Online Appendix to
"What Can We Learn From Sign-Restricted VARs?"

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This online appendix contains supplemental material for the article "What Can We Learn From Sign-Restricted VARs?".

A Model Details

Standard arguments give the model solution

$$\begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} = \underbrace{\frac{1}{1+\phi_\pi \kappa} \begin{pmatrix} \sigma^d & \phi_\pi \sigma^s & -\sigma^m \\ \kappa \sigma^d & -\sigma^s & -\kappa \sigma^m \\ \phi_\pi \kappa \sigma^d & -\phi_\pi \sigma^s & \sigma^m \end{pmatrix}}_{\Theta} \times \begin{pmatrix} \varepsilon_t^d \\ \varepsilon_t^s \\ \varepsilon_t^m \end{pmatrix} \tag{1}$$

Thus, as long as $\sigma^k > 0$ for all shocks $k \in \{d, s, m\}$, and with $\kappa > 0$ as well as $\phi_\pi > 1$, we can conclude that Θ is invertible, as claimed.

To construct panel (a) in Figure 1, I parameterize the model (IS) - (TR) in line with standard practice in the New Keynesian literature: I set $\phi_\pi = 1.5$ (ensuring equilibrium determinacy), $\kappa = 0.2$ (roughly corresponding to around a quarter of prices adjusting in each time period), and $(\sigma^d, \sigma^s, \sigma^m) = (1.60, 0.95, 0.23)$ (as in Wolf (2020)).[1] For the model with bigger monetary shocks in panel (b) of Figure 1, I scale the monetary policy shock volatility by a factor of 30.

[1] For κ, note that standard arguments give $\kappa = (1-\theta)\frac{1-\theta\beta}{\theta}(\gamma + \varphi)$, where $1-\theta$ is the probability of a price re-set, $1/\gamma$ is the elasticity of intertemporal substitution, and φ is the Frisch elasticity of labor supply. $\theta = 0.75$ and $\varphi = \gamma = \beta = 1$ give around $\kappa \approx 0.2$.

B Additional Sign Restrictions

This section discusses the extent to which additional sign restrictions—either on multiple, simultaneously identified shocks or over time for a single shock—can meaningfully tighten identified sets. I begin with an analytical discussion and then provide a numerical illustration.

B.1 Analytical Discussion

Multiple shocks. In Section II.A I claimed that multiple shock identification according to (7) would materially tighten identified sets if monetary shocks are a prominent source of business-cycle fluctuations. For the formal argument, suppose that any shock vector $p \neq (0,0,1)'$ was in the identified set of monetary policy shocks. It follows that one of the other two rows of the corresponding orthogonal rotation matrix P has a non-zero entry in the third column: that is, at least one of the identified "demand" and "supply" shocks is actually partially a monetary policy shock. Call this vector $p^{\perp}$, and consider the impulse response

$$\Theta \cdot p^{\perp}$$

Clearly, for any such $p^{\perp}$, we can ensure that $\Theta_{2,\bullet} \cdot p^{\perp}$ and $\Theta_{3,\bullet} \cdot p^{\perp}$ have the opposite sign, simply by making σ^m large enough. Thus, in the limit $\sigma^m \to \infty$, the model is point-identified, with $P = I$ as the only rotation matrix in the identified set.

Dynamic restrictions. Dynamic sign restrictions are restrictions on the entries of the sequence of impulse response matrices $\{\Theta_h\}_{h=0}^{H}$. Two extreme cases transparently illustrate the potential power of such dynamic restrictions: suppose that the horizon-h impulse response to shock k is given as

$$\Theta_{h,\bullet,k} = \varrho_k^h \Theta_{0,\bullet,k}$$

If $\varrho_k \geq 0$ for all k then nothing has changed relative to the static analysis of the program (5): if a shock vector p is consistent with that program, it remains consistent with the sign restrictions in (4) being imposed for multiple horizons. If, however, $\varrho_k < 0$ for $k \in \{d, s\}$ yet $\varrho_m > 0$, then the identified set gets strictly narrower; intuitively, unless p is sufficiently close to $(0,0,1)'$, the reversal in the demand and supply shock impulse responses will imply that dynamic sign restrictions are violated at horizons $h > 0$.

(a) multiple shocks, volatile monetary policy

(b) baseline calibration

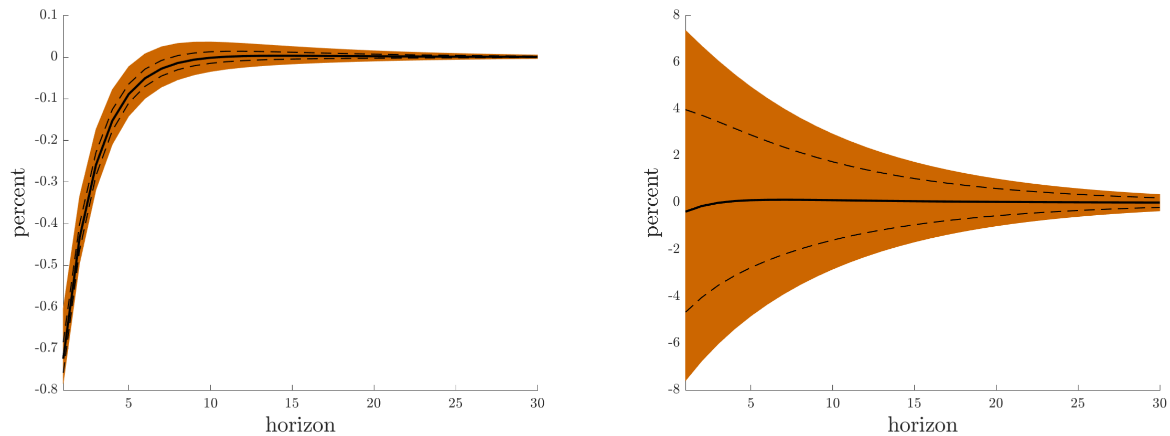

Figure 1: Identified set (orange) with Haar prior-induced posterior bands (black, for 16th, 50th and 84th percentiles) for the output impulse response, identified by imposing the sign restrictions in (7) at horizons $h = 0, 1, \ldots, 6$. Based on 10,000 accepted draws from the Haar prior.

B.2 Simulation Results

I here provide a graphical illustration of identified sets in a dynamic version of the baseline model. This analysis allows me to both (i) illustrate the analytical arguments in the previous section and (ii) argue that, in standard model calibrations, additional restrictions on other impulse responses tend to have little bite. My illustrations rely on the following natural dynamic extension of the baseline model (IS) - (TR):

$$y_t = E_t(y_{t+1}) - (i_t - E_t(\pi_{t+1})) + \omega_t^d \qquad \text{(IS')}$$

$$\pi_t = \kappa y_t + \beta E_t(\pi_{t+1}) - \omega_t^s \qquad \text{(NKPC')}$$

$$i_t = \phi_i i_{t-1} + (1 - \phi_i)(\phi_\pi \pi_t + \omega_t^m) \qquad \text{(TR')}$$

where $\omega_t^k = \rho^k \omega_{t-1}^k + \sigma^k \varepsilon_t^k$, $k \in \{d, s, m\}$. I consider an example model parameterization with $\phi_i = 0.8$, $\rho^d = \rho^s = 0.9$ and $\rho^m = 0$. All other coefficients are set as in the baseline.

The left panel of Figure 1 begins by illustrating the potential power of multiple shock identification. I scale $\{\sigma^d, \sigma^s\}$ down by a factor of 30, and report the identified set (as well as the Haar-induced posterior distribution) for the output response to a monetary policy shock, identified by imposing the multishock restrictions in (7) all the way up to horizon $H = 6$. As expected, the identified set is very tight around the true impulse response.

The right panel shows that, with more empirically relevant values of relative shock volatilities, the picture changes dramatically: the identified set is now again very wide (exactly as in Section II.A), even though the researcher has simultaneously identified multiple shocks *and* restricted their impulse responses for several periods.

References

Wolf, C. K. (2020). SVAR (Mis) identification and the Real Effects of Monetary Policy Shocks. *American Economic Journal: Macroeconomics*, *12*(4), 1–32.