Online Appendix
Negotiations with Limited Specifiability

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A Proofs for the Results in the Main Text

A.1 Proofs for Section 4

Proof of Theorem 1

We use Theorem 2 (in Section 4.2) to prove Theorem 1. The proof of Theorem 2 does not depend on Theorem 1.

"If" part: Since Theorem 2 implies that $x^* \in X^M$ is a SPE outcome, we show that x^* is a unique SPE outcome. Consider the shortest terminal history h under which each player i announces (Yes, P_i), where $\bigcap_{i \in N} P_i = \{x^*\}$ (such a history h exists by assumption). At the history $h^{t(h)-1}$, player $i_1 = \rho(h^{t(h)-1})$ can guarantee herself a payoff of $u_{i_1}(x^*)$, her maximum possible SPE payoff (note that any $y \in X$ with $u_i(y) > u_i(x^*)$, if it exists, is not individually rational for some other player). Since $u_{i_1}(x^*) > d_{i_1}$ by assumption, x^* is the unique outcome in the subgame starting at $h^{t(h)-1}$ in any SPE. Next, at $h^{t(h)-2}$, player $i_2 = \rho(h^{t(h)-2})$ can guarantee herself a payoff of $u_{i_2}(x^*)$, her maximum possible SPE payoff. Since $u_{i_2}(x^*) > d_{i_2}$ by assumption, x^* is the unique outcome in the subgame starting at $h^{t(h)-2}$ in any SPE. Solving backwards in this way, for each $j \in \{1, \dots, t(h)\}$, at any $h^{t(h)-j}$, player $i_j = \rho(h^{t(h)-j})$ can guarantee herself a payoff of $u_{i_j}(x^*)$, her maximum possible SPE payoff. Hence, x^* is the unique SPE outcome in the subgame starting at the initial history h^0.

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“Only if” part: Suppose that x^* is a unique SPE outcome. First, note that $X^M = \{x^*\}$ because Theorem 2 implies that if $X^M (\neq \emptyset)$ is not a singleton then the negotiation has multiple SPE outcomes. Next, if there is $x \in X \setminus \{x^*\}$ such that $u_i(x) = \max_{v \in \mathrm{IR}(U,d)} v_i$ for some $i \in N$, then $x \in X^M$, a contradiction. Hence, for each $i \in N$, x^* is the unique alternative that generates her maximum individually-rational payoff. Thus, $u(x^*) > v$ for all $v \in \mathrm{IR}(U,d) \setminus \{u(x^*)\}$, as desired. □

Proof of Theorem 2

Let $x^{(0)} := x \in X^M$. For each $j \in N$, let $x^{(j)} \in X$ be player j's worst individually-rational and Pareto-efficient alternative. For each $j \in \{0\} \cup N$, fix $(P_i^{(j)})_{i \in N}$ with $\{x^{(j)}\} = \bigcap_{i \in N} P_i^{(j)}$ (such a profile exists by assumption). Denote $P_i = P_i^{(0)}$ for each $i \in N$. Note that it is possible that $x = x^{(j)}$ for some $j \in N$.

Let h^* be the shortest terminal history under which every player i announces (Yes, P_i) at any subhistory at which she speaks (such h^* exists by assumption). Let $Q_0 := \{h \in H \setminus Z \mid h \sqsubseteq h^*\}$ be the set of non-terminal subhistories of h^*. For each $j \in N$, let Q_j be the set of non-terminal histories $h \in (H \setminus Z) \setminus Q_0$ such that player j deviates from announcing (Yes, P_j) first: $j = \min I^\tau(h)$ with $\tau = \min\{t' \in \mathbb{N} \mid (R_i^{t'}(h), P_i^{t'}(h)) \neq (\text{Yes}, P_i)$ for some $i \in N\}$.

We define the following strategy profile s^*. For each $i \in N$ and $h \in H_i$, let

$$s_i^*(h) := \begin{cases} (\text{Yes}, P_i) & \text{if } h \in H_i \cap Q_0 \\ s_i^{(j)}(h) & \text{if } h \in H_i \cap Q_j \text{ for some } j \in N \end{cases},$$

where $s_i^{(j)}(h)$ is defined as

$$s_i^{(j)}(h) := \begin{cases} (\text{Yes}, \tilde{P}_i(h)) & \text{if } h \in H_i \cap Q_{j,1} \\ (\text{Yes}, P_i^{(j)}) & \text{if } h \in H_i \cap Q_{j,2} \\ (\text{No}, P_i^{(j)}) & \text{if } h \in H_i \cap (Q_j \setminus (Q_{j,1} \cup Q_{j,2})) \end{cases}$$

with the following properties. Intuitively, the set $Q_{j,1}$ contains any non-terminal history $h \in Q_j$ such that there are a group of players who can collectively terminate the negotiation with an outcome $\tilde{x} \in X \setminus \{x^{(j)}\}$ at h, which gives a strictly greater payoff to each of them. Note that $Q_{j,1}$ could be empty (e.g., in negotiations with a common-interest alternative). Formally, $h \in Q_{j,1}$ if and only if (i) $h \in Q_j$ and (ii) there are a sequence $\tilde{h}(k^*) := ((N_\ell, ((\text{Yes}, \tilde{P}_m(h)))_{m\in N_\ell}))_{\ell=1}^{k^*}$ with $k^* \in \mathbb{N}$ and $\tilde{x} \in X \setminus \{x^{(j)}\}$ such that, denoting $\tilde{h}(k) := ((N_\ell, ((\text{Yes}, \tilde{P}_m(h)))_{m\in N_\ell}))_{\ell=1}^{k}$ for each $k \in \{1, \ldots, k^*\}$, $N_1 = \rho(h)$, $N_{\ell+1} = \rho(h, \tilde{h}(\ell))$ for all $\ell \in \{1, \ldots, k^*-1\}$, $\varphi^{\text{con}}(h, \tilde{h}(k^*)) = \tilde{x} \in X\setminus\{x^{(j)}\}$, and that $u_\ell(\tilde{x}) > u_\ell(x^{(j)})$ for all $\ell \in \bigcup_{k=1}^{k^*} N_k$. Note that $\tilde{x}$ depends only on h. This is because of the following: Since $x^{(j)}$ is Pareto efficient, $\bigcup_{k=1}^{k^*} N_k \neq N$ for any choice of $\tilde{h}(k^*)$. Also, any player in $N \setminus (\bigcup_{k=1}^{k^*} N_k)$ has to be ok with $\tilde{x}$ at h. Hence, $\tilde{x}$ does not depend on $\tilde{h}(k^*)$ but only on h. We denote this $\tilde{x}$ by $\tilde{x}(h)$.

The set $Q_{j,2}$ has any non-terminal history $h \in Q_j \setminus Q_{j,1}$ with the following properties: Either (i) every player ℓ who spoke at the end of h announced $(\text{No}, P_\ell^{(j)})$, i.e., $h = (h^{t(h)-1}, (I^{t(h)}(h), ((\text{No}, P_\ell^{(j)}))_{\ell\in I^{t(h)}(h)}))$; or (ii) every player ℓ has been announcing $(\text{Yes}, P_\ell^{(j)})$ since the most recent announcement of "No" at time $t^{\text{No}}(h) \leq t(h)-1$, i.e., $h = (h^{t^*}, ((I^k(h), ((\text{Yes}, P_\ell^{(j)}))_{\ell\in I^k(h)}))_{k=t^*+1}^{t(h)})$ with $t(h) - 1 \geq t^* := t^{\text{No}}(h)$.

We now show that each player $i \in N$ following s_i^* is a best response to s_{-i}^* in any subgame, which implies that the SPE s^* induces the history h^* and the outcome x. To show this, first consider a subgame starting at $h \in H_i \cap (Q_j \setminus Q_{j,1})$ for some $j \in N$. The continuation strategy profile $s^*|_h := (s_i^*|_h)_{i\in N}$ induces $x^{(j)}$, where $s_i^*|_h$ is the restriction of s_i^* on $\{h' \in H_i \mid h' \sqsupseteq h\}$. If player i announces "No" at $h' \in H_i$ with $h' \sqsupseteq h$, then it is impossible for any $x' \in X$ with $u_i(x') > u_i(x^{(j)})$ to be an outcome under s_{-i}^*. Suppose to the contrary that some alternative $x' = \varphi^{\text{con}}(h'')$ with $u_i(x') > u_i(x^{(j)})$ and $h'' \sqsupseteq h'$ is an outcome under s_{-i}^*. Since player i cannot terminate the negotiation with outcome x' at h' by saying "No," $h'' \sqsupset h'$. Since $u(x^{(j)}) \in \text{PE}(U)$, some player k with $u_k(x') \leq u_k(x^{(j)})$ is ok with x' at h''. This is impossible because such player k, who follows s_k^*, must not be okay with x' at h''.

For any strategy of player i such that she announces "Yes" after each history at which it is her turn to move in the the subgame starting at h, either every player k keeps announcing $(\text{Yes}, P_k^{(j)})$ to agree upon $x^{(j)}$ or some player announces "No" in the subgame starting at h. In the subgame starting at history h at which some player announces "No," no $x' \in X$ with $u_i(x') > u_i(x^{(j)})$ can be an outcome under $s_{-i}^*|_h$.

Second, consider $h \in H_i \cap Q_{j,1}$. If player i follows $s_i^*|_h$, then $s^*|_h$ induces $\tilde{x}(h)$. Suppose, on the other hand, player i deviates at a history $h' \in H_i$ with $h' \sqsupseteq h$. If she announces "No" at h', then any alternative x' with $u_i(x') > u_i(x^{(j)})$ (which includes $\tilde{x}(h)$) cannot be an outcome. If her announcement is "Yes," then either every player k keeps announcing $(\text{Yes}, \tilde{P}_k)$ to agree upon $\tilde{x}(h)$ or some player announces "No" at some point. In the latter case, any induced history $h \in H_i$ no longer belongs to $Q_{j,1}$.

Third, consider the subgame starting at $h \in H_i \cap Q_0$. Player i gets a payoff of $u_i(x)$ by following $s_i^*|_h$, as $s^*|_h$ induces h^*. Her deviation induces a non-terminal history $h' \in Q_i \setminus Q_{i,1}$, as any other player k follows s_k^*. Thus, player i's maximum possible payoff in the subgame starting at h' is $u_i(x^{(i)}) \leq u_i(x)$. $\square$

A.2 Proofs for Section 5

Proof of Proposition 5

As discussed in the main text, the proof consists of two steps.

First Step: Fix $i \in N$ satisfying the condition in the statement of the proposition. Let $Y_i := \{x \in X \mid u_i(x) < v^{[i,m]} \text{ and } u_{-i}(x) \geq v^{[-i,m]}\}$. Suppose to the contrary that there is $y \in X^{\text{SPE}} \cap Y_i$. Let s^* be a SPE that induces y, and let h^* be the finite terminal history induced by s^*. Let h be a subhistory of h^* such that $i \in \rho(h)$ and that if $h' \sqsubset h^*$ satisfies $i \in \rho(h')$ then $h' \sqsubseteq h$. Letting $(R_{-i}, P_{-i}) = (R_{-i}^{t(h)}(h), P_{-i}^{t(h)}(h))$ and $(R_i, P_i) = s_i^*(h)$, we have $P_1 \cap P_2 = \{y\}$. Since y is unilaterally improvable for player i, there exists $(y', P_i') \in X \times \mathcal{P}_i$ such that $u(y') > u(y)$ and $\{y'\} = P_i' \cap P_{-i}$. Choose one such (y', P_i'), and consider i's deviation to announce (Yes, P_i') at h. On the one hand, in the subgame starting at $(h, (\text{Yes}, P_i'))$, one strategy player $-i$ can take is to keep announcing (Yes, P_{-i}). The consensual termination rule terminates the negotiation under such a strategy profile at $(h, (\text{Yes}, P_i'), (\text{Yes}, P_{-i}))$ with the outcome y'. Thus, player $-i$'s payoff conditional on the history $(h, (\text{Yes}, P_i'))$ under $s^*|_{(h,(\text{Yes},P_i'))}$ is at least $u_{-i}(y')$. On the other hand, since s^* is a SPE, player i's deviation to announce (Yes, P_i') cannot lead to a payoff strictly higher than $u_i(y)$. These facts imply that $s^*|_{(h,(\text{Yes},P_i'))}$ leads to an outcome in $\{y' \in X \mid u_i(y') \leq u_i(y) \text{ and } u_{-i}(y') > u_{-i}(y)\}$.

Thus, there is an infinite sequence $(y^k)_{k \in \mathbb{N}}$ such that $y^{k+1} \in \{y' \in X \mid u_i(y') \leq u_i(y^k) < v^{[i,m]} \text{ and } u_{-i}(y') > u_{-i}(y^k) \geq v^{[-i,m]}\}$ for each $k \in \mathbb{N}$. By construction, $y^{k+1} \neq y^\ell$ for all $\ell \leq k$. This contradicts the assumption that X is finite.

Second Step: Pick $x \in X$ with $\underline{u} \leq u(x) < (v^{[1,m]}, v^{[2,m]})$. Suppose to the contrary that x is sustained by a SPE s^*. Let h^* be the finite terminal history induced by s^*. Let h be a subhistory of h^* such that $i \in \rho(h)$ and that if $h' \sqsubset h^*$ satisfies $i \in \rho(h')$ then $h' \sqsubseteq h$. Let $(R_i, P_i) = s_i^*(h)$ and $(R_{-i}, P_{-i}) = (R_{-i}^{t(h)}(h), P_{-i}^{t(h)}(h))$. We have $P_1 \cap P_2 = \{x\}$. Let $(P_1^{(-i)}, P_2^{(-i)})$ be such that $P_1^{(-i)} \cap P_2^{(-i)} = \{x^{(-i)}\}$, where

$x^{(-i)} \in X$ satisfies $u(x^{(-i)}) = (v^{[i,M]}, v^{[-i,m]})$.

At the history $h' = (h, (\text{No}, P_i^{(-i)}))$, if player $-i$ announces $(\text{Yes}, P_{-i}^{(-i)})$, then player i can receive the best SPE payoff $v^{[i,M]}$ by announcing $(\text{Yes}, P_i^{(-i)})$. Thus, player $-i$ can secure herself a payoff of $v^{[-i,m]}$ at h'. By the equilibrium condition, letting $y \in X$ be the outcome induced by $s^*|_{h'}$, we have $u_i(y) \leq u_i(x) < v^{[i,m]}$ and $u_{-i}(y) \geq v^{[-i,m]} (> u_{-i}(x))$. Now, one must be able to construct an infinite sequence defined in the first step, which leads to a contradiction. □

Proof of Proposition 6

Part 1: This part follows from Theorem 2.

Part 2: Fix $(i, x) \in N \times X$ such that $d_i \leq u_i(x) < v^{[i,m]}$ and $u_{-i}(x) \geq v^{[-i,m]}$. Fix (P_1, P_2) with $\{x\} = P_1 \cap P_2$. Let $(y^i, y^{-i}) = (x, x^{(-i)})$. That is, $u_{-i}(y^{-i}) = v^{[-i,m]}$ and $u_i(y^{-i}) = v^{[i,M]}$. Choose $(P_1^{(-i)}, P_2^{(-i)})$ such that $P_1^{(-i)} \cap P_2^{(-i)} = \{y^{-i}\}$. Note that the profiles of proposals (P_1, P_2) and $(P_1^{(-i)}, P_2^{(-i)})$ exist by assumption.

Let $Q_i \subseteq H \setminus Z$ be the set of non-terminal histories with the following two properties: First, $h^0 \in Q_i$. Second, any $h \in (H \setminus Z) \setminus \{h^0\}$ is in Q_i if and only if, for any $h' \in H \setminus Z$ with $h' \sqsubset h$ and $-i \in \rho(h')$, the history $h^{t(h')+1}$ satisfies the following:

$$h^{t(h')+1} = \begin{cases} (h', (\text{Yes}, P_{-i})) & \text{if } h' = h^0 \\ (h', (\text{Yes}, P_{-i})) & \text{if } h' \neq h^0 \text{ and } (R_i^{t(h')}(h'), P_i^{t(h')}(h')) = (\text{Yes}, P_i) \\ (h', (\text{No}, P_{-i})) & \text{if } h' \neq h^0 \text{ and } (R_i^{t(h')}(h'), P_i^{t(h')}(h')) \neq (\text{Yes}, P_i) \end{cases}.$$

We let $Q_{-i} := (H \setminus Z) \setminus Q_i$.

Consider the following strategy profile s^*. For player $i \in N$ and $h \in H_i$, let

$$s_i^*(h) := \begin{cases} (\text{Yes}, P_i) & \text{if } h \in H_i \cap Q_i \\ (\text{Yes}, P_i^{(-i)}) & \text{if } h \in H_i \cap Q_{-i} \text{ and } \varphi^{\text{con}}(h, (\text{Yes}, P_i^{(-i)})) = y^{-i} \\ (\text{No}, P_i^{(-i)}) & \text{if } h \in H_i \cap Q_{-i} \text{ and } \varphi^{\text{con}}(h, (\text{Yes}, P_i^{(-i)})) \neq y^{-i} \end{cases}.$$

Note that, at $h \in H_i \cap Q_i$, since $\mathcal{P}_i$ is limited and x is not unilaterally improvable for i, there is no $(y, P_i') \in X \times \mathcal{P}_i$ with $\varphi^{\text{con}}(h, (\text{Yes}, P_i'), (\text{Yes}, P_{-i})) = y$ and $u(y) > u(x)$.

For player $-i$, let $h \in H_{-i}$. If $h = h^0 \in H_{-i}$, then let $s_{-i}^*(h) := (\text{Yes}, P_{-i})$. If

$h \in Q_i \setminus \{h^0\}$, then let

$$s^*_{-i}(h) := \begin{cases} (\text{Yes}, P_{-i}) & \text{if } (R_i^{t(h)}(h), P_i^{t(h)}(h)) = (\text{Yes}, P_i) \\ (\text{Yes}, P_{-i}) & \text{if } t(h) \geq 2 \text{ and } h = (h^{t(h)-2}, (\cdot, P_{-i}), (\text{Yes}, \tilde{P}_i)) \text{ for some} \\ & (\tilde{P}_{-i}, y) \text{ with } \tilde{P}_i \cap P_{-i} = \{y\} \text{ and } u_{-i}(y) > u_{-i}(x) \\ (\text{No}, P_{-i}) & \text{otherwise} \end{cases}.$$

Next, we define $s^*_{-i}(h)$ for $h \in Q_{-i}$. First, define $Q^*_{-i} \subseteq Q_{-i}$ by $h \in Q^*_{-i}$ if and only if there are $\tilde{P}_{-i}$ and $\tilde{x}$ such that $\varphi^{\text{con}}(h, (\text{Yes}, \tilde{P}_{-i})) = \tilde{x}$ and $u_{-i}(\tilde{x}) > u_{-i}(y^{-i})$. Since $\tilde{x}$ depends only on h, we write $\tilde{x}(h) := \tilde{x}$. If $h \in Q^*_{-i}$, then $s^*_{-i}(h) := (\text{Yes}, \tilde{P}_{-i})$. Second, $h \in Q_{-i} \setminus Q^*_{-i}$, then let

$$s^*_{-i}(h) := \begin{cases} (\text{Yes}, P_{-i}^{(-i)}) & \text{if } P_i^{t(h)}(h) = P_i^{(-i)} \\ (\text{No}, P_{-i}^{(-i)}) & \text{otherwise} \end{cases}.$$

We show that s^* is a SPE, i.e., for each $j \in N$, following s^*_j is a best response to s^*_{-j} in any subgame. The strategy profile s^* induces the history $((\text{Yes}, P_1), (\text{Yes}, P_2), (\text{Yes}, P_1))$ or $((\text{Yes}, P_2), (\text{Yes}, P_1), (\text{Yes}, P_2))$, and the outcome x in both cases. We show i's best-response condition first, and then $-i$'s best-response condition.

Player i's best-response condition: Take $h \in H_i$. If $h \in Q_{-i}$, then $s^*|_h$ induces player i's best SPE outcome y^{-i}. Suppose $h \in Q_i$. The continuation strategy profile $s^*|_h$ induces x. Notice that any non-terminal history in H_i induced by a continuation strategy profile $(s_i, s^*_{-i})|_h$ is in $H_i \cap Q_i$. If player i proposes $(\text{Yes}, \tilde{P}_i)$ so that player $-i$ can terminate the negotiation with $y \in X$ such that $\{y\} = \tilde{P}_i \cap P_{-i}$ and $u_{-i}(y) > u_{-i}(x)$, then player i receives a payoff $u_i(y) \leq u_i(x)$. Otherwise, a possible outcome is either x or the disagreement outcome. Hence, s^*_i is a best response to s^*_{-i} in the subgame starting at $h \in H_i$.

Player $-i$'s best-response condition: Take $h \in H_{-i}$. First, suppose $h \in Q_{-i} \setminus Q^*_{-i}$. The continuation strategy profile $s^*|_h$ induces y^{-i}. Any continuation strategy profile $(s_{-i}, s^*_i)|_h$ induces either y^{-i} or the disagreement outcome.

Second, suppose $h \in Q^*_{-i}$. The continuation strategy profile $s^*|_h$ induces $\tilde{x}(h)$, and player $-i$ gets a payoff of $u_{-i}(\tilde{x}(h)) > u_{-i}(y^{-i})$. If player $-i$ does not terminate the negotiation with $\tilde{x}(h)$ at h, then the outcome following the continuation play is either y^{-i} or the disagreement outcome.

Finally, suppose $h \in Q_i$. Assume $h = h^0 \in H_{-i}$. The continuation strategy profile $s^*|_h$ induces the outcome x. If $-i$ announces $(R'_{-i}, P'_{-i}) \neq (\text{Yes}, P_{-i})$ at h^0, then $s_i^*(R'_{-i}, P'_{-i}) = (\text{No}, P_i^{(-i)})$. At the history $((R'_{-i}, P'_{-i}), (\text{No}, P_i^{(-i)})) \in H_{-i} \cap Q_{-i}$, player $-i$ can obtain at most $u_{-i}(y^{-i})$.

Now, assume $h \neq h^0$. We consider three cases: (i) $(R_i^{t(h)}(h), P_i^{t(h)}(h)) = (\text{Yes}, P_i)$; (ii) $h = (h^{t(h)-2}, (\cdot, P_{-i}), (\text{Yes}, \tilde{P}_i))$ with $t(h) \geq 2$, $\tilde{P}_i \cap P_{-i} = \{y\}$, and $u_{-i}(y) > u_{-i}(x)$; and (iii) otherwise. In case (i), the continuation strategy profile $s^*|_h$ induces x. If player $-i$ uses s_{-i} and announces $s_{-i}(h) \neq (\text{Yes}, P_{-i})$ at h, then player i announces $s_i^*(h, s_{-i}(h)) = (\cdot, P_i^{(-i)})$. If this announcement terminates the negotiation then the outcome is y^{-i}. If not, then in the subgame starting at the resulting history $h' = (h, s_{-i}(h), s_i^*(h, s_{-i}(h))) \in H_{-i} \cap (Q_{-i} \setminus Q_{-i}^*)$, player $-i$ can obtain at most $u_{-i}(y^{-i})$.

In case (ii), player $-i$ can obtain a payoff of $u_{-i}(y)$ by terminating the negotiation. Indeed, following s_{-i}^* terminates the negotiation. For the case in which she does not terminate the negotiation, consider $h' = (h, (R_{-i}^{t(h)+1}(h'), P_{-i}^{t(h)+1}(h')))$. If $(R_{-i}^{t(h)+1}(h'), P_{-i}^{t(h)+1}(h')) = (\text{No}, P_{-i})$, then player $-i$ can obtain a payoff of at most $u_{-i}(x)$ at $h'' = (h', s_i^*(h'))$. If $(R_{-i}^{t(h)+1}(h'), P_{-i}^{t(h)+1}(h')) \neq (\text{No}, P_{-i})$, then in the subgame starting at $h'' = (h', s_i^*(h')) = (h', (\text{No}, P_i^{(-i)}))$, player $-i$ can obtain a payoff of at most $u_{-i}(y^{-i})$.

In case (iii), the continuation strategy profile $s^*|_h$ induces x. Suppose that player $-i$ does not announce (No, P_{-i}) at h. If she terminates the negotiation, then she can get a payoff of at most $u_{-i}(x)$. If she uses a strategy s_{-i} so as not to terminate the negotiation, then the resulting history $h' = (h, s_{-i}(h))$ is in $H_i \cap Q_{-i}$. If player i terminates the negotiation at h', player $-i$ gets $u_{-i}(y^{-i})$. If not, the resulting history $h'' = (h', s_i^*(h'))$ is in $H_{-i} \cap (Q_{-i} \setminus Q_{-i}^*)$. Player $-i$ can obtain a payoff of at most $u_{-i}(y^{-i})$ in the subgame starting at h''. Hence, s_{-i}^* is a best response to s_i^* in the subgame starting at $h \in H_{-i}$.

Part 3: Fix $x \in X$ with $(v^{[1,m]}, v^{[2,m]}) > u(x) (\geq \underline{u})$ satisfying the conditions of the statement. Fix (P_1, P_2) with $\{x\} = P_1 \cap P_2$. For each $i \in \{1, 2\}$, fix (P_1^i, P_2^i) with $\{y^i\} = P_1^i \cap P_2^i$. Also, for each $i \in N$, let $(P_1^{(i)}, P_2^{(i)})$ be such that $\{x^{(i)}\} = P_1^{(i)} \cap P_2^{(i)}$. Recall that each $x^{(i)} \in X$ satisfies $u_i(x^{(i)}) = v^{[i,m]}$ and $u_{-i}(x^{(i)}) = v^{[-i,M]}$. Our proof of this part consists of three steps. In the first step, in order to define a strategy profile s^* that induces the alternative x, we partition the set of non-terminal histories $H \setminus Z$. In the next step, using the partition, we define the strategy profile s^*. In the last step, we show that each s_i^* is a best response to s_{-i}^*.

Partitioning $H \setminus Z$: Fix $j = \rho(h^0)$. We partition $H \setminus Z$ into Q_1, Q_2, and $Q_0 := \{h^0, ((\text{Yes}, P_j)), ((\text{Yes}, P_j), (\text{Yes}, P_{-j}))\}$. Here, we define each Q_i to be the set of non-terminal histories under which player i deviates from announcing (Yes, P_i) first. Formally, $h \in Q_j$ if and only if $h \in H \setminus Z$ satisfies either (i) $h^1 \neq ((\text{Yes}, P_j))$ or (ii) $h^2 = ((\text{Yes}, P_j), (\text{Yes}, P_{-j}))$ and $(R_j^3(h), P_j^3(h)) \neq (\text{Yes}, P_j)$. Likewise, $h \in Q_{-j}$ if and only if $h \in H \setminus Z$ satisfies $h^1 = ((\text{Yes}, P_j))$ and $h^2 \neq ((\text{Yes}, P_j), (\text{Yes}, P_{-j}))$.

For each $i \in N$, we further partition Q_i into Q_i^{on} and $Q_i^{\text{off}} := Q_i \setminus Q_i^{\text{on}}$. Define Q_i^{on} so that $h \in Q_i^{\text{on}}$ if and only if $h \in Q_i$ satisfies either of the following two properties: First, $h^{t(h)-1} \in Q_0$. Second, for any proper subhistory $h' \sqsubset h$ with $h' \in Q_i \cap H_{-i}$,

$$h^{t(h')+1} = \begin{cases} \left(h', (\text{Yes}, P_{-i}^i)\right) & \text{if } h^{t(h')-1} \in Q_i \text{ and } P_i^{t(h')}(h) = P_i^i \\ \left(h', (\text{No}, P_{-i}^i)\right) & \text{if } h^{t(h')-1} \notin Q_i \text{ or } P_i^{t(h')}(h) \neq P_i^i \end{cases}.$$

Defining s^*: We define the strategy profile s^*. For any $h \in H_i \cap Q_0$, let $s_i^*(h) := (\text{Yes}, P_i)$. For any $h \in H_i \cap Q_i^{\text{off}}$, let

$$s_i^*(h) := \begin{cases} (\text{Yes}, P_i^{(-i)}) & \text{if } \varphi^{\text{con}}(h, (\text{Yes}, P_i^{(-i)})) = x^{(-i)} \\ (\text{No}, P_i^{(-i)}) & \text{otherwise} \end{cases}.$$

For any $h \in H_i \cap Q_{-i}^{\text{off}}$, let

$$s_i^*(h) := \begin{cases} (\text{Yes}, \tilde{P}_i) & \text{if } \varphi^{\text{con}}(h, (\text{Yes}, \tilde{P}_i)) = \tilde{x} (=: \tilde{x}(h)) \text{ and } u_i(\tilde{x}) > u_i(x^{(i)}) \text{ for some } (\tilde{P}_i, \tilde{x}) \\ (\text{Yes}, P_i^{(i)}) & \text{if } (R_{-i}^{t(h)}(h), P_{-i}^{t(h)}(h)) = (\text{No}, P_{-i}^{(i)}) \text{ or } \varphi^{\text{con}}(h, (\text{Yes}, P_i^{(i)})) = x^{(i)} \\ (\text{No}, P_i^{(i)}) & \text{otherwise} \end{cases}.$$

For any $h \in H_i \cap Q_i^{\text{on}}$, let $s_i^*(h) := (\text{Yes}, P_i^i)$. Let $h \in H_i \cap Q_{-i}^{\text{on}}$. If there are $\tilde{P}_i$ and $\tilde{x}(=: \tilde{x}(h))$ with $\varphi^{\text{con}}(h, (\text{Yes}, \tilde{P}_i)) = \tilde{x}$ and $u_i(\tilde{x}) > u_i(y^{-i})$, then let $s_i^*(h) := (\text{Yes}, \tilde{P}_i)$.

Otherwise, let

$$s_i^*(h) := \begin{cases} (\text{Yes}, P_i^{-i}) & \text{if } P_{-i}^{t(h)}(h) = P_{-i}^{-i} \\ (\text{No}, P_i^{-i}) & \text{if } P_{-i}^{t(h)}(h) \neq P_{-i}^{-i} \end{cases}.$$

Showing that s_i^* is a best response to s_{-i}^*: We show that each s_i^* is a best response to s_{-i}^* in any subgame starting at $h \in H_i$. First, let $h \in Q_i^{\text{off}}$. The continuation strategy profile $s^*|_h$ induces i's best SPE outcome $x^{(-i)}$.

Second, let $h \in Q_{-i}^{\text{off}}$. Suppose that there is no $(\tilde{P}_i, \tilde{x})$ with $\varphi^{\text{con}}(h, (\text{Yes}, \tilde{P}_i)) = \tilde{x}$ and $u_i(\tilde{x}) > u_i(x^{(i)})$. The continuation strategy profile $s^*|_h$ induces the outcome $x^{(i)}$. Any continuation strategy profile $(s_i, s_{-i}^*)|_h$ induces either $x^{(i)}$ or the disagreement outcome. Next, suppose that there is $(\tilde{P}_i, \tilde{x}(h))$ with $\varphi^{\text{con}}(h, (\text{Yes}, \tilde{P}_i)) = \tilde{x}(h)$ and $u_i(\tilde{x}(h)) > u_i(x^{(i)})$. The continuation strategy profile $s^*|_h$ induces $\tilde{x}(h)$. If player i does not terminate the negotiation at h with $\tilde{x}(h)$, then the outcome following the continuation play is either $x^{(i)}$ or the disagreement outcome.

Third, let $h \in Q_i^{\text{on}}$. We start with showing that $s^*|_h$ induces y^i. Since $h \in Q_i^{\text{on}} \cap H_i$, $h = (h^{t(h)-1}, (R_{-i}^{t(h)}(h), P_{-i}^i))$. If $R_{-i}^{t(h)}(h) = \text{Yes}$, then $(h, (\text{Yes}, P_i^i))$ induces y^i. If $R_{-i}^{t(h)}(h) = \text{No}$, then $(h, (\text{Yes}, P_i^i), (\text{Yes}, P_{-i}^i))$ induces y^i.

We now show that s_i^* is a best response to s_{-i}^* in the subgame starting at $h \in H_i \cap Q_i^{\text{on}}$. Assume $R_{-i}^{t(h)}(h) = \text{No}$. If player i announces (Yes, P_i) such that $P_i \cap P_{-i}^i = \{y\}$ and $u_{-i}(y) > u_{-i}(y^i)$ for some y, then player $-i$ terminates the negotiation with y by announcing (Yes, P_{-i}^i). However, since y^i is not unilaterally improvable for i, $u_i(y) \leq u_i(y^i)$. If player i announces $(\hat{R}_i, \hat{P}_i) \neq (\text{Yes}, P_i)$, then player $-i$'s following s_{-i}^* induces $(h, (\hat{R}_i, \hat{P}_i), (\cdot, P_{-i}^i)) \in H_i \cap Q_i^{\text{on}}$.

Assume that $R_{-i}^{t(h)}(h) = \text{Yes}$. Thus, $P_i^{t(h)-1}(h) = P_i^i$ so that player $-i$ is ok with y^i at $h^{t(h)-1}$. If player i terminates the negotiation, then the outcome must be y^i. In fact, $s_i^*(h) = (\text{Yes}, P_{-i}^i)$ terminates the negotiation. If she uses s_i so as not to terminate the negotiation at h, then $(h, s_i(h), (\cdot, P_{-i}^i)) \in H_i \cap Q_i^{\text{on}}$.

Fourth, let $h \in Q_{-i}^{\text{on}}$. Suppose that there is no $(\tilde{P}_i, \tilde{x})$ with $\varphi^{\text{con}}(h, (\text{Yes}, \tilde{P}_i)) = \tilde{x}$ and $u_i(\tilde{x}) > u_i(y^{-i})$. The continuation strategy profile $s^*|_h$ induces y^{-i}. If player i uses s_i and induces $(h, s_i(h)) \in H_{-i} \cap Q_{-i}^{\text{off}}$, then the outcome following the continuation play is either $x^{(i)}$ or the disagreement outcome. If player i's announcement induces $(h, s_i(h)) \in H_{-i} \cap Q_{-i}^{\text{on}}$, then player $-i$ is ok with y^{-i} at $(h, s_i(h), (\text{Yes}, P_{-i}^i))$. If this history is non-terminal (i.e, in $H_i \cap Q_{-i}^{\text{on}}$), then there is no $(\tilde{P}_i, \tilde{x})$ with $\varphi^{\text{con}}(h, (\text{Yes}, \tilde{P}_i)) =$

$\tilde{x}$ and $u_i(\tilde{x}) > u_i(y^{-i})$.

Suppose that $\varphi^{\text{con}}(h, (\text{Yes}, \tilde{P}_i)) = \tilde{x}$ and $u_i(\tilde{x}) > u_i(y^{-i})$ for some $(\tilde{P}_i, \tilde{x})$. Since $\tilde{x}$ depends only on h, write $\tilde{x}(h) := \tilde{x}$. Player i can obtain a payoff of $u_i(\tilde{x}(h)) > u_i(y^{-i})$ by following s_i^*. Suppose that player i does not terminate the negotiation with $\tilde{x}(h)$ at h, inducing $h' = (h^{t(h)-1}, (\text{Yes}, P_{-i}^{t(h)}), (R_i^{t(h)+1}(h'), P_i^{t(h)+1}(h')))$. If $h' \in H_{-i} \cap Q_{-i}^{\text{off}}$, then player i can obtain a payoff of at most $u_i(x^{(i)})$ in the subgame starting at h'. Suppose that $h' \in H_{-i} \cap Q_{-i}^{\text{on}}$. Then, in the subgame starting at $\big(h, (\cdot, P_i^{-i}), (\text{Yes}, P_{-i}^{-i})\big) \in H_i \cap Q_{-i}^{\text{on}}$, player i can obtain a payoff of at most $u_i(y^i)$.

Fifth, let $h \in Q_0$. The continuation strategy profile $s^*|_h$ induces x. If player i follows s_i and announces $s_i(h) \neq (\text{Yes}, P_i)$ at h, then, since $\mathcal{P}_i$ is limited and x is not unilaterally improvable for i, player i's maximum payoff in the continuation play against s_{-i}^* is $u_i(y^i) \leq u_i(x)$.

Overall, for each $i \in N$, s_i^* is a best response to s_{-i}^* in the subgame starting at any history. Thus, s^* is a SPE. It induces the history $((\text{Yes}, P_j), (\text{Yes}, P_{-j}), (\text{Yes}, P_j))$ with $j = \rho(h^0)$ and the outcome x. □

B Supplementary Remarks on Propositions 5 and 6

B.1 Counterexample to Proposition 5 for Infinite X

Define $X = \{(10,3), (7,10), (4,5), (0,0)\} \cup \{x^n \mid n \in \mathbb{N}\}$, where

$$x^n = \begin{cases} \Big(4 + 2(\frac{1}{\sqrt{2}})^{n-1}, 5 - (\frac{1}{\sqrt{2}})^{n-1}\Big) & \text{if } n = 2k-1 \text{ for some } k \in \mathbb{N} \\ \Big(4 + 4(\frac{1}{\sqrt{2}})^{n-2}, 5 - \frac{3}{2}(\frac{1}{\sqrt{2}})^{n}\Big) & \text{if } n = 2k \text{ for some } k \in \mathbb{N} \end{cases}.$$

Let players' payoff functions be $u(x) = x$ for each $x \in X$. Let $d = (0,0)$. Since $(x^n)_{n\in\mathbb{N}}$ converges to $(4,5)$, $\text{IR}(U,d)$ is compact. Figure 10 depicts the feasible payoff set. Notice that $\{(4,5)\} \cup \{x^{2k-1} \mid k \in \mathbb{N}\} \subseteq \{x \in X \mid u_1(x) < v^{[1,m]} \text{ and } u_2(x) \geq v^{[2,m]}\}$. The proposer rule is such that player 1 moves in odd periods while player 2 moves in even periods.

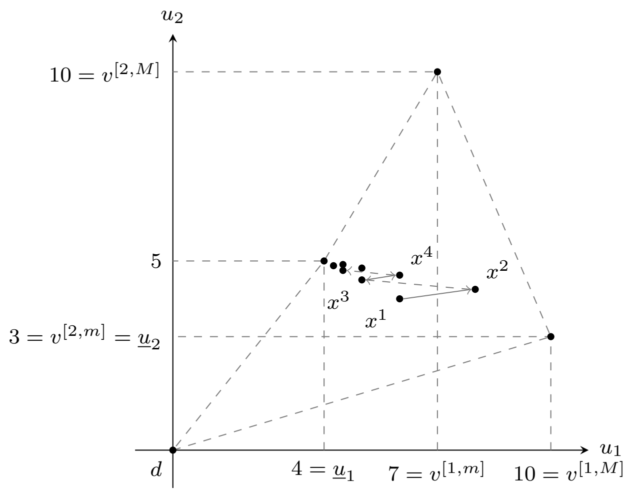

Figure 10: Counterexample to Proposition 5 for infinite X: The solid arrows indicate deviations by player 1 and the dashed arrows indicate punishments against such deviations.

Define the specification rule as follows:

$$\mathcal{P}_1 = \{\{x^n, (0,0)\} \mid n \in \mathbb{N}\} \cup \{\{(10,3),(4,5)\},\{(7,10),(0,0)\}\} \text{; and}$$
$$\mathcal{P}_2 = \{\{x^{2k-1}, x^{2k}\} \mid k \in \mathbb{N}\} \cup \{\{(10,3),(7,10)\},\{(7,10),(4,5)\},\{(10,3),(0,0)\}\} .$$

For any $k \in \mathbb{N}$, the alternative x^{2k-1} is unilaterally improvable for player 1. The only pair (P_1, P_2) such that $P_1 \cap P_2 = \{x^{2k-1}\}$ is $(P_1, P_2) = (\{x^{2k-1}, (0,0)\}, \{x^{2k-1}, x^{2k}\})$, and $P_1' = \{x^{2k}, (0,0)\}$ satisfies the property that $P_1' \cap P_2 = \{x^{2k}\}$ and $x^{2k} > x^{2k-1}$. The alternative $(4,5)$ is also unilaterally improvable for player 1. The only pair (P_1, P_2) such that $P_1 \cap P_2 = \{(4,5)\}$ is $(P_1, P_2) = (\{(10,3),(4,5)\},\{(7,10),(4,5)\})$, and $P_1' = \{(7,10),(0,0)\}$ satisfies the property that $P_1' \cap P_2 = \{(7,10)\}$ and $(7,10) > (4,5)$.

For a fixed $k \in \mathbb{N}$, we construct a SPE s^* that induces x^{2k-1}. The idea of the construction is as follows. First, any deviation by player 2 is punished by the outcome $(10,3)$. In order to incentivize player 1 to comply with the specified strategy, we define a sequence of punishments that we illustrate in Figure 10. If player 1 deviates when the game is supposed to end with outcome x^K under a given history with $K \in \{2k-1, 2k+1, ...\}$, then players' future strategies are such that the game ends with outcome x^{K+2}. This specification provides player 1 with an appropriate incentive for any finite length of histories because there are infinitely many alternatives.

We define s^* recursively as follows. Let $s_1^*(h^0) := (\text{Yes}, \{x^{2k-1}, (0,0)\})$ and, for $h \in H_2$ such that $t(h) = 1$,

$$s_2^*(h) := \begin{cases} (\text{Yes}, \{x^{2k-1}, x^{2k}\}) & \text{if } h = (\text{Yes}, \{x^{2k-1}, (0,0)\}) \\ (\text{Yes}, \{x^{2k+1}, x^{2k+2}\}) & \text{if } h \neq (\text{Yes}, \{x^{2k-1}, (0,0)\}) \end{cases}.$$

Next, let $H_i(h) := \{h' \in H_i \mid h' \sqsubset h\}$ for each $i \in N$ and $h \in H \setminus Z$. Suppose that, for some $h \in H \setminus Z$ with $t(h) \geq 2$, $s_i^*(h')$ is defined for every $h' \in H_i(h)$ for each $i \in N$. For such $h \in H \setminus Z$, we define $\ell(h) \in \mathbb{N}_0$ by

$$\ell(h) := 2 \left| \left\{ h' \in H_1(h) \middle| s_1^*(h') \neq (R_1^{t(h')+1}(h), P_1^{t(h')+1}(h)) \right\} \right|.$$

We also define

$$Q(h) := \left\{ h' \in H_2(h) \middle| s_2^*(h') \neq (R_2^{t(h')+1}(h), P_2^{t(h')+1}(h)) \right\}.$$

Now, we define $(s_1^*(h), s_2^*(h))$. Consider player 1. If $h \in H_1$, then let

$$s_1^*(h) := \begin{cases} (\text{Yes}, \{x^{2k-1+\ell(h)}, (0,0)\}) & \text{if } Q(h) = \emptyset \\ (\text{No}, \{(10,3), (4,5)\}) & \text{if } Q(h) \neq \emptyset \text{ and player 1 is ok} \\ & \text{with } (4,5) \text{ at } (h, (\text{Yes}, \{(10,3), (4,5)\})) \\ (\text{Yes}, \{(10,3), (4,5)\}) & \text{otherwise} \end{cases}.$$

If $h \in H_2$, then let

$$s_2^*(h) := \begin{cases} (\text{Yes}, \{x^{2k-1+\ell(h)}, x^{2k+\ell(h)}\}) & \text{if } Q(h) = \emptyset \\ (\text{Yes}, P_2) & \text{if } Q(h) \neq \emptyset \text{ and there is } (P_2, x) \text{ with} \\ & \varphi^{\text{con}}(h, (\text{Yes}, P_2)) = x (=: x(h)) \in X \text{ and } u_2(x) > 3 \\ (\text{Yes}, \{(10,3), (7,10)\}) & \text{otherwise} \end{cases}.$$

We show that s^* is a SPE. It induces a terminal history

$$\left((\text{Yes}, \{(0,0), x^{2k-1}\}), (\text{Yes}, \{x^{2k-1}, x^{2k}\}), (\text{Yes}, \{(0,0), x^{2k-1}\})\right).$$

Consider player 1. Let $h \in H_1$. First, suppose that $Q(h) \neq \emptyset$. Then, $s^*|_h$ induces player 1's best SPE outcome $(10, 3)$. Next, suppose that $Q(h) = \emptyset$. The strategy profile $s^*|_h$ induces the outcome $x^{2k-1+\ell(h)}$. The definition of s_2^* implies that, irrespective of 1's announcement at h, 2's announcement in the subsequent period is $(\text{Yes}, \{x^{2k-1+\ell(h)}, x^{2k+\ell(h)}\})$. Hence, following $s_1^*|_h$ to obtain a payoff of $u_1(x^{2k-1+\ell(h)})$ is a best response to $s_2^*|_h$.

Consider player 2. Let $h \in H_2$. First, suppose $Q(h) \neq \emptyset$. Suppose that there is $(P_2, x(h)) \in \mathcal{P}_2 \times X$ with $\varphi^{\text{con}}(h, (\text{Yes}, P_2)) = x(h)$ and $u_2(x(h)) > 3$. If he terminates the negotiation with $x(h)$ at h, then he gets a payoff of $u_2(x(h))$. In fact, following $s_2^*|_h$ terminates the negotiation. If he does not terminate the negotiation at h with outcome $x(h)$, then the outcome can be either $(10, 3)$ or the disagreement outcome. Second, suppose $Q(h) = \emptyset$. The continuation strategy profile $s^*|_h$ induces $x^{2k-1+\ell(h)}$. If he deviates at h, the outcome can be either $(10, 3)$ or the disagreement outcome. Overall, $s_2^*|_h$ is a best response to $s_1^*|_h$ in the subgame starting at $h \in H_2$. □

B.2 An Example of a SPE Alternative in $X \setminus X^M$ that is Unilaterally Improvable for a Player

We provide an example in which, $x \in X \setminus X^M$ with $u(x) \geq \underline{u}$ is unilaterally improvable for some player i but is a SPE outcome when there is $y \in X \setminus X^M$ with $u(y) \geq \underline{u}$ that is not unilaterally improvable for i.

Let $X = \{x^1, \ldots, x^6\}$ be such that $x^1 = (10, 2)$, $x^2 = (7, 10)$, $x^3 = (5, 3)$, $x^4 = (6, 4)$, $x^5 = (4, 5)$, and $x^6 = (0, 0)$. Let $u(x) = x$ for each $x \in X$. Set $d = (0, 0)$. The left panel of Figure 11 depicts the feasible payoff set. We define the specification rule as follows: $\mathcal{P}_1 = \{\{x^1, x^6\}, \{x^2, x^6\}, \{x^3, x^6\}, \{x^4, x^6\}, \{x^5, x^6\}\}$ and $\mathcal{P}_2 = \{\{x^1, x^2\}, \{x^3, x^4\}, \{x^1, x^5\}, \{x^4, x^6\}\}$. The proposer rule is such that player 1 speaks in odd periods while player 2 does in even periods.

While each of x^4 and x^5 is not unilaterally improvable for player 1, x^3 is unilaterally improvable for player 1. We define a SPE s^* that induces x^3, together with states θ_1, θ_3, and θ_5 and a transition rule among those states.

Before we formally introduce the SPE s^*, we explain the intuition behind its construction with the right panel of Figure 11. On the equilibrium path, the players make announcements so that they agree on x^3. If player 1 deviates first (e.g., by proposing $\{x^4, x^6\}$ to try to agree on x^4), then player 1 is punished by a continuation

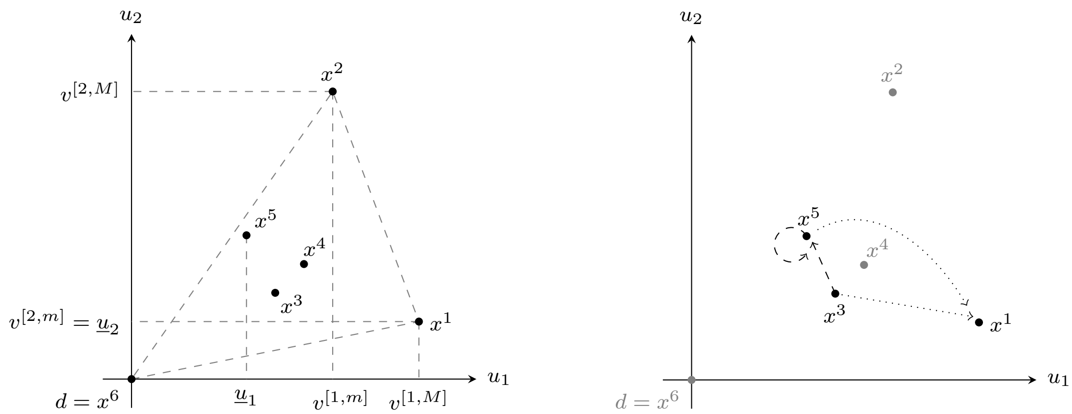

Figure 11: An example in which an alternative is a SPE outcome while it is unilaterally improvable for a player: The left panel depicts the feasible payoff set. Alternative x_3 is a SPE outcome and it is unilaterally improvable for player 1. In the right panel, the dashed arrows indicate punishments for player 1 while the dotted arrows do for player 2.

play in which x^5 is sustained. In the continuation play, while player 2 sticks to agreeing on x^5 even if player 1 deviates, player 2's deviation triggers a punishment in which x^1 is implemented. Player 1's deviation to announcing $\{x^2, x^6\}$ does not lead to an agreement on x^2 because player 2 fears that his responding "Yes" to $\{x^2, x^6\}$ is punished by x^1. If player 2 deviates first from x^3, a punishment involves the alternative x^1 in a similar manner.

Notice that x^3 is in $\{x \in X \mid u_1(x) < v^{[1,m]} \text{ and } u_2(x) \geq v^{[2,m]}\}$. This example shows that some alternative can be sustained as a SPE outcome in this region without condition 2 of Proposition 6 when there is another alternative (x^5 in the current example) which is not unilaterally improvable and which can be used as a punishment for player 1.

Now, let us formally define s^*. Our construction uses a state space that we define to be $\{\theta_1, \theta_3, \theta_5\}$. The initial state is θ_3. Suppose that the state is θ_3 at a non-terminal history $h \in H \setminus Z$. Then, let $s_1^*(h) := (\text{Yes}, \{x^3, x^6\})$ if $h \in H_1$. If $h \in H_2$, then let $s_2^*(h) := (\text{Yes}, \{x^3, x^4\})$. The state transition at θ_3 is defined as follows. If player 1 does not follow this strategy, then the state changes to θ_5. If player 2 does not follow this strategy, the state changes to θ_1. Otherwise, the state stays at θ_3.

Suppose that the state is θ_5. Let $h \in H_1$. If there is $(P_1, x) \in \mathcal{P}_1 \times X$ such that $\varphi^{\text{con}}(h, (1, (\text{Yes}, P_1))) = x \in \{x^1, x^2, x^3, x^4\}$, then we let $s_1^*(h) := (\text{Yes}, P_1)$.[1]

[1]With a slight abuse of notation, in describing an element of a history, we identify the set

Write $x(h) := x$ as x depends only on h. If $\varphi^{\text{con}}(h, (1, (\text{Yes}, \{x^5, x^6\}))) = x^6$, then $s_1^*(h) := (\text{No}, \{x^5, x^6\})$. If $\varphi^{\text{con}}(h') = \text{Continue}$ and $\varphi^{\text{con}}(h', (2, (\text{Yes}, \{x^1, x^2\}))) = x^2$ with $h' = (h, (1, (\text{Yes}, \{x^2, x^6\})))$, then $s_1^*(h) := (\text{Yes}, \{x^2, x^6\})$. Otherwise, let $s_1^*(h) := (\text{Yes}, \{x^5, x^6\})$. For $h \in H_2$, let

$$s_2^*(h) := \begin{cases} (\text{Yes}, \{x^1, x^2\}) & \text{if } \varphi^{\text{con}}(h, (2, \text{Yes}, \{x^1, x^2\})) = x^2 \\ (\text{No}, \{x^1, x^5\}) & \text{if } \varphi^{\text{con}}(h, (2, \text{Yes}, \{x^1, x^2\})) \neq x^2 \text{ and } P_1^{t(h)-1}(h) \neq \{x^5, x^6\} \\ (\text{Yes}, \{x^1, x^5\}) & \text{otherwise, i.e., if } P_1^{t(h)-1}(h) = \{x^5, x^6\} \end{cases}.$$

Note that the second condition includes the case where $\varphi^{\text{con}}(h, (2, \text{Yes}, \{x^1, x^5\})) = x^1$. The state transition at θ_5 is defined as follows. If player 2 does not follow this strategy, then the state changes to θ_1. Otherwise the state stays at θ_5.

Let the state be θ_1, which we define to be absorbing. For each $h \in H_1$, let

$$s_1^*(h) := \begin{cases} (\text{No}, \{x^1, x^6\}) & \text{if } \varphi^{\text{con}}(h, (1, \text{Yes}, \{x^1, x^6\})) = x^6 \\ (\text{Yes}, \{x^1, x^6\}) & \text{otherwise} \end{cases}.$$

Let $h \in H_2$. If there is $(P_2, x) \in \mathcal{P}_2 \times X$ with $\varphi^{\text{con}}(h, (2, (\text{Yes}, P_2))) = x \in \{x^2, x^3, x^4, x^5\}$, then let $s_2^*(h) := (\text{Yes}, P_2)$. Since x depends only on h, write $x(h) := x$. Otherwise, let $s_2^*(h) := (\text{Yes}, \{x^1, x^5\})$.

We show that s^* is a SPE. Since s^* induces its outcome x^3 and the terminal history $((1, (\text{Yes}, \{x^3, x^6\})), (2, (\text{Yes}, \{x^3, x^4\})), (1, (\text{Yes}, \{x^3, x^6\})))$, showing that s^* is a SPE completes the proof.

Consider player 1. Suppose that the state is θ_1 at $h \in H_1$. The strategy profile $s^*|_h$ induces x^1, which is player 1's best SPE outcome. Thus, when the state is θ_1 at $h \in H_1$, $s_1^*|_h$ is a best response to $s_2^*|_h$ in the subgame starting at h.

Let the state be θ_5 at $h \in H_1$. Suppose there is $(P_1, x(h)) \in \mathcal{P}_1 \times \{x^1, x^2, x^3, x^4\}$ with $\varphi^{\text{con}}(h, (1, (\text{Yes}, P_1))) = x(h)$. Player 1, following $s_1^*|_h$, gets a payoff of $u_1(x(h))$. If player 1's announcement does not terminate the negotiation, then the possible outcome following the continuation play is either x^5 or the disagreement outcome.

Now, assume that there is no $P_1 \in \mathcal{P}_1$ such that $\varphi^{\text{con}}(h, (1, (\text{Yes}, P_1))) \in \{x^1, x^2, x^3, x^4\}$. Suppose further that 1's announcement $(\text{Yes}, \{x^2, x^6\})$ leads to a non-terminal history

of prospers by simply writing the name of the proposer, e.g., we write $(1, (\text{Yes}, P_1))$ instead of $(\{1\}, (\text{Yes}, P_1))$. In what follows, we employ the same abuse of notation whenever we identify the set of prospers in describing a history.

satisfying $\varphi^{\text{con}}(h, (1, (\text{Yes}, \{x^2, x^6\})), (2, (\text{Yes}, \{x_1, x_2\})))= x_2$. Following $s_1^*|_h$ leads to the outcome x^2. If not, the negotiation can terminate with either x^5 or the disagreement outcome. Finally, suppose that player 1's announcement $(\text{Yes}, \{x^2, x^6\})$ does not lead to a non-terminal history satisfying $\varphi^{\text{con}}(h, (1, (\text{Yes}, \{x^2, x^6\})), (2, (\text{Yes}, \{x_1, x_2\}))) = x_2$. Then, the negotiation can terminate with either x^5 or the disagreement outcome. Following $s_1^*|_h$ leads to the outcome x^5. Overall, when the state is θ_5 at $h \in H_1$, $s_1^*|_h$ is a best response to $s_2^*|_h$ in the subgame starting at h.

Suppose that the state is θ_3 at $h \in H_1$. If player 1 follows $s_1^*|_h$ then she obtains a payoff of $u_1(x^3)$. If not, the state moves to θ_5. After player 1's deviation, the possible outcome is either x^5 or the disagreement outcome. Player 1 can obtain at most $u_1(x^5) < u_1(x^3)$. Thus, when the state is θ_3 at $h \in H_1$, $s_1^*|_h$ is a best response to $s_2^*|_h$ in the subgame starting at h.

Consider player 2. Suppose that the state is θ_1 at $h \in H_2$. First, if there is $(P_2, x(h)) \in \mathcal{P}_2 \times \{x^2, x^3, x^4, x^5\}$ with $\varphi^{\text{con}}(h, (2, (\text{Yes}, P_2))) = x(h)$, then player 2 obtains a payoff of $u_2(x(h)) > \max\{u_2(x^1), u_2(x^6), d_2\}$ by terminating the negotiation with $x(h)$ by announcing (Yes, P_2) at h. In fact, following $s_2^*|_h$ terminates the negotiation. Suppose instead that player 2 follows a strategy $s_2|_h$ that does not terminate the negotiation at h. Then, since player 1 follows $s_1^*|_h$, the outcome of the negotiation induced by the continuation strategy profile $(s_1^*, s_2)|_h$ is either x^1, x^6, or the disagreement outcome. Second, suppose that there is no $(P_2, x(h)) \in \mathcal{P}_2 \times \{x^2, x^3, x^4, x^5\}$ with $\varphi^{\text{con}}(h, (2, (\text{Yes}, P_2))) = x(h)$. If player 2 follows $s_2^*|_h$ then the outcome is x^1. If he does not follow $s_2^*|_h$, since player 1 follows $s_1^*|_h$, the possible outcome is either x^1, x^6, or the disagreement outcome. Thus, when the state is θ_1 at $h \in H_2$, $s_2^*|_h$ is a best response to $s_1^*|_h$ in the subgame starting at h.

Suppose that the state is θ_3, i.e., $h = (1, (\text{Yes}, \{x^3, x^6\}))$. If player 2 follows $s_2^*|_h$, then he obtains a payoff of $u_2(x^3)$. If not, the state moves to θ_1. Since there is no $P_2 \in \mathcal{P}_2$ such that $\varphi^{\text{con}}(\tilde{h}, (2, (\text{Yes}, P_2))) \in \{x^2, x^3, x^4, x^5\}$ at any $\tilde{h} \in H_2$ after player 2's deviation at h, player 2 can obtain at most $u_2(x^1)$. Hence, when the state is θ_3 at $h \in H_2$, $s_2^*|_h$ is a best response to $s_1^*|_h$ in the subgame starting at h.

Let the state be θ_5 at $h \in H_2$. If $\varphi^{\text{con}}(h, (2, (\text{Yes}, \{x^1, x^2\}))) = x^2$ then player 2 can get her maximum SPE payoff by following $s_2^*|_h$. Suppose $\varphi^{\text{con}}(h, (2, \text{Yes}, \{x^1, x^2\})) \neq x^2$. If he follows $s_2^*|_h$ then the state stays at θ_5, and he obtains a payoff of $u_2(x^5)$. If he deviates from $s_2^*|_h$ then the state moves to θ_1 in which the outcome is either x^1, x^6, or the disagreement outcome. Thus, when the state is θ_5 at $h \in H_2$, $s_2^*|_h$ is a best response to $s_1^*|_h$ in the subgame starting at h. □

C Tightness of Payoff Bounds under Limited Specifiability

Section 5.2.2 shows that, under limited specifiability, whether an alternative x with $u_i(x) \leq v^{[i,m]}$ for some $i \in N$ is sustained as a SPE outcome depends on the structure of the environment and on the way in which the specification rule is imposed. Here, we demonstrate the tightness of payoff lower bounds. First, we show that for any feasible payoff set U, there exists a negotiation whose feasible payoff set is U in which the SPE lower bounds $\underline{u}$ are tight. Second, we show that there exists a negotiation whose feasible payoff set is U in which the SPE payoff set is the IR-Pareto-meet.

Denote $\underline{u}$ by $\underline{u}(U, d)$ to make clear its dependence on (U, d).

Corollary 4 (Existence of games with tight bounds)**.** *Let $U \subseteq \mathbb{R}^2$ and $d \in U$ be such that $\{v \in U \mid v \geq d\}$ is compact and, for each $i \in N$, there is $w \in \{v \in U \mid v \geq d\}$ with $w_i > d_i$. Let ρ be an asynchronous proposer rule.*

1. *Suppose $|U| \geq 2$. There is a two-player negotiation $\Gamma^L = \langle G^L, d, \rho, (\mathcal{P}_i^L)_{i\in N}, \varphi^{\text{con}}\rangle$ with the following properties: (i) $G^L = \langle N, X^L, (u_i^L)_{i\in N}\rangle$ satisfies $U = \{u^L(x) \in \mathbb{R}^2 \mid x \in X^L\}$; (ii) $(\mathcal{P}_i^L)_{i\in N}$ is limited; and (iii) $x \in X^L$ is a SPE outcome of Γ^L if and only if $u^L(x) \geq \underline{u}(U, d)$.*

2. *Suppose $3 \leq |U| < \infty$. There is a two-player negotiation $\Gamma^H = \langle G^H, d, \rho, (\mathcal{P}_i^H)_{i\in N}, \varphi^{\text{con}}\rangle$ with the following properties: (i) $G^H = \langle N, X^H, (u_i^H)_{i\in N}\rangle$ satisfies $U = \{u^H(x) \in \mathbb{R}^2 \mid x \in X^H\}$; (ii) $(\mathcal{P}_i^H)_{i\in N}$ is limited; and (iii) $x \in X^H$ is a SPE outcome of Γ^H if and only if it is in the IR-Pareto-meet.*

To show part 1 of Corollary 4, we first construct a negotiation associated with a normal-form game in which each player's action corresponds to her own payoff. Let $X^L = \mathbb{R}^2$. The payoff functions are defined as follows:

$$u^L(v_1, v_2) = \begin{cases} (v_1, v_2) & \text{if } (v_1, v_2) \in U \\ d & \text{otherwise} \end{cases}.$$[2]

[2] Note that $d \in U$ implies that the feasible payoff set of the environment G^L coincides with U.

Let the specification rule be such that each player i announces her own payoff: $\mathcal{P}_i^L = \{\{v_i\} \times \mathbb{R} \mid v_i \in \mathbb{R}\}$. Since any proposal in $\mathcal{P}_i^L$ (i.e., $\{v_i\} \times \mathbb{R}$) is not a singleton, $\mathcal{P}_i^L$ is limited. No alternative $x \in X^L$ with $u^L(x) \geq \underline{u}(U, d)$ is unilaterally improvable, and thus Propositions 3 and 6 establish the result.

To obtain part 2 of Corollary 4, let $X^H = U$ and $u^H(x) = x$. Suppose first that $|\{v \in U \mid v \geq d\}| \geq 2$. Then, let the specification rule be $\mathcal{P}_i^H = \{\{v, (v^{[i,M]}, v^{[-i,m]})\} \mid v \in U \backslash \{(v^{[i,M]}, v^{[-i,m]})\}\}$ for each $i \in N$.[3] Since any proposal in $\mathcal{P}_i^H$ (i.e., $\{v, (v^{[i,M]}, v^{[-i,m]})\}$) consists of two alternatives, $\mathcal{P}_i$ is limited. Theorem 3 establishes the result because, for any $v \in U$ such that $v_{-i} \geq v^{[-i,m]}$ and $v_i < v^{[i,m]}$, the alternative v is not unilaterally improvable for i. Suppose next that $|\{v \in U \mid v \geq d\}| = 1$, that is, $\{v \in U \mid v \geq d\} = \{v^*\}$. Then, for any specification rule, the negotiation has a unique SPE payoff profile v^*. Choosing distinct $v_1, v_2, v^* \in U$, we can define the following limited specification rule: For each $i \in N$, let $\mathcal{P}_i^H = \bigcup_{w \in \{v^1, v^2, v^*\}} \{\{w, v\} \mid v \in U \setminus \{w\}\}$. This establishes the desired result.

To conclude the discussion on part 2 of Corollary 4, let us go back to the example in Section 1, where U is the convex hull of $\{(0,0), (2,4), (4,2)\}$ and $d = (0,0)$. The arguments made here show that the set of SPE payoffs under limited specifiability is as depicted in the right panel of Figure 1.

D Supplementary Discussions

D.1 Impatience

In the main analysis, we do not assume discounting. Here we consider a model with discounting, and show that the main insight does not change unless discounting is significant. Specifically, consider a model in which $d_i = 0$ for each $i \in N$, and there exists a discount factor $\delta \in (0, 1]$ such that, if an agreement is made at time t with outcome x, each player i receives a payoff of $\delta^{t-1} u_i(x)$.

First, consider a negotiation with the synchronous proposer rule. For any $\delta \in (0, 1]$, an alternative x is a SPE outcome if and only if it is individually rational. The strategy profile we constructed in Proposition 2 is a SPE with discounting as well. As we discussed in footnote 19 in the main text, a related result is obtained in

[3]We assume $|U| \geq 3$ to ensure that the specification rule satisfies the requirement that for each $x \in X^H$, there is $(P_1, P_2) \in \mathcal{P}_1^H \times \mathcal{P}_2^H$ such that $P_1 \cap P_2 = \{x\}$.

Stahl (1986). He examines a dynamic Bertrand competition model with or without discounting where each seller can synchronously change her price announcements. Analogous to our "folk theorem," he shows that any price no more than the monopoly price (and no less than the marginal cost) can be sustained as a SPE outcome for any discount factor.

Next, we consider negotiations with asynchronous moves. Theorem 2 implies that if the IR-Pareto-meet consists of multiple points, then there are multiple SPE alternatives under any specification rule. This is in stark contrast to the uniqueness of SPE in complete-information bargaining models with an asynchronous proposer rule such as Ståhl (1972) and Rubinstein (1982). The reason for this difference is that we do not assume discounting. For the following argument, assume $N = \{1, 2\}$ and that a proposer rule is such that players 1 and 2 propose in odd and even periods, respectively.

To see the connection clearly, first note that, in Rubinstein (1982)'s bargaining model, if $\delta = 1$ and indefinite disagreement results in a payoff of zero, then all possible divisions of the pie can be sustained under SPE. A related result is that if one discretizes the space of offers to make it a finite set, then for sufficiently large $\delta < 1$, all possible divisions of the pie can be sustained under SPE (Van Damme et al. (1990) and Muthoo (1991)).

A parallel result can be obtained in our model. If the Pareto-frontier is characterized by a strictly decreasing and weakly concave continuous function, then there is a unique SPE payoff profile under an unlimited specification rule for any $\delta < 1$. On the other hand, suppose that a feasible payoff set consists of a finite number of points with no payoff ties.[4] Then, any SPE that we have constructed in the main text remains to be a SPE for sufficiently large $\delta < 1$.

Suppose, on the other hand, that δ is close to 0. Then an alternative may be a SPE outcome even if it is not a SPE outcome under no discounting, provided such an alternative requires less time to reach a consensus. The next example illustrates this point.

Example 5. Suppose that $N = \{1, 2\}$, $X = \{(1, 1), (\alpha, \alpha), (\beta, \beta), (0, 0)\}$ with $0 < \beta < \alpha < 1$. Let $d = (0, 0)$. Let $\mathcal{P}_i = \{\{(\alpha, \alpha)\}, \{(1, 1), (0, 0)\}, \{(1, 1), (\beta, \beta)\}, \{(0, 0), (\beta, \beta)\}\}$ for each $i \in N$.

[4] That is, there is no $(i, x, y) \in N \times X \times X$ such that $x \neq y$ and $u_i(x) = u_i(y)$.

We show that if $\delta \in (0, \alpha)$ then (α, α) is a unique SPE outcome. Also, we show that the following strategy profile s^* is a SPE that supports (α, α). For each $i \in N$ and $h \in H_i$,

$$s_i^*(h) = \begin{cases} (\text{Yes}, \{(1,1),(0,0)\}) & \text{if } i \text{ is ok with } (1,1) \text{ at } (h,(i,\text{Yes},\{(1,1),(0,0)\})) \\ (\text{Yes}, \{(1,1),(\beta,\beta)\}) & \text{if } i \text{ is ok with } (1,1) \text{ at } (h,(i,\text{Yes},\{(1,1),(\beta,\beta)\})) \\ (\text{Yes}, \{(\alpha,\alpha)\}) & \text{otherwise} \end{cases}.$$

This s^* induces the terminal history $((1, (\text{Yes}, \{(\alpha,\alpha)\})), (2, (\text{Yes}, \{(\alpha,\alpha)\})))$ and the outcome (α, α).

Now, we show that (α, α) is a unique SPE outcome given that a SPE exists. If player i announces $(\text{Yes}, \{(\alpha,\alpha)\})$ at any history h at which it is her turn to move, then player $-i$ can terminate the negotiation by announcing $(\text{Yes}, \{(\alpha,\alpha)\})$ at $(h, (i, (\text{Yes}, \{(\alpha,\alpha)\})))$. In the subgame starting at $(h, (i, (\text{Yes}, \{(\alpha,\alpha)\})))$, player $-i$'s best discounted SPE payoff is $\alpha = u_{-i}(\alpha, \alpha)$ which she can obtain by announcing $(\text{Yes}, \{(\alpha,\alpha)\})$. If she announces $P_{-i} \in \{\{(1,1),(0,0)\}, \{(1,1),(\beta,\beta)\}\}$ at $(h, (i, (\text{Yes}, \{(\alpha,\alpha)\})))$, then players need at least two more periods to agree on $(1,1)$. Since $\delta^2 < \delta < \alpha$, the outcome after player i's announcement $(\text{Yes}, \{(\alpha,\alpha)\})$ at h is (α, α) in any SPE. Now, at any history at which it is player i's turn to move, player i can secure herself a payoff of $\delta u_i(\alpha, \alpha) = \delta\alpha$. If player i instead announces $P_i \in \{\{(1,1),(0,0)\}, \{(1,1),(\beta,\beta)\}\}$, then it would need at least two more periods to agree on $(1,1)$. Thus, player i can obtain at most a payoff of $\delta^2 < \delta\alpha$. Hence, (α, α) is the unique SPE outcome, provided that a SPE exists.

Next, we show that s^* is indeed a SPE. At any history after which i becomes ok with $(1,1)$ if she announces (Yes, P_i) where $P_i \in \{\{(1,1),(0,0)\}, \{(1,1),(\beta,\beta)\}\}$, we show that it is of i's best interest to become ok with $(1,1)$ by announcing (Yes, P_i). If $\varphi^{\text{con}}(h, (i, (\text{Yes}, P_i))) = (1,1)$, then i can obtain her best feasible payoff. If not, player $-i$ can terminate the negotiation with $(1,1)$ at $(h, (i, (\text{Yes}, P_i)), (-i, (\text{Yes}, P_{-i})))$, which brings $-i$ her best feasible payoff in the subgame starting at $(h, (i, (\text{Yes}, P_i)))$. Note that $P_{-i} \in \{(1,1),(0,0)\}$ or $\{(1,1),(\beta,\beta)\}$. Thus, the outcome after $(h, (i, (\text{Yes}, P_i)))$ is $(1,1)$, and player i obtains a discounted payoff of δ in the subgame starting at $(h, (i, \text{Yes}, P_i))$. If player i instead announces $(R_i, \{(\alpha,\alpha)\})$ at h, then player $-i$ will terminate the negotiation at $(h, (i, (R_i, \{(\alpha,\alpha)\})), (-i, (\text{Yes}, \{(\alpha,\alpha)\})))$. Hence, player i's discounted payoff is $\delta\alpha < \delta$. At any other history, the previous argument showing the uniqueness establishes that following $s_i^*|_h$ and announcing (Yes, $\{(\alpha, \alpha)\}$) is a best response. □

The relevance of our arguments surrounding the comparison of specifiability depends on the relative magnitudes of the salience of impatience and the variety of available alternatives. In light of our discussion in which we argue that impatience may not play a key role in determining the negotiation outcome in our applications such as COP meetings, we believe that our results have economically meaningful content in the applications we have in mind (cf., the "Bargaining" part of Section 6).

D.2 Stochastic Announcements

Here, we allow each player to make their announcements stochastically. We provide an example in which the SPE payoff set becomes larger and complicated when players are allowed to use behavioral strategies.

Example 6. Consider the finite normal-form game given by the left panel of Figure 12. Assume an asynchronous proposer rule and limited specifiability under which each player's proposals correspond to her own actions. Each player i is allowed to use a behavioral strategy: a mapping from the set of histories at which she speaks to the set of probability distributions on $\{\text{Yes}, \text{No}\} \times \mathcal{P}_i$. Let $d = (0, 0)$.

The set of pure-strategy SPE payoffs is given by $\{(3,1),(2,3),(1,0),(0,4),(0,2)\}$. Now, the right panel of Figure 12 depicts the di-convex span of the set of pure-strategy SPE payoffs.[5] For each point in the di-convex span, one can construct a SPE to support such a payoff profile. The specification of the feasible payoff set and the construction of the SPE closely follows the one in the context of long cheap talk in Aumann and Hart (2003).

To understand the idea, consider the payoff profile $(1, 2)$. To sustain this payoff profile, at time 1, player 1 mixes between (No, U) and (No, D) with probability 1/2 for each, where the former induces the continuation payoff $(1, 1)$ and the latter induces $(1, 3)$. Now, consider sustaining a point on the six solid line-segments in the right panel of Figure 12, except for the points in $\{(3,1),(2,3),(1,0),(0,4),(0,2)\}$. As an example, take a point $(2, 1)$ on the line segment from $(1, 1)$ to $(3, 1)$. To sustain this continuation payoff profile, player 2 mixes in his turn between (No, L) and (No, R),

[5] See Aumann et al. (1968) for the definition of di-convex span.

	L	C	R
U	$2,3$	$3,1$	$0,4$
M	$0,2$	$0,2$	$0,2$
D	$1,0$	$1,0$	$1,0$

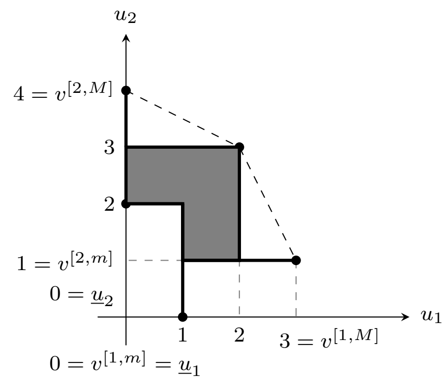

Figure 12: Stochastic announcements under an asynchronous proposer rule and limited specifiability: The left panel is the normal-form game with which the negotiation is associated. The right panel depicts the di-convex span of the SPE payoff set of the negotiation under deterministic announcements (the shaded area and the bold and solid line segments). Any payoff profile in the di-convex span is a SPE payoff profile under stochastic announcements.

where the former is assigned probability $1/2$ and induces the continuation payoff $(1, 1)$, while the latter is assigned probability $1/2$ and induces $(3, 1)$. If $(3, 1)$ is reached, then the players play as in the pure SPE that supports it. □

E Various Negotiation Protocols

The generality of our negotiation model enables one to formally compare various negotiation protocols. To demonstrate wide applicability of the framework and to guide future work, here we provide some possible rules of interest.

E.1 Termination Rules

In the main analysis of this paper, we restricted attention to the consensual termination rule. One can vary termination rules to examine how such variations change the set of SPE outcomes.

1. *Coalitional consensual rules.* Our consensual rule assumes unanimity in the sense that all players have to be ok with x to terminate the negotiation with the outcome being x. One can alternatively consider a rule in which there is a non-empty set of winning coalitions $\mathcal{C} \subseteq 2^N \setminus \{\emptyset\}$ such that the negotiation terminates at h if there is $C \in \mathcal{C}$ such that (i) all players in C are ok with x

at their respective latest opportunity after the latest "No" and (ii) at least one player in C speaks at $t(h)$. Our consensual rule corresponds to the case with $\mathcal{C} = \{N\}$.

2. *Plurality rule.* A plurality rule can be expressed as a termination rule. For example, consider unlimited specifiability, and a proposer rule in which each player proposes exactly once. A plurality rule can be described by a termination rule that ignores the Yes/No responses and terminates the negotiation right after all players have had chances to move, with an alternative that is announced the greatest number of times (with some tie-breaking rule).

3. *Deadline.* One can introduce a deadline by appropriately defining a termination rule. For example, one can construct a new rule φ^T such that $\varphi^T(h) = \varphi^{\text{con}}(h)$ if $t(h) \leq T$ and $\varphi^T(h) = \text{Continue}$ if $t(h) > T$. This means that under φ^T, any negotiation process that lasts more than T periods necessarily ends up in the disagreement outcome. We conjecture that the existence of a deadline further enhances commitment power. For example, with two players under an asynchronous proposer rule and an unlimited specification rule, one can show that there always exists a SPE in which the second-last mover obtains the best payoff in $\text{IR}(U, d)$.

4. *Other rules in the literature.* Many of the negotiation rules considered in the literature can be expressed as a particular instance of our termination rule. For example, one can consider a termination rule that does not depend in any way on the Yes/No responses. Such a termination rule can be thought of as representing the situation in which each party only announces their own proposals/actions. As a concrete example, we formalize Bhaskar (1989)'s "quick-response game" in Section E.4.

5. *k-consensual rules.* One can consider a class of termination rules such that a -necessary- condition to terminate is that every player is ok with the given outcome. A special case of this class is our consensual rule. In general, we can consider a termination rule that terminates with an alternative x at history h if at each of the latest k opportunities for each player i after the latest "No," i is ok with x.

6. *An alternative definition of being ok.* We could define a termination rule in a way that player i is ok with x at h when it is a unique element of the intersection of players' proposals after her own most recent announcement of "No," instead of the most recent announcement of "No" that she has observed or made. This alternative termination rule is different from the consensual termination rule, and we did not use it due to the following example. Suppose that $N = \{1, 2\}$, $X = \{a, b, c\}$, $\mathcal{P}_1 = \{\{a\}, \{b\}, \{c\}\}$, $\mathcal{P}_2 = \{\{a, b\}, \{a, c\}\}$, and $u_2(a) < d_2$. Consider the asynchronous proposer rule in which player 1 moves at odd periods and player 2 moves at even periods. We can show that player 1 can always achieve a as an outcome irrespective of player 2's strategy, which implies that it is possible that 2's SPE payoff is not individually rational. To see this, suppose that 1 announces $(\text{No}, \{a\})$ at the first period. Then, irrespective of 2's announcement in the second period, 2 becomes ok with a under the alternative termination rule. Player 1 can then announce $(\text{Yes}, \{a\})$ to terminate the negotiation with the outcome a.

E.2 Stochastic Rules

The main part of the paper dealt with deterministic negotiation protocols and pure strategies. We considered behavioral strategies in Section D.2. Here we consider stochastic negotiation protocols.

1. *Stochastic proposer rules.* There are various bargaining models in which proposers are randomly chosen each period (pioneered by Baron and Ferejohn (1989)). One can consider stochastic proposer rules to nest such random proposer models. Also, in some models such as Ambrus and Lu (2015) and Kamada and Kandori (2019), the moving players are stochastically determined by Poisson processes in continuous time. We can approximate such processes by considering stochastic proposer rules that allow for the empty set of proposers.

2. *Stochastic specification rules.* Stochastic specification rules may represent the situations where, for example, an interest group imposing a feasibility constraint on available proposals occasionally becomes conciliatory and such an event happens at random times (in the eyes of the negotiating parties).

3. *Stochastic termination rules.* One example of a stochastic termination rule is

that the negotiation ends exogenously with probability p each period. There are various possibilities for the resulting outcome upon ending. One possibility is that the disagreement outcome results, and formally the termination rule returns "Continue" for any history after the exogenous ending. Another is that an outcome x results, where x is the alternative with which the number of players who were ok at the time of exogenous ending is the greatest (with some tie-breaking rule). Another example of a stochastic termination rule is that the negotiation ends with probability p at each history at which every player is ok with some alternative. This last possibility is again in the class of termination rules such that a -necessary- condition to terminate is that every player is ok with the terminating outcome, as in k-consensual rules explained in Section E.1.

E.3 Other Possible Extensions

Here we describe more complicated extensions of our model that may capture additional features of some negotiations in reality.

1. *Side-payments.* In footnote 1, we discussed the difference between conditional and unconditional INDC in the context of the Paris Agreement, where a country may make a proposal that specifies monetary transfers from other countries. We can incorporate side-payments that are made conditional on agreeing upon some alternatives, by expanding X so that the description of each of its elements has side-payments to each player.[6]

2. *Agreeing on a subset of alternatives.* It would also be interesting to consider the possibility that players can agree on a subset of X (not just on a *single* alternative), and the final outcome resulting from such agreements is exogenously or endogenously specified. On the one hand, if such an alternative is specified exogenously,[7] then one can model such a negotiation by modifying the termination rule. On the other hand, our model may not nest the endogenous case: Suppose that the negotiation is associated with a normal-form game and each player's specification rule corresponds to announcing a subset of her own action space. As in Renou (2009), one could consider an extension of our game with the possibility of an agreement on a smaller normal-form game, which the

[6] See Jackson and Wilkie (2005) for a related model with side-payments.

[7] That is, there exists a pre-specified mapping from 2^X to X.

players play after it is agreed upon. In this way, we could replace the commitment stage of Renou (2009)'s game with a negotiation phase, and examine the effect of the detail of such a negotiation phase on the equilibrium outcome of the whole game.

3. *Mediator.* In reality, negotiations are sometimes conducted in the presence of a third party such as a mediator or an arbitrator. It would be an interesting avenue for further research to study how different forms of third party interventions lead to different negotiation outcomes by extending our basic framework. To get an idea, consider a mediator who helps the negotiating parties agree on a Pareto-efficient alternative. In order to add such a mediator in our model, consider a set of players $N \cup \{m\}$ where m represents the mediator. The mediator would not announce any Yes/No response but announce only a proposal. One reasonable possibility of the mediator's preferences would be that she prefers an alternative to another if the former Pareto dominates the latter for players in N. We would modify the consensual termination rule so that, for an alternative x to be agreed upon, m would not need to be ok with x but all other players in N would have to be.

E.4 Quick-Response Termination Rule

Here we introduce the *quick-response termination rule*, and demonstrate that our framework can nest Stahl (1986), Bhaskar (1989), and Muto (1993).

We define the quick-response termination rule $\varphi^{\mathrm{QR}} : \mathcal{H} \to X \cup \{\text{Continue}\}$ so that it returns $\varphi^{\mathrm{QR}}(h) = x \in X$ for a given history $h \in \mathcal{H}$ if and only if the following three conditions hold. First, there is $i \in \rho(h^{t(h)-1})$ such that

$$R_i^{t(h)-1}(h) = R_i^{t_i}(h), \text{ where } t_i = \max\{t' \in \mathbb{N}_0 \mid t' < t(h) - 1 \text{ and } i \in \rho(h^{t'})\}.$$

That is, at least one player repeats her announcement twice in a row at the end of h. Second, for any $j \in N$, there is a proper subhistory h_j of h with $t_i < t(h_j)$ and $j \in \rho(h_j)$. That is, every player has announced at least once by the end of h after i's announcement at t_i. Third, $\{x\} = \bigcap_{j \in N} P_j^{\tau_j(h)}(h)$, where $\tau_j(h) := \max\{t(h') \in \mathbb{N}_0 \mid h' \sqsubset h \text{ and } j \in \rho(h')\}$. That is, the intersection of the players' most recent announcements is equal to the singleton set $\{x\}$. Note that the quick-response termination rule ignores the Yes/No responses.

First, we observe that the quick-response termination rule generalizes Bhaskar (1989)'s Bertrand duopoly game in a general two-player context. Consider a negotiation associated with a two-player normal-form game $\langle N, (A_i)_{i\in N}, (u_i)_{i\in N}\rangle$. Let a proposer rule ρ be asynchronous such that player 1 moves in odd periods and player 2 moves in even periods. Players can only announce their actions, that is, $\mathcal{P}_i = \{\{a_i\} \times A_{-i} \mid a_i \in A_i\}$ for each $i \in N$. Then, the quick-response termination rule $\varphi^{\mathrm{QR}} : \mathcal{H} \to A \cup \{\text{Continue}\}$ terminates the negotiation as follows.

$$\varphi^{\mathrm{QR}}(h) = \begin{cases} (a_1, a_2) & \text{if } h = \left(h^{t(h)-3}, (\cdot, a_i), (\cdot, a_{-i}), (\cdot, a_i)\right) \\ \text{Continue} & \text{otherwise} \end{cases}.$$

Thus, the quick-response termination rule φ^{QR} terminates the negotiation whenever some player i has announced the same action a_i in two consecutive periods of hers. This is exactly the rule of the quick-response game described in Bhaskar (1989). Since Muto (1993) studies the same rule as Bhaskar (1989)'s, this also shows that Muto (1993)'s rule can be described as the quick-response termination rule, too.

Second, Stahl (1986) assumes the synchronous proposer rule. As above, players can only announce their actions. Then, the quick-response termination rule φ^{QR} : $\mathcal{H} \to A \cup \{\text{Continue}\}$ is given as follows.

$$\varphi^{\mathrm{QR}}(h) = \begin{cases} (a_i)_{i\in N} & \text{if } h = \left(h^{t(h)-2}, ((\cdot, a_i'))_{i\in N}, ((\cdot, a_i))_{i\in N}\right) \text{ with } a_i' = a_i \text{ for some } i \in N \\ \text{Continue} & \text{otherwise} \end{cases}.$$

That is, the quick-response termination rule φ^{QR} terminates the negotiation whenever some player i has announced the same action a_i in two adjacent periods. This is exactly the rule of the model in Stahl (1986).

To summarize, our framework can also accommodate other termination rules such as the quick-response termination rule, which we demonstrated encompasses Stahl (1986), Bhaskar (1989), and Muto (1993).

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