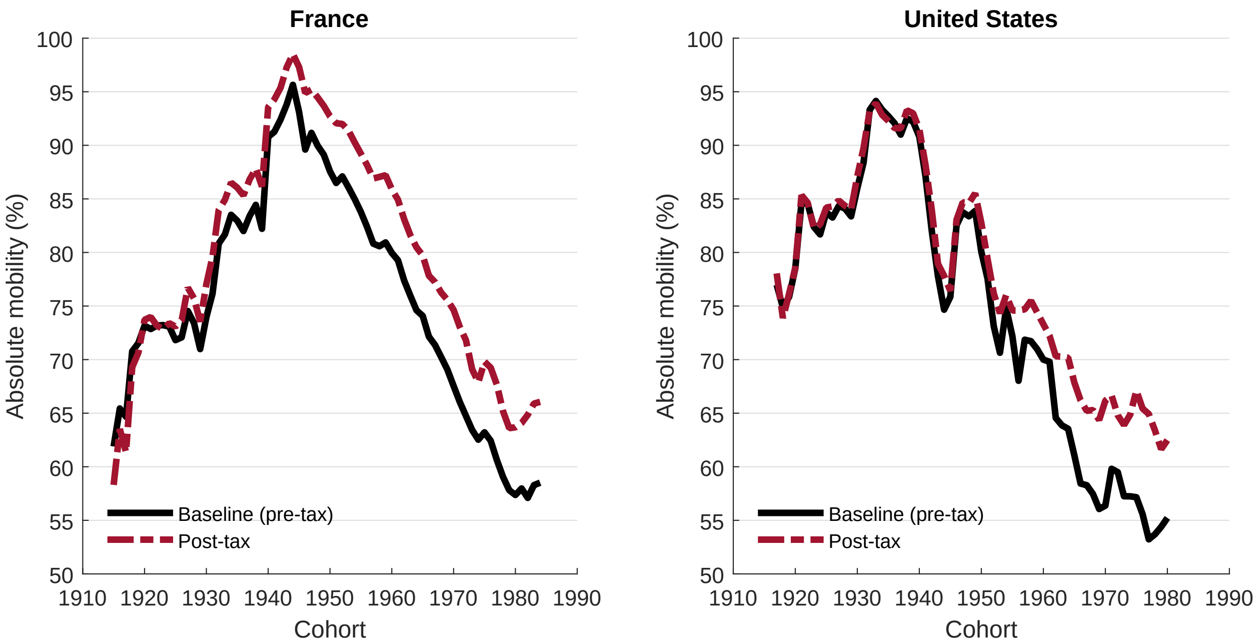

Figure E.6: Absolute intergenerational mobility in France (left) and the United States (right) for pre-tax (black, similar to the baseline estimates) and post-tax income (red).

E.4 Robustness of Absolute Mobility to Over- and Under-estimation of Growth and Inequality

Here we present an additional way to systematically assess the sensitivity of absolute mobility to the accuracy of the marginal distributions used. We would like to know what the impact on levels and trends of absolute mobility would be if inequality changes and income growth are systematically over- or under-estimated.

First, we consider the impact of such mis-measurement on trends. We conduct a similar exercise to the results shown in Figure C.5. For each country we ask how the overall decrease in absolute mobility changes if the changes in inequality and income growth are systematically mis-estimated in each cohort by some value going from -10% to $+10\%$. For every value we recalculate the resulting evolution of absolute mobility and determine the total decrease in absolute mobility over a long time period (in most countries between the 1950 cohort to the latest cohort available; in some countries we begin in an earlier cohort if available). To enable this exercise we use the bivariate log-normal approximation in which the mis-measurement is easily translated into changes in the model parameters (see Section III in the main text).

The results are presented in Figure E.7. These charts show that systematic mis-estimation of up to 10% in either growth or changes in inequality leads to an effect of 0–5 percentage points in the overall change in absolute mobility in the long run. In most countries the impact is limited to 2 percentage points.

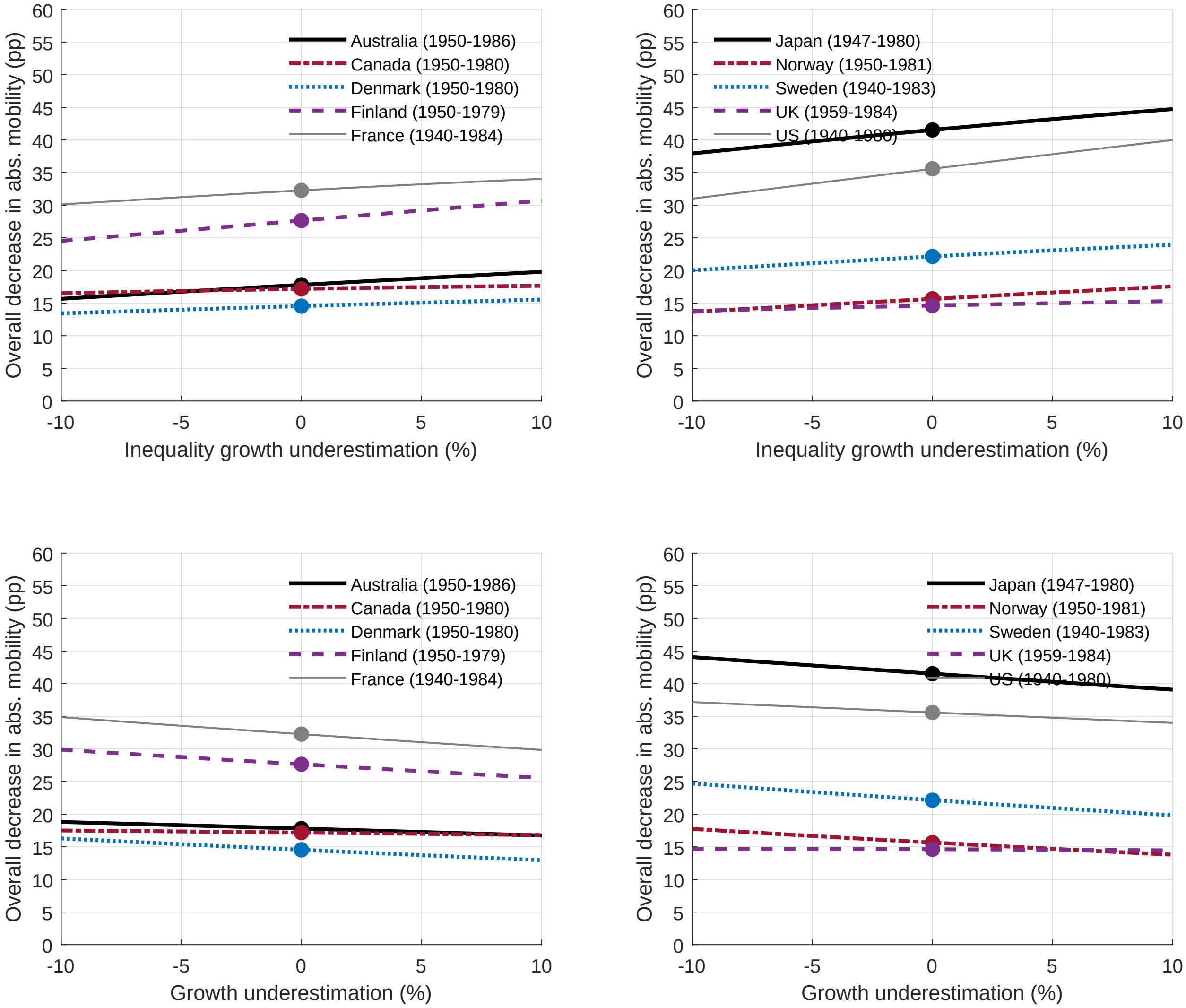

Figure E.7: The decrease in absolute mobility with mis-estimation of inequality changes (top) and income growth (bottom). In each country the magnitude of the decrease in absolute mobility since 1950 (slightly earlier or later in some countries, depending on the data availability) was calculated, while letting the inequality in the children's generation (quantified as the Gini coefficient and parametrized using σ_c in the bivariate log-normal approximation – Eq. (3) in the main text) or the average income growth (parametrized using μ_c in Eq. (3)) change by values ranging from -10% to $+10\%$. The circles show the baseline decrease in absolute mobility for each country.

We can also address the impact of such systematic mis-measurement on absolute mobility levels. We consider the cases in Figure E.7 that were the most sensitive to mis-measurement: Japan and the United States in the case of changes in inequality; Japan and France in the case of income growth. The impact on levels is presented in Figure E.8. It is based on the same calculation as above, when looking at the absolute mobility estimates year-by-year, and not only in the long run.

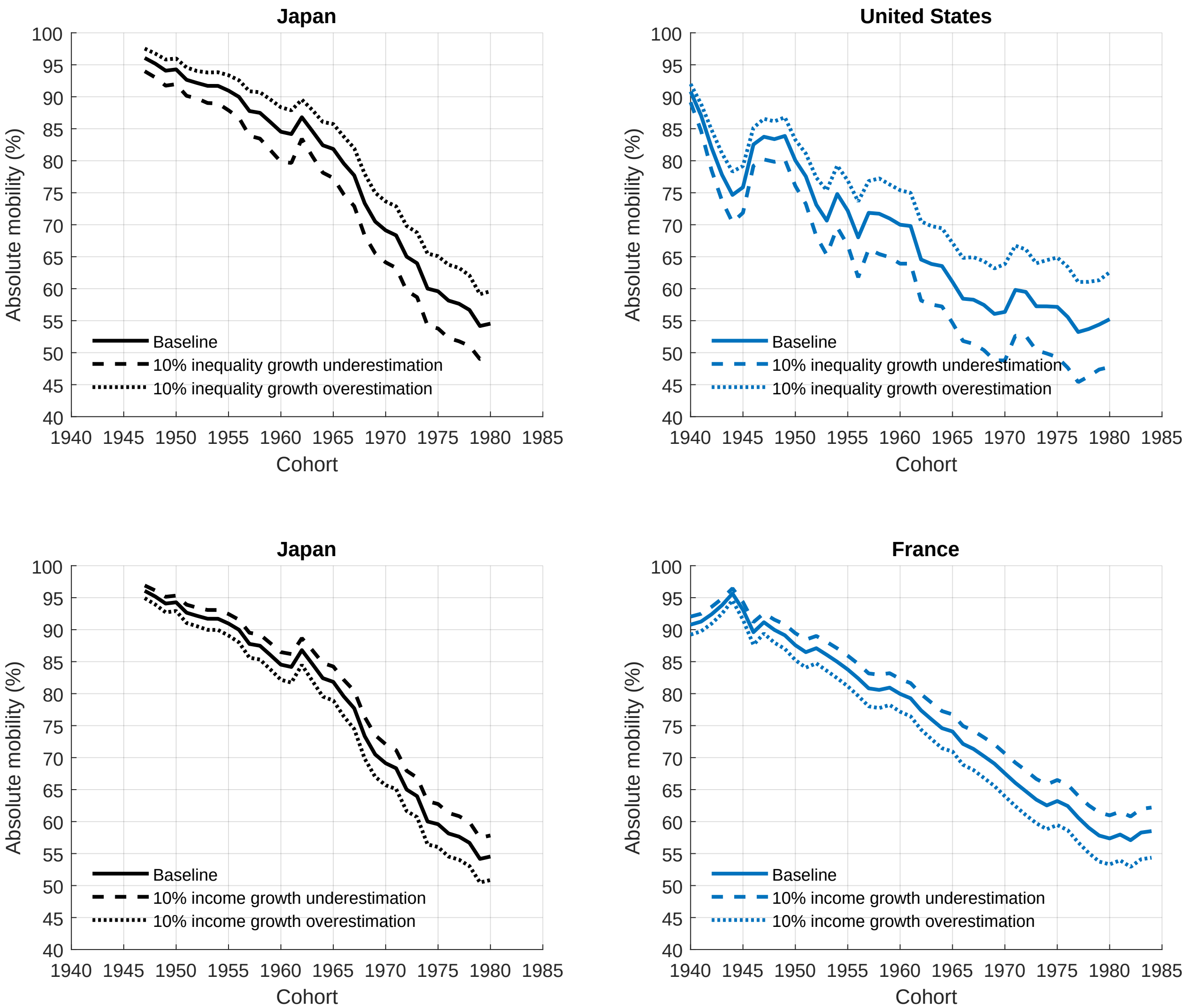

Figure E.8: Absolute mobility with mis-estimation of inequality in Japan and the United States (top) and growth in Japan and France (bottom). In each case, in addition to the baseline estimates, we calculated the evolution of absolute mobility while assuming systematic mis-estimation of $\pm 10\%$ in either case.

As expected, the trends in all cases are nearly identical. In terms of levels, mis-estimation of changes in inequality by up to 10% (quantified, for example, as the Gini coefficient or the top 10% income share) may lead to a sizable impact of up to 8 percentage points.

F Data Specification

The data used for producing the marginal income distributions are taken from The World Inequality Database (2017). We consider only years in which data were sufficiently detailed to use the generalized Pareto curve interpolation method (Blanchet, Fournier and Piketty, 2021), as detailed in Tab. F.1.

Table F.1: The availability of the income distribution in The World Inequality Database (2017)

Country	Time period
Australia	1950–2016
Canada	1950–2010
Denmark	1950–2010
Finland	1950–2009
France	1915–2014
Japan	1947–2010
Norway	1950–2011
Sweden	1903–2013
United Kingdom	1959–2014
United States	1917–2010

Data for the rank correlation in each of the above countries were taken from different sources (see Tab. F.2). As noted, for the countries in which the interegenerational elasticity was reported, rather than the rank correlation, we use the relationship $\rho = \beta\sigma_p/\sigma_c$, where σ_p and σ_c are the standard deviations of the parent and child marginal income distributions and β is the estimated intergenerational income elasticity. The rank correlation is approximated by $\rho_S \approx (6 \arcsin(\rho/2))/\pi$ (see Trivedi and Zimmer (2007)).

Table F.2: Rank correlation values used in the absolute mobility analysis

Country	Rank correlation	Source
Australia	0.22	Leigh (2007)
Canada	0.24	Corak, Lindquist and Mazumder (2014)
Denmark	0.19	Jäntti et al. (2006)
Finland	0.19	Jäntti et al. (2006)
France	0.30	Lefranc and Trannoy (2005)
Japan	0.30	Ueda (2009)
Norway	0.21	Bratberg, Anti Nilsen and Vaage (2005)
Sweden	0.20	Jäntti et al. (2006)
United Kingdom	0.30	Jäntti et al. (2006)
United States	0.30	Chetty et al. (2014)

G Comparison and Reconciliation with Other Sources

A number of recent studies have attempted to estimate absolute intergenerational mobility in some of the countries studied in this paper. Our results are consistent with these estimates in almost all cases, but in some cases they differ. Figure G.1 presents a comparison between our estimates to other sources in seven countries – Canada, Denmark, Finland, Norway, Sweden, the United Kingdom and the United States. For Canada (Ostrovsky, 2017), Finland (Manduca et al., 2020), Norway (Manduca et al., 2020), Sweden (Liss, Korpi and Wennberg, 2019) and the United States (Chetty et al., 2017), the differences between our baseline estimates and the other studies are small. They are within the statistical and methodological sensitivities described above. Notably, in the case of the United States, the only country for which long run series exist, our estimates are particularly close to Chetty et al. (2017).

In the cases of Denmark and the United Kingdom there is a substantial difference between the baseline estimates and the results reported in Manduca et al. (2020) for Denmark and in Blanden, Machin and Rahman (2019) for the United Kingdom. In these two cases Figure G.1 includes an additional estimate of absolute mobility, where the marginal income distributions have the same shape as in the baseline estimates, but their average is taken from Manduca et al. (2020) and Blanden, Machin and Rahman (2019). The relative mobility is also assumed as similar to the relative mobility used for the baseline estimates. In both cases, the additional estimate almost eliminates the difference between the baseline and the estimates from Manduca et al. (2020) and Blanden, Machin and Rahman (2019).

This shows that the major source of discrepancy in Denmark and the United Kingdom is a difference in the estimation of income growth. In both cases this could be because, as explained above, the baseline estimates use the income of the entire adult population and not only of 30-year-olds. Denmark and the United Kingdom might be unique within the group of countries considered in how income growth among 30-year-olds differed from the growth among the entire population. Yet, we note that the case of Denmark is exceptional even when compared to other Nordic countries, as also discussed in Manduca et al. (2020). This makes the results for Denmark robust in terms of their long run trend, but less so in terms of their level.

In the cases of Finland and Norway Manduca et al. (2020) find results that are generally close in level to the baseline estimates, yet less so in terms of trend. Manduca et al. (2020) find generally stable levels of absolute mobility in those countries. Yet, we note that most of the decrease in absolute mobility in those countries has occurred for the cohorts that represent the adult population in the 1950s and early 1960s. This is earlier than the cohorts in which substantial differences are found between the baseline estimates in this paper and in Manduca et al. (2020). Thus, it is likely that in the long run, when also taking into account earlier cohorts, the qualitative picture described in this paper, would still be valid.

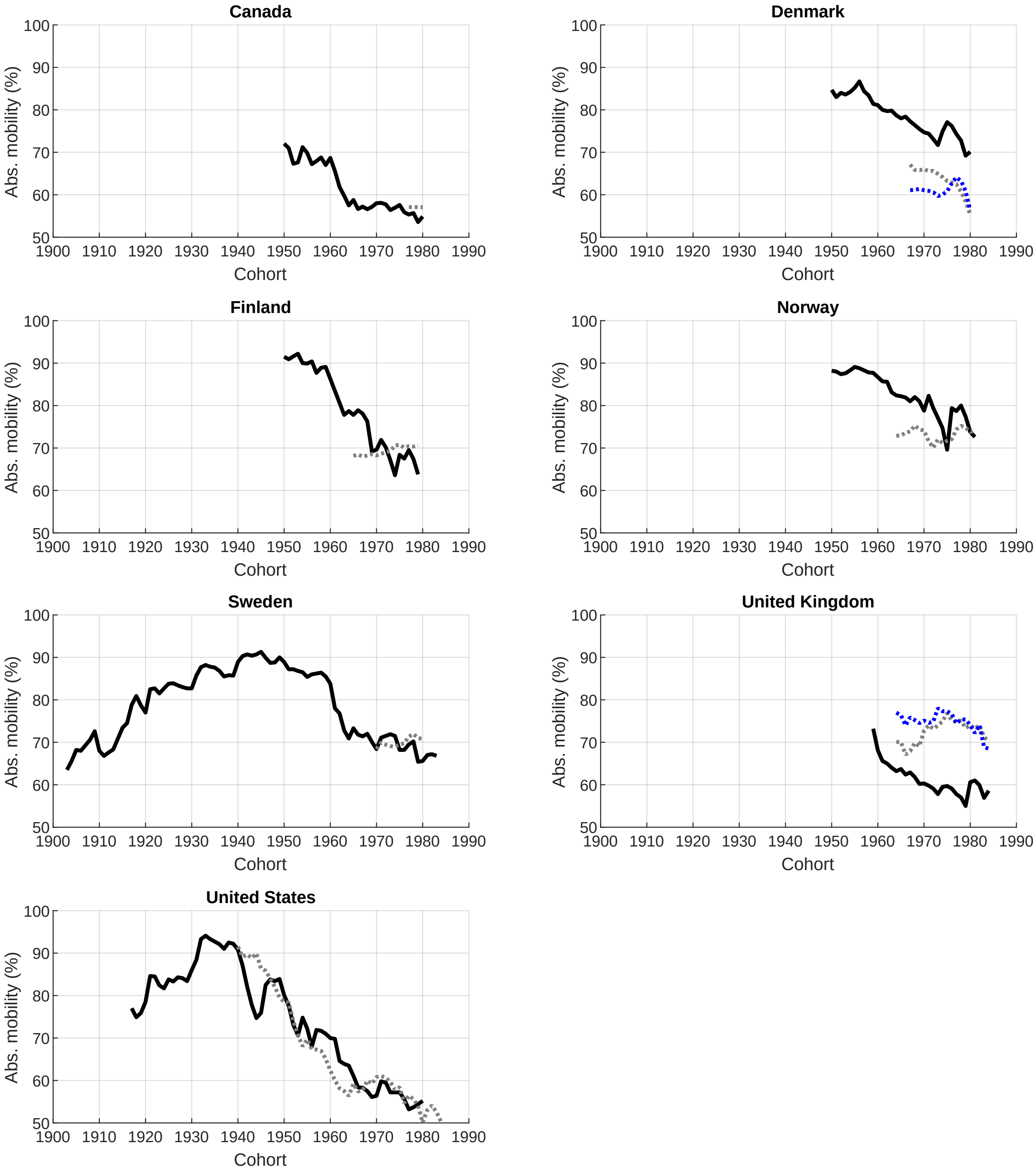

Figure G.1: A comparison between the baseline absolute mobility estimates (black) with other empirical evidence in different countries. The dotted gray lines are absolute mobility estimates from different sources: Canada (Ostrovsky, 2017), Denmark (Manduca et al., 2020), Finland (Manduca et al., 2020), Norway (Manduca et al., 2020), Sweden (Liss, Korpi and Wennberg, 2019), the United Kingdom (Blanden, Machin and Rahman, 2019) and the United States (Chetty et al., 2017). The dotted blue lines in the cases of Denmark and the United Kingdom represent the baseline estimate level of inequality and relative mobility, but assuming the same average income as used in Manduca et al. (2020) and Blanden, Machin and Rahman (2019), respectively.

H Detailed Results for the Evolution of Absolute Mobility

The following tables present the main results – the absolute mobility estimates in Australia, Canada, Denmark, Finland, France, Japan, Norway, Sweden, the United Kingdom and the United States, and the decomposition of the absolute mobility trends to the contributions of changes in inequality and income growth. These are graphically presented in Figure 4 in the main text and Figure H.1, respectively.

Table H.1: The evolution of absolute intergenerational mobility in Australia, Canada, Denmark, Finland, France, Japan, Norway, Sweden, the United Kingdom and the United States (see Figure 4 in the main text)

Cohort	Australia	Canada	Denmark	Finland	France	Japan	Norway	Sweden	UK	US
1903								63.5		
1904								65.6		
1905								68.2		
1906								68.0		
1907								69.3		
1908								70.6		
1909								72.6		
1910								68.0		
1911								66.8		
1912								67.6		
1913								68.3		
1914								70.8		
1915					61.9			73.4		
1916					65.4			74.5		
1917					64.7			78.8		77.0
1918					70.8			80.9		74.9
1919					71.5			78.7		75.9
1920					73.2			77.0		78.5
1921					72.9			82.5		84.6
1922					73.2			82.7		84.5
1923					73.2			81.5		82.4
1924					73.1			82.7		81.7
1925					71.8			83.8		83.8
1926					72.1			83.9		83.3
1927					74.5			83.4		84.3
1928					73.4			83.0		84.1
1929					71.0			82.7		83.4
1930					74.0			82.7		86.0
1931					76.2			85.7		88.4
1932					80.8			87.7		93.3
1933					81.7			88.2		94.1
1934					83.5			87.8		93.3
1935					83.0			87.6		92.7
1936					82.0			86.8		92.1
1937					83.4			85.5		91.0
1938					84.4			85.8		92.5
1939					82.2			85.7		92.2
1940					90.8			88.9		90.8
1941					91.3			90.3		87.1
1942					92.4			90.7		82.1
1943					93.8			90.4		77.8
1944					95.6			90.7		74.7
1945					93.1			91.3		75.9
1946					89.6			89.9		82.5
1947					91.2	96.1		88.7		83.8

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Cohort	Australia	Canada	Denmark	Finland	France	Japan	Norway	Sweden	UK	US
1948					90.0	95.2		88.8		83.4
1949					89.1	94.1		90.9		83.9
1950	80.7	72.1	84.7	91.5	87.6	94.3	88.2	88.9		80.1
1951	80.0	71.0	83.0	90.9	86.5	92.6	88.0	87.2		77.6
1952	80.1	67.3	84.0	91.6	87.1	92.2	87.4	87.2		73.1
1953	78.4	67.6	83.6	92.2	86.1	91.7	87.6	86.8		70.6
1954	77.2	71.2	84.2	90.0	85.0	91.7	88.3	86.5		74.8
1955	76.1	69.9	85.2	89.9	83.8	91.0	89.1	85.4		72.2
1956	75.1	67.2	86.7	90.4	82.4	90.0	88.8	86.0		68.0
1957	72.7	67.9	84.4	87.7	80.8	87.8	88.3	86.2		71.9
1958	73.7	68.8	83.4	88.9	80.6	87.5	87.8	86.4		71.7
1959	74.1	67.0	81.4	89.1	80.9	86.0	87.7	85.5	73.2	71.0
1960	73.9	68.7	81.1		80.0	84.5	86.7	83.8	68.1	70.0
1961	71.2	65.6	80.0		79.3	84.2	85.7	78.0	65.6	69.8
1962	70.7	61.8	79.7		77.4	86.8	85.6	76.8	65.0	64.6
1963	69.8	59.7	79.8	77.8	76.0	84.6	83.1	72.8	64.0	63.9
1964	67.8	57.5	78.7	78.7	74.6	82.4	82.4	70.9	63.2	63.5
1965	67.9	58.8	78.0	77.8	74.1	81.8	82.2	73.3	63.7	61.1
1966	69.1	56.6	78.4	78.9	72.1	79.6	81.9	71.8	62.4	58.4
1967	67.1	57.2	77.3	78.1	71.4	77.7	81.0	71.4	62.9	58.3
1968	67.5	56.6	76.4	76.3	70.2	73.4	82.0	72.0	61.8	57.5
1969	64.9	57.1	75.5	69.2	69.1	70.5	81.0	70.1	60.2	56.1
1970	64.4	58.0	74.7	69.6	67.5	69.1	78.8	68.4	60.3	56.4
1971	65.8	58.1	74.4	71.9	66.0	68.3	82.3	71.1	59.8	59.8
1972	65.1	57.7	73.1	70.2	64.7	65.0	79.4	71.5	59.1	59.5
1973	65.3	56.4	71.7	67.1	63.4	64.0	77.1	71.9	57.8	57.2
1974	64.6	56.9	74.9	63.6	62.5	60.0	74.7	71.5	59.5	57.2
1975	63.3	57.6	77.1	68.4	63.2	59.6	69.6	68.2	59.7	57.2
1976	62.0	55.9	76.2	67.5	62.4	58.1	79.4	68.2	59.1	55.6
1977	62.9	55.3	74.3	69.5	60.6	57.6	78.7	69.5	57.8	53.2
1978	66.4	55.7	72.8	67.4	59.0	56.7	80.0	70.2	57.0	53.7
1979	63.8	53.6	69.2	63.8	57.8	54.2	77.4	65.4	55.0	54.4
1980	63.1	54.9	70.1		57.4	54.5	73.8	65.6	60.6	55.2
1981	63.4				58.0		72.6	67.0	61.0	
1982	63.2				57.1			67.2	59.9	
1983	64.3				58.3			66.8	56.9	
1984	63.7				58.5				58.6	
1985	62.8									
1986	62.9									

Note: Rounding may lead to ±0.1 *percentage point difference between the total decrease and the sum of contributions.*

Figure H.1 graphically presents the results of the decomposition of absolute mobility trends to the impact of income growth and inequality changes. It is similar to Figure 5 in the main text, but includes all the countries analyzed.

Table H.2: Growth and inequality changes contribution to the evolution of absolute mobility from 1940 (or later) onward (see Figure H.1)

Country	Decrease in mobility (*pp*)	Growth contribution (*pp*)	Inequality contribution (*pp*)
Australia	22.0	8.2	13.9
Canada	23.9	17.0	6.9
Denmark	17.2	14.3	2.9
Finland	30.2	18.1	12.2
France	35.5	27.9	7.7
Japan	43.2	31.7	11.5
Norway	17.7	12.7	5.1
Sweden	24.9	18.0	6.9
UK	20.0	12.5	7.5
US	39.2	14.5	24.6

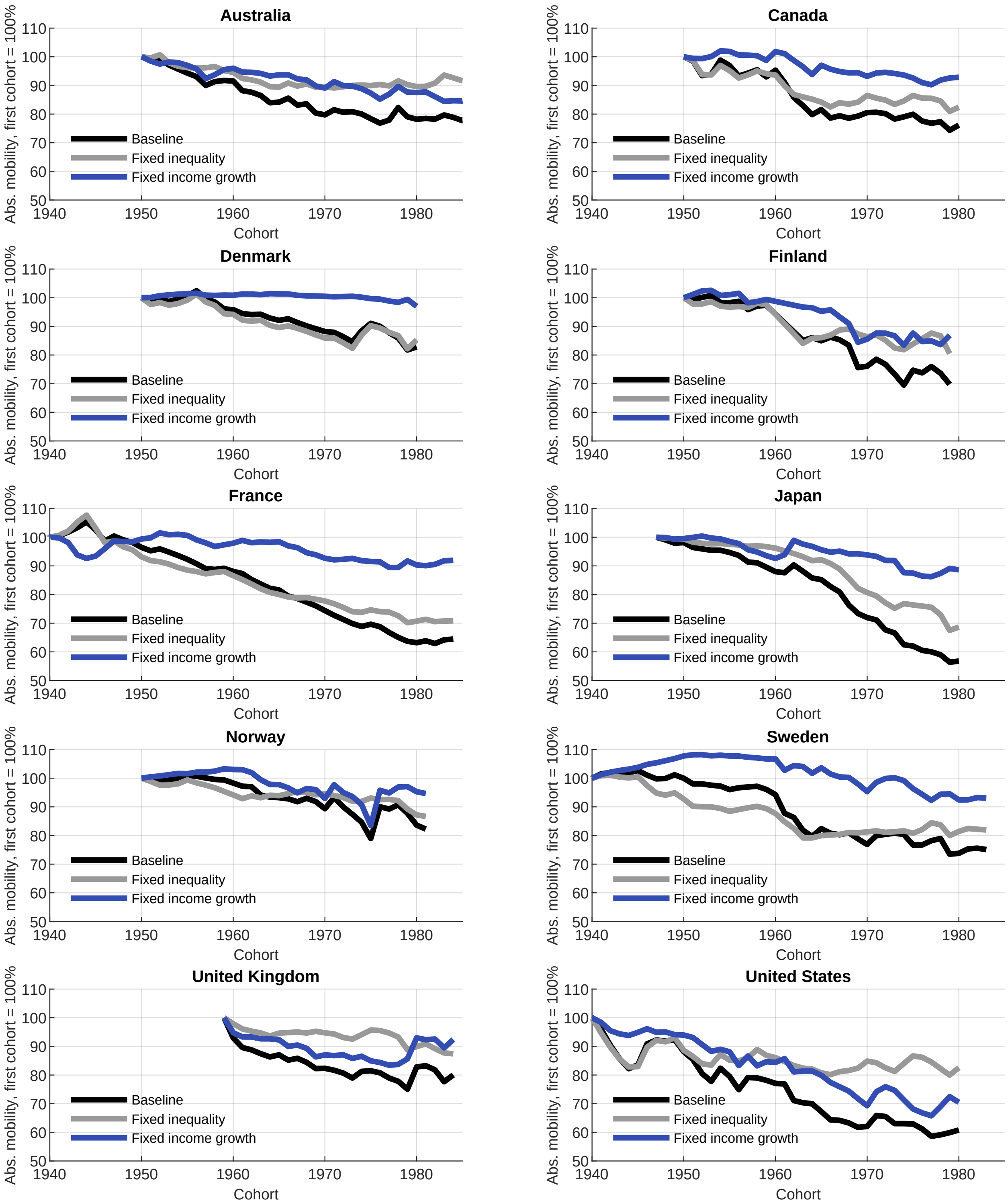

Figure H.1: Counterfactual calculations of absolute mobility in a group of advanced economies. For comparability, we set the absolute mobility to 100% in the earliest cohort. See Tab. H.2 for the contribution of each factor to the overall decrease in mobility in all countries.

References

Aaronson, Daniel, and Bhashkar Mazumder. 2008. "Intergenerational Economic Mobility in the United States, 1940 to 2000." Journal of Human Resources, 43(1): 139–172.

Atkinson, Anthony B. 1983. Social Justice and Public Policy. MIT press.

Atkinson, Anthony B., and François Bourguignon. 1982. "The Comparison of Multi-Dimensioned Distributions of Economic Status." Review of Economic Studies, 49(2): 183–201.

Bartholomew, David J. 1967. Stochastic Models for Social Processes. New York, NY:John Wiley & Sons.

Berman, Yonatan. 2018. "Growth, Inequality and Absolute Mobility in the United States, 1962–2014." Available at SSRN.

Blanchet, Thomas, Juliette Fournier, and Thomas Piketty. 2021. "Generalized Pareto Curves: Theory and Applications." Review of Income and Wealth. Forthcoming.

Blanden, Jo, Stephen Machin, and Sumaiya Rahman. 2019. "Falling Real Wages: The Effect on Absolute Earnings Mobility." Mimeo.

Bonhomme, Stéphane, and Jean-Marc Robin. 2009. "Assessing the Equalizing Force of Mobility Using Short Panels: France, 1990–2000." Review of Economic Studies, 76(1): 63–92.

Bratberg, Espen, Øivind Anti Nilsen, and Kjell Vaage. 2005. "Intergenerational Earnings Mobility in Norway: Levels and Trends." Scandinavian Journal of Economics, 107(3): 419–435.

Chetty, Raj, David Grusky, Maximilian Hell, Nathaniel Hendren, Robert Manduca, and Jimmy Narang. 2017. "The Fading American Dream: Trends in Absolute Income Mobility Since 1940." Science, 356(6336): 398–406.

Chetty, Raj, Nathaniel Hendren, Patrick Kline, Emmanuel Saez, and Nicholas Turner. 2014. "Is the United States Still a Land of Opportunity? Recent Trends in Intergenerational Mobility." American Economic Review, 104(5): 141–147.

Corak, Miles, Matthew J. Lindquist, and Bhashkar Mazumder. 2014. "A Comparison of Upward and Downward Intergenerational Mobility in Canada, Sweden and the United States." Labour Economics, 30: 185–200.

Eberharter, Veronika V. 2014. "The Intergenerational Dynamics of Social Inequality – Empirical Evidence from Europe and the United States." Global Journal of Human-Social Science: E, 14(1): 95–108.

Eurostat. 2018. "Household Composition Statistics." Access Date: May-07-2018.

Garbinti, Bertrand, Jonathan Goupille-Lebret, and Thomas Piketty. 2018. "Income Inequality in France, 1900–2014: Evidence from Distributional National Accounts (DINA)." Journal of Public Economics, 162: 63–77.

Jäntti, Markus, Bernt Bratsberg, Knut Roed, Oddbjorn Raaum, Robin Naylor, Eva Österbacka, Anders Björklund, and Tor Eriksson. 2006. "American Exceptionalism in a New Light: A Comparison of Intergenerational Earnings Mobility in the Nordic Countries, the United Kingdom and the United States." IZA Discussion Paper 1938.

Katz, Lawrence F., and Alan B. Krueger. 2017. "Documenting Decline in US Economic Mobility." Science, 356(6336): 382–383.

Lefranc, Arnaud, and Alain Trannoy. 2005. "Intergenerational Earnings Mobility in France: Is France More Mobile than the US?" Annales d'Economie et de Statistique, 78: 57–77.

Leigh, Andrew. 2007. "Intergenerational Mobility in Australia." The BE Journal of Economic Analysis & Policy, 7(2).

Liss, Erik, Martin Korpi, and Karl Wennberg. 2019. "The American Dream Lives in Sweden: Trends in Intergenerational Absolute Income Mobility." Ratio Working Paper No. 325.

Manduca, Robert, Maximilian Hell, Adrian Adermon, Jo Blanden, Espen Bratberg, Anne C. Gielen, Hans van Kippersluis, Keun Bok Lee, Stephen Machin, Martin D. Munk, Martin Nybom, Yuri Ostrovsky, Sumaiya Rahman, and Outi Sirniö. 2020. "Trends in Absolute Income Mobility in North America and Europe." Mimeo.

Ostrovsky, Yuri. 2017. "Doing as Well as One's Parents?: Tracking Recent Changes in Absolute Income Mobility in Canada." Statistics Canada.

Piketty, Thomas, and Gabriel Zucman. 2014. "Capital is Back: Wealth-Income Ratios in Rich Countries, 1700–2010." Quarterly Journal of Economics, 129(3): 1255–1310.

Piketty, Thomas, Emmanuel Saez, and Gabriel Zucman. 2018. "Distributional National Accounts: Methods and Estimates for the United States." Quarterly Journal of Economics, 133(2): 553–609.

Plackett, Robin L. 1965. "A Class of Bivariate Distributions." Journal of the American Statistical Association, 60(310): 516–522.

Shorrocks, Anthony F. 1978. "The Measurement of Mobility." Econometrica, 46(5): 1013–1024.

Sklar, Abe. 1959. "Fonctions de Répartition à n Dimensions et Leurs Marges." Publications de l'Institut Statistique de l'Université de Paris, 8: 229–231.

Spearman, Charles. 1904. "The Proof and Measurement of Association between Two Things." American Journal of Psychology, 15(1): 72–101.

The World Inequality Database. 2017. "Australia, Canada, Denmark, Finland, France, Japan, Norway, Sweden, UK and USA per Adult National Income." `http://wid.world/data/`, Access Date: Oct-10-2017.

Trivedi, Pravin K., and David M. Zimmer. 2007. "Copula Modeling: An Introduction for Practitioners." Foundations and Trends in Econometrics, 1(1): 1–111.

Ueda, Atsuko. 2009. "Intergenerational Mobility of Earnings and Income in Japan." The BE Journal of Economic Analysis & Policy, 9(1).

United States Bureau of Labor Statistics. 2017. "Current Population Survey Income Data." Access Date: Nov-10-2017.