A Online Appendix for "A Review of Robert Sugden's *Community of Advantage*"

A.1 A Remark on Time-Inconsistency (for Section 3)

Theorem A.1 verifies the assertion in section 3.

Theorem A.1 (Retrospective Pareto Inefficiency)**.** *In the example of section 3, whether Alice is naïve or sophisticated, there exists an $\varepsilon > 0$ small enough such that each of Alice's selves is strictly better off if her time-1 self consumes ε less and her time-3 self consumes ε more than she does when left to her own devices.*

Proof. Naïve Alice fails to foresee that her future selves may frustrate her present self's consumption plan.

Sophisticated Alice's chosen consumption bundle is the outcome of the subgame perfect equilibrium of the game between her three selves: her time-3 self consumes whatever she inherits from her time-2 self; her time-2 self optimally allocates her inheritance from her time-1 self over periods $t = 2$ and $t = 3$ in the knowledge that her time-3 self will consume whatever is left to her; and her time-1 self chooses her consumption at $t = 1$ in the knowledge that her time-2 self will dispose of her inheritance in a manner that she, her time-2 self, finds optimal.

In the present example, whether naïve or sophisticated, Alice ends up consuming

$$x_1 = \Lambda, \quad x_2 = \frac{\delta\beta\,(1+\beta)}{1+\beta\delta}\Lambda, \quad \text{and} \quad x_3 = \frac{(\delta\beta)^2\,(1+\beta)}{1+\beta\delta}\Lambda, \qquad \text{where} \quad \Lambda \equiv \frac{e}{1+\delta\beta+\delta\beta^2}.$$

Any ε-intervention described in the theorem's statement makes the time-2 and the time-3 selves strictly better off. The time-1 self is also strictly better off provided ε is small enough:

$$\frac{\partial}{\partial\varepsilon}\left(\ln(x_1-\varepsilon)+\delta\beta\ln x_2+\delta\beta^2\ln(x_3+\varepsilon)\right)\big|_{\varepsilon=0} = \frac{1-\delta}{\delta\,(1+\beta)\,\Lambda} > 0,$$

where the inequality holds because $\delta < 1$. ∎

A.2 The Context and the Proof of Theorem 1 (for Section 4)

In the **generalized contracting problem**, the set of agents is I, with a typical agent i. The set of feasible contracts is X, with a typical contract x. Each contract names a subset of agents and the terms on which these agents transact. An allocation is a subset $A \subset X$ of contracts; not all subsets need be feasible allocations.

There are **no contracting externalities**; each agent only cares about the contracts that name him. There may or may not be externalities in the Walrasian sense: an agent may care not only about his own clause (think bundle) in a contract (think allocation) but also about others' clauses. In many social choice settings of interest, a contract cannot be naturally partitioned into individual-specific clauses.

Agent i's budget set is denoted by $B_i \subset X$ and is interpreted as the set of contracts that agent i believes to be available to him. A collection of budget sets $(B_i)_{i \in I}$ is **opportunity efficient** (equivalently, satisfies the **opportunity criterion**) if it covers the entire feasible set: $X \subset (\cup_{i \in I} B_i)$. A **budget equilibrium** is a feasible allocation A and supporting budget sets $(B_i)_{i \in I}$ such that each agent i chooses from B_i every contract that names him in A.[A.1] A **budget equilibrium is opportunity efficient** if its supporting budget sets are opportunity efficient.[A.2]

The described environment encompasses three common settings, each featured in theorem 1:

1. In a **pure exchange economy**, X is a set of contracts of the form $x \equiv (x_i)_{i \in I}$, where x_i is agent i's nonnegative consumption bundle, and where $\sum_{i \in I} x_i \leq \sum_{i \in I} e_i$ for some nonnegative endowment profile $(e_i)_{i \in I}$. Each contract is **comprehensive**: it names (implicitly) every agent in I and specifies (explicitly) what each agent consumes. A **Walrasian equilibrium** is a budget equilibrium $\left(A, (B_i)_{i \in I}\right)$ in which (i) A is a singleton; (ii) for some nonnegative price vector p and for all $i \in I$, we have $B_i \equiv \{x \in X \mid p \cdot x_i \leq p \cdot e_i\}$; and (iii) each agent chooses exactly one contract from his budget set.[A.3] Following Kreps (2012, Section 15.5), a **Lindahl equilibrium** replaces (ii) above with the requirement that, for some nonnegative price vector p, for some collection $t \equiv \left(t_{ij}\right)_{i,j \in I}$ of personalized transfer price vectors (with $t_{ij} \cdot x_i$ being the transfer from i to j for the privilege to consume x_i), and for all $i \in I$,

$$B_i \equiv \left\{ x \in X \mid p \cdot x_i + \sum_{j \in I \setminus \{i\}} t_{ij} \cdot x_i \leq p \cdot e_i + \sum_{j \in I \setminus \{i\}} t_{ji} \cdot x_j \right\}.$$

[A.1] The budget equilibrium is introduced by Segal (2007, p. 349), whose Footnote 10 cites prior art.

[A.2] The opportunity efficient budget equilibrium is introduced under the (nonstandard but quite apt) moniker "Lindahl equilibrium" in Definition 2 of Nisan and Segal (2003).

[A.3] Each agent chooses an entire contract x, not just his own bundle x_i. This modeling approach accommodates (without insisting on) externalities and is adopted by Kreps (2012, Section 15.5), for instance.

2. In a **pure public good economy** (Thomson, 1999), X is a set of contracts of the form $x \equiv (x_i)_{i \in I}$, where $x_i \equiv (z_i, y)$ describes agent i's nonnegative consumption z_i of a private good and his nonnegative consumption y of a public good, and where $\sum_{i \in I} x_i + y \leq \sum_{i \in I} e_i$ for some nonnegative endowment profile $(e_i)_{i \in I}$ of the private good. Each contract is comprehensive. A **Lindahl equilibrium** is a budget equilibrium $\left(A, (B_i)_{i \in I}\right)$ in which (i) A is a singleton; (ii) for some personalized prices $(p_i)_{i \in I}$ and for all $i \in I$, we have $B_i \equiv \{x \in X \mid z_i + p_i y \leq e_i\}$, where $\sum_{i \in I} p_i = 1$ is the zero-profit condition for the (subsumed) firm that converts a unit of the private good into a unit of the public good; and (iii) each agent chooses exactly one contract from his budget set.

3. In a **many-to-one two-sided matching problem with contracts**, I contains workers and firms. The contract set X is a finite set of bilateral contracts, each of which names a worker and a firm, plus the null contract. Each nonnull contract specifies the terms (e.g., wages and hours) on which the worker–firm pair match. If no contract in X has terms, the matching problem is **without contracts**. A **stable match** is a budget equilibrium $\left(A, (B_i)_{i \in I}\right)$ in which (i) A need not be a singleton; (ii) $(B_i)_{i \in I}$ are as deployed by Hatfield and Milgrom (2005, Theorem 1) or Segal (2007, Proposition 5); and (iii) each worker chooses exactly one contract from his budget set, and each firm chooses one contract or multiple.

Proof of theorem 1. If $x \in X$ and $x \notin (\cup_{i \in I} B_i)$, then, for all $i \in I$, we have $p \cdot x_i > p \cdot e_i$ for part 1, we have $p \cdot x_i + \sum_{j \neq i} t_{ij} \cdot x_i > p \cdot e_i + \sum_{j \neq i} t_{ji} \cdot x_j$ for exchange economies with externalities in part 2, and we have $z_i + p_i y > e_i$ for exchange economies with a public good in part 2. Adding up the corresponding inequalities gives $p \cdot \sum_{i \in I} (x_i - e_i) > 0$ for part 1 and for exchange economies with externalities in part 2, and gives $\sum_{i \in I} x_i + y > \sum_{i \in I} e_i$ for exchange economies with a public good in part 2. Either inequality contradicts $x \in X$. Conclude that $x \in X$ implies $x \in (\cup_{i \in I} B_i)$.

Part 3 follows from the definition of the matching equilibrium, whose supporting budget sets "partition" and, therefore, cover the feasible set. That the specified budget sets indeed cover the feasible set follows from the discussion preceding theorem 1 of Hatfield and Milgrom (2005, p. 917).[A.4] ■

[A.4] Segal's (2007) Proposition 5 covers the special case of many-to-one matching without contracts. Segal is explicit about the coverage property of the supporting budget sets when calling his budget equilibria partitional.

A.3 The Context and the Proofs of Theorems 2 and 3 (for Section 4)

A **social choice problem** is a special case of the generalized contracting problem. The contract set X is an arbitrary set of comprehensive contracts. Each agent i's preferences over X are described by the **lower contour set** function L_i, where, for any comprehensive contract $x \in X$, the set $L_i(x)$ comprises all contracts in X that agent i does not strictly prefer to x. A budget equilibrium is an singleton allocation $A = \{x\}$ and some budget sets $(B_i)_{i \in I}$, with each agent choosing exactly one contract from his budget set: for all $i \in I$, we have $B_i \subset L_i(x)$, meaning that no contract in agent i's budget set can make him strictly better off than x.

An allocation $x \in X$ is **weakly Pareto efficient** if $X \subset (\cup_{i \in I} L_i(x))$, meaning that no contract in X can make every agent strictly better off.

Proof of theorem 2. The proof follows Nisan and Segal (2003, Proposition 1, p. 12).

For part 1, let $(x, (B_i)_{i \in I})$ be an opportunity efficient budget equilibrium. Combining opportunity efficiency, $X \subset (\cup_{i \in I} B_i)$, with the budget equilibrium, $B_i \subset L_i(x)$ for all $i \in I$, implies weak Pareto efficiency of x: $X \subset (\cup_{i \in I} L_i(x))$.

For part 2, let $x \in X$ be a weakly Pareto efficient allocation: $X \subset (\cup_{i \in I} L_i(x))$. For each agent $i \in I$, define the supporting budget set $B_i \equiv L_i(x)$. By construction, $(x, (B_i)_{i \in I})$ is a budget equilibrium. Weak Pareto efficiency, $X \subset (\cup_{i \in I} L_i(x))$, and the construction of the budget sets imply opportunity efficiency: $X \subset (\cup_{i \in I} B_i)$. ■

Given coalition-feasible sets $(X_S)_{S \subset I}$, an allocation $x \in X$ is in the **weak core** if $X_S \subset (\cup_{i \in S} L_i(x))$ for all $S \subset I$, meaning that no contract in X can make every agent in S strictly better off.

Proof of theorem 3. Fix coalition-feasible sets $(X_S)_{S \subset I}$.

For part 1, let $(x, (B_i)_{i \in I})$ be a strong opportunity efficient budget equilibrium. Combining strong opportunity efficiency, $(\forall S \subset I)\, X_S \subset (\cup_{i \in S} B_i)$, with the budget equilibrium, $B_i \subset L_i(x)$ for all $i \in I$, implies that x is in the weak core: $(\forall S \subset I)\, X_S \subset (\cup_{i \in S} L_i(x))$.

For part 2, let an allocation $x \in X$ be in the weak core: $(\forall S \subset I)\, X_S \subset (\cup_{i \in S} L_i(x))$. For each agent $i \in I$, define the supporting budget set $B_i \equiv L_i(x)$. By construction, $(x, (B_i)_{i \in I})$ is a budget equilibrium. The weak core property, $(\forall S \subset I)\, X_S \subset (\cup_{i \in S} L_i(x))$, and the construction of the budget sets imply strong opportunity efficiency: $(\forall S \subset I)\, X_S \subset (\cup_{i \in S} B_i)$. ■

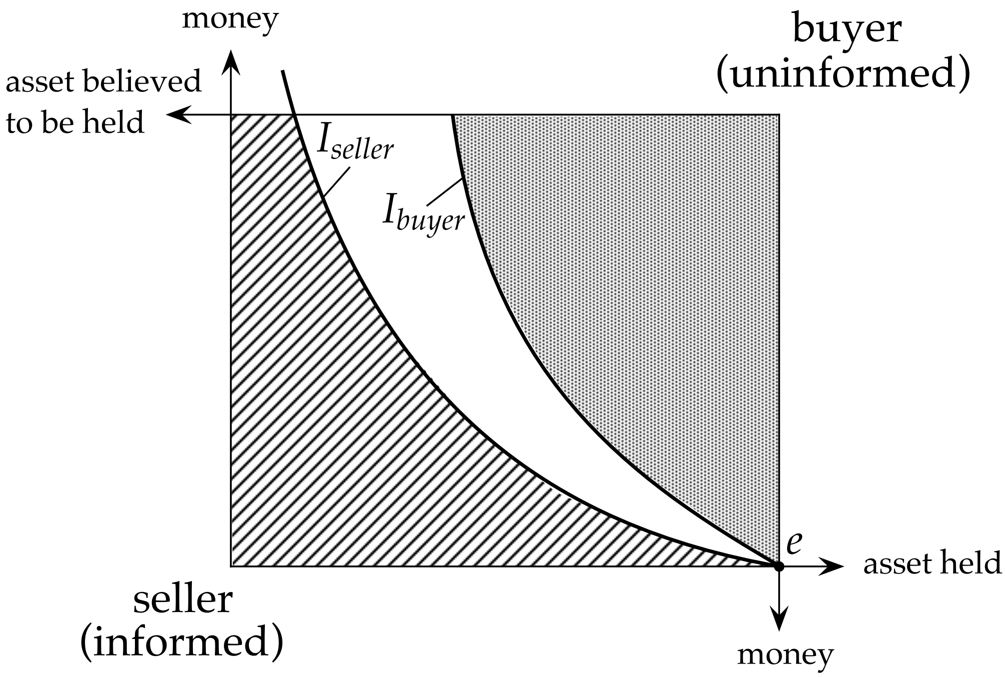

Figure A.1: **Agents' equilibrium budget sets cannot cover the Edgeworth box.** The (informed) seller's lower contour set relative to the endowment point e is the dashed area demarcated by his indifference curve I_{seller}. The (uninformed) buyer's lower contour set relative to the endowment point e is the dotted area demarcated by his indifference curve I_{buyer}, which curves towards the origin due to adverse selection. These lower contour sets are the maximal budget sets the agents can have and still choose the endowment point e.

A.4 Opportunity Criterion with Asymmetric Information (for Section 4)

Section 4 proposes to think about opportunity efficiency under asymmetric information in terms of state-contingent, objective allocations. Here is another way, which focuses on perceived, subjective allocations.

Consider an Edgeworth box economy in Figure A.1. The two goods are a financial asset of uncertain quality and money of certain quality. The seller is endowed with the financial asset and no money. The buyer is endowed with money and no financial asset. The quality of the asset is known to the seller but not the buyer, and is fixed as far as the figure is concerned.

In the figure, the lower horizontal axis measures the quality-adjusted amount of the asset that the seller consumes. The upper horizontal axis measures the expected quality-adjusted amount of the asset that the buyer believes he consumes. The vertical axes measure the money consumed.

Assume that, for the fixed quality of the asset, no trade occurs: each agent consumes his endowment. To construct the maximal budget sets that support this no-trade equilibrium, set each agent's budget set to be the lower contour set with respect to the endowment point. Because, in the figure, these budget sets do not cover the entire Edgeworth box, we say that the opportunity criterion in the economy with asymmetric information fails. The "problem" is with the buyer. He believes that if the seller is willing to sell more, then the reason must be that the seller knows that the asset is worth less. Adverse selection renders the buyer's indifference curve concave, rather than convex (which is the classical case, which reflects the love of variety or risk aversion).

A.5 Competing Definitions of the Opportunity Criterion (for Section 4)

CA defines opportunity efficiency for Walrasian economies and does so in conjunction with equilibrium. CA's definition is Sugden's (2004) except it is cast in terms of trades, rather than allocations. Sugden's (2004) definition is easier to understand. The definition in section 4 maintains the spirit of Sugden (2004) but, in addition to Walrasian economies, applies to economies with externalities and public goods, as well as matching problems, and is easy to visualize and analyze. When Sugden's (2004) definition is decoupled from equilibrium, the two definitions are equivalent when no agent is indifferent between any two allocations, which is "generically" true in social choice problems with finitely many feasible allocations.

For Sugden (2004), an opportunity efficient equilibrium in a pure exchange economy is an allocation and a price vector such that (a) agents' choices from the budget sets demarcated by the price vector add up to a feasible allocation, and (b) any alternative feasible allocation is in the budget set of some agent (c) whose bundle in this alternative allocation differs from his bundle in the equilibrium allocation.[A.5] Part (a) is market clearing. This part conflates equilibrium with opportunity efficiency. Part (b) says that every allocation is in someone's budget set. This part is the essence of opportunity efficiency. Part (c) condemns as wasted for the purpose of freedom accounting any disequilibrium allocation that, even though in some agent's budget set, prescribes to that agent his equilibrium bundle and, thus, offers no distinct opportunity to him. This part has limited appeal. While proposing a notion of duplicate opportunity, part (c) refuses to apply this notion to disequilibrium bundles; an agent's budget set may contain multiple allocations with duplicate bundles as long as these bundles differ from his equilibrium bundle. The appeal of

[A.5] Sugden (2004, p. 1020): "[E]very feasible allocation other than the one that has in fact come about assigns to some consumer a bundle that that consumer had, but did not take, the opportunity to achieve."

part (c) diminishes further in the presence of externalities and vanishes when the set of feasible contracts is the set of social alternatives (e.g., candidates in an election) none of which can be naturally decomposed into private bundles. Parts (a) and (c) are absent from the definition of the opportunity criterion in section 4.

One could aim to strengthen part (c) by condemning as wasted any additional opportunity that delivers an agent the same bundle as some other opportunity in his budget set. Formally, in a pure exchange economy, budget sets satisfy the **no duplicate opportunity criterion** (NDOC) if, for each agent, no two allocations in his budget set assign him the same bundle. Theorem A.2 shows that NDOC is incompatible with the opportunity criterion (as defined in section 4) at equilibrium. In other words, Sugden's (2004) definition of the opportunity criterion cannot be usefully strengthened by requiring consistent application of the no-duplicates condition in part (c).

Theorem A.2 (Impossibility)**.** *One can construct examples of pure exchange economies for which no budget equilibrium satisfies both the opportunity criterion and the no duplicate opportunity criterion (NDOC).*

Proof. Consider a pure exchange economy with two agents and the set of feasible allocations

$$X = \left\{ (x_1, x_2), (x_1, x_2'), (x_1', x_2), (x_1', x_2') \right\},$$

where $x_1 \neq x_1'$ and $x_2 \neq x_2'$. By the symmetry of X, designate an arbitrary allocation, say, (x_1, x_2), as the budget-equilibrium allocation. This allocation must be in each agent's budget set: $B_1 = \{(x_1, x_2), \ldots\}$ and $B_2 = \{(x_1, x_2), \ldots\}$. Then, NDOC requires that $(x_1, x_2') \notin B_1$ and $(x_1', x_2) \notin B_2$. Therefore, for opportunity efficiency to hold, it is necessary that $B_1 = \{(x_1, x_2), (x_1', x_2), \ldots\}$ and $B_2 = ((x_1, x_2), (x_1, x_2'), \ldots)$. Then, NDOC requires that $(x_1', x_2') \notin B_1 \cup B_2$, thereby making construction of an opportunity efficient pair (B_1, B_2) impossible. ■

Finally, both NDOC and part (c) have a certain illiberal flavor. NDOC, in particular, deems illegitimate any contribution to individual freedom that stems from controlling others' bundles.[A.6] Condemning some sources of freedom as unworthy is akin to condemning some sources of utility (e.g., schadenfreude or a taste for mud fights) as too base to merit inclusion in welfare calculus. There is a trace of paternalism in this attitude.

[A.6] NDOC's cousin in axiomatic resource allocation problems is **nonbossiness**, which requires the allocation rule to be such that no agent can change what another agent gets without changing what he himself gets.