# Online Appendix: Political Fragmentation and Government Stability. Evidence from Local Governments in Spain

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# A. Theoretical Appendix

### Party 1 Expected Pay-offs in Different Coalitions

If  $s_1 < 0.5$ , a party 1 will form a coalition in period 1 by making a proposal distribute the available resources  $\theta_1$ . Party 1 will always be able to make a proposal that gathers a majority by offering  $s_3\theta_1$  to party 3. The problem that party 1 faces when forming an initial coalition in the three party case can be written as:

$$\max_{g^1} (g_1^1 + \omega) \Big( 1 + \beta (1 - \mu \mathbb{1}\{\theta_2 \ge g_3^1\}) \Big)$$
(A.1)

s.t. 
$$\sum_{j=1}^{3} g_j^1 \le \theta_1.$$
 (A.2)

Expected payoffs for party 1 in each coalition are given by:

$$V_{mc}^{S} = [\omega + \theta_{1}(1 - s_{3})](1 + \beta)$$
$$V_{mc}^{C} = [\omega + \theta_{1}(1 - s_{3})][(1 + \beta)(1 - \mu) + \mu]$$
$$V_{block}^{S} = [\omega + \theta_{1} - \theta_{2}](1 + \beta),$$

where  $V_{mc}^S$  is the payoff for safe minimum-cost coalitions, which is feasible when  $\theta_2 < s_3\theta_1$ .  $V_{mc}^C$  is the payoff for contestable minimum-cost coalitions, which are always feasible. Finally,  $V_{block}^S$  is the payoff for safe blocking coalitions, which are feasible when  $\theta_2 < \theta_1$ . The constraint in equation 1 can be obtained by combining  $V_{block}^S$  and  $V_{mc}^C$ .

The expressions in the 4 party case replace  $s_*$  instead of  $s_3$ . We can define  $s_*$  formally as  $s_* = s_3 + (s_4 - s_3) \mathbb{1}\{s_1 + s_4 \ge 0.5\}.$ 

*Expression for Prob. of Vote of No confidence*  $\pi(\mathbf{s})$  - *Case with three parties and*  $s_1 < 0.5$ 

In the three party case, the probability of a vote of no confidence when there is no singleparty majority  $\pi(s)$  is given by:

$$\pi(\mathbf{s}) = \mu \left( 1 - \left( \int_0^{\theta_k} \int_0^{\theta_1} g(\theta_1, \theta_2) \ d\theta_2 \ d\theta_1 + \int_{\theta_k}^1 \int_0^{h(\theta_1, s_3)} g(\theta_1, \theta_2) d\theta_2 d\theta_1 \right) \right)$$
(A.3)  
with  $\theta_k = \frac{\mu \omega \beta}{(1 - s_3)(1 + \beta - \mu \beta)},$ 

where  $g(\theta_1, \theta_2)$  is the joint density function of  $(\theta_1, \theta_2)$ ,  $h(\theta_1, s_3)$  is defined in 1, s is a seat share vector satisfying  $s_1 < 0.5$  and  $\theta_k$  is the value of  $\theta$  at the kink resulting from the intersection between constraints (see Figure 1). When  $s_1 \ge 0.5$ , the probability of a vote of no confidence is 0.

### Proof of Proposition 1

In the first place, consider the case in which  $s_1 \ge 0.5$ . This condition implies party 1 forms a single party majority and  $\pi(\mathbf{s}) = 0$ . In this scenario, there are two relevant possibilities depending on whether  $s'_1 \ge 0.5$  or not. If  $s'_1 \ge 0.5$ , we will have that  $\pi(\mathbf{s}') = 0$  for the same reason. If, however  $s'_1 < 0.5$ , then we know  $\pi(\mathbf{s}') \ge 0$  because for a section of  $(\theta_1, \theta_2)$  space, the probability of a vote of no confidence is different from 0. This completes the proof for the  $s_1 \ge 0.5$  case.

In the case with  $s_1 < 0.5$ , the probability of a vote of no confidence will be larger than 0 under both s and s'. Two cases need attention when comparing these probabilities. Define  $s_* \equiv s'_3 + (s'_4 - s'_3)\mathbb{1}\{s'_1 + s'_4 \ge 0.5\}$ . If  $s_* = s_3$ , then integral A.3 is identical for  $s^3$  and  $s^4$ , so that  $\pi(\mathbf{s}) = \pi(\mathbf{s}')$ . If, however,  $s_* < s_3$ , then the region of  $(\theta_1, \theta_2)$  space corresponding to safe coalitions is smaller under s' than under s. As indicated in the right-panel of figure 1, this occurs because the linear constraint  $h(\theta_1, s_*)$  will have the same intercept and a smaller slope than constraint  $h(\theta_1, s_3)$  (see equation 1 in the main text). Given that, by assumption,  $g(\theta_1, \theta_2)$  has positive density everywhere in the unit square, the change in the regions of integration translate into  $\pi(\mathbf{s}') > \pi(\mathbf{s})$  if  $s_* < s_3$ .

#### Equilibrium with two Parties

The case with 2 parties is very straightforward as, necessarily, party 1 is always able to form a single-party majority in period 1 by approving a transfer of  $\theta_1$  to itself. Because no alternative majority can be formed, the probability of a vote of no confidence is 0 regardless of shares  $s_1$  and  $s_2$  or the values of  $(\theta_1, \theta_2)$ .

An increase in the number of parties from 2 to 3 can result in an increase in the probability of a vote of no confidence if and only if  $s_1 < 0.5$  in the 3 party case.

### Equilibrium with five Parties

We now discuss the equilibrium when with 5 parties. If  $s_1 \ge 0.5$ , then party 1 forms a single-party majority, approves paying itself  $\theta_1$ , and the probability of a vote of no confidence in period 2 is 0. When  $s_1 < 0.5$ , the contestable minimum cost coalition will result in an expected pay-off of  $V_{mc}^C = (\omega + (1 - s_*)\theta_1)(1 + \beta(1 - \mu))$ , with  $s_*$  corresponding to the combined seat share of the additional parties that party 1 needs to form a minimum winning coalition. This number will depend on the vector of seat shares, as detailed in table A.1.

The safe minimum cost coalition will be available to party 1 if and only if  $\theta_2 < s_*\theta_1$  with  $s_*$  taking the values illustrated in table A.1. The associated pay-off will be  $V_{mc}^S = (\omega + \theta_1(1 - s_*))(1 + \beta)$ .

	Case	$s_*$
Panel A		
$s_1 + s_3 \ge 0.5$	$s_1 + s_5 \ge 0.5$	$s_5$
	$s_1 + s_4 \ge 0.5 \ \& \ s_1 + s_5 < 0.5$	$s_4$
	$s_1 + s_4 + s_5 \ge 0.5$ & $s_4 + s_5 < s_3$ & $s_1 + s_4 < 0.5$	$s_4 + s_5$
	$s_1 + s_4 < 0.5 \ \& \ s_4 + s_5 \ge s_3$	$s_3$
Panel B		
$s_1 + s_3 < 0.5$	$s_1 + s_3 + s_5 \ge 0.5$ & ( $s_1 + s_4 + s_5 < 0.5$ or $s_4 + s_5 > s_3$ )	$s_3 + s_5$
	$s_1 + s_4 + s_5 \ge 0.5$	$s_4 + s_5$

TABLE A.1 Values of  $s_*$  - 5 Party Case ( $s_1 < 0.5$ )

When considering blocking coalitions there are two cases that warrant separate attention,  $s_1 + s_3 \ge 0.5$  and  $s_1 + s_3 < 0.5$ . In the first case, party 1 only needs one party to form a winning coalition, and can therefore offer  $\theta_2$  to one party (e.g. party 3) to form a blocking coalition. This is analogous to the case with 3 or 4 parties and yields a pay-off of  $V_{block}^S = (\omega + (\theta_1 - \theta_2))(1 + \beta)$ , which is feasible if  $\theta_1 > \theta_2$ . When  $s_1 + s_3 < 0.5$ , party 1 needs two parties to form a coalition, and hence will have to pay  $\theta_2$  to both for that coalition to be blocking. In this case, the pay-off from forming a blocking coalition is  $V_{block}^S = (\omega + (\theta_1 - 2\theta_2))(1 + \beta)$ , and is only feasible if  $\theta_1 > 2\theta_2$ .

In both cases we can determine when blocking coalitions are played in  $(\theta_1, \theta_2)$  space by using condition  $V_{mc}^C \ge V_{block}^S$  to derive incentive compatibility constraints  $\theta_2 \le h(\theta_1, s_*)$ , and the feasibility conditions for a blocking coalition as participation constraints.<sup>26</sup> The incentive compatibility constraints will be given by:

$$h(\theta_{1},s) = \begin{cases} \frac{\mu\omega\beta}{1+\beta} + \frac{s_{*}(1+\beta-\mu\beta)+\mu\beta}{1+\beta}\theta_{1} \text{ if } s_{1}+s_{3} \ge 0.5\\ \frac{\mu\omega\beta}{2(1+\beta)} + \frac{s_{*}(1+\beta-\mu\beta)+\mu\beta}{2(1+\beta)}\theta_{1} \text{ if } s_{1}+s_{3} < 0.5 \end{cases}$$

We can use these to write the probability of a vote of no confidence in the case with 5 parties as:

$$\pi_2(\Theta, \mathbf{s}) \equiv \begin{cases} 0 & \text{if} \\ 0 & \text{if} \\ \mu & 0 \end{cases} \begin{cases} s_1 + s_3 \ge 0.5 \text{ and } \theta_2 \le h(\theta_1, s_*) \text{ and } \theta_2 < \theta_1 \\ 0 & \text{or} \\ s_1 + s_3 < 0.5 \text{ and } \theta_2 \le h(\theta_1, s_*) \text{ and } \theta_2 < \theta_1/2 \\ 0 & \text{or} \end{cases}$$

We can use this expression to prove the equivalent of proposition 1 in the 4 to 5 party case. Assume two seat share vectors  $\mathbf{s} = (s_1, s_2, s_3, s_4)$  and  $\mathbf{s}' = (s'_1, s'_2, s'_3, s'_4, s'_5)$  such that  $s_j \ge s'_j \quad \forall j = \{1, 2, 3, 4\}$  and  $s'_5 > 0$ . For a given joint distribution  $g(\theta_1, \theta_2)$  with positive

<sup>&</sup>lt;sup>26</sup>Because  $s'_*$  and  $s_*$  are both smaller than 0.5, we can guarantee that safe minimum cost coalitions will never be feasible if blocking coalitions are not feasible.

density in the unit square, we have that  $\pi(s') \ge \pi(s)$ . To prove this, it suffices to show that  $s'_* \le s_*$ , where  $s_*$  is the seat share of the ally party 1 needs when building a minimum cost coalition in the 4 party case, and  $s'_*$  corresponds to the same figure in the 5 party case (see table A.1).<sup>27</sup> Because  $h(\theta_1, s_*)$  is increasing in  $s_*$ , and a blocking coalition needs to satisfy  $\theta_2 \ge h(\theta_1, s_*)$ , a decrease in  $s_*$  will reduce the size of the region in  $(\theta_1, \theta_2)$  space for which this condition is satisfied. For a fixed  $g(\theta_1, \theta_2)$  with positive support in the unit square, the will translate in a higher probability of a vote of no confidence. To show  $s_* \ge s'_*$  it suffices to go over table A.1, compare them to expression  $s_* = s_3 + (s_4 - s_3)\mathbb{1}\{s_1 + s_4 \ge 0.5\}$  for the four party case, and note that  $s_j \ge s'_j$   $\forall j = \{1, 2, 3, 4\}$ , by assumption.

In this sense, going from 4 to 5 parties appears to be no different to going from 3 to 4 parties. However, adding a fifth party introduces an additional mechanism. Not only can the cost of a minimum cost coalition fall when adding a fifth party ( $s_* \ge s'_*$ ), but also the cost of forming a blocking coalition can increase. This occurs because in the 5 party case we might have that  $s_1 + s_3 < 0.5$  which implies party 1 needs *two* other parties to form a minimum coalition. To make this a blocking coalition, party 1 needs to pay  $\theta_2$  to each party. This doubles the cost of forming a blocking a blocking coalition, affecting both its feasibility and desirability.<sup>28</sup>

<sup>&</sup>lt;sup>27</sup>If the minimum winning coalition requires two parties (e.g. 3 and 5), then this figure will be the combined share of both parties.

 $<sup>^{28}</sup>$  It is also possible to show that an adapted version of the lemma in section 2 is satisfied in the five party case. Proof available upon request.

#### **B.** Construction of the instrument for fragmentation

To instrument for the number of parties in the council, we use an indicator D equal to one if, in a given election, a given party in a municipality obtained a vote-share above the 5% threshold. Given that the electoral rules exclude parties with less than 5% from the allocation of seats, parties above the threshold have a positive probability of being in the council, whereas parties below the threshold never receive a seat. Thus, the number of parties with seats in the council in a given municipality will be related to how many parties were able to cross this threshold. Our fuzzy-RD design is based on this intuition. It uses variation in the number of parties that crossed the 5% threshold to instrument for the number of parties in council, focusing on observations within a small bandwidth h from 5%.

The instrument is defined for each election, municipality and party. As an illustration, consider an example in which, after an election, vote-shares are determined in a way that there are only two parties that obtained vote-shares sufficiently close to the 5% threshold to be within the bandwidth h.

There are three possible cases, depicted in the figure below: both parties receive less than 5% (case 1), both receive more (case 2), or parties locate at either side of the 5% threshold (case 3). In case 1, our instrument D takes value 0 for both parties A and B. Similarly, in case 2 it is 1 for both parties, while in case 3 it equals 1 for party A and 0 for party B.

It is clear that the number of parties that enter the council is partially determined by the number of parties that manage to get at least 5% of the votes and are, hence, eligible to obtain a seat. In case 2, for example, if the vote-shares of party A and B are sufficiently high, the D'Hondt method will allocate both parties a seat, so that the council will have two additional parties. On the contrary, in situations like case 1, there will be two parties less in the council.



## C. Data Appendix

## C.1. List of Data Sources

# Town Panel

We create a "town panel", that is a list of municipalities-by-year unique identifiers, gathering information on the official naming of municipalities, as well as municipality, province and region codifications. For years after 1999, we use the official list from the *Instituto Nacional de Estadistica* (Instituto Nacional de Estadistica, 2001-2019). This information is not available in earlier years, for which we use the election results dataset (see below) as a basis for our towns panel instead.

This town panel is used as a basis for all subsequent merges with the other datasets used in the paper.

#### Elections

We use municipal election data from *Ministerio del Interior* (the Spanish Ministry of Internal Affairs), relative to all election years between 1979 and 2011 (Ministerio del Interior, 1979 - 2015). This source contains information about all parties running for office, as well as information on votes received by each party, number of citizens with the right to vote, voters, turnout, number of blank ballots, number of non-valid ballots. Notice that, in the original data source, around 400 elections are missing in 1979 and 1983.

# Seats

We received data through personal communication with *Ministerio del Interior* on the seat distribution across parties in all municipality councils relative to all election terms between 1979 and 2011. The data contain information on the number of seats that each party received, as well as the total number of seats in the municipality council. We make this data accessible in the replication folder but it can also be requested by submitting an information request at *Portal de transparencia* (transparencia.gob.es).

#### Mayors

We use yearly information on mayors in office for all municipalities from *Ministerio de Política Territorial y Función Pública* for the years 1979-2014 (Ministerio de Politica Territorial y Funcion Publica, 1979 - 2014). The data contain information about the party affiliation of the mayor, as well as the date in which the mayor entered office.

We aggregate the data at the election level. In the case in which the identity of the mayor changes within a term, we keep the information relative to all mayors who have served. Our main dependent variable, *Mayor Unseated*, is an indicator equal to one if, at some point during the term, the identity of the mayor changes and her party affiliation is different from the one of her predecessor.

# Ideology

We obtain information on ideology by merging our dataset to the 1999-2014 Chapel Hill Expert Survey (CHES) trend file. This dataset was constructed by Polk et al. (2017) and Bakker et al. (2015) and contains ideology measures of parties represented in the national Parliament between 1999 and 2014.<sup>29</sup> These parties are *PP*, *BNG*, *CC*, *CHA*, *CiU*, *EA*, *EH*, *ERC*, *IU*, *PA*, *PAR*, *PNV*, *PSOE*, and *UV*.

To define our measures of ideological distance, we use the variable *lrgen* in the CHES dataset, which measures the general ideology of each party on a scale from 1 (far left) to 10 (far right), after standardizing it and taking the absolute value. In addition to using the continuous variable, we also generate an indicator *far* equal to 1 if the distance between the largest party and the *marginal party*, defined as the party closest to the 5% entry threshold, is above the  $75^{th}$  percentile of the distance distribution. Similarly, we define *close* if the distance is below the  $25^{th}$  percentile. *Same*, instead, is an indicator for these two parties being both on the left or both on the right of the mean ideology among all parties represented in the Spanish Parliament between 1999 and 2014.

## Map shape file

To construct Figure 2, we accessed the shape file of all Spanish municipalities available at the National Geographical Institute webpage (Centro Nacional de Informacion Geografica, 2020).

## C.2. Sample selection

## Fragmentation and stability

The dataset for the analysis of the effect of fragmentation on stability is a party-level panel of municipalities, observed for all election years between 1979 and 2011 and containing all information from data sources described above. We restrict the sample to municipalities with population above 250 residents since the ones below are subject to a different voting rule, based on individual candidates rather than on party lists.

We drop 254 observations related to cases of elections where no seats were assigned (e.g., the whole municipality abstained), or the number of seats is incorrectly coded as an even number. Additionally, we drop 15,506 observations in which the mayor is coded to come from a generic civic list (*Asociación de electores*), or from a generic, unreported party. The reason for this is that in these cases our method to detect no confidence votes cannot be applied. Finally, we drop various cases of inconsistencies in the data, such as missing mayor information, turnout higher than 1, elections with only one party running and obtaining all votes. All these cases are coded in the dataset, available for download at the author's websites, in the variable *tag*. The final sample consists of 143,400 party-municipality observations from 42,259 unique municipal elections.

<sup>&</sup>lt;sup>29</sup>This data can be accessed at https://www.chesdata.eu/1999-2014-chapel-hill-expert-survey-ches-trend-file.

# **D.** Additional figures and results



FIGURE D.1 Evolution of the number of parties in Parliament over time

*Notes:* The vertical axis measures the average number of parties for all countries in the sample calculated in 8-year windows between 1947 and 2019. Time variable represented in the horizontal axis. Source: authors' elaboration based on the *parlgov* dataset (experimental version) by Döring and Manow (2019). The dataset contains information on national election results for 39 countries, including all EU and most OECD countries until 2019.



FIGURE D.2 Number of parties in Municipal Councils

*Notes:* Cumulative distribution of the number of parties represented in Spanish municipal councils between 1979 and 2014.



 $Figure \ D.3$  Density of the running variable around the threshold

*Notes:* Frequency histogram of the running variable used in the RDD on the effect of fragmentation on stability, in bins of 0.25 percentage points. A McCrary (2008) test of the null hypothesis of no discontinuous jump in the density at the threshold fails to reject the null with a p-value of 0.96. A Cattaneo, Jansson and Ma (2017) test, instead, yields a p-value of 0.57.



FIGURE D.4 Covariate Balancing Plots

*Notes:* Averages of different municipal characteristics near the threshold. Population and surface are in logarithms. PSOE mayor is an indicator for the mayor belonging to the socialist party PSOE and, similarly, PP mayor is an indicator for a mayor from the Popular Party. Council size is the number of available seats in the municipality. Parties with votes measures the number of parties that ran and obtained votes in the municipal election. Valid votes is the total number of valid votes cast (including blanks) divided by the total number of votes. Blank votes is the total number of blank votes divided by the total number of votes. Dots are averages in 0.25 percentage points bins of the running variable and lines are nonparametric local linear regressions estimates.

 $Figure \ D.5$  Predicted changes in stability as a function of the Entry Threshold



*Notes:* This figure reports the predicted number of parties as well as the predicted probability of unseating the mayor as a function of entry thresholds, holding the distribution of votes constant. We retrieve the number of parties for any variation in the admission threshold between 0% (no admission threshold) and 10% of valid votes, by applying the D'Hondt rule on observed election results in our sample. Then, we apply the coefficient estimated in Table 2 to retrieve, for each potential admission threshold, the change in probability of no-confidence vote compared to the case of a 5% entry threshold, observed in the data.

FIGURE D.6 Robustness to Bandwidth choice



*Notes:* 2SLS estimates of the effect of fragmentation on the probability of unseating the mayor for different bandwidth choices (eq. 2). The horizontal axis represents the bandwidth used in estimation. The solid line shows the estimated coefficient values, the dashed lines are 95% confidence intervals, whereas the dotted lines are 90% confidence intervals. Controls: surface and population (in logs). FE: number of available seats and election year fixed effects. The vertical dotted line represents the CCT optimal bandwidth. Standard errors are clustered at the municipality level.

FIGURE D.7 Robustness to Bandwidth choice: Additional Specifications



*Notes:* 2SLS estimates of the effect of fragmentation on the probability of unseating the mayor for different bandwidth choices (eq. 2). Panel A corresponds to estimates obtained for the subset of municipalities with 17 or more seats in the council. Panel B corresponds to estimates obtained without weighting for the number of parties running for election. The horizontal axis represents the bandwidth used in estimation. The solid line shows the estimated coefficient values, the dashed lines are 95% confidence intervals, whereas the dotted lines are 90% confidence intervals. Controls: surface and population (in logs). FE: number of available seats and election year fixed effects. The vertical dotted line represents the CCT optimal bandwidth. Standard errors are clustered at the municipality level.



 $Figure \ D.8$  Reduced form estimates for different placebo values of the threshold

*Notes:* Reduced-form estimates of the effect of crossing the admission threshold on the probability of unseating the mayor for different placebo values of the entry threshold. The dependent variable is always an indicator taking value 1 if the mayor was unseated by a vote of no confidence during the legislature. Each point in the horizontal axis represent different values of the admission threshold, from 1 to 10%. For instance, the first point shows point estimates and 95% confidence intervals of the discontinuity present at the 1% vote-share threshold. The bandwidth is 1.7 percentage points at either side of the threshold in all specifications to be consistent with the baseline estimate. Standard errors clustered at the municipality level. The result for the 5% vote-share admission threshold is highlighted.

Covariate Balancing Checks					
	(1)	(2)	(3)		
	Popul.	Surface	PSOE Mayor		
Above threshold	-0.063	-0.051	-0.017		
	(0.056)	(0.047)	(0.020)		
Mean of dep.var.	8.868	4.950	0.441		
Bandwidth	0.017	0.017	0.017		
Obs.	11293	11109	11293		
	PP Mayor	Election year	Council size		
Above threshold	0.020	0.144	-0.047		
	(0.017)	(0.391)	(0.192)		
Mean of dep.var.	0.24	1997.17	14.68		
Bandwidth	0.017	0.017	0.017		
Obs.	11293	11293	11293		
	Parties w. votes	Valid votes	Blank votes		
Above threshold	-0.013	0.002	-0.000		
	(0.061)	(0.001)	(0.001)		
Mean of dep.var.	5.394	0.988	0.013		
Bandwidth	0.017	0.017	0.017		
Obs.	11292	11292	11292		

TABLE D.1	
OVARIATE BALANCING	Снеск

*Notes:* 2SLS estimates of the effect of the number of parties on different covariated. Population and surface are in logarithms. PSOE mayor is an indicator for the mayor belonging to the socialist party PSOE and, similarly, PP mayor is an indicator for a mayor from the Popular Party. Council size is the number of available seat in the municipality. Parties with votes measures the number of parties that ran and obtained votes in the municipal election. Valid votes is the total number of valid votes cast (including blanks) divided by the total number of votes. Blank votes is the total number of blank votes divided by the total number of votes. Estimation by local linear regression using a fixed bandwidth equal to the CCT optimal bandwidth used in table 2. No controls or FE are included. Standard errors are clustered at the municipality level.

TABLE D.2							
First-Stage Results							
	(1)(2)(3)(4)N. PartiesN. PartiesN. PartiesN. Parties						
Above threshold	0.244 (0.044)	0.262 (0.036)	0.258 (0.035)	0.251 (0.035)			
F-stat.	30.39	53.27	53.18	50.71			
Mean of dep.var.	3.426	3.424	3.426	3.424			
Bandwidth	0.017	0.017	0.017	0.017			
Obs.	11293	11109	11293	11109			
Fixed Effects	Ν	Ν	Y	Y			
Controls	Ν	Y	Y	Y			

*Notes:* OLS estimates of the first-stage equation 3. The optimal bandwidth is calculated using the CCT method on equation 2. Controls and FE are included as specified in each column. Controls: surface and population (in logs). FE: number of available seats and year fixed effects. Standard errors are clustered at the municipality level.

	(1) P(Majority)	(2) P(Majority)	(3) P(Majority)	(4) P(Majority)
N. Parties	-0.092 (0.064)	-0.118 (0.058)	-0.101 (0.059)	-0.118 (0.061)
Mean of dep.var	0.628	0.628	0.628	0.628
Bandwidth	0.018	0.018	0.018	0.018
Obs.	11540	11353	11540	11353
Fixed Effects	Ν	Ν	Y	Y
Controls	Ν	Y	Ν	Y

TABLE D.3 2SLS Estimates - Fragmentation and Single-Party Majorities

*Notes:* 2SLS estimates of the effect of number of parties on the probability that the largest party has the absolute majority of seats. The dependent variable is an indicator taking value 1 if one party has strictly more than half of the seats in the municipality council. Controls and FE are included as indicated in each column. Controls: surface and population (in logs). FE: number of available seats and year fixed effects. The optimal bandwidth is calculated using the CCT method. Standard errors are clustered at the municipality level.

2SLS Estimates - Fragmentation and Stability by Single-Party Majorities					
	(1)	(1) (2) (3		(4)	
	Mayor Unseated	Mayor Unseated	Mayor Unseated	Mayor Unseated	
A. 2SLS Results	(No Single-Party	v Majorities)			
N. Parties	0.083	0.092	0.099	0.096	
	(0.041)	(0.049)	(0.048)	(0.050)	
Mean of dep.var.	0.092	0.093	0.092	0.093	
Bandwidth	0.017	0.017	0.017	0.017	
Obs.	4187	4111	4187	4111	
<b>B. 2SLS Results</b>	(Single-Party Ma	ajorities)			
N. Parties	0.004	0.004	0.003	0.003	
	(0.014)	(0.012)	(0.011)	(0.011)	
Mean of dep.var.	0.002	0.002	0.002	0.002	
Bandwidth	0.017	0.017	0.017	0.017	
Obs.	7106	6998	7106	6998	
Fixed Effects	Ν	Ν	Y	Y	
Controls	Ν	Y	Ν	Y	

TABLE D.4

*Notes:* 2SLS estimates of the effect of the number of parties on the probability of unseating the mayor (equation 2). The dependent variable is an indicator taking value 1 if the mayor was unseated by a vote of no confidence during the legislature. Panel A: only legislatures where no single party has more than half the seats. Panel B: only legislatures where there is a party with at least half the seats. Controls and FE are included as indicated in each column. Controls: surface and population (in logs). FE: number of available seats and year fixed effects. The optimal bandwidth is calculated using the CCT method. Standard errors are clustered at the municipality level.

				-	
	(1) Mayor uns.	(2) Mayor uns.	(3) Mayor uns.	(4) Mayor uns.	(5) Mayor uns.
D	0.011	0.006	0.010	0.017	0.018
	(0.011)	(0.012)	(0.011)	(0.011)	(0.012)
$D \times distance$		0.006			
		(0.009)			
$D \times 1(far)$			0.006		
			(0.015)		
$D \times 1(close)$				-0.019	
				(0.011)	
$D \times 1(same)$					-0.014
					(0.013)
Mean of Dep.var.	0.026	0.026	0.026	0.026	0.026
Bandwidth	0.017	0.017	0.017	0.017	0.017
Obs.	3000	3000	3000	3000	3000
<b>Fixed Effects</b>	Y	Y	Y	Y	Y
Controls	Y	Y	Y	Y	Y

TABLE D.5Reduced-form estimates of the entry of a marginal party, by ideology

*Notes:* Reduced-form estimates of the effect of crossing the entry threshold on the probability of unseating the mayor. The dependent variable is an indicator taking value 1 if the mayor was unseated by a vote of no confidence during the legislature. In column 2 we include, in addition to the indicator D for crossing the threshold, an interaction with a continuous measure of ideological distance between the largest party and the *marginal party* (defined as the party closest to the 5% threshold). In column 3 and 4 we include interactions with indicators for this distance being above the 75<sup>th</sup> percentile or below the  $25^{th}$  percentile of the distance's distribution, respectively. In column 5 we include an interaction with an indicator for these two parties being on the same size of the ideological spectrum (i.e. both to the left or both to the right of the mean ideology). The bandwidth is calculated using the CCT method. Controls and FE are included as indicated in each column. Controls: surface and population (in logs). FE: number of available seats and year fixed effects. Standard errors clustered at the municipality level.

Robustness Checks II - Removing one election at a time					
	1979	1983	1987	1991	1995
N. Parties	0.055 (0.027)	0.051 (0.027)	0.049 (0.028)	0.057 (0.027)	0.039 (0.023)
Mean of dep.var. Bandwidth Obs.	0.037 0.017 10456 1999	$\begin{array}{c} 0.037 \\ 0.017 \\ 10290 \\ 2003 \end{array}$	0.038 0.017 9850 2007	0.034 0.017 9687 2011	0.036 0.017 9913
N. Parties	0.047 (0.025)	0.069 (0.028)	0.037 (0.029)	0.062 (0.028)	
Mean of dep.var. Bandwidth Obs.	$0.035 \\ 0.017 \\ 9837$	$0.036 \\ 0.017 \\ 9760$	$0.032 \\ 0.017 \\ 9615$	$0.037 \\ 0.017 \\ 9464$	

TABLE D.6

*Notes*: In each column, we report 2SLS estimate of the effect of fragmentation on stability obtained from estimating equation 2 excluding one full election term at a time, as specified by the column header. The CCT bandwidth is kept constant at the full sample value of 1.7 percentage points. No controls or fixed effects are included. Standard errors clustered at the municipality level.