How Sticky Wages in Existing Jobs Can Affect Hiring

Appendices

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A. Log-Linearized Dynamics of the Models

In this appendix, we derive the log-linearized dynamics of key variables around the steady state for all 5 model specifications: 2 models under flexible wages (with fixed and variable effort) and 3 models under sticky wages (fixed, individual, and common effort). Production is subject to technology shocks (z) and the utility from leisure is subject to preference shocks (ξ). We assume that z and ξ follows AR(1) process in logs. For variable x, $\hat{x}$ denotes the percentage deviation from its steady state $\tilde{x}$.

A.1. Log-Linearized Equations

Sticky Wage with Individual Effort Choice Given the first-order Taylor approximation of worker's match surplus, $H(w,\mathbf{s}) - H(w^{*\prime},\mathbf{s}) = \dfrac{\partial H(w^*,\mathbf{s})}{\partial w}\big(w - w^{*\prime}\big)$. The value of match surplus of a worker is expressed as:

$$\begin{aligned} H(w,\mathbf{s}) = w - b + \xi\psi\frac{(1-e)^{1-1/\gamma}-1}{1-1/\gamma} + \beta(1-\delta)\lambda\mathbb{E}\big[\epsilon(w^{*\prime},\mathbf{s}^\prime)(w-w^{*\prime})|\mathbf{s}\big] \\ + \beta\big(1-\delta-p(\theta)\big)\mathbb{E}\big[H(w^{*\prime},\mathbf{s}^\prime)|\mathbf{s}\big] \end{aligned} \tag{A.1}$$

where $\epsilon(w^*,\mathbf{s})$ denotes the increase in the worker's surplus (for a newly-negotiated match) from a marginal increase of wage:

$$\epsilon(w^*,\mathbf{s}) = \frac{\partial H(w^*,\mathbf{s})}{\partial w} = 1 - \xi\psi(1-e)^{-1/\gamma}\Lambda(w^*,\mathbf{s}) + \beta(1-\delta)\lambda\mathbb{E}\big[\epsilon(w^{*\prime},\mathbf{s}^\prime)|\mathbf{s}\big], \tag{A.2}$$

where $\Lambda(w^*,\mathbf{s}) = \partial e(w^*,\mathbf{s})/\partial w$ denotes the effort change induced by a wage increase. Analogously, the value of match surplus for the firm is:

$$J(w,\mathbf{s}) = \alpha z k^{1-\alpha} e - w + \beta(1-\delta)\lambda\mathbb{E}\big[\mu(w^{*\prime},\mathbf{s}^\prime)(w-w^{*\prime})|\mathbf{s}\big] + \beta(1-\delta)\mathbb{E}\big[J(w^{*\prime},\mathbf{s}^\prime)|\mathbf{s}\big], \tag{A.3}$$

where $\mu(w^*,\mathbf{s})$ denotes the decrease in firm's surplus (for a newly-negotiated match) due to a marginal increase in wage:

$$\mu(w^*,\mathbf{s}) = -\frac{\partial J(w^*,\mathbf{s})}{\partial w} = 1 - \alpha z k^{1-\alpha}\Lambda(w^*,\mathbf{s}) + \beta(1-\delta)\lambda\mathbb{E}\big[\mu(w^{*\prime},\mathbf{s}^\prime)|\mathbf{s}\big]. \tag{A.4}$$

The log-linearized dynamics of $H(w,\mathbf{s})$, $J(w,\mathbf{s})$, and $\mu(w^*,\mathbf{s})/\epsilon(w^*,\mathbf{s})$ are:

$$\begin{aligned}\widehat{H} = \frac{1}{\widetilde{H}}\Bigg\{&\widetilde{w}\widehat{w} + \widetilde{B}\widehat{\xi} - \psi(1-\widetilde{e})^{-1/\gamma}\widetilde{e}\widehat{e} + \beta(1-\delta)\lambda\widetilde{\epsilon}\widetilde{w}\big(\widehat{w} - E\widehat{w}^{*\prime}\big)\\ &+\left(\frac{\eta}{1-\eta}\right)\left(\left[\frac{(1-\delta)\eta\kappa}{\widetilde{q}} - \kappa\widetilde{\theta}\right]\widehat{\theta} - \left[\frac{(1-\delta)\kappa}{\widetilde{q}} - \kappa\widetilde{\theta}\right]E\big[\widehat{\mu}' - \widehat{\epsilon}'\big]\right)\Bigg\},\end{aligned} \tag{A.5}$$

$$\widehat{J} = \frac{1}{\widetilde{J}}\left\{\alpha\widetilde{y}\big(\widehat{z} + (1-\alpha)\widehat{k} + \widehat{e}\big) - \widetilde{w}\widehat{w} - \beta(1-\delta)\lambda\widetilde{\mu}\widetilde{w}\big(\widehat{w} - E\widehat{w}^{*\prime}\big) + \frac{(1-\delta)\eta\kappa}{\widetilde{q}}\widehat{\theta}\right\}, \tag{A.6}$$

$$\widehat{\mu} - \widehat{\epsilon} = -\frac{\widetilde{\gamma}\alpha\widetilde{y}}{\eta\widetilde{J}}\left[\widehat{z} + (1-\alpha)\widehat{k} - \widehat{\xi} - \frac{1}{\widetilde{\gamma}}\widehat{e}\right] + \beta(1-\delta)\lambda E\big[\widehat{\mu}' - \widehat{\epsilon}'\big], \tag{A.7}$$

where $\widetilde{B} = \psi\dfrac{(1-\widetilde{e})^{1-1/\gamma} - 1}{1-1/\gamma}$ and $\widetilde{\epsilon} = \widetilde{\mu} = \dfrac{\eta\widetilde{J}}{\widetilde{\gamma}\alpha\widetilde{y} + \eta\widetilde{J}\big(1-\beta(1-\delta)\lambda\big)}$.

The F.O.C.'s of wage and effort bargaining yield:

$$\widehat{\mu} - \widehat{\epsilon} = \widehat{z} + (1-\alpha)\widehat{k} - \widehat{\xi} - \frac{1}{\widetilde{\gamma}}\widehat{e} = \left(\frac{\eta\widetilde{J}}{\widetilde{\gamma}\alpha\widetilde{y} + \eta\widetilde{J}}\right)\beta(1-\delta)\lambda E\big[\widehat{\mu}' - \widehat{\epsilon}'\big]. \tag{A.8}$$

This implies that $\widehat{\mu} = \widehat{\epsilon}$ for all $\mathbf{s}$. Given the generalized Nash bargaining for matches whose wage are re-negotiated, the increased surplus of a worker is proportional to the decreased surplus of a firm.[1] Therefore, when the wage is renegotiated, i.e., $\widehat{w} = \widehat{w}^*$, the change in effort equals the Frisch elasticity multiplied by the change in marginal product of labor:

$$\widehat{e}(\widehat{w}^*) = \widetilde{\gamma}\Big(\widehat{z} + (1-\alpha)\widehat{k} - \widehat{\xi}\Big). \tag{A.9}$$

The F.O.C. for effort bargaining yields:

$$\widehat{J} + \frac{1}{\widetilde{\gamma}}\widehat{e} = \widehat{H} + \widehat{z} + (1-\alpha)\widehat{k}. \tag{A.10}$$

Substituting (A.5) and (A.6) into (A.10) with $\widehat{\mu} = \widehat{\epsilon}$ yields the individual effort choice as:

$$\begin{aligned}\left(\frac{\widetilde{\gamma}\alpha\widetilde{y} + \eta\widetilde{J}}{\widetilde{\gamma}}\right)\widehat{e} =& \frac{\widetilde{w}}{1-\tau_i}\widehat{w} - \frac{\tau_i\widetilde{w}}{1-\tau_i}E\widehat{w}^{*\prime} - \eta\kappa\widetilde{\theta}\widehat{\theta}\\ &+\Big((1-\eta)\widetilde{B} - \eta\widetilde{J}\Big)\widehat{\xi} + \eta\left(\widetilde{J} - \alpha\widetilde{y}\right)\left(\widehat{z} + (1-\alpha)\widehat{k}\right),\end{aligned} \tag{A.11}$$

[1] Note that the first-order condition for the wage bargaining does not necessarily imply $\mu(w^*,\mathbf{s}) = \epsilon(w^*,\mathbf{s})$ for all $\mathbf{s}$. It only implies that the surplus of each party changes in a proportion to a first-order approximation: $\widehat{\mu} = \widehat{\epsilon}$.

where $\tau_i = \dfrac{\beta(1-\delta)\lambda\widetilde{\mu}}{1+\beta(1-\delta)\lambda\widetilde{\mu}} = \left(\dfrac{\eta\widetilde{J}}{\widetilde{\gamma}\alpha\widetilde{y}+\eta\widetilde{J}}\right)\tau.$

Integrating (A.11) over the wage distribution $G(w)$ and applying the definitions for aggregate effort, $\overline{e} = \displaystyle\int e(w)dG(w)$, and aggregate wage, $\overline{w} = \displaystyle\int wdG(w)$, and $\widehat{k} = -\big(\widehat{n}+\widehat{\overline{e}}\big)$ yields the following expression for aggregate effort:

$$\begin{aligned}\Xi\widehat{\overline{e}} =& \frac{\widetilde{w}}{1-\tau_i}\widehat{\overline{w}} - \frac{\tau_i\widetilde{w}}{1-\tau_i}E\widehat{w}^{*\prime} - \eta\kappa\widetilde{\theta}\widehat{\theta} \\ &+ \Big((1-\eta)\widetilde{B}-\eta\widetilde{J}\Big)\widehat{\xi} + \eta\left(\widetilde{J}-\alpha\widetilde{y}\right)\left(\widehat{z}-(1-\alpha)\widehat{n}\right),\end{aligned} \tag{A.12}$$

where $\Xi = \big(1-\eta(1-\alpha)\big)\alpha\widetilde{y} + \left(\dfrac{1}{\widetilde{\gamma}}+1-\alpha\right)\eta\widetilde{J}.$

Log-linearizing the first-order condition for the wage bargaining and substituting (A.5) and (A.6) for $\widehat{J}$ and $\widehat{H}$, respectively, with $\widehat{\mu}-\widehat{\epsilon}=0$ yields the Nash-bargained wage:

$$\begin{aligned}\widehat{w}^* = (1-\tau_i)\Bigg\{&\alpha\left(\frac{\widetilde{y}}{\widetilde{w}}\right)(\eta+\widetilde{\gamma})\big(\widehat{z}+(1-\alpha)\widehat{k}\big) \\ &+\eta\kappa\left(\frac{\widetilde{\theta}}{\widetilde{w}}\right)\widehat{\theta} + \left(\frac{\widetilde{\gamma}\alpha\widetilde{y}+(1-\eta)\widetilde{B}}{\widetilde{w}}\right)\widehat{\xi}\Bigg\} + \tau_i E\widehat{w}^{*\prime}.\end{aligned} \tag{A.13}$$

Substituting (A.12) for $\widehat{\overline{e}}$ in $\widehat{k} = -(\widehat{n}+\widehat{\overline{e}})$ shows that the Nash-bargaining wage depends on its future expectation $E\widehat{w}^{*\prime}$, the aggregate wage $\widehat{\overline{w}}$, and the wage rate under the flexible wage $\widehat{w}^*_F$ (described below):

$$\widehat{w}^* = \left(1-\frac{\varphi_1}{\widetilde{\gamma}\Xi}\right)\widehat{\overline{w}} + \frac{\varphi_1}{\widetilde{\gamma}\Xi}\Big\{(1-\tau_i)\widehat{w}^*_F + \tau_i E\widehat{w}^{*\prime}\Big\}, \tag{A.14}$$

where $\varphi_1 = \big(1+\widetilde{\gamma}(1-\alpha)\big)\big(\widetilde{\gamma}\alpha\widetilde{y}+\eta\widetilde{J}\big)$. The law of motion for total wage payment, $n'\overline{w}' + (1-\delta)\lambda n\overline{w} + mw^{*\prime}$, and that for employment, $\widehat{n}' = (1-\delta)\widehat{n}+\delta\widehat{m}$, yields the aggregate wage as a weighted average of newly-negotiated wage and its lagged value $\widehat{\overline{w}}_{-1}$:

$$\widehat{\overline{w}} = \big(1-\lambda(1-\delta)\big)\widehat{w}^* + \lambda(1-\delta)\widehat{\overline{w}}_{-1} = (1-(1-\delta)\lambda)\widehat{w}^* + (1-\delta)\lambda\widehat{\overline{w}}_{-1}.$$

Substituting (A.14) for $\widehat{w}^*$ expresses the aggregate wage in terms of its future expectation $E\widehat{w}^{*\prime}$, its lagged value, and the wage under the flexible wage $\widehat{w}^*_F$:

$$\widehat{\overline{w}} = \frac{\varphi_1}{\varphi_1-\varphi_2}(1-(1-\delta)\lambda)\Big\{(1-\tau_i)\widehat{w}^*_F + \tau_i E\widehat{w}^{*\prime}\Big\} + \frac{\widetilde{\gamma}\Xi}{\varphi_1-\varphi_2}(1-\delta)\lambda\widehat{\overline{w}}_{-1}, \tag{A.15}$$

where $\varphi_2 = (1-\delta)\lambda(1-\alpha)(\eta+\widetilde{\gamma})\widetilde{\gamma}\alpha\widetilde{y}.$

Finally, log-linearizing the free entry condition using (A.6) yields the forward-looking difference equation for $\widehat{\theta}$:

$$\frac{\eta\kappa}{\widetilde{q}}\widehat{\theta} = \beta E\left[\alpha\widetilde{y}\Big(\widehat{z}' + (1-\alpha)\widehat{k}' + \widehat{e}'\Big) - \frac{\widetilde{w}}{1-\tau_i}\widehat{w}^{*\prime} + \frac{\tau_i\widetilde{w}}{1-\tau_i}\widehat{w}^{*\prime\prime} + \frac{\eta\kappa}{\widetilde{q}}(1-\delta)\widehat{\theta}'\right]. \tag{A.16}$$

Substituting (A.13) and (A.9) into (A.16) yields the dynamics of the labor market tightness, v/u:

$$\frac{\eta\kappa}{\widetilde{q}}\widehat{\theta} = \beta E\left[(1-\eta)\alpha\widetilde{y}\big(\widehat{z}' + (1-\alpha)\widehat{k}'\big) + \frac{\eta\kappa}{\widetilde{q}}(1-\delta-\widetilde{p})\widehat{\theta}' + (1-\eta)\widetilde{B}\widehat{\xi}'\right]. \tag{A.17}$$

Sticky Wage with Common Effort Choice Match surpluses under sticky wages with common effort choice are identical to those in the individual effort model: (A.1) and (A.3). So are their log-linearized equations: (A.5) and (A.6). However, the effects of the wage bargaining on the match surpluses differ slightly from (A.2) and (A.4), reflecting the assumption that individual wage bargains do not influence the common effort choice, i.e. $\Lambda(w^*, \mathbf{s}) = \frac{\partial e(w^*,\mathbf{s})}{\partial w} = 0$. Thus, the surplus gain to a worker and loss to a firm from a wage increase are simply:

$$\epsilon(w^*, \mathbf{s}) = \frac{\partial H(w^*, \mathbf{s})}{\partial w} = 1 + \beta(1-\delta)\lambda\mathbb{E}\big[\epsilon(w^{*\prime}, \mathbf{s}')|\mathbf{s}\big], \tag{A.18}$$

$$\mu(w^*, \mathbf{s}) = -\frac{\partial J(w^*, \mathbf{s})}{\partial w} = 1 + \beta(1-\delta)\lambda\mathbb{E}\big[\mu(w^{*\prime}, \mathbf{s}')|\mathbf{s}\big]. \tag{A.19}$$

It is clear that $\epsilon(w^*, \mathbf{s}) = \mu(w^*, \mathbf{s})$ for all $\mathbf{s}$, with $\widetilde{\epsilon} = \widetilde{\mu} = \dfrac{1}{1-\beta(1-\delta)\lambda}$; so $\widehat{\mu} - \widehat{\epsilon} = 0$ for all $\mathbf{s}$.[2]

The log-linearized first-order condition for common effort bargaining is given by:

$$\frac{1}{\widetilde{\gamma}}\widehat{e} = \widehat{z} + (1-\alpha)\widehat{k} + \int \Big[\widehat{H} - \widehat{J}\Big]dG, \tag{A.20}$$

and $\widehat{H} - \widehat{J}$ is obtained from (A.5) and (A.6) as:

$$\begin{aligned}\widehat{H} - \widehat{J} = \frac{1}{\eta\widetilde{J}}\Big\{&(1-\eta)(\widetilde{w}-b)\widehat{\xi} + \big(1+\beta(1-\delta)\lambda\widetilde{\mu}\big)\widetilde{w}\widehat{w} - \beta(1-\delta)\lambda\widetilde{\mu}E\widehat{w}^{*\prime} \\ &- \eta\kappa\widetilde{\theta}\widehat{\theta} - \alpha\widetilde{y}\widehat{e} - \eta\alpha\widetilde{y}\big(\widehat{z} + (1-\alpha)\widehat{k}\big) - \frac{\eta\kappa}{\widetilde{q}}(1-\delta-\widetilde{p})E\big[\widehat{\mu}' - \widehat{\epsilon}'\big]\Big\}.\end{aligned} \tag{A.21}$$

[2] Thus, the F.O.C. of wage bargaining holds exactly (not to a first-order approximation) in the common effort model.

Integrating (A.21) over the wage distribution, substituting the resulting expression into (A.20), and applying the equilibrium condition, $\widehat{e} = \widehat{\overline{e}}$, yields the log-linearized expression for aggregate effort. That expression is identical to (A.12) except now $\tau = \beta(1-\delta)\lambda$.

The log-linearized first-order condition for the wage bargaining is given by $\widehat{H} = \widehat{J}$. Substituting (A.5), (A.6) and (A.12) for $\widehat{H}$, $\widehat{J}$ and $\widehat{\overline{e}}$, respectively, yields the log-linearized expression for the bargained wage:

$$\widehat{w}^* = \left(1 - \frac{\varphi_3}{\widetilde{\gamma}\Xi}\right)\widehat{\overline{w}} + \frac{\varphi_3}{\widetilde{\gamma}\Xi}\Big\{(1-\tau)\widehat{w}^*_{FV} + \tau E\widehat{w}^{*\prime}\Big\}, \tag{A.22}$$

where $\varphi_3 = \big(1 + \widetilde{\gamma}(1-\alpha)\big)\eta\widetilde{J}$.

Substituting (A.22) into $\widehat{\overline{w}} = (1-(1-\delta)\lambda)\widehat{w}^* + (1-\delta)\lambda\widehat{\overline{w}}_{-1}$ and rearranging terms yields the log-linearized expression for the aggregate wage:

$$\widehat{\overline{w}} = \frac{\varphi_3}{\varphi_3+\varphi_4}(1-(1-\delta)\lambda)\Big\{(1-\tau)\widehat{w}^*_F + \tau E\widehat{w}^{*\prime}\Big\} + \frac{\widetilde{\gamma}\Xi}{\varphi_3+\varphi_4}(1-\delta)\lambda\widehat{\overline{w}}_{-1}, \tag{A.23}$$

where $\varphi_4 = (1-\delta)\lambda\big(1-\eta(1-\alpha)\big)\widetilde{\gamma}\alpha\widetilde{y}$.

The dynamics of v/u ratio in the common effort model is characterized by the same equation as (A.17). In fact, the log-linearized expressions for v/u ratio are identical regardless of the specifications of wage and effort bargaining. Note that, despite the identical expressions for v/u ratio across models, its quantitative properties differ considerably because the models exhibit different dynamics of wages and effort.

Flexible Wage with Variable Effort If wages are perfectly flexible, i.e., $w = w^*$, match surpluses and their derivatives with respect to wage are:

$$H(w^*, \mathbf{s}) = w^* - b + \xi\psi\frac{(1-e)^{1-1/\gamma} - 1}{1-1/\gamma} + \beta(1-\delta-p(\theta))\mathbb{E}\big[H(w^{*\prime}, \mathbf{s}')|\mathbf{s}\big], \tag{A.24}$$

$$J(w^*, \mathbf{s}) = \alpha z k^{1-\alpha} e - w^* + \beta(1-\delta)\mathbb{E}\big[J(w^{*\prime}, \mathbf{s}')|\mathbf{s}\big], \tag{A.25}$$

$$\epsilon(w^*, \mathbf{s}) = 1 - \xi\psi(1-e)^{-1/\gamma}\Lambda(w^*, \mathbf{s}), \tag{A.26}$$

$$\mu(w^*, \mathbf{s}) = 1 - \alpha z k^{1-\alpha}\Lambda(w^*, \mathbf{s}). \tag{A.27}$$

Note that with $\lambda = 0$ these expressions are identical to (A.1), (A.3), (A.2) and (A.4).

The F.O.C. for the effort choice, combined with (A.26) and (A.27), is:

$$\widehat{\mu} - \widehat{\epsilon} = \widehat{z} + (1-\alpha)\widehat{k} - \widehat{\xi} - \frac{1}{\widetilde{\gamma}}\widehat{e} = -\frac{\eta\widetilde{J}}{\widetilde{\gamma}\alpha\widetilde{y}}\Big(\widehat{\mu} - \widehat{\epsilon}\Big),$$

which implies $\widehat{\mu} = \widehat{\epsilon}$ for all $\mathbf{s}$, and $\widehat{e} = \widetilde{\gamma}\big(\widehat{z} + (1-\alpha)\widehat{k} - \widehat{\xi}\big)$. Substituting $\widehat{k} = -(\widehat{n} + \widehat{e})$ yields:

$$\widehat{e} = \frac{\widetilde{\gamma}}{1+\widetilde{\gamma}(1-\alpha)}\Big(\widehat{z} - (1-\alpha)\widehat{n} - \widehat{\xi}\Big). \tag{A.28}$$

The (log-linearized) first-order condition for the bargained wage becomes:

$$\begin{aligned}\widehat{w}_F^* = \alpha\left(\frac{\widetilde{y}}{\widetilde{w}}\right)\left(\frac{\eta+\widetilde{\gamma}}{1+\widetilde{\gamma}(1-\alpha)}\right)\big(\widehat{z} - (1-\alpha)\widehat{n}\big) + \eta\kappa\left(\frac{\widetilde{\theta}}{\widetilde{w}}\right)\widehat{\theta} \\ - \left(\frac{\widetilde{\gamma}\alpha\widetilde{y}\big(1-\eta(1-\alpha)\big)}{\big(1+\widetilde{\gamma}(1-\alpha)\big)\widetilde{w}} + \frac{(1-\eta)\widetilde{B}}{\widetilde{w}}\right)\widehat{\xi}\end{aligned} \tag{A.29}$$

Flexible Wage with Fixed Effort Under flexible wages and fixed effort, the worker and firm match surpluses are identical to (A.24) and (A.25) respectively, with $e = \widetilde{e}$ and $\epsilon(w^*, \mathbf{s}) = \mu(w^*, \mathbf{s}) = 1$ for all $\mathbf{s}$. Then, the Nash bargained wage is:

$$\widehat{w}_F^* = \eta\alpha\left(\frac{\widetilde{y}}{\widetilde{w}}\right)\big(\widehat{z} - (1-\alpha)\widehat{n}\big) + \eta\kappa\left(\frac{\widetilde{\theta}}{\widetilde{w}}\right)\widehat{\theta} - \left(\frac{(1-\eta)\widetilde{B}}{\widetilde{w}}\right)\widehat{\xi}, \tag{A.30}$$

which is identical to the expression for $\widehat{w}_F^*$ in (A.29) when $\widetilde{\gamma} = 0$.

Sticky Wage with Fixed Effort Under the standard sticky-wage with effort fixed, match surpluses for worker and firm are identical to (A.1) and (A.3) respectively, with $e = \widetilde{e}$. Because $\epsilon(w^*, \mathbf{s}) = 1 + \beta(1-\delta)\lambda\mathbb{E}[\epsilon(\mathbf{s}')|\mathbf{s}]$ and $\mu(w^*, \mathbf{s}) = 1 + \beta(1-\delta)\lambda\mathbb{E}[\mu(\mathbf{s}')|\mathbf{s}]$, we have $\epsilon(w^*, \mathbf{s}) = \mu(w^*, \mathbf{s})$ for all $\mathbf{s}$, where $\widetilde{\epsilon} = \widetilde{\mu} = \dfrac{1}{1-\beta(1-\delta)\lambda} = \dfrac{1}{1-\tau}$. This implies that the increase (decrease) in the match value for a worker (firm) due to wage increase is simply 1.

The log-linearized first-order condition for wage bargaining is:

$$\widehat{w}^* = (1-\tau)\widehat{w}_F^* + \tau E\widehat{w}^{*\prime}. \tag{A.31}$$

where $\widehat{w}_F^*$ is the bargained wage under flexible wage with fixed effort in (A.30). Then, the aggregate wage becomes:

$$\widehat{\overline{w}} = (1-(1-\delta)\lambda)\Big\{(1-\tau)\widehat{w}_F^* + \tau E\widehat{w}^{*\prime}\Big\} + (1-\delta)\lambda\widehat{\overline{w}}_{-1}. \tag{A.32}$$

Other Aggregate Variables Given the aggregate wage $\widehat{\overline{w}}$, effort $\widehat{\overline{e}}$, and the labor-market tightness $\widehat{\theta}$, the dynamics of other aggregate variables are:

$$\widehat{z} = \rho_z\widehat{z}_{-1} + \widehat{\varepsilon}_z$$

$$
\begin{aligned}
\widehat{\xi} &= \rho_\xi \widehat{\xi}_{-1} + \widehat{\varepsilon}_\xi \\
\widehat{y} &= \widehat{z} + \alpha\left(\widehat{n} + \widehat{\overline{e}}\right) \\
\widehat{v} &= \widehat{\theta} + \widehat{u}_{-1} \\
\widehat{u} &= -\frac{\widetilde{n}}{1-\widetilde{n}}\widehat{n} \\
\widehat{n} &= (1-\delta)\widehat{n}_{-1} + \delta\widehat{m} \\
\widehat{m} &= (1-\eta)\widehat{\theta} + \widehat{u}_{-1} \\
\widehat{TFP} &= \widehat{z} + \alpha\widehat{\overline{e}}.
\end{aligned}
$$

A.2. Dynamics of Aggregate Wage in Sticky Wage Models

We now derive the dynamics of aggregate wage. By iterating (A.14), the Nash bargained wage in the individual effort model can be written as:

$$
\begin{aligned}
\widehat{w}^* =& \sum_{j=0}^{\infty}\left(\frac{\varphi_1\tau_i}{\widetilde{\gamma}\Xi}\right)^j\left\{\frac{\varphi_1}{\widetilde{\gamma}\Xi}(1-\tau_i)\widehat{w}^*_{F,t+j} + \left(1-\frac{\varphi_1}{\widetilde{\gamma}\Xi}\right)\widehat{\overline{w}}_{t+j}\right\} \\
=& \varphi\left(\frac{1}{\tau_i}-1\right)\frac{E\widehat{w}^*_{F,t}}{1-\varphi\mathbb{F}} + \left(1-\frac{\varphi}{\tau_i}\right)\frac{E\widehat{\overline{w}}_t}{1-\varphi\mathbb{F}},
\end{aligned}
\tag{A.33}
$$

where $\varphi = \dfrac{\varphi_1\tau_i}{\widetilde{\gamma}\Xi}$ and $\mathbb{F}$ is a forward operator, i.e., $\mathbb{F}x_t = Ex_{t+1}$. Analogously, iterations of (A.22) yields the Nash-bargaining wage under common effort (where $\varphi_1\tau_i = \varphi_3\tau$):

$$
\begin{aligned}
\widehat{w}^* =& \sum_{j=0}^{\infty}\left(\frac{\varphi_3\tau}{\widetilde{\gamma}\Xi}\right)^j\left\{\frac{\varphi_3}{\widetilde{\gamma}\Xi}(1-\tau)\widehat{w}^*_{F,t+j} + \left(1-\frac{\varphi_3}{\widetilde{\gamma}\Xi}\right)\widehat{\overline{w}}_{t+j}\right\} \\
=& \varphi\left(\frac{1}{\tau}-1\right)\frac{E\widehat{w}^*_{F,t}}{1-\varphi\mathbb{F}} + \left(1-\frac{\varphi}{\tau}\right)\frac{E\widehat{\overline{w}}_t}{1-\varphi\mathbb{F}}.
\end{aligned}
\tag{A.34}
$$

While the two expressions appear nearly identical, τ_i in the individual effort model differs from the τ in the common effort model.

The Nash-bargaining wage under the fixed effort is readily obtained by setting the Frisch elasticity of effort to zero, $\widetilde{\gamma} = 0$:

$$
\widehat{w}^* = (1-\tau)\sum_{j=0}^{\infty}\tau^j E\widehat{w}^*_{F,t+j} = (1-\tau)\frac{E\widehat{w}^*_{F,t}}{1-\tau\mathbb{F}}. \tag{A.35}
$$

Substituting (A.33) into $\widehat{\widehat{w}}_t = (1-(1-\delta)\lambda)\widehat{w}_t^* + (1-\delta)\lambda\widehat{\widehat{w}}_{t-1}$, and multiplying both sides by $1-\varphi\mathbb{F}$, yields the aggregate wage in the individual effort model:

$$\widehat{\widehat{w}}_t = \pi_1^i \widehat{\widehat{w}}_{t-1} + \pi_2^i E\widehat{\widehat{w}}_{t+1} + (1-\pi_1^i-\pi_2^i)\widehat{w}_{F,t}^*,$$

where $\pi_1^i = \dfrac{(1-\delta)\lambda}{1+\varphi(1-\delta)\lambda-(1-(1-\delta)\lambda)(1-\varphi/\tau_i)}$ and $\pi_2^i = \dfrac{\varphi}{1+\varphi(1-\delta)\lambda-(1-(1-\delta)\lambda)(1-\varphi/\tau_i)}$.

Analogous to (A.34) and (A.35), the aggregate wage under common effort is:

$$\widehat{\widehat{w}}_t = \pi_1^c \widehat{\widehat{w}}_{t-1} + \pi_2^c E\widehat{\widehat{w}}_{t+1} + (1-\pi_1^c-\pi_2^c)\widehat{w}_{F,t}^*,$$

where $\pi_1^c = \dfrac{(1-\delta)\lambda}{1+\varphi(1-\delta)\lambda-(1-(1-\delta)\lambda)(1-\varphi/\tau)}$, $\pi_2^c = \dfrac{\varphi}{1+\varphi(1-\delta)\lambda-(1-(1-\delta)\lambda)(1-\varphi/\tau)}$.

Similarly, the aggregate wage in the standard sticky wage with fixed effort is

$$\widehat{\widehat{w}}_t = \pi_1 \widehat{\widehat{w}}_{t-1} + \pi_2 E\widehat{\widehat{w}}_{t+1} + (1-\pi_1-\pi_2)\widehat{w}_{F,t}^*,$$

where $\pi_1 = \dfrac{(1-\delta)\lambda}{1+\tau(1-\delta)\lambda}$ and $\pi_2 = \dfrac{\tau}{1+\tau(1-\delta)\lambda}$.

Since $\dfrac{\varphi}{\tau} = \dfrac{\varphi_3}{\widetilde{\gamma}\Xi} < 1$ and $\dfrac{\varphi}{\tau_i} = \dfrac{\varphi_1}{\widetilde{\gamma}\Xi} > 1$, $\tau_i < \varphi < \tau$. It follows that $\pi_1^c > \pi_1^i$ and $\pi_2^c > \pi_2^i$. It is also clear that $\pi_2 > \pi_2^c$, as $\tau[1+\varphi(1-\delta)\lambda-(1-(1-\delta)\lambda)(1-\varphi/\tau)] - \varphi[1+\tau(1-\delta)\lambda] = (1-\delta)\lambda(\tau-\varphi) > 0$. The relative sizes of π_1 and π_1^i depends on the sign of $(1-\delta)\lambda(\tau-\varphi)+(1-(1-\delta)\lambda)(1-\varphi/\tau_i)$, which cannot be determined analytically. However, under our benchmark calibration (as well as for a wide range of parameter values), this sign is positive, which implies that $\pi_1^i > \pi_1$. Combining all these results yields $\pi_1^c > \pi_1^i > \pi_1$: the weight on the lagged value is the largest under the common effort choice.

B. Additional Results for the Model Economies

In this appendix section we provide extended results for models with fixed versus variable effort under flexible wages to parallel those discussed in the text under sticky wages. We also provide additional business cycle moments from the models, specifically the standard deviations of variables and their correlations with real GDP.

First consider shocks to technology. The model economies are subjected to persistent productivity shocks (auto-correlation of 0.95), with standard deviation chosen so that each model matches the standard deviation of HP-filtered TFP in the data. Because TPF reflects effort responses, this implies that the volatility of the shocks differ across models. For the models with fixed effort the standard deviation of the innovation to technology is 0.86%. For the models with effort fluctuations this standard deviation is respectively 0.69%, 1.06%, and 2.83% under flexible wages, sticky wages with individual effort, and sticky wages with common effort. In addition to the figures discussed in the text, Figure A1 compares the responses of wages, output, and employment under flexible wages for fixed effort and variable effort that reflects a common-effort choice. The models do not differ so sharply under flexible wages. Allowing the effort margin provides an added dimension for labor hours to respond, with effort declining in response to the decline in productivity. As a result output (and TFP) respond by about 20 percent more to the shock. Employment actually declines a little less. This reflects that the decline in effort, by increasing labor's marginal product, essentially substitutes for some of the decline in employment.

Table A1 extends Table 2 from the text to provide results for the flexible wage models. The data column and the rightmost four columns are carried forward from Table 2. The top panel provides model moments in response to productivity shocks. Looking at the two flexible wage columns we see that, regardless of whether effort is fixed or responsive, the models produce wage rates that are far too cyclical and employment that is far too acyclical relative to the data.

Table A2 gives additional model moments results across the various models as well as for quarterly statistics for the U.S. economy for 1959:I to 2017:IV (first column) for model economies hit by productivity shocks. The additional moments are standard deviation of a series and its correlation with real output. In addition to output, the variables reported are employment, average hourly earnings, and TFP. Real output, employment, and average earnings are for the U.S. business sector as reported by the BLS program on Labor Productivity and Costs (https://www.bls.gov/lpc/). TFP is constructed from these data, hours (from same source), and the business-sector capital stock from the U.S. Department of Commerce. The capital series is annual; we interpolate quarterly values. TFP reflects a correction for procyclical capital utilization. In the table we report the newly bargained wage, w^*, for the model, but not the data, as there is no aggregate data series corresponding to w^*. In the text we report cyclicality for an estimate of the new-hire wage based on Basu and

House (2016). Much of the volatility of such a series reflects sampling error in the estimates. So the moments in Table A2, standard deviations and correlations with output, would be biased upwards and downwards respectively by these errors. (The cyclical elasticities, based on projections on aggregate output, reported in text Table 2 and Table A1, should not in principle be biased by those errors.) The table provides model results under flexible and sticky wages; but we focus discussion on the models with sticky wages. The standard deviation of employment in the data is 0.75 that for output.[3] But under fixed effort, or with individually chosen effort, model-predicted employment is much less volatile, with standard deviations one-sixth to one-eighth that in output. By contrast, the standard deviation of employment under our preferred model with common effort is 0.58 that in output, much closer to the data than any other models. Directly related, the model with a common effort response matches much better the relative standard deviation of TFP (0.68 versus 0.54 in the data). Finally, while none of the models matches the low correlation of aggregate wages with output in the data (0.16), the common-effort model does much better (0.65) than all other models (between 0.8 and 1).

For comparison, the last column, "G-T (2009)," reports the moments for the standard sticky-wage model with constant effort, but with wages for newly-hired workers tied explicitly to the sticky contracted wages of existing workers as, for instance, in Gertler and Trigari (2009). We calibrate the parameters of this model to be comparable to our benchmark, for instance, with the replacement ratio of 0.75 and a Calvo parameter of $\lambda = 3/4$.

The standard deviation of wages from the G-T economy is 0.45. That is larger than that for newly-hired or aggregate wages from our common effort model. The correlation of wages with output from the G-T economy is 0.74, which falls between those of newly-hired, 0.92, and aggregate wages, 0.65, from our common effort model. The volatility of employment from the G-T economy, 0.30, is about half that from the common effort model. In sum, our model with common effort generates much more inertia in aggregate wages and twice as volatile employment compared to the G-T economy, even though newly-hired wages are completely flexible.

Figures A2 and A3 display model responses to persistent preference shocks. These sup-

[3] All hours fluctuations in the models reflect employment. For the data we also examined statistics on total hours (employment times workweek). Its standard deviation relative to that in output is 0.93; its correlation with output is 0.86.

plement those in Figure 5 under the sticky wages. The preference shocks have an auto-correlation 0.95. Figure A2 compares flexible versus sticky wage responses under fixed effort. Except for the aggregate wage, the economies behave essentially the same. In particular, wage stickiness has no impact on the responses of employment or output (not pictured). Figure A3 compares responses under flexible wages when effort is fixed versus responsive. Wages behave very differently across the models. Wages go up in response to the negative labor supply under fixed effort, but go down when effort can respond, reflecting the decline in effort driven by the decline in desired labor supply. The decline in effort causes a much larger decline in output than under constant effort, but a somewhat smaller decline in employment, again, reflecting that the decline in effort partially substitutes for a decline in employment.

The bottom panel of Table A1 extends the results from the bottom panel of text Table 2 to give results for the flexible wage models. Results under sticky wages are repeated in the rightmost three columns. The model with flexible wages and variable effort actually responds similarly to that under productivity shocks. Wages and TFP are much more procyclical than in the data, while employment is much more acyclical.

In Table A3 we report the added business cycle moments from our simulated models in the face of preference shocks. The model economies are subjected to preference shocks with auto-correlation 0.95 and a standard deviation of its innovations to 5.2 percent. That standard deviation is set so that the standard deviation of output from the flexible wage with variable effort model matches that in the data, (2.01 percent). The table reports the standard deviation for output; for other variables it reports its standard deviation relative to that for output and correlation with output. The table provides model results under flexible and sticky wages; but again we focus discussion on the models with sticky wages. The fixed effort model generates very different fluctuations under preference versus productivity shocks. The volatility of employment now exceeds that in output. Wages are extremely countercyclical, exhibiting correlations with respect to output of -0.83 to -0.92. The moments for models with variable effort closely resemble those generated by productivity shocks (Table A2). In particular, for the model with common effort the volatility of employment versus output is 0.65, compared to 0.58 under productivity shocks. Both fall a little short of the corresponding value in the data of 0.75. But the match to the data is much closer than any other models.

C. SIPP Data for Measuring Wage Flexibility

The SIPP is a longitudinal survey of households designed to be representative of the U.S. population. It consists of a series of overlapping longitudinal panels. Each panel is three or more years in duration. Each is large, containing samples of about 20,000 households. Households are interviewed every four months. At each interview, information on work experience (employers, hours, earnings) are collected. Each year from 1984 through 1993 a new panel was begun. New, somewhat longer, panels were initiated in 1996, 2001, 2004, and 2008. In our analysis we employ the 6 panels from 1990 through 2001. (The 1984-1989 panels contain less reliable information on employer changes. The 2004 and 2008 panels carry forward employment information, including the wage rate, if the respondent deems changes to be small.) The SIPP interviews provide employment status and weeks worked for each of the prior four months. But earnings information is only collected for the interview month; so we restrict attention to the survey month observations.

For our purposes the SIPP has some distinct advantages. Compared to a matched CPS sample, we are able to calculate workers' wage changes across multiple surveys and at intervals of four months, rather than 12. It also provides better information for defining employer turnover. The SIPP has both a larger and more representative sample than the PSID or NLS panels and, most importantly, individuals are interviewed every four months.

We restrict our sample to persons of ages 20 to 60. Individuals must not be in the armed forces, not disabled, and not be attending school full-time. We only consider wage rates for workers who usually work more than 10 hours per week and report monthly earnings of at least \$100 and no more than \$25,000 in December 2004 CPI dollars. Any reported hourly wage rates that are imputed, top-coded, or below \$4 in December 2004 dollars are set equal to missing. Although the SIPP panels draw representative samples, in constructing all reported statistics we employ SIPP sampling weights that account for sample attrition. We also weight individuals by their relative earnings in the sample period, as this is consistent with the influence of workers for aggregate labor statistics.

We calculate frequency of wage changes over the 4-month interval between surveys for workers who remain with the same employer for their main job. For the 1990 to 1993 panels we define workers as stayers if the SIPP employer ID remains the same across surveys. We employ the 1990-1993 SIPP revised employer ID's, which have been edited at the Census to be consistent with information available in the non-public Census version of the data. Such edits have not been undertaken for 1996 and later panels. For the later panels a number of changes in employer ID appear (based on wages, et cetera) to not represent an employer change. For the later panels we define stayers based on responses to when the reference job began. More exactly, we define the worker as a new hire if they report that their job began within the last four months, or if in the prior survey they report that the reference job had ended by the survey. (This latter case is relatively rare.) We additionally call the worker a new hire if the employer ID and the industry of employment both change across surveys. We similarly calculate frequency of wage changes across eight-month intervals for those workers we classify as stayers over that 8-month interval. In calculating the Calvo parameter we use 4-month frequencies calculated just for 8-month stayers, so that the 4 and 8-month changes are calculated for the same sample.

Employed respondents report monthly earnings. In addition, just over half report an hourly rate of pay. For each worker we also calculate a weekly wage by dividing monthly earnings by the number of weeks worked in the month. We define a worker's wage as not changing if any of these three measures remains the same across the surveys.

The SIPP provides the worker's 3-digit industry code, allowing us to map SIPP workers to KLEMS industries. The total sample, combining observations from the 1990 to 2001 panels is large. For calculating 4-month and 8-month frequencies of wage changes it equals 350,044 observations; of these, 294,678 map to one of our KLEMS industries.

Table A1: Cyclicality Under Various Model Specifications

	Data	Models under Productivity Shocks (z)					
		Flexible Wages		Sticky Wages			
		Fixed Effort	Variable Effort	Fixed Effort	Individual Effort	Common Effort	G-T (2009)
Employment*	0.59 (0.03)	0.13	0.09	0.13	0.15	0.53	0.21
Unemployment Rate	-4.84 (0.19)	-1.93	-1.30	-1.93	-2.18	-7.99	-3.21
TFP	0.39 (0.03)	0.92	0.94	0.92	0.91	0.66	0.85
Newly-Hired Wage	0.46 (0.10)	0.81	0.87	0.71	1.06	0.37	0.33
Aggregate Wage	-0.03 (0.05)	0.81	0.87	0.38	0.75	0.20	0.33

	Data	Models under Preference Shocks (ξ)				
		Flexible Wages		Sticky Wages		
		Fixed Effort	Variable Effort	Fixed Effort	Individual Effort	Common Effort
Employment*	0.59 (0.03)	1.56	0.17	1.56	0.25	0.61
Unemployment Rate	-4.84 (0.19)	-23.4	-2.49	-23.4	-3.70	-9.11
TFP	0.39 (0.03)	0.00	0.89	0.00	0.84	0.61
Newly-Hired Wage	0.46 (0.10)	-0.90	0.75	-0.76	0.86	0.24
Aggregate Wage	-0.03 (0.05)	-0.90	0.75	-0.64	0.62	0.12

Notes: Coefficients are projection of $\ln(X)$ on log aggregate output, where X takes roles of employment, unemployment rate, wages, and TFP. "G-T (2009)" refers to the standard staggering-wage model (with fixed effort), such as Gertler and Trigari (2009), where wages of newly-hired are partially sticky (see text for the calibration of this model). All logged variables are quarterly and HP-filtered with smoothing parameter 1,600. Data are based on 1959:I-2017:IV. The civilian unemployment rates (16 years of age and older) are quarterly averages of the monthly series from the BLS (based on the Current Population Survey). For wages (both aggregate and newly-hired), the estimates are based on a shorter time period of 1978:I-2015:II when the newly-hired wages from Basu et al. (2016) are available. The cyclicality of employment and TFP for this corresponding shorter time period are 0.68 (0.04) and 0.30 (0.03), respectively. *The projection coefficient of total hours, as opposed to employment, on aggregate output is 0.77 (0.03).

Table A2: Business Cycle Moments of Models in Response to Productivity Shock

	Data		Flexible Wages				Sticky Wages							
			Fixed Effort		Variable Effort		Fixed Effort		Individual Effort		Common Effort		G-T (2009)	
	S.D.	*cor*	S.D.	*cor*	S.D.	*cor*	S.D.	*cor*	S.D.	*cor*	S.D.	*cor*	S.D.	*cor*
Output	2.01	1.00	1.21	1.00	1.18	1.00	1.21	1.00	1.23	1.00	1.66	1.00	1.27	1.00
Employment	0.75	0.79	0.18	0.73	0.12	0.70	0.18	0.73	0.16	0.89	0.58	0.92	0.30	0.72
Aggregate Wage	0.49	0.16	0.81	1.00	0.87	1.00	0.46	0.83	0.76	0.99	0.31	0.65	0.45	0.74
Newly Hired Wage	–	–	0.81	1.00	0.87	1.00	0.71	1.00	1.26	0.84	0.40	0.92	0.45	0.74
TFP	0.54	0.72	0.92	1.00	0.95	1.00	0.92	1.00	0.91	1.00	0.68	0.98	0.88	0.97

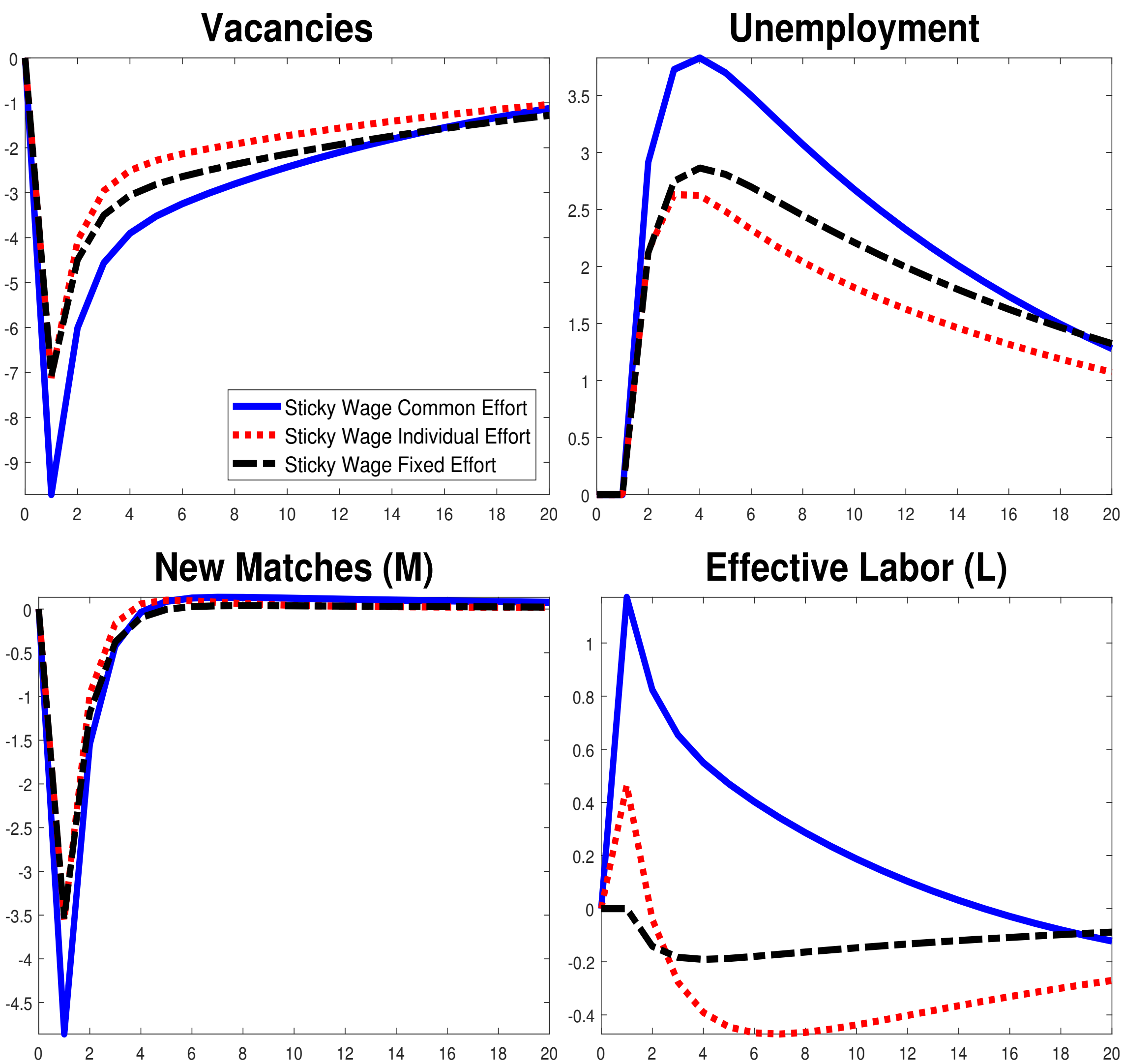
Notes: The standard deviations (S.D.) are relative to output except for output itself. The correlations with output are denoted by *cor*. "G-T (2009)" refers to the standard staggering-wage model (with fixed effort), such as Gertler and Trigari (2009), where the wages of newly-hired are partially sticky (see text for the calibration of this model). All variables are logged and H-P filtered with smoothing parameter 1,600. In all simulations, the auto-correlations of aggregate productivity shock are 0.95 and the standard deviation for innovation is chosen for the model to match the volatility of measured TFP in the U.S. data (see footnote 21 in the text). The S.D. of total hours and its correlation with output are respectively, 0.99 and 0.87. The data statistics are based on 1959:I - 2017:IV. The auto-correlations in the data (H-P filtered) are 0.86, 0.94, 0.68, and 0.74 respectively for output, employment, aggregate wage, and TFP. Those in our preferred benchmark model are 0.88, 0.83, 0.96, and 0.82, respectively.

Table A3: Business Cycle Moments of Models in Response to Preference Shock

	Data		Flexible Wages				Sticky Wages					
			Fixed Effort		Variable Effort		Fixed Effort		Individual Effort		Common Effort	
	S.D.	*cor*	S.D.	*cor*	S.D.	*cor*	S.D.	*cor*	S.D.	*cor*	S.D.	*cor*
Output	2.01	1.00	0.40	1.00	2.00	1.00	0.41	1.00	1.71	1.00	0.89	1.00
Employment	0.75	0.79	1.56	1.00	0.23	0.73	1.56	1.00	0.28	0.90	0.65	0.93
Aggregate Wage	0.49	0.16	1.08	-0.83	0.76	0.99	0.70	-0.92	0.62	0.99	0.20	0.61
Newly Hired Wage	–	–	1.08	-0.83	0.76	0.99	1.04	-0.73	1.05	0.82	0.26	0.89
TFP	0.54	0.72	–	–	0.90	0.99	–	–	0.83	1.00	0.63	0.97

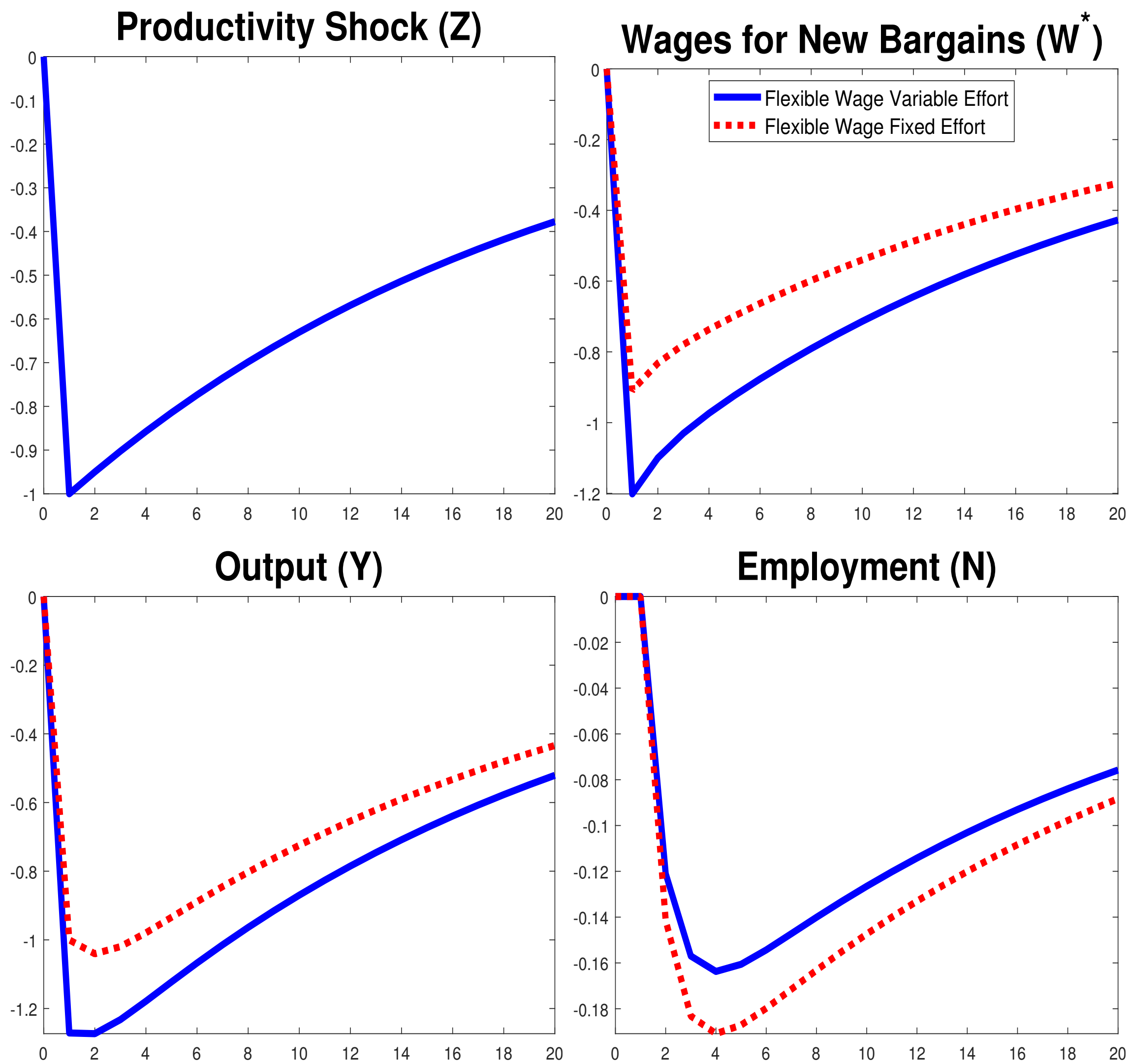
Notes: The standard deviations (S.D.) are relative to output except for output itself. The correlations with output are denoted by *cor*. "G-T (2009)" refers to the standard staggering-wage model (with fixed effort), such as Gertler and Trigari (2009), where the wages of newly-hired are partially sticky (see text for the calibration of this model). All variables are logged and H-P filtered with smoothing parameter 1,600. In all simulations, the auto-correlations of preference shock are 0.95 (before H-P filtered) and the standard deviation for innovation is chosen for the output volatility for the model with flexible wage and variable effort to match the volatility of GDP in the U.S. data.

Figure A1: Sticky Wage Models: Negative Productivity Shock



Notes: Productivity decreases by 1% in period 1 with auto-correlation of 0.95. The dash-dot line (–.) represents the sticky wage model with fixed effort. The dotted line represents the sticky wage model with individual effort choice. The solid line represents the sticky wage model with common effort choice. All models feature $\alpha = 0.64$, $\widetilde{\gamma} = 0.5$, $\lambda = 3/4$, and $\widetilde{e} = 0.5$.

Figure A2: Flexible Wage Models: Negative Productivity Shock



Notes: Productivity decreases by 1% in period 1 with auto-correlation of 0.95. The dotted line represents the model with fixed effort. The solid line represents the model with variable effort. The x axis represents periods (in quarters) and y axis represents percentage deviation from the steady state. All models feature $\alpha = 0.64$, $\widetilde{\gamma} = 0.5$, and $\widetilde{e} = 0.5$.