Online Appendix
"SVAR Identification From Higher Moments: Has the Simultaneous Causality Problem Been Solved?"

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I. Bootstrap Test of Shock Independence

We herein describe a simple test of mutual shock independence. The test is related to the permutation test in Risk, James and Matteson (2015), Matteson and Tsay (2017), and Davis and Ng (2021). We use an alternative bootstrap approach that is familiar to SVAR practitioners, and we use a different test statistic that directs power against economically salient alternatives.

If $\varepsilon_{j,t}$ are independent, then the squared shocks $\varepsilon_{j,t}^2$ should also be independent, and hence, mutually uncorrelated. We focus on this particular implication of independence, as this will direct power towards economically salient alternatives where the shocks have shared volatility factors, as in Equation 3 in the main paper. Thus, while other independence metrics are possible, we here use the following intuitive test statistic:

$$\hat{S} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j \neq i} \widehat{\text{Corr}}(\hat{\varepsilon}_{i,t}^2, \hat{\varepsilon}_{j,t}^2)^2},$$

i.e., the root mean squared sample cross-correlation of the squared estimated shocks.

We reject the null hypothesis of shock independence if $\hat{S}$ exceeds a bootstrap critical value. The bootstrap takes into account estimation error in the VAR coefficients $\hat{\mathbf{A}}_\ell$ and ICA estimate $\hat{\mathbf{H}}$. Given significance level α, the critical value equals the $1-\alpha$ quantile of the bootstrapped test statistics $\hat{S}^*$, which are computed in the same way as above but on the bootstrap data sets.

Importantly, we impose the null hypothesis in the bootstrap sampling scheme by slightly modifying the conventional recursive residual VAR bootstrap (Kilian and Lütkepohl, 2017, Ch. 12.2.1): We resample each shock $\hat{\varepsilon}_{j,t}$ independently of the other shocks, instead of resampling all components in the vector $\hat{\varepsilon}_t$ jointly. Note that this test can be performed without labeling the shocks, since their ordering does not matter.

II. Empirical Application

We now apply the bootstrap test to quarterly U.S. macroeconomic data. Our specification largely follows Gouriéroux, Monfort and Renne (2017, Sec. 3.2), except that we omit an exogenous oil price variable from the SVAR for simplicity. The three observed variables in the VAR are inflation, the output gap, and the nominal short-term interest rate.[1] We use 6 lags, as selected by the Akaike Information Criterion for the 1959-2019 sample. We use the OLS estimator of the VAR coefficients $\mathbf{A}_\ell$ and the quasi-MLE ICA estimator of $\mathbf{H}$ proposed by Gouriéroux, Monfort and Renne (2017).[2]

Table 1 shows that we can reject mutual independence of the shocks at a 5% significance level on the post-1973 and post-1985 samples, though not on the longest 1959–2019 sample. On the post-1985 sample, the test statistic $\hat{S}$ equals 0.166, which amounts to an economically non-trivial root mean squared cross-correlation $\widehat{\text{Corr}}(\hat{\varepsilon}_{i,t}^2, \hat{\varepsilon}_{j,t}^2)$. In contrast, we fail to reject independence at conventional levels under the procedure featured in Davis and Ng (2021).

Our results demonstrate that mutual shock independence can be tested, and should not be viewed as an automatic, unobjectionable assumption in practice. They also support Matteson and Tsay's (2017) call for additional research into the Type II error properties of tests for independence.

[1] These are obtained from the Federal Reserve Bank of St. Louis FRED database. Inflation is the log change in the GDP deflator (GDPDEF), the output gap is the log difference between actual (GDPC1) and potential output (GDPPOT), and the interest rate is the 3-month Treasury rate (TB3MS).

[2] The quasi-likelihood uses the same shock densities as Gouriéroux, Monfort and Renne. We use Matlab's `fmincon` numerical optimization procedure, started at several different points using the `GlobalSearch` algorithm. Bootstrap draws are initialized at a single starting point (the MLE of $\mathbf{H}$).

TABLE 1—TEST OF SHOCK INDEPENDENCE IN U.S. DATA.

Sample	Our test			Davis and Ng (2021)
	Test statistic	5% CV	10% CV	p-value
1959:IV-2019:IV	0.054	0.117	0.097	0.492
1973:I-2019:IV	0.168	0.120	0.100	0.641
1985:I-2019:IV	0.166	0.132	0.116	0.922

Note: Test statistic and corresponding 5% and 10% bootstrap critical values for testing mutual independence of shocks. SVAR variables: inflation, output gap, nominal interest rate. ICA estimator of **H**: quasi-MLE of Gouriéroux, Monfort and Renne (2017). Final column applies the testing procedure featured in Davis and Ng (2021) to the SVAR shocks.

III. Simulation Study

To illustrate the properties of our testing procedure, we run a simulation study calibrated to our empirical application. The DGP is given by a 3-variable VAR model with a sample length of 200 and known lag length of 6. We take the intercept vector **c**, autoregressive matrices $\mathbf{A}_\ell$, and **H** matrix from our estimation results on the 1973:I-2019:IV sample. The unobserved shocks $\varepsilon_{1,t}, \varepsilon_{2,t}$ and $\varepsilon_{3,t}$ are given by the product of an i.i.d. shared volatility process τ_t and the i.i.d., mutually independent, mean zero, and unit variance disturbances $\zeta_{1,t}, \zeta_{2,t}$, and $\zeta_{3,t}$:

$$\varepsilon_{j,t} = \tau_t \zeta_{j,t}, \quad \tau_t \sim \frac{\chi^2(k)}{k}, \quad j = 1, 2, 3,$$

where $\zeta_{j,t}$ is independent of τ_t for $j = 1, 2, 3$. The chi-square distribution for τ_t is parameterized as a gamma distribution to accommodate a continuous degrees of freedom parameter. The null hypothesis of mutual shock independence holds in the limit $k = \infty$, while finite values for k amount to violations of independence. We consider a grid of step size 0.1 for $1/k$ on the unit interval.

We consider three different specifications for the processes $\zeta_{1,t}, \zeta_{2,t}$, and $\zeta_{3,t}$. These are a subset of the DGPs considered by Fiorentini and Sentana (2020):

- DGP1: Three homogeneous t-distributions with 5 degrees of freedom.
- DGP2: Three homogeneous Laplace distributions.
- DGP3: Three heterogeneous discrete location scale mixtures of two normal distributions (DLSMN). Processes are distributed according to

$$\zeta_{1,t} \sim \text{DLSMN}(0.8, 0.06, 0.52),$$
$$\zeta_{2,t} \sim \text{DLSMN}(1.2, 0.08, 0.4),$$
$$\zeta_{3,t} \sim \text{DLSMN}(-1, 0.2, 0.2).$$

See Fiorentini and Sentana (2020) for details on the parameterization.

We simulate 1,000 data sets per DGP. We use OLS to estimate the VAR coefficients and the quasi-MLE ICA estimator for the **H** matrix. Critical values are computed using 500 bootstrap draws.

Figure 1 shows that power appears to increase monotonically for larger departures from the null at both the 5% and 10% significance levels. For the null model, the test slightly over-rejects for DGP1 (16% rejection at 10% nominal level and 11% rejection at 5% nominal level) and is approximately correctly sized for DGP2 and DGP3.

REFERENCES

Davis, Richard, and Serena Ng. 2021. "Time Series Estimation of the Dynamic Effects of Disaster-Type Shocks." Manuscript, Columbia University.

Fiorentini, Gabriele, and Enrique Sentana. 2020. "Discrete mixtures of normals pseudo maximum likelihood estimators of structural vector autoregressions." CEPR Discussion Paper DP15411.

Gouriéroux, Christian, Alain Monfort, and Jean-Paul Renne. 2017. "Statistical inference for independent component analysis: Application to structural VAR models." *Journal of Econometrics*, 196(1): 111–126.

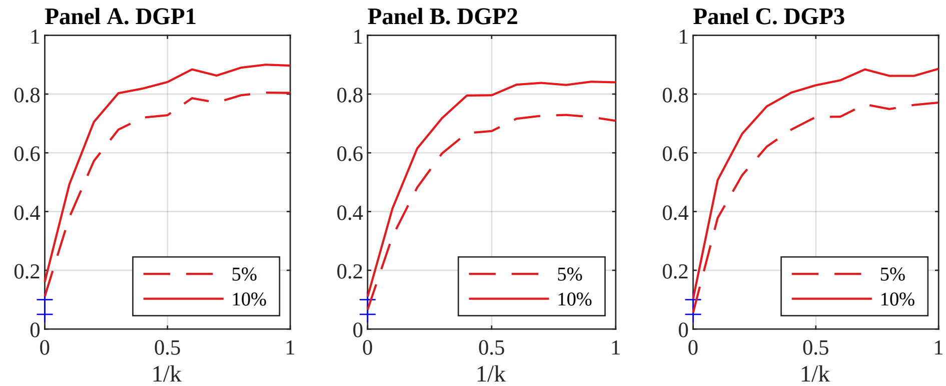

FIGURE 1. EMPIRICAL REJECTION PROBABILITIES AS A FUNCTION OF $1/k$.

Note: Panels plot empirical rejection probabilities at the 10% significance (solid line) and 5% significance (dashed line) nominal levels. Blue reference lines on the vertical axis mark 5% and 10% rejection probabilities.

Kilian, Lutz, and Helmut Lütkepohl. 2017. *Structural Vector Autoregressive Analysis.* Cambridge University Press.

Matteson, David S., and Ruey S. Tsay. 2017. "Independent Component Analysis via Distance Covariance." *Journal of the American Statistical Association*, 112(518): 623–637.

Risk, Benjamin B., Nicholas A. James, and David S. Matteson. 2015. "steadyICA: ICA and Tests of Independence via Multivariate Distance Covariance." R package version 1.0.