Properties of New Keynesian Model: Analytic Applications

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Outline

• Fisherian vs anti-Fisherian Debate:
  – How do you get inflation down (or, up)?
    • Fisherian answer: cut the nominal rate of interest.
    • Anti-Fisherian answer: raise the interest rate.

• How do we think about these seemingly contradictory answers?
  – NK model gives us a way to think about this.

• Draw on Erceg and Levin, 2003 JME paper, “Imperfect credibility and inflation persistence”

• Forward Guidance Puzzle

• How does the Taylor Principle work to stabilize inflation in the equilibrium local to steady state?
Fisherian versus Anti Fisherian Policy

• **Fisherian effect**
  – Interest rate and inflation move in the same direction.

• **Anti-Fisherian effect**
  – Interest rate and inflation move in opposite direction.
Intuition

• Monetary policy rule (inflation target, $\bar{\pi}_t$):

$$r_t = \pi_t + \phi (\pi_t - \bar{\pi}_t)$$

• Temporary cut in $\bar{\pi}_t$ (anti-Fisherian effect)
  – actual inflation, $\pi_t$, responds very little because price setters focus on long-run conditions
    • remember that inflation depends on current and future values of marginal cost
  – $r_t$ rises and $r_t - \pi_{t+1}$ rises too.
  – output, inflation fall

$$\text{cov}(\pi_t, r_t) < 0$$

• Permanent cut in $\bar{\pi}_t$ (Fisherian effect)
  – $\pi_t$ drops strongly
  – $r_t$ falls
  – not much change in $r_t - \pi_{t+1}$ so little change in output.

\[ \text{cov}(\pi_t, r_t) > 0 \]
Linearized Equilibrium Conditions

• Monetary policy rule:

\[ r_t = \pi_t + \phi (\pi_t - \bar{\pi}_t). \]

• Law of motion of inflation target, \( \bar{\pi}_t \):

\[ \bar{\pi}_t = \delta \bar{\pi}_{t-1} + \varepsilon_t. \]

• Phillips curve and output gap:

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]
\[ x_t = E_t x_{t+1} - [r_t - E_t \pi_{t+1}]. \]
Solving Linearized Equilibrium Conditions

• (Linearized) Equilibrium Conditions of Model:
  \[ r_t = \pi_t + \phi (\pi_t - \bar{\pi}_t), \quad \bar{\pi}_t = \delta \bar{\pi}_{t-1} + \varepsilon_t \]

• Undetermined coefficients method, \( a_1, a_2, a_3 \):

  \[ \pi_t = a_1 \bar{\pi}_t, \quad x_t = a_2 \bar{\pi}_t, \quad r_t = a_3 \bar{\pi}_t \]

• Substitute the solution into the equations and require that they hold for all possible \( \bar{\pi}_t \):

  \[ a_3 \bar{\pi}_t = a_1 \bar{\pi}_t + \phi (a_1 - 1) \bar{\pi}_t \]

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  \[ a_1 \bar{\pi}_t = \beta \delta a_1 \bar{\pi}_t + \kappa a_2 \bar{\pi}_t \]

  \[ a_2 \bar{\pi}_t = a_2 \delta \bar{\pi}_t - [a_3 - a_1 \delta] \bar{\pi}_t. \]

• Want to know: \( a_1, a_3 \) when \( \delta = 0 \) and \( \delta = 1 \).
Solving the Model: Getting the $a$’s

• Substitute the solution into the equations:

\[
\begin{align*}
  a_3 &= a_1 + \phi (a_1 - 1) \\
  a_1 &= \beta \delta a_1 + \kappa a_2 \\
  a_2 &= a_2 \delta - [a_3 - a_1 \delta].
\end{align*}
\]

• Now start rearranging stuff

\[
\begin{align*}
  a_3 &= (1 + \phi) a_1 - \phi \\
  a_1 &= \frac{\kappa}{1 - \beta \delta} a_2 \\
  a_2 &= a_2 \delta - (1 + \phi - \delta) a_1 + \phi
\end{align*}
\]

• $\delta = 1$ result now obvious ($a_1 = a_3 = 1$); $\delta = 0$ easy.
Solving the Model: Getting the $a$’s

- **Model:**
  
  \[ r_t = \pi_t + \phi (\pi_t - \bar{\pi}_t), \quad \bar{\pi}_t = \delta \bar{\pi}_{t-1} + \varepsilon_t \]

  \[ \pi_t = \beta \pi_{t+1} + \kappa x_t \]

  \[ x_t = x_{t+1} - [r_t - \pi_{t+1}] \cdot \]

- **Undetermined coefficients, $a_1, a_2, a_3$:**
  
  \[ \pi_t = a_1 \bar{\pi}_t, \quad x_t = a_2 \bar{\pi}_t, \quad r_t = a_3 \bar{\pi}_t \]

- **Solution derivation**
  
  \[ a_1 = \frac{\phi}{\left[ \frac{1 - \beta \delta}{\kappa} + 1 \right] (1 - \delta) + \phi}, \quad a_3 = (1 + \phi) a_1 - \phi. \]

- **Permanent case, $\delta = 1$:**
  
  \[ a_1 = a_3 = 1 \]

- **Temporary case, $\delta = 0$:**
  
  \[ -a_1 = \frac{\phi}{\frac{1}{\kappa} + 1 + \phi} > 0, \quad a_3 = -\frac{\phi / \kappa}{\frac{1}{\kappa} + 1 + \phi} < 0. \]
It all depends on persistence

\[ \pi_t = a_1 \bar{\pi}_t, \quad x_t = a_2 \bar{\pi}_t, \quad r_t = a_3 \bar{\pi}_t \]

- **Fisherian result:**
  - A permanent increase in \( \bar{\pi}_t \), (\( \delta = 1 \)) leads to a rise in \( \pi_t \) and a rise in \( r_t \).

- **Anti-Fisherian result:**
  - A purely temporary increase in \( \bar{\pi}_t \) leads to a rise in \( \pi_t \) and a fall in \( r_t \).

- **Impact of a rise in \( \bar{\pi}_t \) on \( \pi_t \) and \( r_t \) (sign of \( a_3 \)) depends on persistence of the rise and the other parameters of the model**
Erceg and Levin Combine the two Effects to Explain Volcker Recession

- Volcker reduced the inflation target, $\bar{\pi}_t$, permanently.
- But, what matters is people’s beliefs, and they were convinced the reduced target was only temporary.
  - The 1970s had witnessed numerous episodes in which the Fed reduced the target ‘permanently’, only to raise it again soon after.
- So, the public thought Volcker was ‘business as usual’ and interpreted the decline in the target as temporary.
The Volcker Recession

- Interest rates went way up and output, down.

- Forecasts of inflation remained stubbornly high.

- Eventually, everyone realized that $\bar{\pi}_t$ was down permanently.
  - Fisherian effects kicked in and both interest rates and inflation fell.
  - Output returned to potential.

- Bond markets now indicate that people believe that there hasn’t been a persistent increase in $\bar{\pi}_t$, i.e. the Fed will bring inflation down.
Forward Guidance Puzzle

• When interest rates became very low after 2008, monetary policy authorities resorted to ‘forward guidance’:
  – Announcements that interest rates in the future will be low.

• Studying forward guidance in models, researchers stumbled on what came to be called the ‘forward guidance puzzle’:
  – Announcements about a cut in the interest rate in the distant future have a bigger impact than a current reduction in the interest rate.

• People felt this was implausible (though we have no empirical evidence on the issue) and so called it a puzzle.
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Characterizing the Puzzle

- Phillips curve and output gap:
  \[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]
  \[ x_t = E_t x_{t+1} - [r_t - E_t \pi_{t+1}] . \]

- Two scenarios, each followed by Taylor rule in \( t + j + s, s \geq 0 \).
  - \( j \) Period Forward guidance:
    - \( r_{t+s} = 0 \) for \( s = 0, ..., j-1 \), \( r_{t+j} = \theta \).
  - Immediate policy:
    - \( r_t = \theta, r_{t+s} = 0 \), for \( s = 1, 2, ..., j \).

- Taylor rule:
  \[ r_t = \underbrace{\phi \pi_t} \quad \text{Taylor principle, } \phi > 1 \]
Characterizing the Puzzle

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- Two scenarios, each followed by Taylor rule in \( t + j + s, s \geq 0 \).
  - \( j \) Period Forward guidance:
    - \( r_{t+s} = 0 \) for \( s = 0, \ldots, j - 1 \), \( r_{t+j} = \theta \).
  - Immediate policy:
    - \( r_t = \theta, r_{t+s} = 0, \) for \( s = 1, 2, \ldots, j \).

- Taylor rule:
  \[
  r_t = \underbrace{\phi}_{\text{Taylor principle, } \phi > 1} \pi_t.
  \]

- Result: impact on date \( t \) variables greater from forward guidance than from immediate policy.
One-period Forward Guidance ($j = 2$)

- Announcement at time $t$: $r_t = 0$, $r_{t+1} = \theta$, Taylor rule thereafter.
- Because (i) there are no shocks, (ii) the model is purely forward looking and (iii) Taylor rule with $\phi > 1$ in place after $t + 1$:
  \[ r_{t+s} = x_{t+s} = \pi_{t+s} = 0, \quad s > 1. \]
- In period $t + 1$
  \[ x_t = E_t x_{t+1} - [r_t - E_t \pi_{t+1}] . \]
  \begin{align*}
  r_{t+1} &= \theta \\
  x_{t+1} &= x_{t+2} - [r_{t+1} - \pi_{t+2}] = 0 - [r_{t+1} - 0] = -r_{t+1} \\
  \pi_{t+1} &= \beta \pi_{t+2} + \kappa x_{t+1} = \kappa x_{t+1} = -\kappa r_{t+1}
  \end{align*}
- So, in $t + 1$:
  \[ r_{t+1} = \theta, \quad x_{t+1} = -\theta, \quad \pi_{t+1} = -\kappa \theta. \]
- What happens in period $t$?
One-period Forward Guidance \((j = 2)\)

- Effect: in the period \(t + 1\), of \(t + 1\) policy action announced in \(t\):
  \[
  r_{t+1} = \theta, \ x_{t+1} = -\theta, \ \pi_{t+1} = -\kappa \theta. 
  \]

- In period \(t\):
  \[
  r_t = 0 \\
  x_t = x_{t+1} - [r_t - \pi_{t+1}] = - \left( \begin{array}{c} \text{direct effect} \\ 1 + \kappa \end{array} \right) \cdot \begin{pmatrix} r_{t+1} \\ \pi_{t+1} \end{pmatrix} \\
  \pi_t = \beta \pi_{t+1} + \kappa x_t = - \beta \kappa \cdot (r_{t+1} = \theta) + \kappa x_t 
  \]
  so,
  \[
  \pi_t = - (1 + \kappa) r_{t+1} \\
  \pi_t = - \beta \kappa r_{t+1} + \kappa \cdot \begin{pmatrix} x_t \end{pmatrix} \\
  \rightarrow \pi_t = - [1 + \beta + \kappa] \kappa \theta, \ x_t = - (1 + \kappa) \theta 
  \]
Immediate Policy

• Announcement at time $t$: $r_t = \theta \neq 0$, $r_{t+1} = 0$ and Taylor rule thereafter.
• Because the model is completely forward looking,
  \[ r_{t+s} = x_{t+s} = \pi_{t+s} = 0, \ s > 0. \]

Then,

\[
\begin{align*}
r_t &= \theta \\
x_t &= x_{t+1} - [r_t - \pi_{t+1}] = 0 - [r_t - 0] = -r_t \\
\pi_t &= \beta \pi_{t+1} + \kappa x_t = \beta \times 0 + \kappa x_t
\end{align*}
\]

• So,

\[
\rightarrow r_t = \theta, \ x_t = -\theta, \ \pi_t = -\kappa \theta.
\]

which is smaller than with one-period forward guidance:

\[
\begin{align*}
\pi_t &= -[1 + \beta + \kappa] \kappa \theta, \quad x_t = -(1 + \kappa) \theta
\end{align*}
\]
Intuition

• Consider $j$ Period Forward Guidance.
  – Announcement at time $t$: $r_{t+j} = \theta \neq 0$ and $r_{t+s} = 0$ for $s = 0, 1, \ldots, j - 1$. Switch to Taylor rule after $t + j$.

• IS equation (recall, $r_t = \ldots = r_{t+j-1} = 0$):

$$
x_{t+j} = x_{t+j+1} - (r_{t+j} - \pi_{t+j+1}) = -r_{t+j}
$$

$$
x_{t+j-1} = x_{t+j} - (r_{t+j-1} - \pi_{t+j}) = - (r_{t+j-1} - \pi_{t+j}) - r_{t+j}
$$

\[ \vdots \]

$$
x_t = - (r_t - \pi_{t+1}) - (r_{t+1} - \pi_{t+2}) - \ldots - (r_{t+j-1} - \pi_{t+j}) - r_{t+j}
$$
Intuition, cnt’d

• IS equation (recall, $r_t = ... = r_{t+j-1} = 0$):

$$x_t = -(r_t - \pi_{t+1}) - (r_{t+1} - \pi_{t+2})$$

$$- ... - (r_{t+j-1} - \pi_{t+j}) - r_{t+j}$$

• Change in $r_{t+j}$ has a direct effect on $x_t$ and an indirect effect.
  – **Direct**: change in $r_{t+j}$ moves $x_{t+j}$ and (by consumption smoothing channel) that leads to an equal change in earlier output gaps, including $x_t$.
    • This channel holds fixed the real interest rates,
      $$(r_{t+s} - \pi_{t+s+1}) , s = 0, ..., j - 1.$$  
  – **Indirect**: change in $r_{t+j}$ affects $(r_{t+s} - \pi_{t+s+1}) , 0 \leq s \leq j - 1$ in each date between now and $t + j$ by reducing inflation in each date.
    • The impact on $x_t$ of the indirect effect is the cumulative sum (increasing in $j$) of the changes in the real interest rate.
Forward Guidance: Conclusion


- Sparked a large literature to ‘solve’ the problem.

  • This offers what is perhaps the simplest resolution: in practice, announcements about policy actions far in the future have little impact on behavior because they are not credible.
  • In my presentation, I assumed 100% credibility.
How Does the Taylor Principle Work to Stabilize Inflation?

• Model

\[ x_t = E_t x_{t+1} - \left[ r_t - E_t \pi_{t+1} - r^*_t \right] \]
\[ r_t = \phi_{\pi} \pi_t, \quad \phi_{\pi} > 1 \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]
\[ \Delta a_t = \rho \Delta a_{t-1} + \epsilon_t \]
\[ r^*_t = E_t (a_{t+1} - a_t) = \rho \Delta a_t. \]

• Unique non-explosive solution:

\[ \pi_t = \gamma_1 \Delta a_t, x_t = \gamma_2 \Delta a_t, r_t = \gamma_3 \Delta a_t \]

- \( \gamma_i \)'s ~ undetermined coefficients.
Solving the Model

- Model and solution

\[ x_t = E_t x_{t+1} - [r_t - E_t \pi_{t+1} - r^*_t] \]
\[ r_t = \phi_\pi \pi_t, \quad \phi_\pi > 1 \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]
\[ \Delta a_t = \rho \Delta a_{t-1} + \epsilon_t \]
\[ r^*_t = E_t (a_{t+1} - a_t) = \rho \Delta a_t \]

- Solution is of form

\[ \pi_t = \gamma_1 \Delta a_t, x_t = \gamma_2 \Delta a_t, r_t = \gamma_3 \Delta a_t \]
Solving the Model

• Substitute solution into model:

\[
\begin{align*}
\gamma_2 &= \rho \gamma_2 - \gamma_3 + \rho \gamma_1 + \rho \\
\gamma_3 &= \phi \pi \gamma_1 \\
\gamma_1 &= \beta \gamma_1 \rho + \kappa \gamma_2
\end{align*}
\]

• Real rate: \( \tilde{r}_t = r_t - E_t \pi_{t+1} = \gamma_4 \Delta a_t \),

\[
\gamma_4 = \gamma_3 - \gamma_1 \rho.
\]
Solving the Model

• Each to verify:

\[ r_t - E_t \pi_{t+1} = \psi \Delta a_t, x_t = \frac{(1 - \beta \rho)}{\kappa (\phi_\pi - \rho)} \psi \Delta a_t, \pi_t = \frac{\psi}{\phi_\pi - \rho} \Delta a_t \]

where

\[ \psi \equiv \frac{\rho}{\frac{(1-\beta \rho)(1-\rho)}{\kappa (\phi_\pi - \rho)} + 1} \]

• For \( \phi_\pi \) sufficiently large,

\[ \psi \approx \rho, \ r_t - E_t \pi_{t+1} \approx r_t^*, \ \pi_t \approx 0, \ x_t \approx 0. \]
Solving the Model

• Big value of $\phi_\pi$ stabilizes equilibrium around first best.
  – However, requires very large value of $\phi_\pi$.
  – For practical values, Taylor rule too weak, $\psi < \rho$ and $\gamma_2 > 0$.

• Taylor principle:
  – real rate of interest increases when $\pi_t$ high ($\psi > 0$ and $\phi > \rho$).
  – effects bigger with bigger $\phi_\pi$. 
Solving the Model

• The equations:

\[ r_t = \pi_t + \phi (\pi_t - \bar{\pi}_t) \]
\[ \pi_t = \beta \pi_{t+1} + \kappa x_t \]
\[ x_t = x_{t+1} - \left[ r_t - \pi_{t+1} \right]. \]

• Substitute the solution in here:

\[ a_3 = a_1 + \phi (a_1 - 1) \]
\[ a_1 = \beta \delta a_1 + \kappa a_2 \]
\[ a_2 = a_2 \delta - [a_3 - a_1 \delta]. \]

• Rearranging:

\[ a_3 = (1 + \phi) a_1 - \phi \]
\[ a_1 = \frac{\kappa}{1 - \beta \delta} a_2 \]
\[ a_2 = a_2 \delta - [a_3 - a_1 \delta] = a_2 \delta - (1 + \phi - \delta) a_1 + \phi \]
\[ \rightarrow a_2 = - \frac{1 + \phi - \delta}{1 - \delta} a_1 + \frac{\phi}{1 - \delta}. \]
Solving the Model

• Working on the second equation,

\[
a_1 \frac{1 - \beta \delta}{\kappa} = -\frac{(1 + \phi - \delta)}{1 - \delta} a_1 + \frac{\phi}{1 - \delta}
\]

then,

\[
a_1 = \frac{\phi}{1 - \delta} \frac{1 - \beta \delta}{\kappa} + \frac{1 + \phi - \delta}{1 - \delta}
= \frac{\phi}{1 - \delta} \frac{1 - \beta \delta}{\kappa} + 1 + \frac{\phi}{1 - \delta} = \left[ \frac{1 - \beta \delta}{\kappa} + 1 \right] (1 - \delta) + \phi
\]

• Then,

\[
a_3 = \frac{(1 + \phi) \phi}{\left[ \frac{1 - \beta \delta}{\kappa} + 1 \right] (1 - \delta) + \phi} - \phi
\]

• So, when \( \delta = 1 \): \( a_1 = a_3 = 1 \). When \( \delta = 0 \), get formulas for \( a_1, a_3 \) in main presentation.