

Deduction Dilemmas: The Taiwan Assignment Mechanism

Online Appendix

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A Beyond Taiwan: Heterogeneous Priorities

In this appendix, we examine two examples of deduction mechanisms without Assumption 1 and show that Propositions 1 and 3 no longer apply. The first example we consider shows that opportunities for manipulation need not increase with higher levels of deduction.

Example 3. There are five schools $S = \{a, b, c, d, e\}$ and five students $I = \{i_1, i_2, i_3, i_4, i_5\}$. Let $q_s = 1$ for all $s \in S$. The priority scores of students for schools are as follow: $\pi_a(i_1) = 100$, $\pi_a(i_3) = 98$, $\pi_a(i_4) = 97$, $\pi_b(i_2) = 99$, $\pi_b(i_3) = 98$, $\pi_c(i_3) = 90$, $\pi_c(i_4) = 88.5$, $\pi_c(i_5) = 100$, $\pi_d(i_4) = 100$, $\pi_d(i_5) = 90$, and $\pi_e(i_3) = 100$. The preference of students are as follows:

P_{i_1} :	a	\emptyset			
P_{i_2} :	b	\emptyset			
P_{i_3} :	a	b	c	e	\emptyset
P_{i_4} :	a	c	d	\emptyset	
P_{i_5} :	d	c	\emptyset		

We consider two deduction rules: $\lambda = (0, 1, 2, 2, 2, 2)$ and $\lambda' = (0, 2, 7, 7, 7, 7)$. Given λ , P and π , the implied priority scores profile π^λ is: $\pi_a^\lambda(i_1) = 100$, $\pi_a^\lambda(i_3) = 98$, $\pi_a^\lambda(i_4) = 97$, $\pi_b^\lambda(i_2) = 99$, $\pi_b^\lambda(i_3) = 97$, $\pi_c^\lambda(i_3) = 88$, $\pi_c^\lambda(i_4) = 87.5$, $\pi_c^\lambda(i_5) = 99$, $\pi_d^\lambda(i_4) = 98$, $\pi_d^\lambda(i_5) = 90$, and $\pi_e^\lambda(i_3) = 98$. Given λ' , P and π , the implied priority scores profile $\pi^{\lambda'}$ is: $\pi_a^{\lambda'}(i_1) = 100$, $\pi_a^{\lambda'}(i_3) = 98$, $\pi_a^{\lambda'}(i_4) = 97$, $\pi_b^{\lambda'}(i_2) = 99$, $\pi_b^{\lambda'}(i_3) = 96$, $\pi_c^{\lambda'}(i_3) = 83$, $\pi_c^{\lambda'}(i_4) = 86.5$, $\pi_c^{\lambda'}(i_5) = 98$, $\pi_d^{\lambda'}(i_4) = 93$, $\pi_d^{\lambda'}(i_5) = 90$, and $\pi_e^{\lambda'}(i_3) = 93$.

In this problem, TM^λ selects:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 \\ a & b & e & d & c \end{pmatrix}.$$

In this problem, $TM^{\lambda'}$ selects:

$$\nu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 \\ a & b & e & c & d \end{pmatrix}.$$

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Under TM^λ , student i_4 can manipulate her preferences by ranking c at the top and the outcome is

$$\nu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 \\ a & b & e & c & d \end{pmatrix}.$$

On the other hand, no student can benefit from manipulation under $TM^{\lambda'}$.

Next, we show that under Taiwan mechanism without Assumption 1, it is possible for there to be a unique Nash equilibrium outcome in weakly undominated strategies, but that outcome is not stable under augmented priorities. This example slightly modifies the previous one.

Example 4. There are five schools $S = \{a, b, c, d, e\}$ and six students $I = \{i_1, i_2, i_3, i_4, i_5, i_6\}$. Let $q_s = 1$ for all $s \in S$. Suppose all students are strategic. The priority scores of students for schools are as follow: $\pi_a(i_1) = 100$, $\pi_a(i_3) = 99.5$, $\pi_a(i_4) = 97$, $\pi_b(i_2) = 99$, $\pi_b(i_3) = 98.5$, $\pi_c(i_3) = 90$, $\pi_c(i_4) = 86$, $\pi_c(i_5) = 100$, $\pi_d(i_4) = 100$, $\pi_d(i_5) = 90$, $\pi_e(i_3) = 100$, and $\pi_e(i_6) = 99.5$.² The preference of students are as follows:

P_{i_1} :	a	\emptyset			
P_{i_2} :	b	\emptyset			
P_{i_3} :	a	b	c	e	\emptyset
P_{i_4} :	a	c	d	\emptyset	
P_{i_5} :	d	c	\emptyset		
P_{i_6} :	e	\emptyset			

We consider the following deduction rule: $\lambda = (0, 1, 2, 2, 2, 2)$. Given λ , π and P , the implied priority scores profile π^λ is: $\pi_a^\lambda(i_1) = 100$, $\pi_a^\lambda(i_3) = 99.5$, $\pi_a^\lambda(i_4) = 97$, $\pi_b^\lambda(i_2) = 99$, $\pi_b^\lambda(i_3) = 97.5$, $\pi_c^\lambda(i_3) = 88$, $\pi_c^\lambda(i_4) = 85$, $\pi_c^\lambda(i_5) = 99$, $\pi_d^\lambda(i_4) = 98$, $\pi_d^\lambda(i_5) = 90$, $\pi_e^\lambda(i_3) = 98$ and $\pi_e^\lambda(i_6) = 99.5$.

Note that, i_1 and i_2 can obtain their top choices by submitting their true preferences and they may not be assigned to their top choices if they do not rank them as top choice. Hence, in any weakly undominated strategy i_1 and i_2 rank their true top choice at the top. Independent of the other students ranking, student i_4 and i_5 can get an acceptable school by submitting their true preferences. Moreover, submitting something else is weakly dominated by their true preference profile.

Similarly, if i_6 ranks an unacceptable school at the top she may be assigned to it and ranking \emptyset as top choice is weakly dominated by her true preference profile. That is, for all students except i_3 submitting true preference is weakly undominated strategy. For such a

²The priority scores of all students at all other schools are below 70.

strategy profile i_3 's best response is ranking e as top choice. Otherwise she will be unassigned. The corresponding equilibrium outcome is:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 \\ a & b & e & c & d & \emptyset \end{pmatrix}.$$

Student i_3 is ranked higher than student i_4 under the augmented priorities of school c and she would like to be assigned to c instead of her match under the equilibrium outcome.

B Omitted Simulation Results

Figure 3: Simulation results for all students (15,000 students).

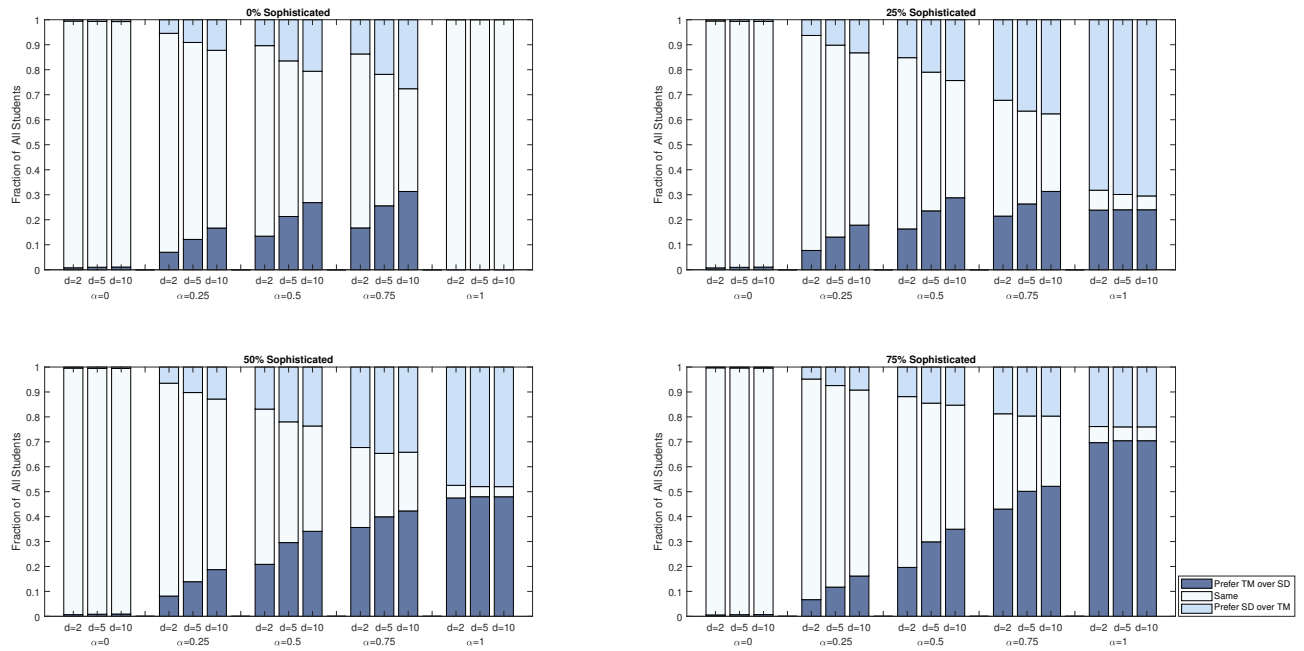


Figure 4: Simulation results for sincere students (5,000 students).

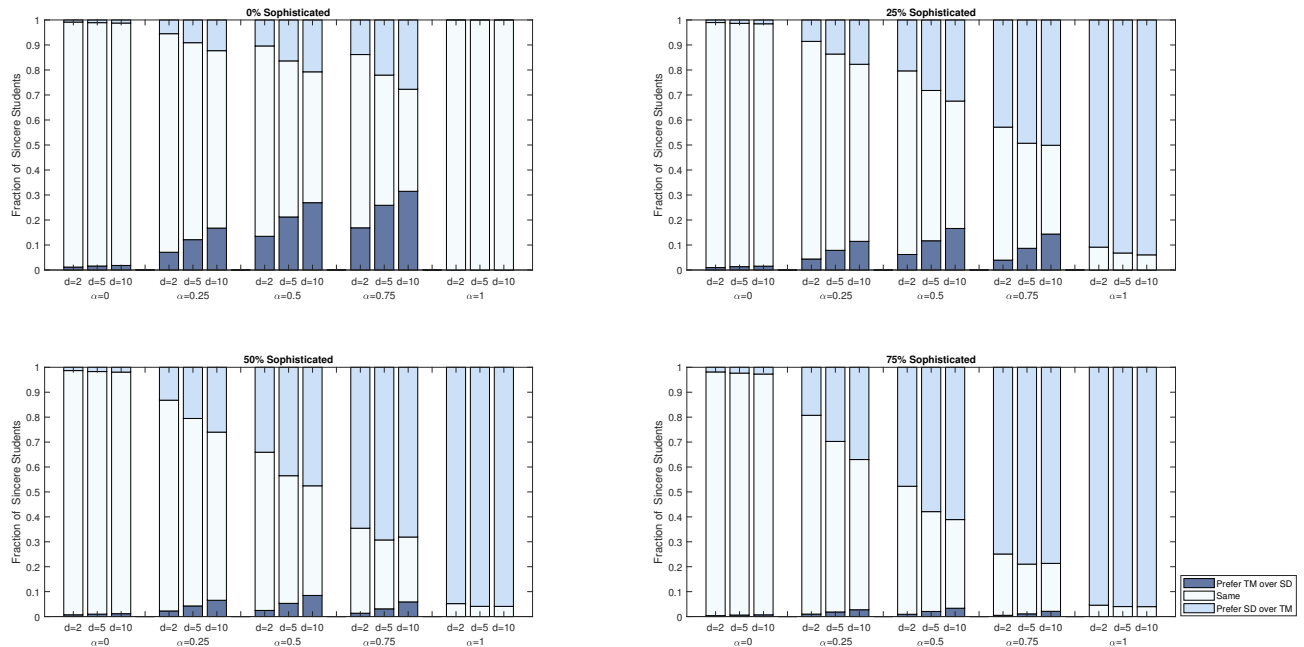


Figure 5: Simulation results for sophisticated students (5,000 students).

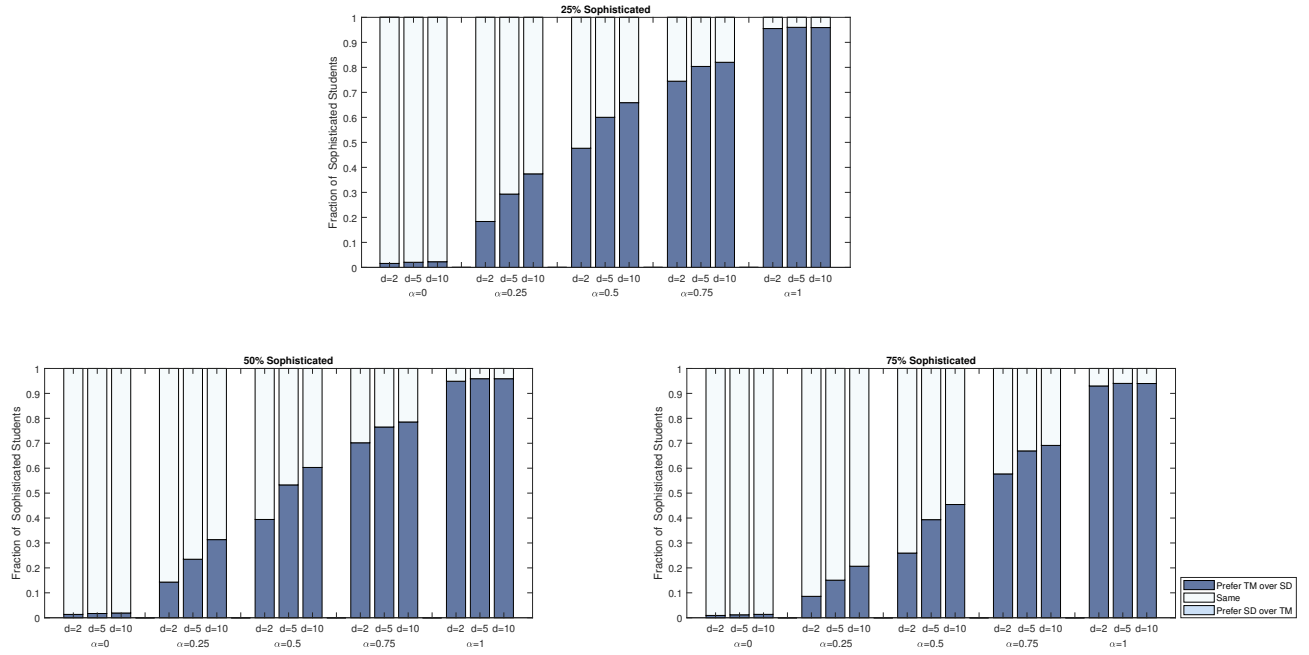


Figure 6: Simulation results for all students (5,000 students).

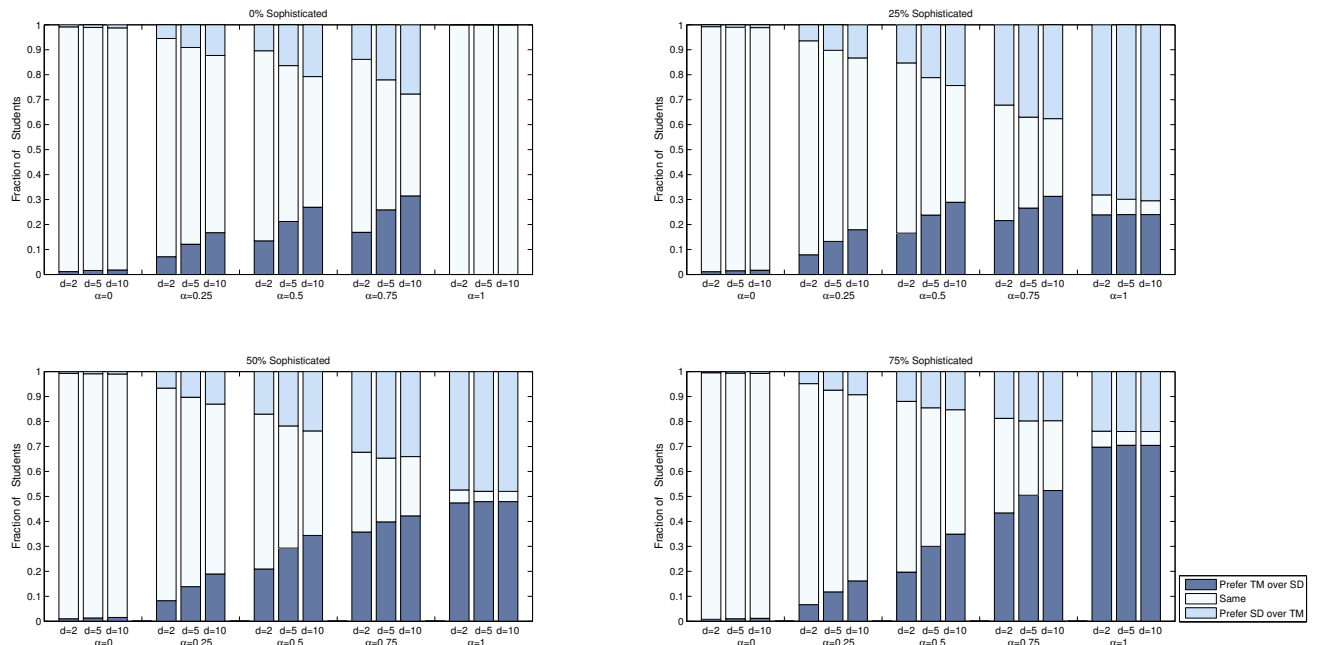


Figure 7: Simulation results for sincere students (10,000 students).

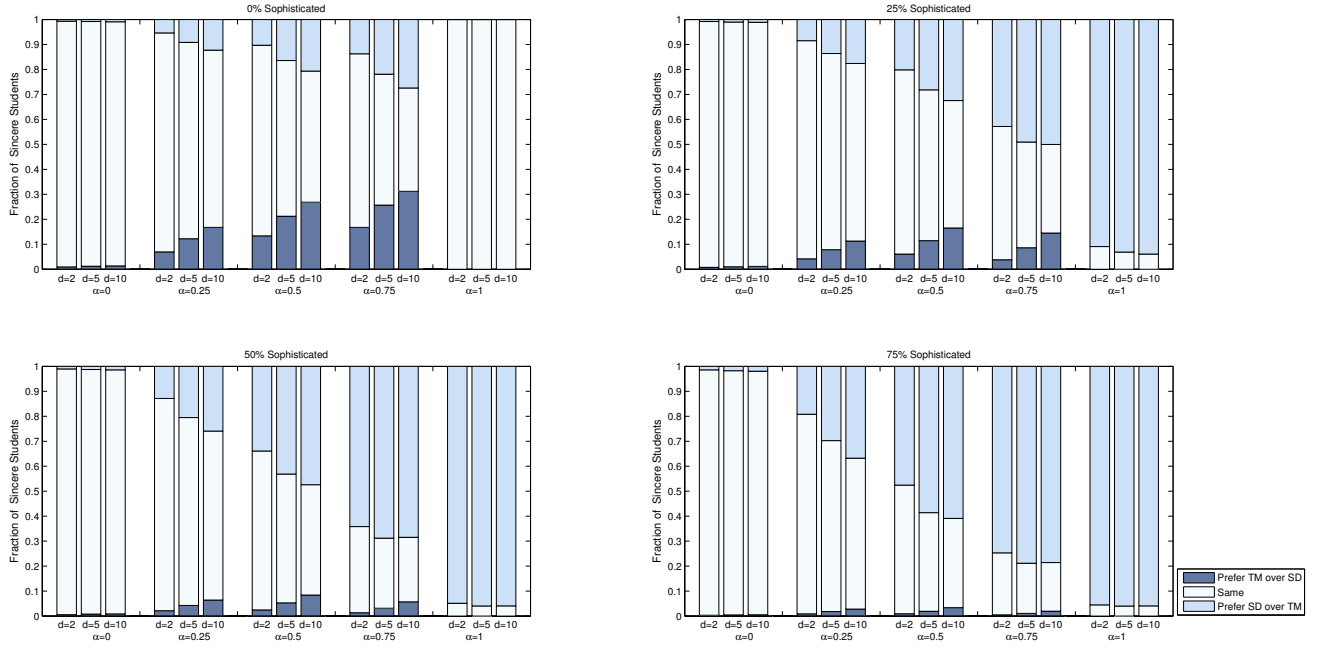


Figure 8: Simulation results for sophisticated students (10,000 students).

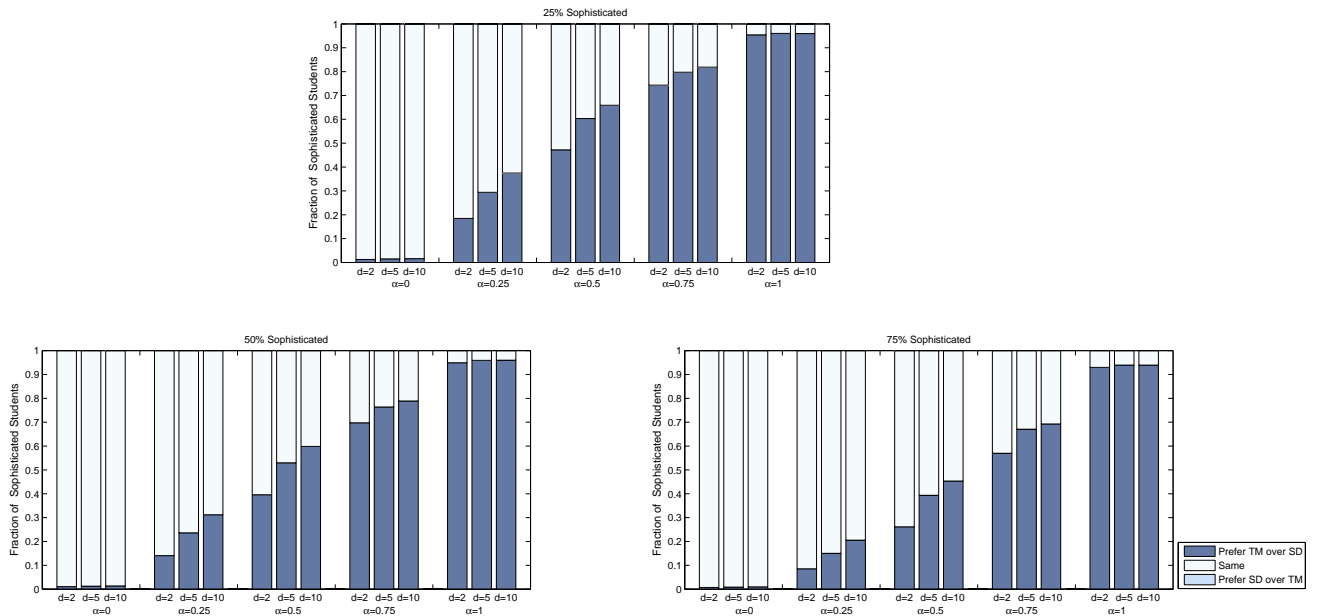


Figure 9: Simulation results for all students (10,000 students).

