Production Clustering and Offshoring

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Online Appendix

A. Proofs of Propositions

Proposition 2
PROOF:
Let’s assume there is an optimal path A for \( \tau_0 \), an optimal path B for \( \tau_1 \), \( \tau_0 > \tau_1 \), and \( MC(A, \tau_0, C) < MC(B, \tau_1, C) \), where C is a vector of the costs of production. Note that \( MC(A, \tau_0, C) \geq MC(A, \tau_1, C) \) and by optimality of B: \( MC(A, \tau_1, C) \geq MC(B, \tau_1, C) \). It follows that \( MC(A, \tau_0, C) \geq MC(B, \tau_1, C) \), which contradicts the initial assumption.

Lemma 1
PROOF:
Let path A with transportation quantity \( TQ(A) \) be chosen for \( \tau = \tau_0 \) and path B with transportation quantity \( TQ(B) \) be chosen for \( \tau = \tau_1 \) and \( \tau_0 > \tau_1 \). Now assume that the transportation quantity is an increasing function of \( \tau \) and hence \( TQ(A) > TQ(B) \). Then given the choice that the firm made under \( \tau_1 \): \( NTMC(B) + TQ(B) \tau_1 < NTMC(A) + TQ(A) \tau_1 \) and under \( \tau_0 \): \( NTMC(B) + TQ(B) \tau_0 > NTMC(A) + TQ(A) \tau_0 \). Adding \( TQ(B) (\tau_0 - \tau_1) \) to the first inequality, I get: \( NTMC(B) + TQ(B) \tau_0 < NTMC(A) + TQ(A) \tau_1 + TQ(B) (\tau_0 - \tau_1) < NTMC(A) + TQ(A) \tau_1 + TQ(A) (\tau_0 - \tau_1) = NTMC(A) + TQ(A) \tau_0 \) or \( NTMC(A) + TQ(B) \tau_1 < NTMC(A) + TQ(A) \tau_1 \), which contradicts the condition on optimality of A under \( \tau_0 \).

Proposition 3
PROOF:
Let’s assume \( \tau_0 > \tau_1 \). Let A be an optimal path for \( \tau = \tau_0 \) and transportation quantity \( TQ(A) > 0 \). Then by Proposition 2 \( MC(A, \tau_0) > MC(A, \tau_1) \). Let B an optimal path for \( \tau_1 \), then by definition of optimal path \( MC(A, \tau_1) \geq MC(B, \tau_1) \), and hence \( MC(A, \tau_0) > MC(B, \tau_1) \).

Proposition 4
PROOF:
Let A be an optimal path for \( \tau_0 \), B an optimal path for \( \tau_1 \), \( \tau_0 > \tau_1 \), and \( A \neq B \). By definition of optimality and because of the uniqueness of optimal paths, \( MC(A, \tau_0) < MC(B, \tau_0) \) and \( MC(A, \tau_1) > MC(B, \tau_1) \). From Lemma 1 \( \tau_1 TQ(A) < \tau_1 TQ(B) \). Assume \( NTMC(A) < NTMC(B) \), then
$MC (A, \tau_1) = NTMC (A) + \tau_1 TQ (A) < NTMC (B) + \tau_1 TQ (B) = MC (B, \tau_1)$, which contradicts the optimality of $B$ under $\tau_1$.

**B. Incomplete Trees**

To write down the problem for an arbitrary tree, I need to enumerate production nodes. Every node has a unique index $\{i, b\}$ that represents at what stage $i$ the part is produced and to what branch $b$ it belongs. Production costs for a part from branch $b$, produced on stage $i$ in country $k$, are then $a_{i,b,k}$.

Stage $i = 1$ corresponds to the most downstream stage of production and $i = N$ denotes the most upstream stage. In case two or more of the intermediate goods are assembled together, each of the corresponding nodes gets the same stage number $i$; in addition, each of these nodes gets branch index $b$, which was not previously assigned to another branch.

I define $n_b$ as the last stage of branch $b$; I call $n_b$ the length of branch $b$. In addition, for each stage $i$ I introduce an assembly set $\Omega_{i,b}$, which is the set of branch indexes $b$ of all parts produced on stage $i + 1$, connected to stage $\{i, b\}$. $\nu_{i,b}$ is a branch of a part produced at stage $i + 1$, connected to stage $\{i, b\}$, a node that $\{i, b\}$ is connected to. $B_i$ is a set of all branches present at stage $i$. I present an example of such enumeration in Figure 1.

$$MC = \min_{\{c_{i,b}\}} \max\{n_b\} \sum_{i=1}^{\max\{n_b\}} \sum_{b \in B_i} \left( \sum_{k=1}^{K} 1 (c_{i,b} = k) a_{i,b,k} + \tau T (c_{i,b}, c_{i-1}^{\nu_{i,b}}) \right).$$

**Figure 1. Incomplete Tree Notation**
This expression differs from (4) due to more complex indexing structure of an incomplete tree. The corresponding Bellman equation is

$$V_{i,b}(c_{i,b}) = \min_{c_{i,b} \in K} \left\{ \sum_{k=1}^{K} 1(c_{i,b} = k) a_{i,b,k} + \sum_{l \in \Omega_{i,b}} [\tau T(c_{i,b}, c_{i+1,l}) + V_{i+1,l}(c_{i+1,l})] \right\}.$$

C. Clustering with Iceberg Trade Costs

![Graphs showing clustering and tree length with iceberg trade costs](image)

**Figure 2. Clustering and Tree Length: Iceberg Trade Costs**

D. Elasticities

E. Endogenous Wages

The problem presented above is the model of absolute advantage as there is no labor market. With a given supply of labor in each country $L_j$ and endogenous wages that are determined through labor market clearing conditions, all countries will produce some parts no matter what production costs are. I normalize the wage in country 1 to $w_1 = 1$. I assume that labor supply is perfectly inelastic and the firm has constant returns to scale production technology. The problem of every firm then looks like

^1As long as trade costs are not too high for a given firm.
Figure 3. Clustering and the Number of Countries: Iceberg Trade Costs

\[
MC = \min_{c_{i,b}} \sum_{i=1}^{N} \sum_{b=1}^{M_{i-1}} \left( w_j \mathbf{1}(c_{i,b} = k) a_{i,b,k} + \tau T \left( c_{i,b}, c_{i-1}, \frac{b}{M} \right) \right),
\]

and a firm’s labor demand per unit produced is

\[
L_{D_k} \equiv \sum_{i=1}^{N} \sum_{b=1}^{M_{i-1}} \mathbf{1}(c_{i,b} = k) a_{i,b,k} \quad \text{for} \quad \forall k \in \{1, \ldots, K\}.
\]

Here for simplicity I assume that transportation services are performed by independent transport companies and do not affect domestic and foreign labor markets.

**Lemma A1:** Demand of the firm from country \(i\) \(L_{Di}\) for labor in country \(k\) is a nonincreasing function of \(w_k\).

**Proof:**

Let the wage in country \(k\) decrease, while all other wages remain constant: \(w_k^A > w_k^B\) and \(w_{j\neq k}^A = w_{j\neq k}^B = w_{j\neq k}\). Let \(A\) and \(B\) be optimal paths under wage schedules \(w^A\) and \(w^B\). In case \(A = B\), \(L_{D_k}^A = L_{D_k}^B\). Now consider the case \(A \neq B\). Then because of the optimality of \(A\) and \(B\): (a) \(MC(A, w^A) < MC(A, w^B)\).
and (b) \( MC(B, w^B) < MC(A, w^B) \). Let \( \Delta^{VT} \equiv \tau VT(A) - \tau VT(B) \), and 
\[ \Delta^L \equiv \sum_{j \neq k} w_j \left( L^A_{Dj} - L^B_{Dj} \right). \]
Then (a) and (b) can be rewritten as: 
\[ L^A_k w^A_k - L^B_k w^B_k + \Delta^L + \Delta^{VT} < 0, \]
and \[ L^A_k w^A_k - L^B_k w^B_k + \Delta^L + \Delta^{VT} > 0, \]
subtracting the first inequality from the second obtains: 
\[ (L^A_D - L^B_D) (w^B_k - w^A_k) > 0, \]
and then \[ L^A_D < L^B_D. \]

Note that if the firm changes its optimal path, then \( L_D \) is decreasing in \( w_k \).

**PROPOSITION A1:** There exists a wage schedule that clears the labor market. In a two-country case, this schedule is unique.

**PROOF:**

Existence:

The world economy can be considered as an exchange economy with \( M \) agents, where labor supply in country \( k \) is the endowment of good \( k \) and wage in country \( k \) is the price of this good. Then from Lemma A1 demand of each agent for each good is nondecreasing in price of this good, so by proposition 17.C.1 in Mas-Colell et al. (1995) an equilibrium exists.

Uniqueness for the case of two countries:

A firm’s relative demand for labor \( \frac{L^A}{L^B} \) is a nonincreasing function of the relative wage \( w \). Every firm takes the wage as given, but decisions of the firm determine the wage through market clearing condition. Here once again I apply the revealed preferences argument. Let’s assume there is path \( A \) with \( \sum_{i=1}^N c_i a_{Wi} = R_{WA} \) and \( \sum_{i=1}^N (1-c)_i a_{Ei} = R_{EA} \) that was chosen for \( w = w_0 \) and there is path

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**Figure 4. Direct and Indirect Clustering: Iceberg Trade Costs**

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B with $\sum_{i=1}^{N} c_i a_{Ni} = R_{WB}$ and $\sum_{i=1}^{N} (1 - c)_i a_{Si} = R_{EB}$ that was chosen for $w = w_1; w_1 > w_0$ and $R_{WB} > R_{WA}$. Let function $NPC(Y)$ be a value of nonproduction costs for path $Y$, then given the choice that the firm made under $w_0$: $NPC(A) + w_0 R_{WA} + R_{NA} < NPC(B) + w_0 R_{WB} + R_{EB}$ and under $w_1$: $NPC(A) + w_1 R_{WA} + R_{EA} > NPC(B) + w_1 R_{WB} + R_{EB}$. Adding $R_{WA}(w_1 - w_0)$ to the both parts of the first inequality, I get: $NPC(A) + w_1 R_{WA} + R_{EA} < NPC(B) + w_0 R_{WB} + R_{EB} + R_{WA}(w_1 - w_0) < NPC(B) + w_1 R_{WB} + R_{EB} < NPC(B) + w_1 R_{WB} + R_{EB} or NPC(A) + w_1 R_{WA} + R_{EA} < NPC(B) + w_1 R_{WB} + R_{EB}$, which contradicts the condition of optimality of $B$ under $w_1$.

For the case of multiple countries, proof of uniqueness of the equilibrium is nontrivial: decrease in the wage in one country can increase demand for labor in another country through the bridge FDI channel, similar to Proposition 8. As a result, the gross substitute property does not hold, and the uniqueness cannot be proven using the approach of Allen, Arkolakis and Li (2015).

REFERENCES


Figure 5. Reshoring: Iceberg Trade Costs
Figure 6. Trade Elasticities and Tree Order

Figure 7. Trade Elasticities and Tree Order: Iceberg Trade Costs
Figure 8. Trade Elasticities and Tree Length

Figure 9. Trade Elasticities and Tree Length: Iceberg Trade Costs
Figure 10. Trade Elasticities and the Number of Countries

Figure 11. Trade Elasticities and the Number of Countries: Iceberg Trade Costs