Supplementary Online Appendix for “Optimal Taxation with Endogenous Default under Incomplete Markets” by D. Pouzo and I. Presno.

**Stylized Facts: Emerging vs. Industrialized Economies**

Throughout the paper, we mention that our theoretical model is capable of replicating qualitatively several stylized facts observed for a wide range of economies. In this section, we present these stylized facts regarding the domestic government debt-to-output ratio and central government revenue-to-output ratio of several countries: industrialized economies (IND), emerging economies (EME) and a subset of these: Latin American (LAC).

In our dataset (described in section SM.B), no default event is observed for IND, whereas EME/LAC (LAC in particular) do exhibit several defaults. Thus, we take the former group as a proxy for economies with access to risk-free debt and the latter group as a proxy for economies without commitment to repay. It is worth to point out however that we are not presuming that IND economies are a type of economy that would never default. In turn, we are just using the fact that in our dataset IND economies do not show default events to use them as a proxy for the type of economy modeled in AMSS, that is, one with risk-free government debt.

Several stylized facts stand out in our dataset. First, in EME/LAC economies default is more likely than in IND economies and within the former group, the default risk is much higher for highly indebted economies. Second, EME and LAC economies exhibit tighter debt ceilings than IND, as also reported by Reinhart et al. [2003]. Third, economies with higher default risk tend to exhibit more volatile tax revenues than those with low default risk, and this fact is particularly notable for the group of EME/LAC economies. Bauducco and Caprioli [2014] documents a similar finding.

As shown in section V, our theory predicts that endogenous borrowing limits are more active for a high level of indebtedness. That is, when the government debt is high relative to output, the probability of default next period is higher, thus implying tighter borrowing limits and higher bond spreads. As the government’s ability to smooth its needs for funds using debt is hindered, the volatility of taxes turns out to be higher. But when debt is low, default is an unlikely event, thereby implying less stringent borrowing limits, lower spreads and therefore lower volatility in taxes. Hence, implications in the upper tail of the domestic debt-to-output ratio distribution can be different from those in the “central part” of it. Therefore, the mean and even the variance of the distribution may not be too informative, as they are affected by the central part of the distribution. Quantiles are better suited for recovering the information in the tails of the distribution.

Figure A.1 plots the percentiles of the domestic government debt-to-output ratio and of a measure of default risk for three groups: IND (black triangle), EME (blue square) and LAC (red circle). The X-axis plots the time series averages of domestic government debt-to-output ratio, and the Y-axis plots the values of the measure of default risk. For each group, the last point on the right corresponds to the 95th percentile, the second to last to the 90 percentile and so on; these are comparable between groups as all of them represent a percentile of the corresponding distribution. EME and LAC have

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36 For government revenue-to-output ratios, we used the data from Kaminsky et al. [2004], and for the domestic government debt-to-output ratios, we used the data from Panizza [2008]. We thank Ugo Panizza and Carmen Reinhart for kindly sharing their datasets with us. See appendix SM.B for a detailed description of the data.

37 We refer the reader to Koenker [2005] for a thorough treatment of quantiles and quantile-based econometric models.

38 This type of graph is not the conventional QQplot as the axis has the value of the random variable which achieves a certain quantile and not the quantile itself. For our purposes, this representation is more convenient.

39 The measure of default risk is constructed as the spread using the EMBI+ real index from J.P. Morgan for countries for which it is available and using the 3-7 year real government bond yield for the rest, minus the return of a US Treasury bond of similar maturity. Although bond returns are not entirely driven by default risk but also respond to other factors related to risk appetite, uncertainty and liquidity, for our purpose they constitute a valid conventional proxy of default risk. Furthermore, our spreads are an imperfect measure of default risk for domestic debt since EMBI+ considers mainly foreign debt. However, it is still informative since domestic defaults are positively correlated with defaults on sovereign debt, at least for the period from 1950’s onwards. See figure 10 in Reinhart and Rogoff [2008].
lower domestic debt-to-output ratio levels than IND; in fact the domestic debt-to-output ratio value of around 50 percent that pertains to the 95 percentile for EME and LAC, corresponds roughly to only the 85 percentile for IND. Thus, economies that are prone to default (EME and LAC) exhibit tighter debt ceilings than economies that do not default (in this dataset, represented by IND).

Figure A.1 also shows that for the IND group, the default risk measure is low and roughly constant for different levels of debt-to-output ratios. On the other hand, the default risk measure for the EME group is not only higher, but increases substantially for high levels of indebtedness. We consider this as evidence that for EME economies higher default risk is more prevalent for high levels of debt-to-output ratios.

![Figure A.1. The percentiles of the domestic government debt-to-output ratio and of a measure of default risk for three groups: IND (black triangle), EME (blue square) and LAC (red circle)](image)

**Figure A.1. The percentiles of the domestic government debt-to-output ratio and of a measure of default risk for three groups: IND (black triangle), EME (blue square) and LAC (red circle)**

<table>
<thead>
<tr>
<th>Debt/GDP</th>
<th>EME</th>
<th>IND</th>
<th>Default Risk</th>
<th>EME</th>
<th>IND</th>
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<td>2.9</td>
<td>75</td>
<td>2.5</td>
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Table A.1—(A) Measure of default risk (%) for EME and IND groups for different levels of debt-to-output ratio (%); (B) standard deviation of central government revenue over GDP (%) for EME and IND groups for different levels of default risk.

Table A.1(A) compares the measure of default risk between IND and EME for low and high debt-to-output ratio levels. That is, for both groups (IND and EME) we select economies with a debt-to-output ratio below the 25th percentile (low debt-to-output) for which we compute the average risk measure. We proceed analogously with those economies with debt-to-output ratio above the 75th percentile (high debt-to-output). For the case of low debt-to-output levels, the EME group presents higher (approximately twice as high) default risk than the IND group. For high debt-to-output ratio economies, however, this difference is quadrupled. Thus, economies that are prone to default (EME and LAC) exhibit higher default
risk than economies that do not default (in this dataset, represented by IND), and, moreover, the default risk is much higher for economies in the former group that have high levels of debt-to-output ratio.

Table A.1(B) compares the standard deviation of the central government revenue-to-output ratio between IND and EME for low and high default risk levels. It indicates that for IND there is little variation of the volatility across low and high levels of default risk. For EME, however, the standard deviation of the central government revenue-to-output ratio is dramatically higher for economies with high default risk.\(^{40}\) It is worth pointing out that all the EME with high default risk levels defaulted at least once during our sample period. Thus, economies with higher default risk exhibit more volatile tax revenues than economies with low default risk. This is particularly notable for the group of EME/LAC economies.

These stylized facts establish a link between (a) default risk/default events, (b) debt ceilings and (c) volatility of tax revenues. In particular, the evidence suggests that economies that show higher default risk, also exhibit lower debt ceilings and more volatile tax revenues. The theory behind our model helps shed light upon the forces driving these facts.\(^{41}\)

**Description of the Data**

In this section we describe how we constructed the figures presented in section SM.A.


For section SM.A we constructed the data as follows. First, for each country, we computed time average, or time standard deviations or any quantity of interest (in parenthesis is the number of observations use to construct these). Second, once we computed these averages, we group the countries in IND, EME and LAC. We do this procedure for (a) central government domestic debt (as % of output) ; (b) central government expenditure (as % of output) ; (c) central government revenue (as % of output) , and (d) Real Risk Measure. The data for (a) is taken from Panizza [2008] ; the data for (b-c) is taken from Kaminsky et al. [2004] ; finally the data for (d) is taken from www.globalfinancialdata.com.

For Greece and Portugal we use central government public debt because central government domestic debt was not available. For Sweden, Ecuador and Thailand we use general government expenditure because central government expenditure was not available. For Albania, Bulgaria, Cyprus, Czech Rep., Hungary, Latvia, Poland and Russia no measure of government expenditure was available and thus these countries were excluded from the sample for the calculations of this variable. The same caveats apply to the central government revenue sample. For Argentina, Brazil, Colombia, Ecuador, Egypt, Mexico, Morocco, Panama, Peru, Philippines, Poland, Russia, Turkey and Venezuela we used the real EMBI+ as a measure of real risk. For the rest of the countries we used differences in government note yields of 1-5 years maturity relative to the United States, depending on availability.

\(^{40}\) We looked also at the inflation tax as a proxy for tax policy; results are qualitatively the same.

\(^{41}\) It is important to note that we are not arguing any type of causality; we are just illustrating co-movements. In fact, in the model below all these features are endogenous outcomes of equilibrium.