ONLINE APPENDIX

Labor Market Power

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This Appendix is organized as follows. Section A provides additional tables and figures referenced in the text. Section B provides our micro-foundation for nested-CES preferences used in the main text and references in Section 1. Section C contains details about the data and sample selection criteria. Section D contains summary statistics used and references in the paper and additional concentration measures. Section E contains derivations of all mathematical expressions in the text, including proofs of Propositions. Section F provides additional details regarding the calibration. Section G provides additional discussion of our empirical results and robustness on a number of dimensions.

A Additional tables and figures

Description	Model	Data (KPZW)
Replication Targets		
Log change in VAPW (VAPW=$\widetilde{Z}\tilde{z}_{ij}n_{ij}^{\tilde{\alpha}-1}$)	0.13	0.13
Mean firm size	61.83	61.49
Replication Parameters		
Size cutoff (Employees)	9.00	
Shock size ($dlog(\tilde{z}_{ij})$)	0.19	

Table A1: Wage pass-through experiment details

Notes: Summary statistics for replication of Patrick Kline, Neviana Petkova, Heidi Williams and Owen Zidar (2019) regressions. We randomly sample one percent of firms in our benchmark economy. We draw firms with employment greater than $\underline{n}$. We increase the productivity of treated firms by a factor $dlog\widetilde{z}_{ij}$. The values of $\underline{n}$ and Δ are calibrated to match the KPWZ (1) mean firm size of 61 employees, (2) increase in post-tax value added per worker of 13 percent. We keep aggregates fixed and solve the new market equilibrium. We treat the untreated and treated observations for each firm as a panel with two observations per firm of wages $\left\{w_{ij0}, w_{ij1}\right\}$ and value added per worker, $\left\{\frac{y_{ij0}}{n_{ij0}}, \frac{y_{ij1}}{n_{ij1}}\right\}$. We then regress the wages in levels on VAPW in levels and a firm-specific fixed effect. The regression coefficient is converted into an elasticity using untreated mean wages and mean value added per worker.

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	Model	Data
Wage bill Herfindahl – Payroll weighted average	0.17	0.17
Wage bill Herfindahl – Unweighted average	0.33	0.48
Wage bill Herfindahl correlation with market employment	-0.89	-0.25
Employment Herfindahl – Payroll weighted average	0.16	0.15
Employment Herfindahl – Unweighted average	0.32	0.45
Employment Herfindahl correlation with Wage-bill Herfindahl	1.00	0.98

Table A2: Concentration and competition, model versus data

Notes: Data is from 2014 LBD, tradeable sectors. Model is for tradeable calibration. The market level wage-bill Herfindahl is given by: $HHI_j^{wn} := \sum_{i\in j}\left(s_{ij}^{wn}\right)^2$, $s_{ij}^{wn} = \frac{w_{ij}n_{ij}}{\sum_{i\in j} w_{ij}n_{ij}}$. When aggregating, we weight by the market's payroll share $s_j = \frac{\sum_{i\in j} w_{ij}n_{ij}}{\int \sum_{i\in j} w_{ij}n_{ij}dj}$ so that $HHI^{wn} = \int s_j HHI_j^{wn} dj$. The market level employment Herfindahl is given by: $HHI_j^{n} := \sum_{i\in j}\left(s_{ij}^{n}\right)^2$, $s_{ij}^{n} = \frac{n_{ij}}{\sum_{i\in j} n_{ij}}$. We weight the market level employment Herfindahls similarly.

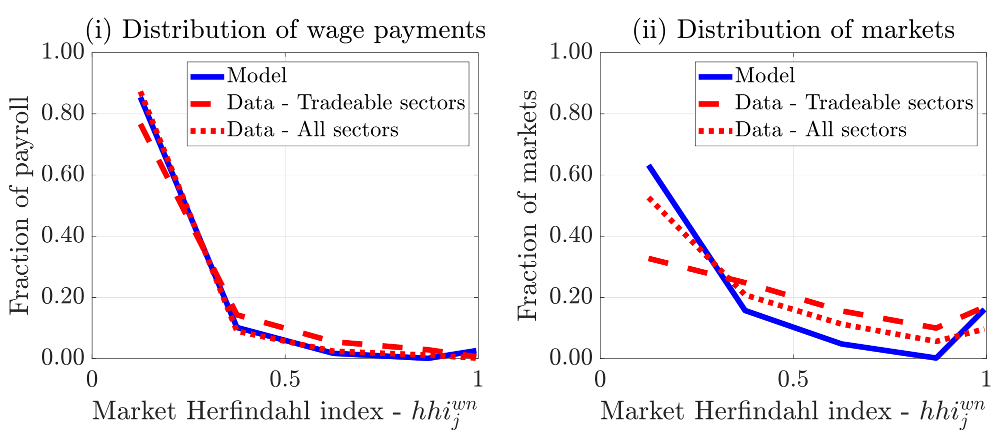

Figure A1: Cross market distribution of concentration model v. data

Notes: This figure plots the market-level distribution of the payroll Herindahl index (HHI_j^{wn}). Model corresponds to the all sectors model. Bins are determined by the following bounds: $\{0, 0.25, 0.50, 0.75, 0.99, 1\}$. The horizontal axis gives the center of each bin. Panel (i) plots the fraction of total payroll in each bin. Panel (ii) plots the fraction of markets in each bin. Data is Census LBD. See Appendix C for additional details.

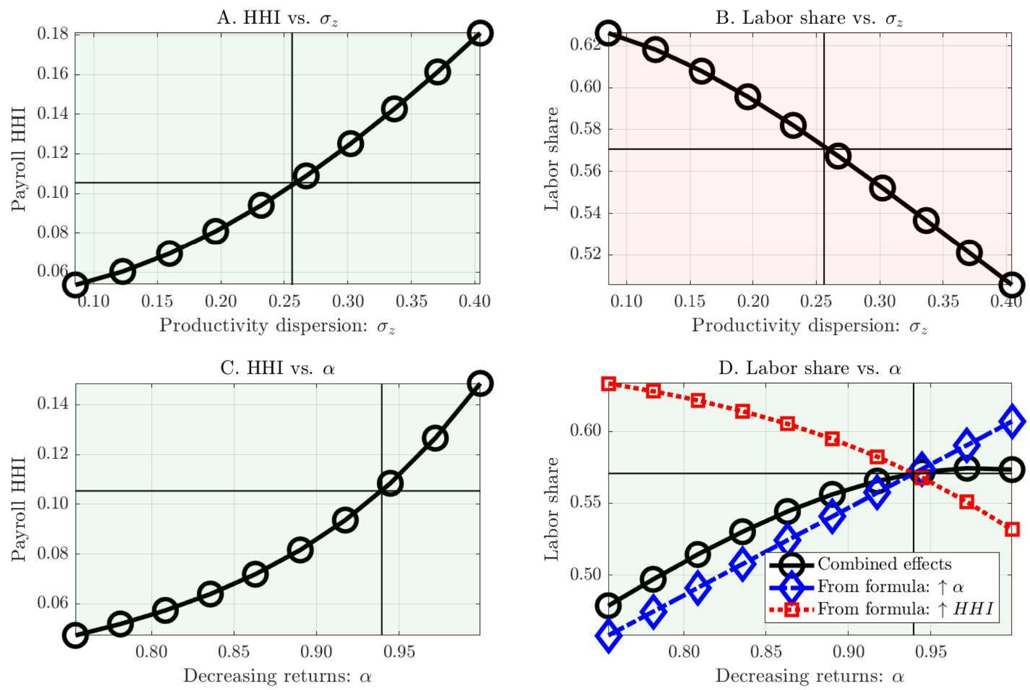

Figure A2: Identification of productivity dispersion σ, and DRS α

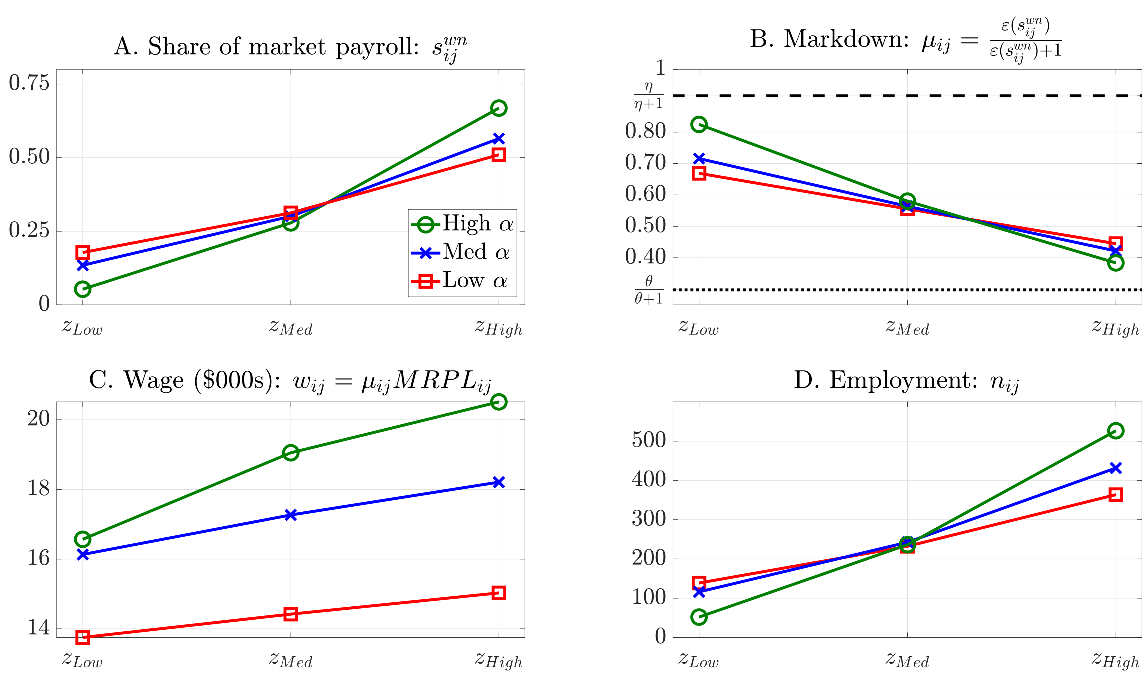

Figure A3: Oligopsonistic equilibrium with varying decreasing returns α

Notes: Figure constructed from model under estimated parameters (Table 3). Low, medium and high productivities of the firms correspond to the 10^{th}, 50^{th} and 90^{th} percentiles of the productivity distribution.

B Microfounding the nested CES labor supply system

In this section we provide a micro-foundation for the nested CES preferences used in the main text. The arguments used here adapt those in Frank Verboven (1996). We begin with the case of monopsonistic competition to develop ideas and then move to the case of oligopsonistic labor markets studied in the text. We then show that the same supply system occurs in a setting where workers solve a dynamic discrete choice problem and firms compete in a dynamic oligopoly.

B.1 Static discrete choice framework

Agents. There is a unit measure of ex-ante identical individuals indexed by $l \in [0,1]$. There is a large but finite set of J sectors in the economy, with finitely many firms $i \in \{1,\ldots,M_j\}$ in each sector.

Preferences. Each individual has random preferences for working at each firm ij. Their disutility of labor supply is *convex* in hours worked h_l. Worker l's disutility of working h_{lij} hours at firm ij are:

$$\nu_{lij} = e^{-\xi_{lij}} h_{lij} \qquad , \qquad \log \nu_{lij} = \log h_{lij} - \xi_{lij},$$

where the random utility term ξ_{lij} is distributed *iid* across individuals according from a multi-variate Gumbel distribution:

$$F(\xi_{i1}, ..., \xi_{NJ}) = \exp\left[-\sum_{ij} e^{-(1+\eta)\xi_{ij}}\right].$$

The term ξ_{lij} is a worker-firm specific term which reduces labor disutility and hence could capture (i) an inverse measure of commuting costs, or (ii) a positive amenity.

Decisions. Each individual must earn $y_l \sim F(y)$, where earnings $y_l = w_{ij} h_{lij}$. After drawing their vector $\{\xi_{lij}\}$, each worker solves

$$\min_{ij} \{\log h_{lij} - \xi_{lij}\} \equiv \max_{ij} \{\log w_{ij} - \log y_l + \xi_{lij}\}.$$

This problem delivers the following probability that worker l chooses to work at firm ij, which is independent of y_l:

$$Prob_l\left(w_{ij}, w_{-ij}\right) = \frac{w_{ij}^{1+\eta}}{\sum_{ij} w_{ij}^{1+\eta}}. \tag{B1}$$

Aggregation. Total labor supply to firm ij, is then found by integrating these probabilities, multiplied by the hours supplied by each worker l:

$$
\begin{aligned}
n_{ij} &= \int_0^1 Prob_l\left(w_{ij}, w_{-ij}\right) h_{lij} dF\left(y_l\right) \qquad , \qquad h_{lij} = y_l / w_{ij} \\
n_{ij} &= \frac{w_{ij}^{\eta}}{\sum_{i\in j} w_{ij}^{1+\eta}} \underbrace{\int_0^1 y_l dF\left(y_l\right)}_{:=Y}
\end{aligned} \tag{B2}
$$

Aggregating this expression we obtain the obvious result that $\sum_{i\in j} w_{ij} n_{ij} = Y$. Now define the following indexes:

$$
\boldsymbol{W} := \left[\sum_{i\in j} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \boldsymbol{N} := \left[\sum_{i\in j} n_{ij}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} .
$$

Along with (B2), these indexes imply that $\boldsymbol{WN} = Y$. Using these definitions along with $\boldsymbol{WN} = Y$ in (B2) yields the CES supply curve:

$$
n_{ij} = \left(\frac{w_{ij}}{\boldsymbol{W}}\right)^{\eta} \boldsymbol{N}.
$$

We therefore have the result that the supply curves that face firms in this model of individual discrete choice are equivalent to those that face the firms when a representative household solves the following income maximization problem:

$$
\max_{\{n_{ij}\}} \sum_{i\in j} w_{ij} n_{ij} \quad s.t. \quad \left[\sum_{i\in j} n_{ij}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} = \boldsymbol{N}.
$$

Since at the solution, the objective function is equal to $\boldsymbol{WN}$, then the envelope condition delivers a natural interpretation of $\boldsymbol{W}$ as the equilibrium payment to total labor input in the economy for one additional unit of aggregate labor disutility. That is, the following equalities hold:

$$
\frac{\partial}{\partial \boldsymbol{N}} \sum_{i\in j} w_{ij} n_{ij}^{*}(w_{ij}, w_{-ij}) = \Lambda = \boldsymbol{W} = \frac{\partial}{\partial \boldsymbol{N}} \boldsymbol{WN}.
$$

Nested logit and nested CES. Consider changing the distribution of preference shocks as follows:

$$
F(\xi_{i1}, ..., \xi_{NJ}) = \exp\left[-\sum_{j=1}^{J}\left(\sum_{i=1}^{M_j} e^{-(1+\eta)\xi_{ij}}\right)^{\frac{1+\theta}{1+\eta}}\right].
$$

We recover the distribution (B1) above if $\eta = \theta$. Otherwise, if $\eta > \theta$ the problem is convex and the conditional covariance of within sector preference draws differ from the economy wide variance of preference draws. We discuss this more below.

In this setting, choice probabilities can be expressed as the product of the probability of supplying labor to firm i conditional on supplying labor to market j, and the probability of supplying labor to market j:

$$Prob_l\left(w_{ij}, w_{-ij}\right) = \underbrace{\frac{w_{ij}^{1+\eta}}{\sum_{i=1}^{M_j} w_{ij}^{1+\eta}}}_{Prob_l(\text{Choose firm } i\,|\,\text{Choose market } j)} \times \underbrace{\frac{\left[\sum_{i=1}^{M_j} w_{ij}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{l=1}^{J}\left[\sum_{k=1}^{M_l} w_{kl}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}}_{Prob_l(\text{Choose market } j)}.$$

Following the same steps as above, we can aggregate these choice probabilities and hours decisions to obtain firm level labor supply:

$$n_{ij} = \frac{w_{ij}^{\eta}}{\sum_{i=1}^{M_j} w_{ij}^{1+\eta}} \frac{\left[\sum_{i=1}^{M_j} w_{ij}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{l=1}^{J}\left[\sum_{k=1}^{M_l} w_{kl}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} Y. \tag{B3}$$

We can now define the following indexes:

$$\boldsymbol{W}_j = \left[\sum_{i=1}^{M_j} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \boldsymbol{N}_j = \left[\sum_{i=1}^{M_j} n_{ij}^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}},$$
$$\boldsymbol{W} = \left[\sum_{j=1}^{J} \boldsymbol{W}_j^{1+\theta}\right]^{\frac{1}{1+\theta}} \quad , \quad \boldsymbol{N} = \left[\sum_{j=1}^{J} \boldsymbol{N}_j^{\frac{1+\theta}{\theta}}\right]^{\frac{\theta}{1+\theta}}.$$

Using these definitions and similar results to the above we can show that $\boldsymbol{W}_j\boldsymbol{N}_j = \sum_{i=1}^{M_j} w_{ij}n_{ij}$, and $Y = \boldsymbol{W}\boldsymbol{N} = \sum_{j=1}^{J} \boldsymbol{W}_j\boldsymbol{N}_j$.

Consider the thought experiment of adding more markets J (which is necessary to identically map these formulas to our model). While the min of an infinite number of draws from a Gumbel distribution is not defined (it asymptotes to $-\infty$), the distribution of choices across markets is defined at each point in the limit as we add more markets J (Hannes Malmberg, 2013). As a result, the distribution of choices will have a well defined limit, and with the correct scaling as we add more markets (we can scale the disutilities at each step and not affect the market choice), as described in Malmberg (2013), the limiting wage indexes will be defined as above. We can then express (B3) as:

$$n_{ij} = \left(\frac{w_{ij}}{\boldsymbol{W}_j}\right)^{\eta}\left(\frac{\boldsymbol{W}_j}{\boldsymbol{W}}\right)^{\theta}\boldsymbol{N},$$

which completes the CES supply system defined in the text.

Comment. The above has established that it is straightforward to derive the supply system in the model through a discrete choice framework. This is particularly appealing given recent modeling of labor supply using familiar discrete choice frameworks first in models of economic geography and more recently in labor (Katarina Borovickova and Robert Shimer, 2017; David Card, Ana Rute Cardoso, Jörg

Heining and Patrick Kline, 2018; Thibaut Lamadon, Magne Mogstad and Bradley Setzler, 2019). Since firms take this supply system as given, we can then work with the nested CES supply functions as if they were derived from the preferences and decisions of a representative household. This vastly simplifies welfare computations and allows for the integration of the model into more familiar macroeconomic environments.

The second advantage of this micro-foundation is that it provides a natural interpretation of the somewhat nebulous elasticities of substitution in the CES specification: η and θ. Returning to the Gumbel distribution we observe the following

$$F(\xi_{i1}, ..., \xi_{NJ}) = \exp\left[-\sum_{j=1}^{J} \left(\sum_{i=1}^{M_j} e^{-(1+\eta)\xi_{ij}} \right)^{\frac{1+\theta}{1+\eta}} \right]$$

A higher value of η *increases* the correlation of draws within a market (McFadden, 1978). Within a market if η is high, then an individual's preference draws are likely to be clustered. With little difference in non-pecuniary idiosyncratic preferences for working at different firms, wages dominate in an individual's labor supply decision and wage posting in the market is closer to the competitive outcome. A higher value of θ *decreases* the overall variance of draws across all firms (i.e. it *increases* the correlation across any two randomly chosen sub-vectors of an individual's draws). An individual is therefore more likely to find that their lowest levels of idiosyncratic disutility are in two different markets, increasing across market wage competition.

In the case that $\eta = \theta$, the model collapses to the standard logit model. In this case the following obtains. Take an individual's ξ_{lij} for some firm. The conditional probability distribution of some other draw $\xi_{li'j'}$ is the same whether firm i' is in the same market ($j' = j$) or some other market ($j' \neq j$). Individuals are as likely to find somewhere local that incurs the same level of labor disutility as finding somewhere in another market. In this setting economy-wide monopsonistic competition obtains. When an individual is more likely to find their other low disutility draws in the *same* market, then firms within that market have local market power. This is precisely the case that obtains when $\eta > \theta$.

B.2 Dynamic discrete choice framework

We show that the above discrete choice framework can be adapted to an environment where some individuals draw new vectors $\boldsymbol{\xi}_l$ each period and reoptimize their labor supply. Firms therefore compete in a dynamic oligopoly. Restricting attention to the stationary solution of the model where firms keep employment and wages constant—as in the tradition of K. Burdett and D.T. Mortensen (1998)—we show that the allocation of employment and wages once again coincide with the solution to the problem in the main text. To simplify notation we consider the problem for a market with M firms $i \in \{1, \ldots, M\}$ which may be generalized to the model in the text.

Environment. Every period a random fraction λ of workers each draw a new vector $\boldsymbol{\xi}_l$. Let n_i be employment at firm i. Let $\overline{w}_i$ be the *average wage* of workers at firm i, such that the total wage bill in the firm is $\overline{w}_i n_i$. Let the equilibrium labor supply function $h(w_i, w_{-i})$ determine the amount of hires a firm makes if it posts a wage w_i when its competitors' wages in the market are given by the vector $\boldsymbol{w}_{-i}$.

Value function. Let $V(n_i, \overline{w}_i)$ be the firm's present discounted value of profits, where the firm has discount rate $\beta = 1$. Then $V(n_i, \overline{w}_i)$ satisfies:

$$\begin{aligned} V\left(n_i, \overline{w}_i\right) &= \left(Pz_i - \overline{w}_i\right)\left(1-\lambda\right) n_i + \max_{w_i'} \left\{\left(Pz_i - w_i'\right) h\left(w_i', w_{-i}'\right) + V\left(n_i', \overline{w}_i'\right)\right\} \quad , & \text{(B4)} \\ n'\left(n_i, w_i', \boldsymbol{w}_{-i}'\right) &= \left(1-\lambda\right) n_i + h\left(w_i', \boldsymbol{w}_{-i}'\right) \quad , & \text{(B5)} \\ \overline{w}'\left(n_i, \overline{w}_i, w_i', w_{-i}'\right) &= \frac{\left(1-\lambda\right)\overline{w}_i n_i + h\left(w_i', \boldsymbol{w}_{-i}'\right) w_i'}{\left(1-\lambda\right) n_i + h\left(w_i', \boldsymbol{w}_{-i}'\right)} . & \text{(B6)} \end{aligned}$$

The firm operates a constant returns to scale production function. Of the firm's n_i workers, a fraction $(1 - \lambda)$ do not draw new preferences. The total profit associated with these workers is then average revenue (Pz_i) minus average cost $(\overline{w}_i)$. The firm chooses a new wage w_i' to post in the market. In equilibrium, given its competitor's wages $\boldsymbol{w}_{-i}'$, it hires $h(w_i', \boldsymbol{w}_{-i}')$ workers. The total profit associated with these workers is again average revenue (Pz_i) minus average cost (w_i'). The second and third equations account for the evolution of the firm's state variables.

Optimality. Given its competitor's prices, the first order condition with respect to w_i' is:

$$\left(Pz_i - w_i'\right) h_1\left(w_i', w_{-i}'\right) - h\left(w_i', w_{-i}'\right) + V_n\left(n_i', \overline{w}_i'\right) n_w'\left(n_i, w_i', w_{-i}'\right) + V_{\overline{w}}\left(n_i', \overline{w}_i'\right) \overline{w}_w\left(n_i, \overline{w}_i, w_i', w_{-i}'\right) = 0$$

The relevant envelope conditions are

$$\begin{aligned} V_n\left(n_i, \overline{w}_i\right) &= \left(Pz_i - \overline{w}_i\right)\left(1-\lambda\right) + V_n\left(n_i', \overline{w}_i'\right) n_n'\left(n_i, w_i', \boldsymbol{w}_{-i}'\right) + V_{\overline{w}}\left(n_i', \overline{w}_i'\right) \overline{w}_n'\left(n_i, \overline{w}_i, w_i', \boldsymbol{w}_{-i}'\right) \\ V_{\overline{w}}\left(n_i, \overline{w}_i\right) &= -\left(1-\lambda\right) n_i + V_{\overline{w}}\left(n_i', \overline{w}_i'\right) \overline{w}_{\overline{w}}'\left(n_i, \overline{w}_i, w_i', \boldsymbol{w}_{-i}'\right) \end{aligned}$$

In a stationary equilibrium $\overline{w}_i = w_i'$, and $n_i' = n_i$. One can compute the partial derivatives involved in these expressions, and evaluate the conditions under stationarity to obtain

$$\left(Pz_i - w_i\right) h_1\left(w_i, w_{-i}\right) = h\left(w_i, w_{-i}\right).$$

Rearranging this expression:

$$w_i = \frac{\varepsilon_i(w_i, w_{-i})}{\varepsilon_i(w_i, w_{-i}) + 1} Pz_i \qquad , \qquad \varepsilon_i(w_i, w_{-i}) \quad := \quad \frac{h_1(w_i, w_{-i}) w_i}{h(w_i, w_{-i})}$$

The solution to the dynamic oligopsony problem for a *given* supply system is identical to the solution of the static problem. In this setting, the supply system is obviously that which is obtained from the individual discrete choice problem in the previous section.

Comments. This setting establishes that the model considered in the main text can also be conceived as a setting where individuals periodically receive some preference shock that causes them to relocate, and firms engage in a dynamic oligopoly given these worker decisions. When $\eta > \theta$ the shock causes a worker to consider all firms in one market very carefully to the exclusion of other markets when they are making their relocation decision. When $\eta = \theta$ the individual considers all firms in all markets equally.

C Data

This section provides additional details regarding the data sources used in the paper, sample restrictions, and construction of a number of variables. We use the LBD (Bureau of the Census (2016*a*)) and SSEL (Bureau of the Census (2016*b*)).

C.1 Census Longitudinal Business Database (LBD)

The LBD is built on the Business Register (BR), Economic Census and surveys. The BR began in 1972 and is a database of all U.S. business establishments. The business register is also called the Standard Statistical Establishment List (SSEL). The SSEL contains records for all industries except private households and illegal or underground activities. Most government owned entities are not in the SSEL. The SSEL includes single and multi unit establishments. The longitudinal links are constructed using the SSEL. The LBD does not distinguish C-Corporations from S-Corporations consistently over time, and so we merge the Form 1120 filing status from the SSEL into the LBD. For establishments with missing filing status, we impute their filing status with the modal filing status of reporting establishments owned by the same firm in the same year (i.e. imputations are made within *firmid-year*). The database is annual.

C.2 Sample restrictions

For both the summary statistics and corporate tax analysis, we isolate all plants (lbdnums) with non missing firmids, with strictly positive pay, positive employment, and non-missing county codes for the continental US (we exclude Alaska, Hawaii, and Puerto Rico). The units of payroll were manually changed from dollars to tens-of-thousands of dollars in the SSEL from 1976-1981 and 1983-1989. As a result we must remove data errors associated with this manual coding. We do so by removing firms that are in the upper two percentiles of the wage distribution while simultaneously being in the upper percentile of firm size. We then isolate all lbdnums with non-missing 2 digit NAICS codes equal to 11, 21, 31, 32, 33 or 55. These are the top tradeable 2-digit NAICS codes as defined by Mercedes Delgado, Richard Bryden and Samantha Zyontz (2014). We use the consistent 2012 NAICS codes provided by Teresa C. Fort and Shawn D. Klimek (2018) throughout the paper. We winsorize the wage and employment at the 1% level to remove remaining outliers. Each plant has a unique firmid which corresponds to the owner of the plant.[2] Throughout the paper, we define a firm to be the sum of all establishments in a commuting zone with a common firmid and NAICS3 classification.

Summary Statistics Sample: Our summary statistics include all observations that satisfy the above criteria in 1976 and 2014 (Tables D1 and D2).

Corporate Tax Sample: The corporate tax analysis includes all observations that satisfy the above criteria between 1977 and 2011. We additionally require the firm to have at least 5 employees in order to compute direct elasticities (see Section G.3.2). The LBD begins in 1976, but we require information on

[2] Each firm only has one firmid. The firmid is different from the EIN. The firmid aggregates EINS to build a consistent firm identifier in the cross-section and over time.

lags of the wage bill share, thus 1977 is our first usable year. The tax series ends in 2012 but the 'Year t+1' estimates require information on forward lags, thus our final usable year is 2011. We further restrict the sample to firmid-market-year observations which are corporations. To build a consistent corporation definition over time, we use both the SSEL and LBD. We identify corporations as those with SSEL Form 1120 codes which indicate 'C-Corporation' status and LBD legal form of organization codes that also indicate 'C-Corporation' status. Table C1 provides summary statistics for this sample.

Variable		**Mean**	**Std. Dev.**
Corporate tax rate (percent)	$\tau_{s(j)t}$	7.24	3.01
Change in corporate tax rate	$\Delta\tau_{s(j)t}$	0.03	0.61
Total Pay At Firm (Thousands)	$w_{ijt}n_{ijt}$	3,200	22,110
Employment	n_{ijt}	92	470
Wage bill Herfindahl	HHI_{jt}^{wn}	0.13	0.18
Wage bill share	s_{ijt}^{wn}	0.05	0.14
Wage bill share, Lagged 1 yr	s_{ijt-1}^{wn}	0.05	0.14
Number of firms per market	M_j	487	821
Log number of firms per market	$\log M_j$	4.99	1.74
Log employment	$\log n_{ijt}$	3.38	1.24
Log wage	$\log w_{ijt}$	3.74	0.54
Observations			4,260,000

Table C1: Regression sample summary statistics

Notes: Tradeable C-Corps from 1977 to 2011.

Sample NAICS Codes and Commuting Zones: Table C2 describes the NAICS 3 codes in our sample. Table C3 provides examples of commuting zones and the counties that are associated with those commuting zones.

Table C2: NAICS 3 digit examples

NAICS3	**Description**	**NAICS3**	**Description**
111	Crop Production	322	Paper Manuf.
112	Animal Production and Aquaculture	323	Printing and Related Support Activities
113	Forestry and Logging	324	Petroleum and Coal Products Manuf.
114	Fishing, Hunting and Trapping	325	Chemical Manuf.
115	Support Activities for Agriculture and Forestry	326	Plastics and Rubber Products Manuf.
211	Oil and Gas Extraction	327	Nonmetallic Mineral Product Manuf.
212	Mining (except Oil and Gas)	331	Primary Metal Manuf.
213	Support Activities for Mining	332	Fabricated Metal Product Manuf.
311	Food Manuf.	333	Machinery Manuf.
312	Beverage and Tobacco Product Manuf.	334	Computer and Electronic Product Manuf.
313	Textile Mills	335	Electrical Equipment, Appliance, Component Manuf.
314	Textile Product Mills	336	Transportation Equipment Manuf.
315	Apparel Manufacturing	337	Furniture and Related Product Manuf.
316	Leather and Allied Product Manuf.	339	Miscellaneous Manuf.
321	Wood Product Manuf.	551	Management of Companies and Enterprises

Table C3: Commuting Zone (CZ) examples: Census commuting zones numbers 58 and 47

CZ ID, 2000	County Name	Metro. Area, 2003	County Pop. 2000	CZ Pop. 2000
58	Cook County	Chicago-Naperville-Joliet, IL Metro. Division	5,376,741	8,704,935
58	DeKalb County	Chicago-Naperville-Joliet, IL Metro. Division	88,969	8,704,935
58	DuPage County	Chicago-Naperville-Joliet, IL Metro. Division	904,161	8,704,935
58	Grundy County	Chicago-Naperville-Joliet, IL Metro. Division	37,535	8,704,935
58	Kane County	Chicago-Naperville-Joliet, IL Metro. Division	404,119	8,704,935
58	Kendall County	Chicago-Naperville-Joliet, IL Metro. Division	54,544	8,704,935
58	Lake County	Lake County-Kenosha County, IL-WI Metro. Division	644,356	8,704,935
58	McHenry County	Chicago-Naperville-Joliet, IL Metro. Division	260,077	8,704,935
58	Will County	Chicago-Naperville-Joliet, IL Metro. Division	502,266	8,704,935
58	Kenosha County	Lake County-Kenosha County, IL-WI Metro. Division	149,577	8,704,935
58	Racine County	Racine, WI MSA	188,831	8,704,935
58	Walworth County	Whitewater, WI Micropolitan SA	93,759	8,704,935
47	Anoka County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	298,084	2,904,389
47	Carver County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	70,205	2,904,389
47	Chisago County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	41,101	2,904,389
47	Dakota County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	355,904	2,904,389
47	Hennepin County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	1,116,200	2,904,389
47	Isanti County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	31,287	2,904,389
47	Ramsey County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	511,035	2,904,389
47	Scott County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	89,498	2,904,389
47	Washington County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	201,130	2,904,389
47	Wright County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	89,986	2,904,389
47	Pierce County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	36,804	2,904,389
47	St. Croix County	Minneapolis-St. Paul-Bloomington, MN-WI MSA	63,155	2,904,389

A. Firm-market-level averages	**1976**	**2014**
Total firm pay ($1,000s)	673.30	2018.00
Total firm employment	54.94	34.63
Pay per employee	$ 12,255	$ 58,273
Firm-level observations	375,000	465,000

B. Market-level averages	**1976**	**2014**
Wage-bill HHI, Unweighted	0.50	0.48
Wage-bill HHI, Weighted by market's share of total payroll)	0.22	0.17
Firms per market	28.00	33.86
Percent of markets with 1 firm	16.5%	16.4%
National payroll share of markets with 1 firm	0.58%	0.43%
Market-level observations	13,000	14,000

C. Across market correlations with wage-bill HHI	**1976**	**2014**
Number of firms	-0.26	-0.36
Employment Herfindahl	0.98	0.98
Market Employment	-0.21	-0.25
Market-level observations	13,000	14,000

Table D1: Summary Statistics, U.S. Census Longitudinal Employer Database 1976 and 2014 - Tradeables only

Notes: Tradeable NAICS2 codes (11,21,31,32,33,55). Market defined to be NAICS3 within Commuting Zone. Observations rounded to nearest thousand and numbers rounded to 4 significant digits according to Census disclosure rules. Firm-market-level refers to a 'firmid by Commuting Zone by 3-digit NAICs by Year' observation. Market-level refers to a 'Commuting Zone by 3-digit NAICs by Year' aggregation of observations.

D Labor market concentration 1976 and 2014

Table D1A describes characteristics of the firm-market observations in 1976 and 2014. Average nominal payroll was $673,300 in 1976 and $2,018,000 in 2014. Average firm-market employment was 55 workers in 1976 and 35 workers in 2014. Average nominal wage was $12,255 in 1976 and $58,273 in 2014.

Table D1B shows that different weighting schemes of across-market averages imply different levels and trends. The unweighted average wage-bill Herfindahl for wages is between two and three times larger than its payroll weighted counterpart. Little employment or payroll is located in highly concentrated markets. In both periods, 16 percent of markets have only one employer and so HHIs equal to one. However, these single firm markets only account for roughly one half of one percent of national payroll. In terms of the time-series, unweighted average wage-bill Herfindahl declines marginally between 1976 and 2014. In contrast, payroll weighted wage-bill Herfindahl declines by 23% from 0.22 to 0.17.

Table D1C confirms that the number of firms and total market employment are negatively correlated with concentration. This is important for understanding why weighted and unweighted Herfindahls are so different and will be used as an over-identifying test of the estimated model. Moreover, employment and wage-bill Herfindahls are highly correlated.

Table D2 includes summary statistics of labor market concentration across all industries. Similar to tradeable industries, the market-level unweighted and weighted Herfindahls decline. The unweighted wage-bill Herfindahl declines from 0.36 to 0.34. The payroll weighted wage-bill Herfindahl declines from 0.16 to 0.11.

	(A) Firm-market-level averages	
	1976	**2014**
Total firm pay (000s)	202.10	1000.00
Total firm employment	19.35	22.83
Pay per employee	$ 10,444	$ 43,802
Firm-level observations	3,746,000	5,845,000

	(B) Market-level averages	
	1976	**2014**
Wage-bill Herfindahl (Unweighted)	0.36	0.34
Wage-bill Herfindahl (Weighted by market's share of total payroll)	0.16	0.11
Firms per market	75.71	113.10
Percent of markets with 1 firm	10.4%	9.4%
Market-level observations	49,000	52,000

	(C) Market-level correlations	
	1976	**2014**
Correlation of Wage-bill Herfindahl and number of firms	-0.20	-0.17
Correlation of Wage-bill Herfindahl and Employment Herfindahl	0.98	0.97
Correlation of Wage-bill Herfindahl and Market Employment	-0.15	-0.16
Market-level observations	49,000	52,000

Table D2: Summary Statistics, U.S. Census Longitudinal Employer Database 1976 and 2014 - All industries

Notes: All NAICS. Market defined to be NAICS3 within Commuting Zone. Observations rounded to nearest thousand and numbers rounded to 4 significant digits according to Census disclosure rules. Firm-market-level refers to a 'firmid by Commuting Zone by 3-digit NAICs by Year' observation. Market-level refers to a 'Commuting Zone by 3-digit NAICs by Year' aggregation of observations.

E Mathematical derivations

This section provides detailed derivations of mathematical formulae that appear in the main text. It covers: (i) the household problem (Section E.1) (ii) the firm problem (Section E.2), (iii) market equilibrium (Proposition 1.1) (Section E.3), (iv) general equilibrium and aggregation (Proposition 1.2) (Section E.4), (v) relationship between the labor share and concentration (Proposition 1.3) (Section E.5), (vi) closed form general equilibrium solution and scaling properties used in calibration (Section E.6), (vii) reduced form and structural labor supply elasticities (Section E.7), (viii) pass-through expression (Section E.8), (ix) expressions used in the discussion of corporate taxes (Section E.9).

E.1 Household problem - Section 1.2

- The household's problem is

$$\max_{\{n_{ijt},c_{ijt},K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U\left(\mathbf{C}_t, \mathbf{N}_t\right)$$

where

$$\mathbf{N}_t = \left[\int \mathbf{n}_{jt}^{\frac{\theta+1}{\theta}} dj\right]^{\frac{\theta}{\theta+1}}$$

$$\mathbf{n}_{jt} = \left[\sum_{i\in j} n_{ijt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$$

$$\mathbf{C}_t = \int \sum_{i\in j} c_{ijt} dj$$

subject to the initial endowment $K_0 > 0$, and the following budget constraint in each period, in which it takes all prices as given, these include the wage w_{ijt} at all firms-ij, rental rate R_t and profits Π_t as given:

$$\mathbf{C}_t + K_{t+1} - (1-\delta) K_t = \int \sum_{i\in j} w_{ijt} n_{ijt} dj + R_t K_t + \Pi_t$$

E.1.1 First order conditions

- The first order conditions for consumption and capital give

$$U_C\left(\mathbf{C}_t, \mathbf{N}_t\right) = \beta U_C\left(\mathbf{C}_{t+1}, \mathbf{N}_{t+1}\right)\left[R_{t+1} + (1-\delta)\right]$$

- The first order conditions for consumption and labor supply to firm-ij gives

$$w_{ijt} = \frac{\partial \mathbf{n}_{jt}}{\partial n_{ijt}} \frac{\partial \mathbf{N}_t}{\partial \mathbf{n}_{jt}} \left(-\frac{U_N\left(\mathbf{C}_t, \mathbf{N}_t\right)}{U_C\left(\mathbf{C}_t, \mathbf{N}_t\right)}\right)$$

E.1.2 Deriving supply system

- Define the following terms. The *market wage* $\boldsymbol{w}_{jt}$ is the number that satisfies $\boldsymbol{w}_{jt}\boldsymbol{n}_{jt} = \sum_{i\in j} w_{ijt}n_{ijt}$. The *aggregate wage* $\boldsymbol{W}_t$ is the number that satisfies $\boldsymbol{W}_t\boldsymbol{N}_t = \int \boldsymbol{w}_{jt}\boldsymbol{n}_{jt}dj$.

- We can write the first order condition as:

$$w_{ijt}n_{ijt} = \left(\frac{\partial \boldsymbol{n}_{jt}}{\partial n_{ijt}}\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)\left(\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}}\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)\left(-\frac{U_N\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}{U_C\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}\right)\boldsymbol{N}_t$$

- Using the labor disutility indexes, note that

$$\frac{\partial \boldsymbol{n}_{jt}}{\partial n_{ijt}}\frac{n_{ijt}}{\boldsymbol{n}_{jt}} = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{\eta+1}{\eta}} \quad \text{, therefore,} \quad \sum_{i\in j}\frac{\partial \boldsymbol{n}_{jt}}{\partial n_{ijt}}\frac{n_{ijt}}{\boldsymbol{n}_{jt}} = 1$$

$$\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}}\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t} = \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{\theta+1}{\theta}} \quad \text{, therefore,} \quad \int\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}}\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}dj = 1$$

- Using these results and aggregating the first order condition over $i \in j$, then over $j \in [0,1]$

$$\begin{aligned}
\text{Aggregate over } i \in j: \quad \boldsymbol{w}_{jt}\boldsymbol{n}_{jt} &= \left(\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}}\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)\left(-\frac{U_N\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}{U_C\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}\right)\boldsymbol{N}_t \\
\text{and then aggregate over } j \in [0,1]: \quad \boldsymbol{W}_t\boldsymbol{N}_t &= -\frac{U_N\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}{U_C\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}\boldsymbol{N}_t \\
\boldsymbol{W}_t &= -\frac{U_N\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}{U_C\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}
\end{aligned}$$

- Take the first order condition aggregated over markets $j \in [0,1]$, and substitute in the aggregate inverse labor supply curve. Doing so we can obtain the *market supply curve*:

$$\begin{aligned}
\boldsymbol{w}_{jt}\boldsymbol{n}_{jt} &= \left(\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}}\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)\left(-\frac{U_N\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}{U_C\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}\right)\boldsymbol{N}_t \\
\boldsymbol{w}_{jt}\boldsymbol{n}_{jt} &= \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{\theta+1}{\theta}}\boldsymbol{W}_t\boldsymbol{N}_t \\
\boldsymbol{n}_{jt} &= \left(\frac{\boldsymbol{w}_{jt}}{\boldsymbol{W}_t}\right)^{\theta}\boldsymbol{N}_t
\end{aligned}$$

  which also implies that

$$\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}}\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t} = \frac{\boldsymbol{w}_{jt}\boldsymbol{n}_{jt}}{\boldsymbol{W}_t\boldsymbol{N}_t}.$$

- Substituting this into the first order condition we can obtain the *firm supply curve*:

$$
\begin{aligned}
w_{ijt}n_{ijt} &= \left(\frac{\partial \boldsymbol{n}_{jt}}{\partial n_{ijt}}\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)\left(\frac{\partial \boldsymbol{N}_t}{\partial \boldsymbol{n}_{jt}}\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)\left(-\frac{U_N\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}{U_C\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}\right)\boldsymbol{N}_t \\
w_{ijt}n_{ijt} &= \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{\eta+1}{\eta}}\left(\frac{\boldsymbol{w}_{jt}\boldsymbol{n}_{jt}}{\boldsymbol{W}_t\boldsymbol{N}_t}\right)\boldsymbol{W}_t\boldsymbol{N}_t \\
n_{ijt} &= \left(\frac{w_{ijt}}{\boldsymbol{w}_{jt}}\right)^{\eta}\left(\frac{\boldsymbol{w}_{jt}}{\boldsymbol{W}_t}\right)^{\theta}\boldsymbol{N}_t
\end{aligned}
$$

- We can now compute expressions for the wage indexes $\boldsymbol{w}_{jt}$ and $\boldsymbol{W}_t$.
- Take the labor supply curve to the firm and aggregate

$$
\begin{aligned}
n_{ijt} &= \left(\frac{w_{ijt}}{\boldsymbol{w}_{jt}}\right)^{\eta}\boldsymbol{n}_{jt} \\
w_{ijt}n_{ijt} &= w_{ijt}^{1+\eta}\boldsymbol{w}_{jt}^{-\eta}\boldsymbol{n}_{jt} \\
\sum_{i\in j} w_{ijt}n_{ijt} &= \left[\sum_{i\in j} w_{ijt}^{1+\eta}\right]\boldsymbol{w}_{jt}^{-\eta}\boldsymbol{n}_{jt} \\
\boldsymbol{w}_{jt}\boldsymbol{n}_{jt} &= \left[\sum_{i\in j} w_{ijt}^{1+\eta}\right]\boldsymbol{w}_{jt}^{-\eta}\boldsymbol{n}_{jt} \\
\boldsymbol{w}_{jt} &= \left[\sum_{i\in j} w_{ijt}^{1+\eta}\right]^{\frac{1}{1+\eta}}
\end{aligned}
$$

- Applying the same to the labor supply curve to the market we get

$$
\boldsymbol{W}_t = \left[\int \boldsymbol{w}_{jt}^{1+\theta}dj\right]^{\frac{1}{1+\theta}}
$$

- Therefore we have the set of results used in the body of the paper:

$$
\begin{aligned}
\boldsymbol{W}_t &= -\frac{U_N\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)}{U_C\left(\boldsymbol{C}_t,\boldsymbol{N}_t\right)} \quad ,\boldsymbol{n}_{jt} = \left(\frac{\boldsymbol{w}_{jt}}{\boldsymbol{W}_t}\right)^{\theta}\boldsymbol{N}_t \quad n_{ijt} = \left(\frac{w_{ijt}}{\boldsymbol{w}_{jt}}\right)^{\eta}\left(\frac{\boldsymbol{w}_{jt}}{\boldsymbol{W}_t}\right)^{\theta}\boldsymbol{N}_t \\
\boldsymbol{W}_t &= \left[\int \boldsymbol{w}_{jt}^{1+\theta}dj\right]^{\frac{1}{1+\theta}} \quad ,\boldsymbol{w}_{jt} = \left[\sum_{i\in j} w_{ijt}^{1+\eta}\right]^{\frac{1}{1+\eta}}
\end{aligned}
$$

- Using the above we can invert the labor supply curve to the firm in two steps. At the market level

$$
\boldsymbol{w}_{jt} = \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}}\boldsymbol{W}_t
$$

and then at the firm level

$$w_{ijt} = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}} \boldsymbol{w}_{jt} \quad , \quad w_{ijt} = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}} \boldsymbol{W}_t$$

- This is delivers the set of partial equilibrium conditions specified in the text in Section 1.2

E.2 Proof of Nash equilibrium expressions - Section 1.3

- We can write the arguments of the firm's labor supply curve as the employment at competing firms in the same market which we denote by the vector n_{-ijt}, aggregate employment $\boldsymbol{N}_t$ and the aggregate wage $\boldsymbol{W}_t$
- **Definition** - The *Nash equilibrium labor demand* of each firm $\left\{n^*_{ijt}\right\}_{i\in j}$ must satisfy the following set of conditions:

$$n^*_{ijt} = \arg\max_{n_{ijt}} \widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}} - w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right) n_{ijt} \quad \forall i \in j$$

where the inverse labor supply curve is given by the household optimality condition:

$$w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right) = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}} \boldsymbol{W}_t \quad , \quad \boldsymbol{n}_{jt} = \left[n_{ijt}^{\frac{\eta+1}{\eta}} + \sum_{k\neq i} n_{kjt}^{*\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$$

- The first order condition for each firm is as follows, where we write the marginal revenue product of labor $mrpl_{ijt} = \widetilde{\alpha}\widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}-1}$:

$$mrpl_{ijt} = \frac{\partial w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)}{\partial n_{ijt}} n_{ijt} + w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)$$

$$mrpl_{ijt} = w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right) \left[\frac{\partial w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)}{\partial n_{ijt}} \frac{n_{ijt}}{w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)} + 1\right]$$

- The elasticity is

$$\frac{\partial \log w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)}{\partial \log n_{ijt}} = \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \frac{\partial \log \boldsymbol{n}_{jt}\left(n_{ijt}, n^*_{-ijt}\right)}{\partial \log n_{ijt}}$$

$$\frac{\partial \log w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)}{\partial \log n_{ijt}} = \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{\eta+1}{\eta}}$$

- We can write this in terms of the payroll share of the firm. Using our expression for the labor supply curve to the firm

$$s_{ijt} = \frac{w_{ijt} n_{ijt}}{\sum_{i\in j} w_{ijt} n_{ijt}} = \frac{\left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}} n_{ijt}}{\sum_{i\in j} \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}} \left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}} n_{ijt}} = \frac{n_{ijt}^{\frac{\eta+1}{\eta}}}{\sum_{i\in j} n_{ijt}^{\frac{\eta+1}{\eta}}} = \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{\eta+1}{\eta}}.$$

- This gives

$$\frac{\partial \log w\left(n_{ijt}, n^*_{-ijt}, \boldsymbol{W}_t, \boldsymbol{N}_t\right)}{\partial \log n_{ijt}} = s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta}$$

- Define the *equilibrium inverse labor supply elasticity* ε^*_{ijt} as

$$\varepsilon^*_{ijt} = \left[s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta}\right]^{-1}$$

- Then we can write the wage as

$$\begin{aligned} w^*_{ijt} &= \mu^*_{ijt} mrpl_{ijt} \\ \mu^*_{ijt} &= \frac{1}{s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta} + 1} \\ \mu^*_{ijt} &= \frac{\left[s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta} s_{ijt}\right]^{-1}}{\left[s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta}\right]^{-1} + 1} \\ \mu^*_{ijt} &= \frac{\varepsilon^*_{ijt}}{\varepsilon^*_{ijt} + 1} \end{aligned}$$

- This delivers the set of partial equilibrium conditions specified in the text in Section 1.3

$$\begin{aligned} w^*_{ijt} &= \mu^*_{ijt} mrpl_{ijt} \\ \mu^*_{ijt} &= \frac{\varepsilon_{ijt}}{\varepsilon_{ijt} + 1} \\ \varepsilon^*_{ijt} &= \left[s_{ijt}\frac{1}{\theta} + \left(1 - s_{ijt}\right)\frac{1}{\eta}\right]^{-1} \end{aligned}$$

E.3 Proof of Proposition 1.1

- Collect terms in the labor supply curve that are common to all firms in the market, $\boldsymbol{x}_j := \boldsymbol{w}_j^{\theta-\eta}\boldsymbol{W}^{-\theta}\boldsymbol{N}$.

- We have the following conditions

$$
\begin{aligned}
mrpl_{ij} &= \widetilde{\alpha}\widetilde{z}_{ij}n_{ij}^{\widetilde{\alpha}-1} \\
n_{ij} &= w_{ij}^{\eta} \times \boldsymbol{x}_j \\
w_{ij} &= \mu\left(s_{ij}\right) mrpl_{ij}
\end{aligned}
$$

- Substituting the labor supply curve into the $mrpl_{ij}$ definition, and then the pricing condition into the labor supply curve for w_{ij} we have

$$
\begin{aligned}
mrpl_{ij} &= \left[\widetilde{\alpha}\widetilde{z}_{ij}\mu\left(s_{ij}\right)^{-\eta(1-\widetilde{\alpha})}\boldsymbol{x}_j^{\widetilde{\alpha}-1}\right] \times mrpl_{ij}^{-\eta(1-\widetilde{\alpha})} \\
mrpl_{ij} &= \left[\widetilde{\alpha}\widetilde{z}_{ij}\mu\left(s_{ij}\right)^{-\eta(1-\widetilde{\alpha})}\boldsymbol{x}_j^{\widetilde{\alpha}-1}\right]^{\frac{1}{1+\eta(1-\widetilde{\alpha})}}
\end{aligned}
$$

- Substituting this back into the optimality condition:

$$
w_{ij} = \left[\mu\left(s_{ij}\right)\widetilde{\alpha}\widetilde{z}_{ij}\right]^{\frac{1}{1+\eta(1-\widetilde{\alpha})}} \times \boldsymbol{x}_j^{-\frac{1-\widetilde{\alpha}}{1+\eta(1-\widetilde{\alpha})}} = \left[\mu\left(s_{ij}\right)\widetilde{z}_{ij}\right]^{\frac{1}{1+\eta(1-\widetilde{\alpha})}} \times g\left(\boldsymbol{x}_j\right)
$$

- The definition of the payroll share, combined with the labor supply curve gives

$$
s_{ij} = \frac{w_{ij}n_{ij}}{\sum_{k\in j} w_{kj}n_{kj}} = \frac{w_{ij}\left(\frac{w_{ij}}{\boldsymbol{w}_j}\right)^{\eta}\left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right)^{\theta}\boldsymbol{N}}{\sum_{k\in j} w_{kj}\left(\frac{w_{kj}}{\boldsymbol{w}_j}\right)^{\eta}\left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right)^{\theta}\boldsymbol{N}} = \frac{w_{ij}^{\eta+1}}{\sum_{k\in j} w_{kj}^{\eta+1}} = \frac{w_{ij}^{\eta+1}}{\sum_{k\in j} w_{kj}^{\eta+1}}.
$$

- Under the above expression for w_{ij}:

$$
s_{ij} = \frac{\left[\mu\left(s_{ij}\right)\widetilde{z}_{ij}\right]^{\frac{\eta+1}{1+\eta(1-\widetilde{\alpha})}}}{\sum_{k\in j}\left[\mu\left(s_{kj}\right)\widetilde{z}_{kj}\right]^{\frac{\eta+1}{1+\eta(1-\widetilde{\alpha})}}}.
$$

- Now recall that $\widetilde{z}_{ij}$ is the firm productivity under the firm's optimal capital decision, and $\widetilde{\alpha}$ is the corresponding exponent:

$$
\begin{aligned}
y_i &= z_i\left(k^*\left(n_i, z_i, R\right)^{1-\gamma} n_i^{\gamma}\right)^{\alpha} = \widetilde{z}_i n_i^{\widetilde{\alpha}} \\
\widetilde{z}_i &= \left[1-\left(1-\gamma\right)\alpha\right] z_i^{\frac{1}{1-(1-\gamma)\alpha}}\left(\frac{\left(1-\gamma\right)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} \\
\widetilde{\alpha} &= \frac{\gamma\alpha}{1-\left(1-\gamma\right)\alpha}
\end{aligned}
$$

- Substituting these in

$$s_{ij} = \frac{\left[\mu\left(s_{ij}\right)^{1-(1-\gamma)\alpha} z_{ij}\right]^{\frac{\eta+1}{1-(1-\gamma)\alpha+\eta(1-\alpha)}}}{\sum_{k\in j}\left[\mu\left(s_{kj}\right)^{1-(1-\gamma)\alpha} z_{kj}\right]^{\frac{\eta+1}{1-(1-\gamma)\alpha+\eta(1-\alpha)}}}$$

- This is the expression in Proposition 1.1, which holds for all firms i in market j, and is independent of aggregates.

- Note that in the limit as $\gamma \to 1$, then we can check that we obtain the no-capital expression from above

$$s_{ij} = \frac{\left[\mu\left(s_{ij}\right) z_{ij}\right]^{\frac{\eta+1}{1+\eta(1-\alpha)}}}{\sum_{k\in j}\left[\mu\left(s_{kj}\right) z_{kj}\right]^{\frac{\eta+1}{1+\eta(1-\alpha)}}}$$

- Additionally in the limit with $\alpha \to 1$, we have

$$s_{ij} = \frac{\left[\mu\left(s_{ij}\right) z_{ij}\right]^{\eta+1}}{\sum_{k\in j}\left[\mu\left(s_{kj}\right) z_{kj}\right]^{\eta+1}}$$

- In the limit $\alpha \to 1$ with $\gamma < 1$:

$$s_{ij} = \frac{\left[\mu\left(s_{ij}\right)^{\gamma} z_{ij}\right]^{\frac{\eta+1}{\eta+\gamma}}}{\sum_{k\in j}\left[\mu\left(s_{kj}\right)^{\gamma} z_{kj}\right]^{\frac{\eta+1}{\eta+\gamma}}}$$

E.4 Proof of Proposition 1.2

- We proceed in three steps.

- First, consider an economy with a single nest, with a single elasticity of substitution η, and consider the case of labor as the only input into production with decreasing returns $\alpha \in (0, 1]$.

- Our starting point is the following set of equations, where we can take the markdown as exogenous. These describe firm level (i) output, (ii) labor supply, (iii) labor demand optimality, (iv) marginal revenue product:

$$\begin{aligned}
y_i &= z_i n_i^{\alpha} \\
n_i &= \left(\frac{w_i}{\boldsymbol{w}}\right)^{\eta} \boldsymbol{n} \\
w_i &= \mu_i mrpl_i \\
mrpl_i &= \alpha z_i n_i^{\alpha-1}
\end{aligned}$$

- We then have two aggregation conditions: (i) output, (ii) wage index

$$
\begin{aligned}
\boldsymbol{y} &= \int y_i di \\
\boldsymbol{w} &= \left[\int w_i^{1+\eta} di\right]^{\frac{1}{1+\eta}}
\end{aligned}
$$

- This set of 6 equations are our inputs to the following claim.

- **Claim** - *The aggregates* $\{\boldsymbol{y}, \boldsymbol{w}, \boldsymbol{n}\}$ *can be written:*

$$
\begin{aligned}
\boldsymbol{y} &= \omega z \boldsymbol{n}^{\alpha} \\
\boldsymbol{w} &= \mu \alpha z \boldsymbol{n}^{\alpha-1}
\end{aligned}
$$

  where

$$
\begin{aligned}
z &= \left[\int z_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \\
\mu &= \left[\int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \\
\omega &= \int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu}\right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}} di
\end{aligned}
$$

- **Proof**

- With decreasing returns to scale, we first solve out for the marginal revenue product of labor. Note that here we only multiply and divide by z:

$$
\begin{aligned}
mrpl_i &= \alpha z_i n_i^{\alpha-1} \\
mrpl_i &= \alpha z_i \left(\left(\frac{w_i}{\boldsymbol{w}}\right)^{\eta} \boldsymbol{n}\right)^{\alpha-1} \\
mrpl_i &= \left(\frac{z_i}{z}\right) w_i^{\eta(\alpha-1)} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle \boldsymbol{w}^{\eta(1-\alpha)} \\
mrpl_i &= \left(\frac{z_i}{z}\right) \mu_i^{\eta(\alpha-1)} mrpl_i^{\eta(\alpha-1)} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle \boldsymbol{w}^{\eta(1-\alpha)} \\
mrpl_i &= \left(\frac{z_i}{z}\right)^{\frac{1}{1+\eta(1-\alpha)}} \mu_i^{-\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle^{\frac{1}{1+\eta(1-\alpha)}} \boldsymbol{w}^{\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}}
\end{aligned}
$$

- We can check that in the case of $\alpha = 1$, then $mrpl_i = z_i$.

- Using this in the wage

$$
\begin{aligned}
w_i &= \mu_i mrpl_i \\
w_i &= \left(\frac{z_i}{z}\right)^{\frac{1}{1+\eta(1-\alpha)}} \mu_i^{\frac{1}{1+\eta(1-\alpha)}} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle^{\frac{1}{1+\eta(1-\alpha)}} \boldsymbol{w}^{\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}}
\end{aligned}
$$

- Now aggregating:

$$
\begin{aligned}
\boldsymbol{w} &= \left[\int w_i^{1+\eta} di\right]^{\frac{1}{1+\eta}} \\
\boldsymbol{w} &= \left[\int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di\right]^{\frac{1}{1+\eta}} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle^{\frac{1}{1+\eta(1-\alpha)}} \boldsymbol{w}^{\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}} \\
\boldsymbol{w}^{\frac{1}{1+\eta(1-\alpha)}} &= \left[\int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di\right]^{\frac{1}{1+\eta}} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle^{\frac{1}{1+\eta(1-\alpha)}} \\
\boldsymbol{w} &= \left[\int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \times \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle \\
\boldsymbol{w} &= \mu \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle
\end{aligned}
$$

- This delivers the first result. Note that if $\alpha = 1$, then

$$
\mu = \left[\int \left(\frac{z_i}{z}\right)^{1+\eta} \mu_i^{1+\eta} di\right]^{\frac{1}{1+\eta}}
$$

- Now turning to firm output, under the labor supply curve and labor demand:

$$
\begin{aligned}
y_i &= z_i n_i^{\alpha} \\
y_i &= z_i \left(\left(\frac{w_i}{\boldsymbol{w}}\right)^{\eta} \boldsymbol{n}\right)^{\alpha} \\
y_i &= z_i \left(\left(\frac{\mu_i mrpl_i}{\boldsymbol{w}}\right)^{\eta} \boldsymbol{n}\right)^{\alpha} \\
y_i &= z_i \mu_i^{\alpha\eta} mrpl_i^{\alpha\eta} \left(\frac{1}{\boldsymbol{w}}\right)^{\alpha\eta} \boldsymbol{n}^{\alpha}
\end{aligned}
$$

- Using the previous expression for $mrpl_i$:

$$y_i = z_i \mu_i^{\alpha\eta} \left\{ \left(\frac{z_i}{z}\right)^{\frac{1}{1+\eta(1-\alpha)}} \mu_i^{-\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}} \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle^{\frac{1}{1+\eta(1-\alpha)}} \boldsymbol{w}^{\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}} \right\}^{\alpha\eta} \left(\frac{1}{\boldsymbol{w}}\right)^{\alpha\eta} \boldsymbol{n}^{\alpha}$$

$$y_i = z \left[\left(\frac{z_i}{z}\right)^{1+\frac{\alpha\eta}{1+\eta(1-\alpha)}} \mu_i^{\alpha\eta\left(1-\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}\right)} \left\{ \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle \boldsymbol{w}^{\eta(1-\alpha)} \right\}^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{1}{\boldsymbol{w}}\right)^{\alpha\eta} \right] \boldsymbol{n}^{\alpha}$$

$$y_i = z \left[\left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left\{ \left\langle \alpha z \boldsymbol{n}^{\alpha-1} \right\rangle \boldsymbol{w}^{\eta(1-\alpha)} \right\}^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{1}{\boldsymbol{w}}\right)^{\alpha\eta} \right] \boldsymbol{n}^{\alpha}$$

Given that we have shown that $\boldsymbol{w} = \mu \alpha z \boldsymbol{n}^{\alpha-1}$, we can use this to simplify $\{\cdot\}$:

$$y_i = z \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left\{ \left(\frac{\boldsymbol{w}}{\mu}\right) \boldsymbol{w}^{\eta(1-\alpha)} \right\}^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{1}{\boldsymbol{w}}\right)^{\alpha\eta} \boldsymbol{n}^{\alpha}$$

$$y_i = z \left[\left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu}\right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \right] \boldsymbol{n}^{\alpha}$$

- Then aggregating:

$$\boldsymbol{y} = \int y_i di$$

$$\boldsymbol{y} = \left[\int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu}\right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} di \right] \times \langle z \boldsymbol{n}^{\alpha} \rangle$$

$$\boldsymbol{y} = \omega \times z \boldsymbol{n}^{\alpha}$$

- This delivers the second result. Note that if $\alpha = 1$, then

$$\omega = \int \left(\frac{z_i}{z}\right)^{1+\eta} \left(\frac{\mu_i}{\mu}\right)^{\eta} di$$

- We still need to show that the productivity term z is correct. Notice that up to this point these derivations would hold under any $\boldsymbol{z}$. We pin down $\boldsymbol{z}$, by requiring that if there are no distortions ($\mu_i = 1$ for all firms), then the aggregate markdown is also $\mu = 1$. This requires:

$$1 = \left[\int \left(\frac{z_i}{z}\right)^{1+\eta} di \right]^{\frac{1}{1+\eta}}$$

$$z = \left[\int z_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}$$

which also implies that in an undistorted economy, since $\boldsymbol{\omega} = 1$, then output is simply $\boldsymbol{y} = \boldsymbol{z}\boldsymbol{n}^{\alpha}$.

- This also implies that in the expression for μ and the expression for ω, the productivity terms are well-defined weights:

$$\mu = \left[\int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \quad , \quad \int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} di = 1$$

$$\omega = \int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu}\right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} di \quad , \quad \int \left(\frac{z_i}{z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} di = 1$$

- Using the above results we can now turn to the nested economy.
- **Inner nest**
  - We take the same approach as above, starting with the isomorphic 6 conditions at the firm level and now those expressing *market level* aggregates:
  - **Firm conditions**: We have 4 conditions representing firm output, labor supply, labor demand, and the marginal revenue product of labor:

$$\begin{aligned} y_{ij} &= z_{ij} n_{ij}^{\alpha} \\ n_{ij} &= \left(\frac{w_{ij}}{\boldsymbol{w}_j}\right)^{\eta} \boldsymbol{n}_j \\ w_{ij} &= \mu_{ij} mrpl_{ij} \\ mrpl_{ij} &= \alpha z_{ij} n_{ij}^{\alpha-1} \end{aligned}$$

  - **Market aggregates**: We then have two aggregation conditions: (i) market output, (ii) market wage index

$$\begin{aligned} \boldsymbol{y}_j &= \sum_{i \in j} y_{ij} \\ \boldsymbol{w}_j &= \left[\sum_{i \in j} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \end{aligned}$$

  - Following the same steps as above, it is clear that we can show that:
  - **Claim** - *The market aggregates* $\left\{\boldsymbol{y}_j, \boldsymbol{w}_j, \boldsymbol{n}_j\right\}$ *can be written:*

$$\begin{aligned} \boldsymbol{y}_j &= \omega_j \boldsymbol{z}_j \boldsymbol{n}_j^{\alpha} \\ \boldsymbol{w}_j &= \mu_j \alpha z_j \boldsymbol{n}_j^{\alpha-1} \end{aligned}$$

where

$$\boldsymbol{z}_j = \left[\sum_{i \in j} z_{ij}^{\frac{1+\eta}{1+\eta(1-\alpha)}}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}$$

$$\boldsymbol{\mu}_j = \left[\sum_{i \in j} \left(\frac{z_{ij}}{z_j}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_{ij}^{\frac{1+\eta}{1+\eta(1-\alpha)}}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}$$

$$\boldsymbol{\omega}_j = \sum_{i \in j} \left(\frac{z_i}{\boldsymbol{z}_j}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\boldsymbol{\mu}}\right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}}$$

- **Outer nest**
  - The solution to the inner nests allows us to establish a similar set of 6 conditions
  - **Market level**: We have 3 conditions representing market output, aggregate labor supply, market labor demand:

$$\boldsymbol{y}_j = \omega_j \boldsymbol{z}_j \boldsymbol{n}_j^{\alpha}$$

$$\boldsymbol{n}_j = \left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right)^{\theta} \boldsymbol{N}$$

$$\boldsymbol{w}_j = \mu_j \alpha z_j \boldsymbol{n}_j^{\alpha-1}$$

  - **Economy aggregates**: We have two aggregation conditions: (i) aggregate output, (ii) aggregate wage index

$$\boldsymbol{Y} = \int \boldsymbol{y}_j dj$$

$$\boldsymbol{W} = \left[\int \boldsymbol{w}_j^{1+\theta} dj\right]^{\frac{1}{1+\theta}}$$

- Following the above steps again, we can obtain:

$$\boldsymbol{Y} = \boldsymbol{\Omega} \boldsymbol{Z} \boldsymbol{N}^{\alpha}$$

$$\boldsymbol{W} = \mu \alpha \boldsymbol{Z} \boldsymbol{N}^{\alpha-1}$$

where

$$\boldsymbol{Z} = \left[\int \boldsymbol{z}_j^{\frac{1+\theta}{1+\theta(1-\alpha)}}\right]^{\frac{1+\theta(1-\alpha)}{1+\theta}}$$

$$\boldsymbol{\mu} = \left[\int \left(\frac{\boldsymbol{z}_j}{\boldsymbol{Z}}\right)^{\frac{1+\theta}{1+\theta(1-\alpha)}} \boldsymbol{\mu}_j^{\frac{1+\theta}{1+\theta(1-\alpha)}} dj\right]^{\frac{1+\theta(1-\alpha)}{1+\theta}}$$

$$\boldsymbol{\Omega} = \int \left(\frac{\boldsymbol{z}_j}{\boldsymbol{Z}}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\boldsymbol{\mu}_j}{\boldsymbol{\mu}}\right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}} \boldsymbol{\omega}_j dj$$

- This delivers the main expressions in Proposition 1.2, in the case of an economy with decreasing returns to labor, but without capital.

E.4.1 Adding capital

- The value-added production function of the firm in our model is

$$y_i = z_i \left(k_i^{1-\gamma} n_i^{\gamma}\right)^{\alpha}$$

- The optimal choice of capital solves

$$k_i^* \left(z_i, n_i, R\right) = \arg\max_{k_i} \; z_i \left(k_i^{1-\gamma} n_i^{\gamma}\right)^{\alpha} - Rk_i$$

$$k_i^* \left(z_i, n_i, R\right) = \left(\frac{\left(1-\gamma\right)\alpha z_i}{R}\right)^{\frac{1}{1-(1-\gamma)\alpha}} n_i^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}}$$

- We can substitute this back into output to obtain:

$$y_i = z_i^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{\left(1-\gamma\right)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} n_i^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}}$$

- Note that in terms of factor payment shares, capital is competitively priced:

$$Rk_i = \alpha\left(1-\gamma\right) y_i$$

- Combining these, profits are

$$\pi_i = y_i - Rk_i - w_i n_i$$

$$\pi_i = \left[1-\alpha\left(1-\gamma\right)\right] y_i - w_i n_i$$

$$\pi_i = \left[1-\alpha\left(1-\gamma\right)\right] z_i^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{\left(1-\gamma\right)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} n_i^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}} - w_i n_i$$

- We can write this as

$$\begin{aligned}
\pi_i &= \widetilde{y}_i - w_i n_i \\
\widetilde{y}_i &= \widetilde{z}_i n_i^{\widetilde{\alpha}} \\
\widetilde{z}_i &= \left[1-\alpha\left(1-\gamma\right)\right] z_i^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{\left(1-\gamma\right)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} \\
\widetilde{\alpha} &= \frac{\gamma\alpha}{1-\left(1-\gamma\right)\alpha}
\end{aligned}$$

- Note that this implies that

$$\begin{aligned}
\widetilde{y}_i &= y_i - Rk_i = \left[1-\gamma\left(1-\alpha\right)\right] y_i \\
y_i &= \left[\frac{1}{1-\gamma\left(1-\alpha\right)}\right] \widetilde{y}_i
\end{aligned}$$

- It should therefore be clear from what we have obtained so far in our aggregation results that:

1. **Market level** - At the market level, define $\widetilde{\boldsymbol{y}}_j = \sum_{i\in j} \widetilde{y}_{ij}$, then

$$\begin{aligned}
\widetilde{\boldsymbol{y}}_j &= \boldsymbol{\omega}_j \widetilde{z}_j \boldsymbol{n}_j^{\widetilde{\alpha}} \\
\boldsymbol{w}_j &= \boldsymbol{\mu}_j \widetilde{\boldsymbol{z}}_j \widetilde{\alpha} \boldsymbol{n}_j^{\widetilde{\alpha}-1}
\end{aligned}$$

where $\left\{\boldsymbol{\omega}_j, \boldsymbol{\mu}_j\right\}$ are as before, except with $\widetilde{\alpha}$ in place of α, and we define $\widetilde{\boldsymbol{z}}_j$ as:

$$\begin{aligned}
\widetilde{\boldsymbol{z}}_j &= \left[\sum_{i\in j} \widetilde{z}_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}} \\
\boldsymbol{\mu}_j &= \left[\sum_{i\in j} \left(\frac{\widetilde{z}_{ij}}{\widetilde{\boldsymbol{z}}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \mu_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}} \\
\boldsymbol{\omega}_j &= \sum_{i\in j} \left(\frac{\widetilde{z}_{ij}}{\widetilde{\boldsymbol{z}}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \left(\frac{\mu_{ij}}{\boldsymbol{\mu}_j}\right)^{\frac{\eta\widetilde{\alpha}}{1+\eta(1-\widetilde{\alpha})}}
\end{aligned}$$

2. **Aggregate level** - At the aggregate level, define $\widetilde{\boldsymbol{Y}} = \int \widetilde{y}_j dj$, then

$$\begin{aligned}
\widetilde{\boldsymbol{Y}} &= \boldsymbol{\Omega} \widetilde{\boldsymbol{Z}} \boldsymbol{N}^{\widetilde{\alpha}} \\
\boldsymbol{W} &= \mu \widetilde{\alpha} \widetilde{\boldsymbol{Z}} \boldsymbol{N}^{\widetilde{\alpha}-1}
\end{aligned}$$

where $\{\mathbf{\Omega}, \boldsymbol{\mu}\}$ are as before, except with $\widetilde{\alpha}$ in place of α, and we define $\widetilde{\mathbf{Z}}$ as:

$$\widetilde{\mathbf{Z}} = \left[\int \widetilde{z}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}}\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$
$$\boldsymbol{\mu} = \left[\int \left(\frac{\widetilde{z}_j}{\widetilde{\mathbf{Z}}}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \boldsymbol{\mu}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$
$$\mathbf{\Omega} = \int \left(\frac{\widetilde{z}_j}{\widetilde{\mathbf{Z}}}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \left(\frac{\boldsymbol{\mu}_j}{\boldsymbol{\mu}}\right)^{\frac{\eta\widetilde{\alpha}}{1+\eta(1-\widetilde{\alpha})}} \boldsymbol{\omega}_j dj$$

- Now observe that when we aggregate capital

$$K = \int \sum_{i \in j} k_i dj$$
$$K = \int \sum_{i \in j} \left(\frac{\alpha\,(1-\gamma)\, y_i}{R}\right) dj$$
$$RK = \alpha\,(1-\gamma) \int \sum_{i \in j} y_i dj$$
$$RK = \alpha\,(1-\gamma)\, \mathbf{Y} \quad (*)$$

- Now note that

$$\mathbf{Y} = \int \sum_{i \in j} y_{ij} dj = \int \sum_{i \in j} \left[\frac{1}{1-\gamma\,(1-\alpha)} \widetilde{y}_{ij}\right] dj = \frac{1}{1-\gamma\,(1-\alpha)} \int \sum_{i \in j} \widetilde{y}_{ij} dj = \frac{1}{1-\gamma\,(1-\alpha)} \widetilde{\mathbf{Y}}$$

- Substituting the aggregate output expression $\widetilde{\mathbf{Y}} = \mathbf{\Omega}\widetilde{\mathbf{Z}}\mathbf{N}^{\widetilde{\alpha}}$, into the aggregate labor demand condition:

$$\mathbf{W} = \mu\widetilde{\alpha}\widetilde{\mathbf{Z}}\mathbf{N}^{\widetilde{\alpha}-1}$$
$$\mathbf{W} = \mu\widetilde{\alpha}\left(\frac{\widetilde{\mathbf{Z}}\mathbf{N}^{\widetilde{\alpha}}}{\mathbf{N}}\right)$$
$$\mathbf{W} = \left(\frac{\mu}{\mathbf{\Omega}}\right)\widetilde{\alpha}\left(\frac{\widetilde{\mathbf{Y}}}{\mathbf{N}}\right)$$
$$\mathbf{W} = \left(\frac{\mu}{\mathbf{\Omega}}\right)\left(\frac{\gamma\alpha}{1-(1-\gamma)\,\alpha}\right)\left(\frac{[1-\gamma\,(1-\alpha)]\,\mathbf{Y}}{\mathbf{N}}\right)$$
$$\mathbf{W} = \gamma\alpha\left(\frac{\mu}{\mathbf{\Omega}}\right)\frac{\mathbf{Y}}{\mathbf{N}} \quad (**)$$

- The equations $(*)$ and $(**)$ describe aggregate factor demand for capital and labor and appear in Proposition 1.2.

- The steady-state resource constraint is $\boldsymbol{C} = \boldsymbol{Y} - \delta K$, which requires no proof, and the steady-state Euler equation is $1 = \beta\left[R + (1-\delta)\right]$, these can be combined with optimal capital demand to yield:

$$
\begin{aligned}
\boldsymbol{C} &= \boldsymbol{Y} - \delta K \\
\boldsymbol{C} &= \boldsymbol{Y} - \frac{\delta}{R} RK \\
\boldsymbol{C} &= \left[1 - \frac{\delta}{R}\left(1-\gamma\right)\alpha\right]\boldsymbol{Y} \\
\boldsymbol{C} &= \left[1 - \frac{\delta}{R}\left(1-\gamma\right)\alpha\right]\frac{\widetilde{\boldsymbol{Y}}}{1-\gamma\left(1-\alpha\right)} \\
\boldsymbol{C} &= \left[1 - \frac{\beta\delta}{1-\beta\left(1-\delta\right)}\left(1-\gamma\right)\alpha\right]\frac{\widetilde{\boldsymbol{Y}}}{1-\gamma\left(1-\alpha\right)}
\end{aligned}
$$

  where we denote the constant s_C, which is the consumption share of output. These expressions appear in the main text.

- This implies that given $\boldsymbol{\mu}$, and $\boldsymbol{\Omega}$, we can solve for equilibrium $\left\{\widetilde{\boldsymbol{Y}}, \boldsymbol{W}, \boldsymbol{N}, \boldsymbol{C}\right\}$ from

$$
\begin{aligned}
\boldsymbol{W} &= \boldsymbol{\mu}\widetilde{\alpha}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}-1} \\
\boldsymbol{W} &= \frac{U_N\left(\boldsymbol{C},\boldsymbol{N}\right)}{U_C\left(\boldsymbol{C},\boldsymbol{N}\right)} \\
\boldsymbol{C} &= s_C\frac{\widetilde{\boldsymbol{Y}}}{1-\gamma\left(1-\alpha\right)} \\
\widetilde{\boldsymbol{Y}} &= \boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}}
\end{aligned}
$$

- In the case of $U_N/U_C = \boldsymbol{N}^{\varphi}\boldsymbol{C}^{-\sigma}$, as is the case under GHH ($\sigma = 0$) or CRRA preferences ($\sigma \geq 1$), then all aggregates can be solved in closed form using the following equations from top to bottom:

$$
\begin{aligned}
\boldsymbol{N} &= \left[\left(\frac{s_C}{1-\gamma\left(1-\alpha\right)}\boldsymbol{\Omega}\right)^{-\sigma\varphi}\left(\widetilde{\alpha}\boldsymbol{\mu}\right)^{\varphi}\widetilde{\boldsymbol{Z}}^{(1-\sigma)\varphi}\right]^{\frac{1}{1+\varphi(1-\widetilde{\alpha})+\sigma\varphi\widetilde{\alpha}}} \\
\boldsymbol{W} &= \boldsymbol{\mu}\widetilde{\alpha}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}-1} \\
\widetilde{\boldsymbol{Y}} &= \boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}} \\
\boldsymbol{C} &= \frac{s_c}{1-\gamma\left(1-\alpha\right)}\widetilde{\boldsymbol{Y}} \\
\boldsymbol{Y} &= \frac{1}{1-\left(1-\gamma\right)\alpha}\widetilde{\boldsymbol{Y}} \\
R &= \frac{1}{\beta} - \left(1-\delta\right) \\
K &= \frac{\left(1-\gamma\right)\alpha}{R}\boldsymbol{Y}
\end{aligned}
$$

- Note that in the case of no wealth-effects on labor supply $\sigma = 0$, we have the following result, cited in the text, that the equilibrium aggregate employment and wage are independent of $\boldsymbol{\Omega}$. In this case, the wage and employment are pinned down by

$$\boldsymbol{N} = \left[\tilde{\alpha}\boldsymbol{\mu}\widetilde{\boldsymbol{Z}}\right]^{\frac{\varphi}{1+\varphi(1-\tilde{\alpha})}} \quad , \quad \boldsymbol{W} = \boldsymbol{\mu}\widetilde{\alpha}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}-1}$$

However output, and hence consumption, still clearly depend on $\boldsymbol{\Omega}$.

E.4.2 Production function

- In the main text of the paper we provide the above conditions, but instead with the output as follows, which we now derive.

$$\boldsymbol{Y} = \boldsymbol{\Omega}^{1-(1-\gamma)\alpha}\boldsymbol{Z}\left(K^{1-\gamma}\boldsymbol{N}^{\gamma}\right)^{\alpha}$$

where we also need to specify $\boldsymbol{Z}$.

- First we need to go remove the optimal capital decisions encoded in $\widetilde{z}_{ij}$ and consequently $\widetilde{z}_j$ and $\widetilde{\boldsymbol{Z}}$.
- Recall that

$$\widetilde{z}_{ij} = \left(1-\left(1-\gamma\right)\alpha\right) z_{ij}^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{\left(1-\gamma\right)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} , \widetilde{z}_j = \left[\sum_{i\in j}\widetilde{z}_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}} , \widetilde{\boldsymbol{Z}} = \left[\int \widetilde{z}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

- Substituting $\widetilde{z}_{ij}$ into $\widetilde{z}_j$, we can define market productivity as follows, which gives the following implication:

$$z_j := \left[\sum_{i\in j} z_{ij}^{\frac{1+\eta}{1-(1-\gamma)\alpha+\eta(1-\alpha)}}\right]^{\frac{1-(1-\gamma)\alpha+\eta(1-\alpha)}{1+\eta}} \implies \widetilde{z}_j = \left(1-\left(1-\gamma\right)\alpha\right) z_j^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{\left(1-\gamma\right)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}}$$

- We can do the same for market and aggregate productivity:

$$\boldsymbol{Z} := \left[\int z_j^{\frac{1+\theta}{1-(1-\gamma)\alpha+\theta(1-\alpha)}} dj\right]^{\frac{1-(1-\gamma)\alpha+\theta(1-\alpha)}{1+\theta}} \implies \widetilde{\boldsymbol{Z}} = \left(1-\left(1-\gamma\right)\alpha\right) \boldsymbol{Z}^{\frac{1}{1-(1-\gamma)\alpha}} \left(\frac{\left(1-\gamma\right)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}}$$

- Note that this implies that the relationship between $\boldsymbol{Z}$ and $\widetilde{\boldsymbol{Z}}$ given in the text

$$\boldsymbol{Z} = \left[\frac{R}{\left(1-\gamma\right)\alpha}\right]^{(1-\gamma)\alpha} \left[\frac{\widetilde{\boldsymbol{Z}}}{1-\left(1-\gamma\right)\alpha}\right]^{1-(1-\gamma)\alpha}$$

- Now return to the aggregate production function $\widetilde{\boldsymbol{Y}} = \boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}}$ and substitute in (i) the definitions of $\widetilde{\alpha}$, (ii) $\widetilde{\boldsymbol{Z}}$, and, (iii) $\widetilde{\boldsymbol{Y}} = [1-(1-\gamma)\alpha]\boldsymbol{Y}$, (iv) the aggregate capital demand condition $R = (1-\gamma)\alpha(\boldsymbol{Y}/K)$, this gives the expression in Proposition 1.2:

$$
\begin{aligned}
\widetilde{\boldsymbol{Y}} &= \boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}} \\
\widetilde{\boldsymbol{Y}} &= \boldsymbol{\Omega}\left(1-(1-\gamma)\alpha\right)\boldsymbol{Z}^{\frac{1}{1-(1-\gamma)\alpha}}\left(\frac{(1-\gamma)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}}\boldsymbol{N}^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}} \\
\boldsymbol{Y} &= \boldsymbol{\Omega}\boldsymbol{Z}^{\frac{1}{1-(1-\gamma)\alpha}}\left(\frac{K}{\boldsymbol{Y}}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}}\boldsymbol{N}^{\frac{\gamma\alpha}{1-(1-\gamma)\alpha}} \\
\boldsymbol{Y} &= \boldsymbol{\Omega}^{1-(1-\gamma)\alpha}\boldsymbol{Z}\left(K^{1-\gamma}\boldsymbol{N}^{\gamma}\right)^{\alpha}
\end{aligned}
$$

where we have the productivity terms which are described in the footnote of Proposition 1.2:

$$
\boldsymbol{z}_j = \left[\sum_{i\in j} z_{ij}^{\frac{1+\eta}{1-(1-\gamma)\alpha+\eta(1-\alpha)}}\right]^{\frac{1-(1-\gamma)\alpha+\eta(1-\alpha)}{1+\eta}} \quad , \quad \boldsymbol{Z} = \left[\int z_j^{\frac{1+\theta}{1-(1-\gamma)\alpha+\theta(1-\alpha)}} dj\right]^{\frac{1-(1-\gamma)\alpha+\theta(1-\alpha)}{1+\theta}}
$$

E.5 Proof of Proposition 1.3 - Labor share and concentration

- Rearranging the above conditions immediately gives the labor share

$$
LS := \frac{\int \sum_{i\in j} w_{ij}n_{ij}dj}{\int \sum_{i\in j} y_{ij}dj} = \frac{\boldsymbol{W}\boldsymbol{N}}{\boldsymbol{Y}} = \gamma\alpha\left(\frac{\boldsymbol{\mu}}{\boldsymbol{\Omega}}\right)
$$

- Since $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}$ are independent of the supply block of the model, then they are independent of aggregates and preferences, and so is the labor share. That is, we only have to solve market equilibria to compute the labor share.

- We now show that the expression linking the labor share and aggregate HHI^{wn} holds.

- We can use 'tilde' objects and then scale up at the end:

$$
\widetilde{ls}_{ij} = \frac{w_{ij}n_{ij}}{\widetilde{y}_{ij}} = \frac{w_{ij}n_{ij}}{\widetilde{z}_{ij}n_{ij}^{\widetilde{\alpha}}} = \widetilde{\alpha}\frac{w_{ij}}{\widetilde{\alpha}\widetilde{z}_{ij}n_{ij}^{\widetilde{\alpha}-1}} = \widetilde{\alpha}\frac{w_{ij}}{mrpl_{ij}} = \widetilde{\alpha}\mu_{ij}
$$

- Therefore the inverse of the labor share is:

$$
\widetilde{ls}_{ij}^{-1} = \frac{1}{\widetilde{\alpha}}\mu_{ij}^{-1}
$$

- Note that from our earlier expression for the markdown:

$$
\mu_{ij}^{-1} = s_{ijt}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)+\frac{1}{\eta}+1 = \frac{\eta+1}{\eta}+s_{ijt}\left(\frac{\theta+1}{\theta}-\frac{\eta+1}{\eta}\right)
$$

- Now the *market* inverse labor share is

$$\widetilde{ls}_j^{-1} = \frac{\sum_{i\in j}\widetilde{y}_{ij}}{\sum_{i\in j} w_{ij}n_{ij}} = \sum_{i\in j}\frac{w_{ij}n_{ij}}{\sum_{i\in j} w_{ij}n_{ij}}\frac{\widetilde{y}_{ij}}{w_{ij}n_{ij}} = \sum_{i\in j} s_{ij}\widetilde{ls}_{ij}^{-1} = \frac{1}{\widetilde{\alpha}}\sum_{i\in j} s_{ij}\mu_{ij}^{-1} = \frac{1}{\widetilde{\alpha}}\sum_{i\in j} s_{ij}\left[\frac{\eta+1}{\eta} + s_{ijt}\left(\frac{\theta+1}{\theta} - \frac{\eta+1}{\eta}\right)\right]$$

which gives:

$$\widetilde{ls}_j^{-1} = \frac{1}{\widetilde{\alpha}}\left[\left(1-hhi_j\right)\frac{\eta+1}{\eta} + hhi_j\frac{\theta+1}{\theta}\right]$$

where $hhi_j = \sum_{i\in j} s_{ij}^2$.

- Then the aggregate labor share is as follows, where we use our definition $HHI = \int s_j hhi_j dj$:

$$\begin{aligned}
\widetilde{LS} &= \left[\frac{\int\sum_{i\in j} y_{ij}dj}{\int\sum_{i\in j} w_{ij}n_{ij}dj}\right]^{-1} \\
&= \left[\int s_j\widetilde{ls}_j^{-1}dj\right]^{-1} \\
\widetilde{LS} &= \widetilde{\alpha}\left[HHI\left(\frac{\theta}{\theta+1}\right)^{-1} + \left(1-HHI\right)\left(\frac{\eta}{\eta+1}\right)^{-1}\right]^{-1}
\end{aligned}$$

- Now note that

$$\boldsymbol{Y} = \frac{1}{1-\left(1-\gamma\right)\alpha}\widetilde{\boldsymbol{Y}}$$

therefore

$$LS = \frac{\boldsymbol{WN}}{\boldsymbol{Y}} = \frac{\boldsymbol{WN}}{\frac{1}{1-(1-\gamma)\alpha}\widetilde{\boldsymbol{Y}}} = \left[1-\left(1-\gamma\right)\alpha\right]\widetilde{LS},$$

so under $\widetilde{\alpha} = \alpha\gamma/\left(1-\left(1-\gamma\right)\alpha\right)$ we have

$$LS = \alpha\gamma\left[HHI\left(\frac{\theta}{\theta+1}\right)^{-1} + \left(1-HHI\right)\left(\frac{\eta}{\eta+1}\right)^{-1}\right]^{-1}$$

- This is the expression in Proposition 1.3.

- This also established the following claims: (i) the moments LS and HHI are independent of aggregate preferences and shifters in labor supply and productivity which we study below $\widetilde{A}$ and $\overline{\varphi}$, and since the capital share is $KS = \alpha\left(1-\gamma\right)$, then this is also independent of preferences, therefore (ii) estimating the model using LS, KS, HHI as moments implies that we can estimate the model without specifying aggregate preferences.

E.6 Scaling the economy

- Here we prove our claim in the text that we can choose scaling parameters for productivity and labor supply that can always be chosen to match average firm size and average worker pay without affecting any of the other moments of the economy.

- Suppose we add two constants to the above economy $\widetilde{A}$ and $\overline{\varphi}$ such that the firm production function after optimizing out capital and the aggregate labor supply curve are:

$$\widetilde{y}_{ij} = \widetilde{A}\widetilde{z}_{ij} n_{ij}^{\widetilde{\alpha}}$$
$$\boldsymbol{W} = -\frac{U_N\left(\boldsymbol{C},\boldsymbol{N}\right)}{U_C\left(\boldsymbol{C},\boldsymbol{N}\right)} = \overline{\varphi}^{-1/\varphi}\boldsymbol{N}^{1/\varphi}\boldsymbol{C}^{\sigma}$$

- We claim that we can *always* choose these constants to match two moments of the data: average firm size, and average worker pay.

- Note that the above labor supply curve obtains under either CRRA or GHH preferences, where GHH preferences are the case of $\sigma = 0$:

$$U(\boldsymbol{C},\boldsymbol{N}) = \frac{\boldsymbol{C}^{1-\sigma}}{1-\sigma} - \overline{\varphi}^{-1/\varphi}\frac{\boldsymbol{N}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \quad , \quad U\left(\boldsymbol{C},\boldsymbol{N}\right) = u\left(\boldsymbol{C} - \overline{\varphi}^{-1/\varphi}\frac{\boldsymbol{N}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)$$

- Note that this implies that the changes to the set of equilibrium conditions are that aggregate output, labor demand and labor supply are:

$$\widetilde{\boldsymbol{Y}} = \boldsymbol{\Omega}\widetilde{A}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}}$$
$$\boldsymbol{W} = \boldsymbol{\mu}\widetilde{\alpha}\widetilde{A}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}-1}$$
$$\boldsymbol{N} = \overline{\varphi}\boldsymbol{W}^{\varphi}\boldsymbol{C}^{-\sigma\varphi}$$

- Combined these imply the same expression for the labor share:

$$\boldsymbol{W}\boldsymbol{N} = \left(\frac{\boldsymbol{\mu}}{\boldsymbol{\Omega}}\right)\widetilde{\alpha}\widetilde{\boldsymbol{Y}}$$

  which gives

$$\widetilde{\boldsymbol{Y}} = \frac{\boldsymbol{W}\boldsymbol{N}}{\left(\frac{\boldsymbol{\mu}}{\boldsymbol{\Omega}}\right)\widetilde{\alpha}}$$

- Consider the following two moments: (i) Average firm size $(AveFirmSize)$, (ii) Average worker pay $(AveWorkerPay)$

- The average firm size in the economy is

$$AveFirmSize := \frac{\int \sum_{i\in j} n_{ij} dj}{\int M_j dj} = \frac{\int \sum_{i\in j} \left(\frac{w_{ij}}{\boldsymbol{w}_j}\right)^{\eta} \left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right)^{\theta} \boldsymbol{N} dj}{\int M_j dj}$$

- Define the following:

$$\nu_{ij} := \left(\frac{w_{ij}}{\boldsymbol{w}_j}\right)^{\eta} \left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right)^{\theta}$$

- Using this

$$AveFirmSize = \frac{\int \sum_{i\in j} \nu_{ij} dj}{\int M_j dj} \boldsymbol{N}$$

- Denote $\boldsymbol{\nu} = \int \sum_{i\in j} \nu_{ij} dj$.
- The average worker pay in the economy is

$$\begin{aligned} AveWorkerPay &= \frac{\int \sum_{i\in j} w_{ij} n_{ij} dj}{\int \sum_{i\in j} n_{ij} dj} \\ &= \frac{\boldsymbol{WN}}{AveFirmSize \times \int M_j dj} \\ AveWorkerPay &= \frac{\boldsymbol{W}}{\boldsymbol{\nu}} \end{aligned}$$

- We can re-write these:

$$\begin{aligned} \boldsymbol{N} &= AveFirmSize \times \frac{\int M_j dj}{\boldsymbol{\nu}} \\ \boldsymbol{W} &= AveWorkerPay \times \boldsymbol{\nu} \end{aligned}$$

- **Claim** - The aggregate $\boldsymbol{\nu}$ is independent of all other aggregates including the shifters $\widetilde{A}$ and $\overline{\varphi}$
  - Using our above results, but now including the shifter terms:

$$\begin{aligned} \left(\frac{w_{ij}}{\boldsymbol{w}_j}\right) &= \frac{\mu_{ij} \widetilde{\alpha} \widetilde{z}_{ij} \widetilde{A} n_{ij}^{\widetilde{\alpha}-1}}{\boldsymbol{\mu}_j \widetilde{\alpha} \widetilde{z}_j \widetilde{A} \boldsymbol{n}_j^{\widetilde{\alpha}-1}} \\ \left(\frac{w_{ij}}{\boldsymbol{w}_j}\right) &= \left(\frac{\mu_{ij} \widetilde{z}_{ij}}{\boldsymbol{\mu}_j \widetilde{z}_j}\right)^{\frac{1}{1+\eta(1-\widetilde{\alpha})}} \end{aligned}$$

  and using a similar approach at the market level:

$$\left(\frac{\boldsymbol{w}_j}{\boldsymbol{W}}\right) = \left(\frac{\boldsymbol{\mu}_j \widetilde{\boldsymbol{z}}_j}{\boldsymbol{\mu} \widetilde{\boldsymbol{Z}}}\right)^{\frac{1}{1+\theta(1-\widetilde{\alpha})}}$$

– Therefore we have the following equation which implies that ν_{ij} is independent of aggregates including the shift parameters $\widetilde{A}$ and $\overline{\varphi}$

$$\nu_{ij} = \left(\frac{\mu_{ij}\widetilde{z}_{ij}}{\boldsymbol{\mu}_j\widetilde{z}_j}\right)^{\frac{1}{1+\eta(1-\widetilde{\alpha})}} \left(\frac{\boldsymbol{\mu}_j\widetilde{\boldsymbol{z}}_j}{\boldsymbol{\mu}\widetilde{\boldsymbol{Z}}}\right)^{\frac{1}{1+\theta(1-\widetilde{\alpha})}}$$

- We can then write the following system of equations, which we describe below

$$\begin{aligned}
\boldsymbol{N} &= AveFirmSize \times \frac{\int M_j dj}{\boldsymbol{\nu}} \\
\boldsymbol{W} &= AveWorkerPay \times \boldsymbol{\nu} \\
\widetilde{\boldsymbol{Y}} &= \frac{\boldsymbol{W}\boldsymbol{N}}{\left(\frac{\boldsymbol{\mu}}{\boldsymbol{\Omega}}\right)\widetilde{\alpha}} \\
\boldsymbol{Y} &= \frac{\widetilde{\boldsymbol{Y}}}{1-(1-\gamma)\,\alpha} \\
K &= \frac{(1-\gamma)\,\alpha\boldsymbol{Y}}{R} \\
\boldsymbol{C} &= \boldsymbol{Y} - \delta K
\end{aligned}$$

- Starting with a solution of the market equilibria, we can obtain $\{\boldsymbol{\nu}, \boldsymbol{\Omega}, \boldsymbol{\mu}\}$ which we have proven are independent of aggregates and shifters $\left\{\widetilde{A}, \overline{\varphi}\right\}$. Then given data on $AveFirmSize^{Data}$ and $AveWorkerPay^{Data}$, we can put data into the first two expressions above, and use these to compute $\boldsymbol{N}$ and $\boldsymbol{W}$. Given these we can use the remaining equations to compute all other aggregate quantities $\{\boldsymbol{Y}, \boldsymbol{C}, K\}$. What remains is to choose $\widetilde{A}$ and $\overline{\varphi}$ to be consistent with these aggregates. For this we use the two conditions that have not been used above by themselves: output and labor supply

$$\widetilde{\boldsymbol{Y}} = \boldsymbol{\Omega}\widetilde{A}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}} \quad , \quad \boldsymbol{N} = \overline{\varphi}\boldsymbol{W}^{\varphi}\boldsymbol{C}^{-\sigma\varphi},$$

which imply that

$$\widetilde{A} = \frac{\widetilde{\boldsymbol{Y}}}{\boldsymbol{\Omega}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}}} \quad , \quad \overline{\varphi} = \frac{\boldsymbol{N}}{\boldsymbol{W}^{\varphi}\boldsymbol{C}^{-\sigma\varphi}}.$$

- Proceeding in this way we can *always* choose shifters $\widetilde{A}$ and $\overline{\varphi}$ in order to match data on average firm size and average worker pay.

- Once these are pinned down, then the system of equilibrium conditions without the moment conditions can be solved. Going from top to bottom, the equilibrium is computed:

$$
\begin{aligned}
\boldsymbol{N} &= \left[\overline{\varphi}\left(\frac{s_c}{1-\gamma\left(1-\alpha\right)}\boldsymbol{\Omega}\right)^{-\sigma\varphi}\left(\tilde{\alpha}\boldsymbol{\mu}\right)^{\varphi}\left(\widetilde{A}\widetilde{\boldsymbol{Z}}\right)^{(1-\sigma)\varphi}\right]^{\frac{1}{1+\varphi(1-\tilde{\alpha})+\sigma\varphi\tilde{\alpha}}} \\
\boldsymbol{W} &= \boldsymbol{\mu}\widetilde{\alpha}\widetilde{A}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}-1} \\
\widetilde{\boldsymbol{Y}} &= \boldsymbol{\Omega}\widetilde{A}\widetilde{\boldsymbol{Z}}\boldsymbol{N}^{\widetilde{\alpha}} \\
\boldsymbol{C} &= \frac{s_c}{1-\gamma\left(1-\alpha\right)}\widetilde{\boldsymbol{Y}} \\
\boldsymbol{Y} &= \frac{1}{1-\left(1-\gamma\right)\alpha}\widetilde{\boldsymbol{Y}} \\
R &= \frac{1}{\beta}-\left(1-\delta\right) \\
K &= \frac{\left(1-\gamma\right)\alpha}{R}\boldsymbol{Y}
\end{aligned}
$$

where in the text we use the transformation $\overline{Z}^{\frac{1}{1-(1-\gamma)\alpha}} = \widetilde{A}$ so that $y_{ij} = \overline{Z}z_{ij}\left(k_{ij}^{1-\gamma}n_{ij}^{\gamma}\right)^{\alpha}$.

E.7 Reduced form labor supply elasticities

- We derive the expression linking reduced form and structural labor supply elasticities in Section 2.1
- Consider the inverse labor supply curve to the firm

$$
\begin{aligned}
w_{ijt} &= \left(\frac{n_{ijt}}{\boldsymbol{n}_{jt}}\right)^{\frac{1}{\eta}}\left(\frac{\boldsymbol{n}_{jt}}{\boldsymbol{N}_t}\right)^{\frac{1}{\theta}}\boldsymbol{W}_t \\
\log w_{ijt} &= \frac{1}{\eta}\log n_{ijt} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right)\log\boldsymbol{n}_{jt} + \text{Aggregates}
\end{aligned}
$$

- Consider a first order approximation around the Nash equilibrium, denoted by asterisks, following *any* change to firms in the market

$$
\begin{aligned}
\Delta\log w_{ijt} &= \frac{1}{\eta}\Delta\log n_{ijt} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right)\sum_{k\in j}\left.\frac{\partial\log\boldsymbol{n}_{jt}}{\partial\log n_{kjt}}\right|_{n^*_{-kjt}}\Delta\log n_{kjt} \\
\Delta\log w_{ijt} &= \left(\frac{1}{\eta}+\left(\frac{1}{\theta}-\frac{1}{\eta}\right)\left.\frac{\partial\log\boldsymbol{n}_{jt}}{\partial\log n_{ijt}}\right|_{n^*_{ijt}}\right)\Delta\log n_{ijt} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right)\sum_{k\neq i}\left.\frac{\partial\log\boldsymbol{n}_{jt}}{\partial\log n_{kjt}}\right|_{n^*_{-kjt}}\Delta\log n_{kjt} \\
\Delta\log w_{ijt} &= \left(\frac{1}{\eta}+\left(\frac{1}{\theta}-\frac{1}{\eta}\right)s^*_{ijt}\right)\Delta\log n_{ijt} + \left(\frac{\eta-\theta}{\theta\eta}\right)\sum_{k\neq i}s^*_{kjt}\Delta\log n_{kjt}
\end{aligned}
$$

- The definition of the *reduced form labor supply elasticity* in the text is

$$\epsilon_{ijt} = \frac{\Delta \log n_{ijt}}{\Delta \log w_{ijt}}$$

- Using the above approximation:

$$\begin{aligned}
\epsilon_{ijt} &= \frac{\Delta \log n_{ijt}}{\left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^*_{ijt}\right) \Delta \log n_{ijt} + \left(\frac{\eta-\theta}{\theta\eta}\right) \sum_{k\neq i} s^*_{kjt} \Delta \log n_{kjt}} \\
\epsilon_{ijt} &= \frac{1}{\left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^*_{ijt}\right) + \left(\frac{\eta-\theta}{\theta\eta}\right) \left\{\sum_{k\neq i} s^*_{kjt} \frac{\Delta \log n_{kjt}}{\Delta \log n_{ijt}}\right\}} \\
\epsilon_{ijt} &= \frac{\left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^*_{ijt}\right)^{-1}}{1 + \left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^*_{ijt}\right)^{-1} \left(\frac{\eta-\theta}{\theta\eta}\right) \left\{\sum_{k\neq i} s^*_{kjt} \frac{\Delta \log n_{kjt}}{\Delta \log n_{ijt}}\right\}} \\
\epsilon_{ijt} &= \left\langle \frac{1}{1 + \varepsilon^*_{ijt} \left(\frac{\eta-\theta}{\theta\eta}\right) \left\{\sum_{k\neq i} s^*_{kjt} \frac{\Delta \log n_{kjt}}{\Delta \log n_{ijt}}\right\}} \right\rangle \varepsilon^*_{ijt}
\end{aligned}$$

- This gives the expression in the text.

E.8 Pass-through expression

- We derive the pass-through expression that appears in Section 3.1
- Consider a firm's Nash equilibrium wage

$$w^*_{ijt} = \mu^*_{ijt} mrpl_{ijt}$$

- Here we have

$$\begin{aligned}
mrpl_{ijt} &= \widetilde{\alpha} \widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}-1} \\
mrpl_{ijt} &= \widetilde{\alpha} \frac{\widetilde{y}_{ijt}}{n_{ijt}} \\
mrpl_{ijt} &= \widetilde{\alpha} \left[1 - (1-\gamma)\alpha\right] \frac{y_{ijt}}{n_{ijt}} \\
mrpl_{ijt} &= \widetilde{\alpha} \left[1 - (1-\gamma)\alpha\right] vapw_{ijt}
\end{aligned}$$

- Where $vapw_{ijt}$ is *value-added per worker*. Using this we have the equilibrium system

$$\log w^*_{ijt} = \log vapw_{ijt} + \log \mu^*_{ijt} + \text{Constant.} \quad \text{for all } i \in j$$

- In each labor market, the equilibrium markdown of a firm is a function of its share s_{ijt}, which we can write $s_{ijt} = \left(w_{ijt}/\boldsymbol{w}_{jt}\right)^{\eta+1}$. Therefore we can write $\mu_{ijt} = \mu^*\left(w^*_{ijt}, w_{-ijt}\right)$ as a function of the firms wage and competitor wages.

- Consider a perturbation of any firm in the market's $vapw_{kjt}$, then to a first order around the Nash equilibrium:

$$d\log w_{ijt} = d\log vapw_{ijt} + \left.\frac{\partial \log \mu\left(w_{ij}, w^*_{-ijt}\right)}{\partial \log w_{ij}}\right|_{w^*_{ijt}} d\log w_{ijt} + \sum_{k\neq i} \left.\frac{\partial \log \mu\left(w_{ij}, w^*_{-kjt}\right)}{\partial \log w_{kj}}\right|_{w^*_{ijt}} d\log w_{kjt}$$

- Denote these elasticities m_{ii} and m_{ik}, then we have

$$d\log w_{ijt} = \frac{1}{1-m_{ii}} d\log vapw_{ijt} + \frac{1}{1-m_{ii}} \sum_{k\neq i} m_{ik} d\log w_{kjt}$$

- We can compute these elasticities, computing the following one by one:

$$\frac{\partial \log \mu_{ij}}{\partial \log w_{ij}} = \frac{\partial \log \mu_{ij}}{\partial \log \varepsilon_{ij}} \frac{\partial \log \varepsilon_{ij}}{\partial \log s_{ij}} \frac{\partial \log s_{ij}}{\partial \log w_{ij}}$$

  1. The elasticity of the markdown with respect to the labor supply elasticity is

$$\frac{\partial \log \mu_{ij}}{\partial \log \varepsilon_{ij}} = \frac{\mu_{ij}}{\varepsilon_{ij}}$$

  2. The elasticity of the elasticity with respect to the payroll share is

$$\frac{\partial \log \varepsilon_{ij}}{\partial \log s_{ij}} = -\left(\frac{\eta-\theta}{\theta\eta}\right) \varepsilon_{ij} s_{ij}$$

  3. The elasticity of the payroll share with respect to the wage is

$$\frac{\partial \log s_{ij}}{\partial \log w_{ij}} = (1+\eta)\left(1-s_{ij}\right)$$

- Combined these give

$$\frac{\partial \log \mu_{ij}}{\partial \log w_{ij}} = -\mu_{ij}\left(\frac{\eta-\theta}{\theta\eta}\right)(1+\eta)\, s_{ij}\left(1-s_{ij}\right)$$

- We can also write the markdown only in terms of shares

$$\mu_{ij} = \frac{\theta\eta}{\left(1-s_{ij}\right)\theta + s_{ij}\eta + \theta\eta}$$

- Substituting this into the above:

$$m_{ii} = \frac{\partial \log \mu_{ij}}{\partial \log w_{ij}} = -\frac{(\eta - \theta)(1 + \eta) s_{ij} \left(1 - s_{ij}\right)}{\left(1 - s_{ij}\right)\theta + s_{ij}\eta + \theta\eta}$$

- We can also compute the elasticity of the firms' markdown with respect to *competitor* wages. Proceeding as above

$$\begin{aligned}
m_{ik} = \frac{\partial \log \mu_{ij}}{\partial \log w_{kj}} &= \frac{\partial \log \mu_{ij}}{\partial \log \varepsilon_{ij}} \frac{\partial \log \varepsilon_{ij}}{\partial \log s_{ij}} \left\langle \frac{\partial \log s_{ij}}{\partial \log w_{kj}} \right\rangle \\
\frac{\partial \log \mu_{ij}}{\partial \log w_{kj}} &= \frac{\mu_{ij}}{\varepsilon_{ij}} \left\{ -\left(\frac{\eta - \theta}{\theta\eta}\right) \varepsilon_{ij} s_{ij} \right\} \left\langle -(1 + \eta) s_{kj} \right\rangle \\
\frac{\partial \log \mu_{ij}}{\partial \log w_{kj}} &= \mu_{ij} \left(\frac{\eta - \theta}{\theta\eta}\right)(1 + \eta) s_{ij} \left\langle s_{kj} \right\rangle
\end{aligned}$$

- Then note that we have the following relationship between the two elasticities:

$$\begin{aligned}
\sum_{k \neq i} \frac{\partial \log \mu_{ij}}{\partial \log w_{kj}} &= \mu_{ij} \left(\frac{\eta - \theta}{\theta\eta}\right)(1 + \eta) s_{ij} \left\langle \sum_{k \neq i} s_{kj} \right\rangle \\
&= \mu_{ij} \left(\frac{\eta - \theta}{\theta\eta}\right)(1 + \eta) s_{ij} \left\langle 1 - s_{ij} \right\rangle \\
\sum_{k \neq i} m_{ik} &= -m_{ii}
\end{aligned}$$

- Using this in the pass-through equilibrium expression:

$$\begin{aligned}
d \log w_{ijt} &= \frac{1}{1 - m_{ii}} d \log vapw_{ijt} + \frac{1}{1 - m_{ii}} \sum_{k \neq i} m_{ik} d \log w_{kjt} \\
d \log w_{ijt} &= \frac{1}{1 - m_{ii}} d \log vapw_{ijt} + \frac{\sum_{l \neq i} m_{il}}{1 - m_{ii}} \sum_{k \neq i} \frac{m_{ik}}{\sum_{l \neq i} m_{il}} d \log w_{kjt} \\
d \log w_{ijt} &= \frac{1}{1 - m_{ii}} d \log vapw_{ijt} + \frac{-m_{ii}}{1 - m_{ii}} \sum_{k \neq i} \frac{m_{ik}}{\sum_{l \neq i} m_{il}} d \log w_{kjt}
\end{aligned}$$

- Now note that

$$\begin{aligned}
\frac{m_{ik}}{\sum_{l \neq i} m_{il}} &= \frac{\mu_{ij} \left(\frac{\eta - \theta}{\theta\eta}\right)(1 + \eta) s_{ij} s_{kj}}{\sum_{l \neq i} \mu_{ij} \left(\frac{\eta - \theta}{\theta\eta}\right)(1 + \eta) s_{ij} s_{lj}} \\
\frac{m_{ik}}{\sum_{l \neq i} m_{il}} &= \frac{s_{kj}}{\sum_{l \neq i} s_{lj}} = \frac{s_{kj}}{1 - s_{ij}}
\end{aligned}$$

- Therefore we have

$$d\log w_{ijt} = \frac{1}{1-m_{ii}} d\log vapw_{ijt} + \frac{-m_{ii}}{1-m_{ii}} \sum_{k\neq i} \frac{s_{kj}}{1-s_{ij}} d\log w_{kjt}$$

- Now define $\Omega_{ii} = 1/\left(1-m_{ii}\right)$. Using this:

$$d\log w_{ijt} = \Omega_{ii} d\log vapw_{ijt} + \left(1-\Omega_{ii}\right) \sum_{k\neq i} \frac{s_{kj}}{1-s_{ij}} d\log w_{kjt}$$

- Using the expression for m_{ii} we can obtain an expression for Ω_{ii}:

$$\begin{aligned} m_{ii} &= -\frac{(\eta-\theta)(1+\eta)\, s_{ij}\left(1-s_{ij}\right)}{\left(1-s_{ij}\right)\theta + s_{ij}\eta + \theta\eta} \\ \Omega_{ii} &= \frac{s_{ij}(\eta-\theta) + \theta(\eta+1)}{\left[1+(1+\eta)\left(1-s_{ij}\right)\right] s_{ij}(\eta-\theta) + \theta(\eta+1)} \end{aligned}$$

- This gives the expression in Section 3.1 of the main text.

E.9 Corporate taxes

- Consider a single firm i, and assume constant returns to scale.
- Let the corporate tax rate be given by τ_C . Suppose that the firm can deduct some portion of its capital expenses λ_K. This corresponds to the fraction of capital expenses that are financed by long-term debt.
- Accounting profits of the firm, on which taxes are based, are given by

$$\pi_i^A = z_i k_i^{1-\gamma} n_i^{\gamma} - w_i n_i - \underbrace{\lambda_K R k_i}_{\text{Interest expense}}$$

- The economic profits of the firm are

$$\pi_i^E = z_i k_i^{1-\gamma} n_i^{\gamma} - w_i n_i - R k_i$$

- The after tax profits are given by

$$\pi_i = \pi_i^E - \tau_C \pi_i^A$$

- This gives, the following which reflects the idea that on net the firm pays corporate taxes on its total economic profits, and then is reimbursed for the taxes paid on capital financed by debt.

$$\pi_i = \left(1-\tau_C\right)\left[z_i k_i^{1-\gamma} n_i^{\gamma} - w_i n_i - R k_i\right] + \tau_C \lambda_K R k_i$$

- Dividing by $(1-\tau_C)$, the firm maximizes

$$\frac{\pi_i}{1-\tau_C} = z_i k_i^{1-\gamma} n_i^{\gamma} - w_i n_i - \underbrace{\left(\frac{1-\tau_C \lambda_K}{1-\tau_C}\right)}_{\text{Term is }>1} R k_i$$

- The effective rental rate of capital $\widetilde{R}\left(R, \tau_C, \lambda_K\right)$, that the firm faces is now *higher* than R due to the fact that not all of capital expenses can be deducted, while all of its labor expenses can. This causes the firm to take on a sub-optimal amount of capital. This lowers the marginal revenue product of other factors, including labor. If the firm could deduct *all* of its capital costs, then $\lambda_K = 1$, and the firm's input decisions are undistorted.

- Substituting in the firm's capital decision into their production function gives

$$\frac{\pi_i}{1-\tau_C} = \widetilde{z}\left(z_i, R, \tau_C, \lambda_K\right) n_i - w_i n_i$$

$$\widetilde{z}\left(z_i, R, \tau_C, \lambda_K\right) = \gamma z_i^{\frac{1}{\gamma}} \left(\frac{1-\gamma}{\widetilde{R}\left(R, \tau_C, \lambda_K\right)}\right)^{\frac{1-\gamma}{\gamma}} = \gamma z_i^{\frac{1}{\gamma}} \left(\frac{1-\tau_C}{1-\tau_C \lambda_K} \frac{1-\gamma}{R}\right)^{\frac{1-\gamma}{\gamma}}$$

- The marginal product of labor $\widetilde{z}_i$ is now *lower due to* the presence of corporate taxes and deductibility of interest payments on debt.

F Estimation details and bias exercise

F.1 Distribution of firms across markets

We assume there are 5,000 markets. For computational reasons, we must cap the number of firms per market since the Pareto distribution has a fat tail. We set the cap equal to 200 firms per market. Our results are not sensitive to the number of markets or the cap on firms per market.

Tradeable firm distribution. Figure F1 (left) plots the distribution from which we draw the number of firms per market, M_j. The distribution is a mixture of a discrete mass point at $M_j = 1$ and a Pareto distribution over the support $M_j \in [2, \infty]$. The Pareto's shape, scale, and location parameters are set to minimize the distance with the first three moments of the tradeable firm distribution. The parameters, data moments, and simulated moments are in Table F1.

Economy-wide firm distribution. Figure F1 (right) plots the economy-wide distribution from which we draw the number of firms per market, M_j. The distribution is a mixture of a discrete mass point at $M_j = 1$ and a Pareto distribution over the support $M_j \in [2, \infty]$. The Pareto's shape, scale, and location parameters are set to minimize the distance with the first three moments of the economy-wide firm distribution. The parameters, data moments, and simulated moments are in Table F2.

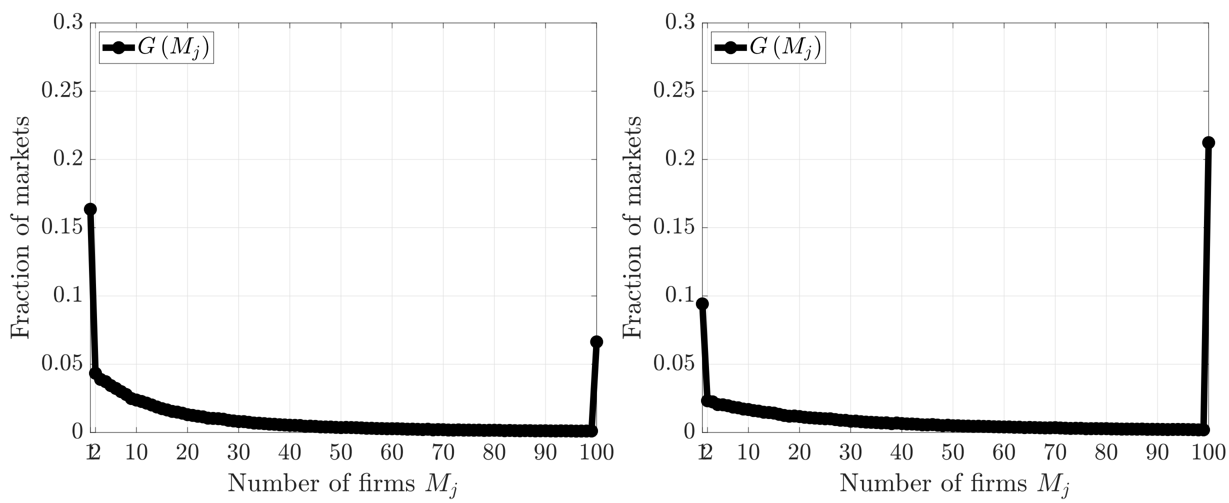

Figure F1: Distribution of the number of firms across sectors. Left: Tradeable industries, Right: all industries

Notes: Parameters given in Table F1 for tradeable and Table F2 for all industries.

F.2 Tax Experiment Details

In each simulation of the model, we conduct a tax experiment where we simulate a common corporate tax change of $\Delta_\tau = \tau'_C - \tau_C = 0.01$ (i.e. a one percentage point increase in the corporate tax), holding

A. Moments Distribution of firms M_j	Mean	Std. Dev	Skewnewss
Data (LBD 2014)	33.86	102.90	10.44
Model	33.80	102.94	22.90
B. Parameters Mass at $M_j = 1$	Pareto Tail	Pareto Scale	Pareto Location
0.16	0.52	18.74	2.00

Table F1: Distribution of firms across markets, $M_j \sim G(M_j)$, tradeable industries

A. Moments Distribution of firms M_j	Mean	Std. Dev	Skewnewss
Data (LBD 2014)	113.10	619.00	26.14
Model	113.14	618.82	36.08
B. Parameters Mass at $M_j = 1$	Pareto Tail	Pareto Scale	Pareto Location
0.09	0.71	38.36	2.00

Table F2: Distribution of firms across markets, $M_j \sim G(M_j)$, all industries

aggregate quantities fixed. We rerun our reduced-form regressions on the simulated data in order to recover average reduced-form labor supply elasticities as a function of wage-bill shares. These market-share-dependent reduced-form labor supply elasticities are the moments used to recover η and θ in Section 2. We describe the details of the exercise below:

1. Simulate the benchmark equilibrium for two periods (date $t = 0, 1$) without taxes. Treat these observations as 'data.' We must simulate two prior periods in order to define the lagged wage-bill share of the firm in the market.

2. C-corps in the model economy (recall there is a share ω_C of C-corps in all markets) have their taxes raised by 1 percentage point.

3. Simulate the 'post-shock' equilibrium, treat as date $t = 2$ 'data.'

4. Estimate the same reduced-form regressions as Section 2 using the $t = 0, 1, 2$ simulated data. Estimate the following regressions for each firm i in region j:

$$\log(n_{ijt}) = \alpha_i + \beta_n \tau_{Ct} + \gamma_{0s} s_{ijt-1} + \beta_{ns} \tau_{Ct} * s_{ijt-1} + \epsilon_{ijt}$$
$$\log(w_{ijt}) = \alpha_i + \beta_w \tau_{Ct} + \omega_{0s} s_{ijt-1} + \beta_{ws} \tau_{Ct} * s_{ijt-1} + u_{ijt}$$

5. Compute the employment and wage elasticities with respect to productivity, $\frac{d\log(n_{ijt})}{d\tau_{Ct}}$ and $\frac{d\log(w_{ijt})}{d\tau_{Ct}}$. Use these expressions to recover the average reduced-form labor supply elasticities using the formula:

$$\widehat{\epsilon}(s_{ijt-1}) = \frac{\beta_n + \beta_{ns} s_{ijt-1}}{\beta_w + \beta_{ws} s_{ijt-1}}$$

6. Use the recovered $\{\widehat{\epsilon}(s_{ijt-1}), s_{ijt-1}\}$ pairs as moments to recover η and θ.

F.3 Biases

To explore the difference between structural and reduced-form labor supply elasticities, we conduct a Monte Carlo exercise where we simulate a perfectly idiosyncratic shock and then compute reduced form elasticities. We average these across firms within payroll share bins and compare these to the structural labor supply elasticity implied by $\varepsilon(s_{ij}) = [\theta^{-1}s_{ij} + \eta^{-1}(1 - s_{ij})]^{-1}$. We repeat this exercise for 5,000 simulations and report the averages in Figure 5. We describe the details of the exercise below:

1. Simulate the benchmark equilibrium, treat as date $t = 1$ 'data.'
2. Randomly select 1 firm in each market and increase their productivity by 1% (or 50%), holding aggregates fixed (assuming partial equilibrium).
3. Simulate 'post-shock' partial equilibrium (industry competitors adjust but aggregates are held fixed), treat as date $t = 2$ 'data.'
4. Isolate firms with pre-shock wage-bill shares s_{ij} in bins with nodes $[0.1, \ldots, 0.9]$. Within each bin, compute the mean of $\Delta \log(n_{ijt})/\Delta \log(w_{ijt})$ using the $t = 1, 2$ simulated data.
5. Figure 5 plots these values at the upper cutoff of these bins. For shares equal to 0 and 1, the solution is exact $\varepsilon(1) = \widehat{\epsilon}(1) = \theta$, $\varepsilon(0) = \widehat{\epsilon}(0) = \eta$.

F.4 Additional threats to consistency.

There are two additional threats to consistency of our simulations. (i) apportionment of state taxes across multi-state production units may mean that state corporate taxes do not affect firms within a state, and (ii) anticipation of tax changes. We discuss these issues in the context of prior analysis by Juan Carlos Suárez Serrato and Owen Zidar (2016) and Xavier Giroud and Joshua Rauh (2019).

First, Suárez Serrato and Zidar (2016) show that the impact of state corporate taxes on local economic activity is extremely similar for both (i) the statutory corporate taxes used here, and (ii) effective corporate taxes—i.e. 'business taxes'—carefully adjusted for apportionment weights.[3] Since establishment sales and company property values are not available to us, we cannot construct accurate apportionment weights and thus we focus on statutory tax rates compiled by Giroud and Rauh (2019). We only require similarly sized firms to face similarly sized shocks. The magnitude of the shock is not important for our identification of η and θ, instead it is their relative employment to wage adjustment that identifies η and θ.

Second, both Suárez Serrato and Zidar (2016) and Giroud and Rauh (2019) establish that including various aspects of changes to fiscal policy around corporate tax adjustments have negligible affects on

[3]See their discussion of Table A21, p.19 (emphasis added): "Column (6) of Table 5 and Appendix Table A21 show that using statutory state corporate tax rates in Equation 21 (instead of business tax rates τ_b) results in similar and significant estimates, indicating that *our measure of business tax rates is not crucial for the results*."

their measured elasticities of local economic activity to state corporate taxes.[4] We interpret this as indirect evidence that the reforms are not paired with other predictable components of fiscal stimulus, such as unemployment insurance, which follow time-invariant threshold rules and are typically triggered in recessions (e.g. Kurt Mitman and Stanislav Rabinovich, 2019).

G Discussion of empirical estimation and robustness

This section is divided into three parts. First, a discussion of how our empirical strategy relates to other papers in the literature. Second, including exit in our regressions and re-estimating the model under exit. Third, a set of robustness exercises around state-level omitted variables, non-wage compensation, variation in capital intensity, and an alternative approach using 'direct' elasticities.

G.1 Discussion

As discussed in Section 1.4, the model predicts that the labor supply elasticity faced by firms varies by their market share. If this relationship were known in the data, it would precisely pin down the elasticities of substitution of labor within and across sectors. Existing work estimating labor supply elasticities to firms has focused either on specific markets (e.g. Douglas A. Webber, 2016) or in well identified responses to small experimental variations in wages (Arindrajit Dube, Jeff Jacobs, Suresh Naidu and Siddharth Suri, 2020; Arindrajit Dube, Doruk Cengiz, Attila Lindner and Ben Zipperer, 2019). A contribution of this paper is to estimate a share-elasticity relationship through a novel quasi-natural experiment using a large cross-section of firms.

The intuition for our procedure is as follows. We first estimate the rate at which labor demand shocks *pass-through* to wages and employment and the reduced-form relationship between these labor supply elasticities and local labor market shares. We then invert this empirical relationship using our model to recover estimates of the structural parameters that control the relative substitutability of labor within and between markets. To identify how pass-through rates vary by market share, we compare how the firm responds to these labor demand shocks differentially across markets within the same state, but in which their shares of the labor market differ.

This procedure requires a shock to labor demand in order to trace out the labor supply curve. We use state corporate tax changes which constitute a shock to firm labor demand via their distortion of accounting profits relative to economic profits, shifting the marginal revenue product of labor.[5] Both Suárez Serrato and Zidar (2016) and Giroud and Rauh (2019) have studied the impact of state-level corporate tax shocks on local economic activity. We address three issues that may arise: (i) apportionment

[4] Giroud and Rauh (2019) establish the plausible exogeneity of state-corporate tax changes. From a public finance perspective they study the effects of state corporate tax changes on employment and wages. Their focus is *within firm*, *across state* responses, and the reallocation of firm employment across states following tax changes. For an exhaustive description of these tax changes we point the interested reader to their paper.

[5] We have not included corporate taxes in our benchmark model. We show that the mapping of our model to the data does not require us to take a stance on the transmission mechanism linking corporate taxes to productivity. Nevertheless, Appendix E.9 shows how corporate tax rates map to shocks to the marginal revenue productivity of labor in our framework.

of state taxes across multi-state production units may mean that state corporate taxes do not affect firms within a state, (ii) taxes are anticipated, (iii) such shocks affect all firms in a region and so can only be used to identify θ.

First, Suárez Serrato and Zidar (2016) show that the impact of corporate taxes on local economic activity is extremely similar for both (i) the statutory corporate taxes that we use and (ii) effective corporate taxes adjusted for apportionment weights.[6] Since establishment sales and company property values are not available to us, we focus on statutory taxes rates compiled by Giroud and Rauh (2019) and based on Suárez Serrato and Zidar (2016) we do not adjust for the apportionment regime of the state.

Second, both Suárez Serrato and Zidar (2016) and Giroud and Rauh (2019) establish that the inclusion of other aspects of changes to fiscal policy around the corporate tax changes does not affect their measured elasticities of local economic activity to corporate taxes.[7]

Third, the fact that (i) only C-corps pay statutory corporate tax rates, (ii) the structure of our model and (iii) Monte Carlo exercises, provide support that we may infer η and θ from a shock that affects some but not all firms. We briefly discuss this in more detail.

G.2 Exits

G.2.1 Empirics

In Table G1, we estimate linear probability models of firm-market exit in year $t+1$ as a function of corporate taxes in year t. In column (1) and (2), we find economically insignificant results. This complements the work of Giroud and Rauh (2019), who aggregate plants at the firm-state level and study how the number of plants per C-Corp in a state responds to corporate tax changes. Since our relevant level of economic activity is at the firm-market level, and since we are interested in exits from the market entirely, we use a different approach. We regress whether a firm exits a firm-market entirely, instead of simply regressing the number of plants in the state on the tax change. Our results do not necessary contradict Giroud and Rauh (2019). Giroud and Rauh (2019)'s results imply that firms may adjust the number of plants in the state. Our results imply that firms do not appear to be exiting markets *entirely* in response to a corporate tax change.

G.2.2 Model re-estimation with exit

Our empirical results suggest that exits are not a threat to our exercise. Nonetheless, we show that our model estimates of η and θ are robust under the assumption that 5% of C-Corps exit. This is an

[6] See their discussion of Table A21, p.19 (emphasis added): "Column (6) of Table 5 and Appendix Table A21 show that using statutory state corporate tax rates in Equation 21 (instead of business tax rates τ_b) results in similar and significant estimates, indicating that *our measure of business tax rates is not crucial for the results*."

[7] Giroud and Rauh (2019) establish the plausible exogeneity of state-corporate tax changes. From a public finance perspective they study the effects of state corporate tax changes on employment and wages. Their focus is *within firm*, *across state* responses, and the reallocation of firm employment across states following tax changes. For an exhaustive description of these tax changes we point the interested reader to their paper.

	(1) Exit$_{ijt+1}$	(2) Exit$_{ijt+1}$
$\tau_{s(j)t}$	-0.000624	-0.000793
	(0.000454)	(0.000628)
	[0.000150]	[0.000166]
Fixed Effects	Market, Year	Firmid, Market, Year
R-squared	0.056	0.184
Observations	4.260e+06	4.260e+06

Table G1: Exit probability

Notes: According to Census requirements, the number of observations is rounded to the nearest 1,000. Standard errors in round parentheses $(\cdot)$ are clustered at State $\times$ Year level. Standard errors in square parentheses $[\cdot]$ are clustered at Market $\times$ Year level. Sample includes tradeable C-Corps from 1977 to 2011.

extreme and counterfactually high exit response to corporate tax hikes. Table G2 reports the results. Our estimates of η and θ are similar to the baseline.

	η	θ
Benchmark	10.85	0.42
Exit rate 5%	10.87	0.48

Table G2: Re-estimation of the model assuming 5% of firms exit the market in response to a corporate tax increase of 1 ppt.

G.3 Regression Robustness

We discuss robustness of our regression specifications and their implications for the relationship between market shares and reduced form labor supply elasticities. We consider (i) state-level omitted variables, (ii) compute direct elasticities at the firm level, (iii) account for systematic variation in non-wage compensation, (iv) account for systematic variation in capital intensity.

G.3.1 State-level omitted variables

Model estimation simply requires *consistent* auxiliary moments that can be simulated. The threat to *consistency* when we estimate equation 13 is that there are other forces moving employment and wages at the state-year level, e.g. tax cuts occur in boom years etc. To control for state-level responses, Giroud and Rauh (2019) include S-Corps as a control for C-Corps. Through the lens of our theory, S-Corps do not provide a suitable control group, since they respond to the treatment as well. Thus, the stable unit treatment value assumption (SUTVA) is violated. To alleviate concerns that our estimates are being driven by omitted state-year level variation, we include specifications which include both state$\times$year fixed effects as well as firmid$\times$market fixed effects. State-level corporate tax changes are subsumed in the fixed effects, and so we are only able to identify the interaction between corporate taxes and wage-bill shares. Table G3 illustrates our results. Comparing columns (1) through (4) in Table G3 to Table 1, we find very similar interactions between taxes and wage-bill shares for both date t and $t+1$ employment and wages. We view these results as suggestive evidence that omitted variables at the state-year level are unlikely to explain our results.

	(1)	**(2)**	**(3)**	**(4)**
	$\log n_{ijt}$	$\log w_{ijt}$	$\log n_{ijt+1}$	$\log w_{ijt+1}$
s^{wn}_{ijt-1}	0.983***	0.0662***	0.777***	0.0559***
	(0.0280)	(0.00957)	(0.0237)	(0.00984)
	(0.0111)	(0.00622)	(0.0116)	(0.00671)
$\tau_{s(j)t} \times s^{wn}_{ijt-1}$	0.0131***	0.00779***	0.0128***	0.00712***
	(0.00304)	(0.00122)	(0.00263)	(0.00131)
	(0.00135)	(0.000795)	(0.00145)	(0.000849)
Fixed effects	Y	Y	Y	Y
R-squared	0.908	0.781	0.889	0.731
Round N	4.260e+06	4.260e+06	4.260e+06	4.260e+06

Table G3: State-year fixed effects

Notes: All specifications include fixed effects for: (i) year, (ii) firmid×market, and (iii) state×year. According to Census requirements, the number of observations is rounded to the nearest 1,000.

G.3.2 Direct elasticities

To provide further evidence that labor supply elasticities decline as a function of a firm's wage-bill share, we directly compute the ratio of wage changes to employment changes at the firm-level and we study their relationship to a firm's wage-bill share. To allow for perfect competition (non-zero employment change with zero wage change), we compute the inverse elasticity at the firm level $\frac{\Delta w_{ijt}}{\Delta n_{ijt}} \frac{n_{ijt}}{w_{ijt}}$ between year t and $t+1$. To measure an elasticity, we require a supply or demand shifter. We use corporate tax changes as demand shifters. These 'direct' elasticities include significant amounts of measurement error. In particular, the measurement error in the denominator results in many extreme outliers. We impose several criteria to deal with this measurement error. First, we require an employment adjustment of at least ± 5 employees.[8] Second, we only use tax changes of at least one percentage point $|\Delta \tau_{s(j)t}| > 1$. Third, we winsorize the dependent variable at the 0.5% level.[9] Fourth, to remove common state-year fluctuations in wages and employment, we include state-year fixed effects as well as firm-market fixed effects.

To isolate the size-dependent labor supply elasticity, we interact the corporate tax changes with the firm's wage-bill share. Because of the high-dimensional fixed effects firm must adjust employment twice by at least ± 5 employees between 1977 to 2011 in order for our fixed effects to be estimated. We run

[8] We also tried cutoffs of $\{3, 7, 10\}$ and our results are robust.

[9] We also tried winsorizing at the 1% and 5% levels and our results are robust.

specifications of the following form:

$$[\epsilon^{Data}(s)]^{-1} = \frac{\Delta w_{ijt}}{\Delta n_{ijt}} \frac{n_{ijt}}{w_{ijt}} = \alpha_{ij} + \gamma_{s(j)t} + \beta_1 \mathbf{1}(s_{ijt-1} \in [.01, .05)) + \beta_2 \mathbf{1}(s_{ijt-1} \in [.05, 1]) \\ + \beta_3 \mathbf{1}(|\Delta\tau_{s(j)t}| > 1) \times \mathbf{1}(s_{ijt-1} \in [.01, .05)) + \beta_4 \mathbf{1}(|\Delta\tau_{s(j)t}| > 1) \times \mathbf{1}(s_{ijt-1} \in [.05, 1]) + \epsilon_{ijt} \tag{G1}$$

Table G4 provides estimates of equation (G1). Column (1) shows that the inverse elasticity for firms with wage bill shares between 1% and 5% is significantly different from those whose wage-bill shares are less than 1%. Their inverse elasticity is .0542 percentage points greater. Based on our estimates in Table 2, if firms with a wage-bill share less than 1% have a labor supply elasticity of 2, then these estimates imply that firms with a wage bill share between 1% and 5% have a labor supply elasticity of roughly 1.80. For those with wage-bill shares greater than 5%, their inverse elasticity increases by 0.0645 percentage points relative to those with a wage-bill share less than 1%. Based on our estimates in Table 2, if firms with a wage-bill share less than 1% have a labor supply elasticity of 2, then these estimates imply that firms with a wage bill share greater than 5% have a labor supply elasticity of roughly 1.77. Lastly, column (2) estimates the inverse labor supply elasticity using changes in employment and wages between t and $t+2$, while keeping the same and all other right-hand-side regressors in equation (G1) the same. We interpret column (2) as a long-run inverse labor supply elasticity. Relative to the omitted group of firms with shares less than 1%, we find that firms with wage-bill shares greater than 5% have an inverse elasticity that is 0.080 percentage points greater. Based on our estimates in Table 2, if firms with a wage-bill share less than 1% have a labor supply elasticity of 2, then these estimates imply that firms with a wage bill share greater than 5% have a labor supply elasticity of roughly 1.72. These estimates are remarkably close to our linear regression estimates in Section 2.2.

	(1) Inverse elasticity t to $t+1$	**(2)** Inverse elasticity t to $t+2$
$1(\lvert\Delta\tau_{s(j)t}\rvert > 1) \times s^{wn}_{ijt-1} \in [.01, .05)$	0.0542**	0.00791
	(0.0223)	(0.0355)
	[0.0228]	[0.0437]
$1(\lvert\Delta\tau_{s(j)t}\rvert > 1) \times s^{wn}_{ijt-1} \in [.05, 1]$	0.0645**	0.0800*
	(0.0310)	(0.0452)
	[0.0287]	[0.0532]
Fixed Effects	Yes	Yes
R-squared	0.191	0.177
N	1.334e+06	1.334e+06

Table G4: Estimation results for equation (G1)

Notes: All specifications include fixed effects for: (i) year, (ii) firmid×market, and (iii) state×year. According to Census requirements, the number of observations is rounded to the nearest 1,000. Standard errors in round parentheses $(\cdot)$ are clustered at State × Year level. Standard errors in square parentheses $[\cdot]$ are clustered at Market × Year level. Sample details described in text.

G.3.3 Non-wage compensation

Can our results be attributed to non-wage benefits that vary by size? We argue no. We find that while benefits covary positively with wage-bill share, they cannot explain the magnitude of size-dependent markdowns we estimate in the data.

To bound the effect of benefits on our markdowns, we must measure the elasticity of benefits with respect to the wage-bill share of the firm. The Census of Manufacturers includes data on worker benefits in recent survey waves. We use the 2012 Census of Manufacturers to estimate how benefits per employee varies with local wage-bill shares. To mitigate spurious correlations between market-share and benefits, we include firm fixed effects. Thus our empirics compare within a firm, across plants, how local wage-bill shares covary with benefits. Table G5 includes our results. We find that a 1 percentage point increase in the wage bill share results in a 0.597% increase in benefits per worker. We repeat the exercise after winsorizing benefits per worker at the 1% level, and we find a very similar elasticity; thus, our low elasticity of benefit per worker with wage-bill share is not driven by outliers.

	(1) Log benefits per employee$_{ijt}$	(2) Log benefits per employee$_{ijt}$, 1% winsorized
s_{ijt}^{wn}	0.00597***	0.00565***
	(0.000261)	(0.000241)
R-squared	0.687	0.688
N	36000	36000

Table G5: Benefits per employee and wage-bill shares

Notes: All specifications include fixed effects for: (i) year, (ii) firmid , and (iii) market. According to Census requirements, the number of observations is rounded to the nearest 1,000. Standard errors clustered at the firmid level. Sample includes tradeable Census of Manufacturers firms in 2012.

Figure G1 plots the elasticity of the model's markdowns as a function of the wage-bill share. For wage-bill shares of 1%, the elasticity of the wage-bill share is close to -16%. Thus non-wage benefits are too small to be responsible for our estimated markdown elasticities.

G.3.4 Capital Intensity

The Census of Manufacturers includes data on assets per employee.[10] We use the 2012 Census of Manufacturers to estimate how assets per employee varies with local wage-bill shares. We include firm (firmid) fixed effects to isolate within-firm, across-plant variation in the way assets per employee covaries with local wage-bill shares. Table G5 includes our results. We find that a 1 percentage point increase in the wage bill share results in a 0.176% increase in assets per employee. Our mean value of assets per employee in this sample of multi-plant firms is $332,500. Thus a 1 percentage point increase in wage-bill

[10] We use beginning-of-year asset values.

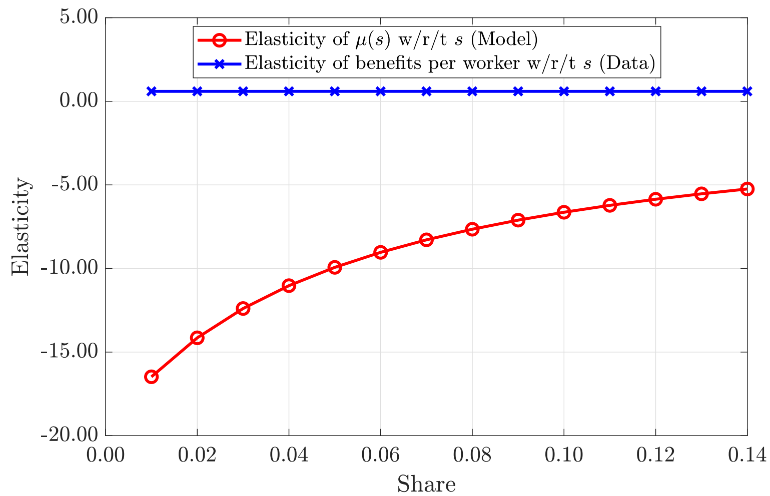

Figure G1: Non-wage benefits and wage-bill shares

share corresponds to an increase in assets of $595 for the average firm. We view this as economically insignificant. Column (2) removes outliers by winsorizing the data at the 1% level. We find a slightly larger elasticity, however, we view these results as supportive evidence that our size-dependent labor supply elasticities cannot be explained by differential capital adjustment.

	(1) Log assets per employee$_{ijt}$	(2) Log assets per employee$_{ijt}$, 1% winsorized
s^{wn}_{ijt}	0.00176**	0.00193***
	(0.000749)	(0.000693)
R-squared	0.639	0.638
Observations	36000	36000

Table G6: Assets per employee and wage-bill shares

Notes: All specifications include fixed effects for: (i) year, (ii) firmid , and (iii) market. According to Census requirements, the number of observations is rounded to the nearest 1,000. Standard errors clustered at the firmid level. Sample includes tradeable Census of Manufacturers firms in 2012.

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