Mediation in reputational bargaining: Online Appendix, Jack Fanning

This online appendix provides a proof of Proposition 4 from the main text, and also a proof of the inefficacy of ongoing mediation.

Proof of Proposition 4

Suppose the Proposition is not true, then there must exist some sequence of bargaining games (z_1^n, z_2^n) with $z_i^n \to 0$ and $z_1^n/z_2^n \in [1/K, K]$ such that there is no mediated equilibrium that benefits both agents. To prove the Proposition, therefore, I will consider an arbitrary sequence and establish that mediation is beneficial for all sufficiently large n. To do this, I will establish a system of mediation that ensures the frequency of demands and counterdemands is unchanged from unmediated bargaining. Clearly, reputations after demand choices are also unchanged, therefore. I refer to generic agent i's updated reputation after demand announcements $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$ as $\hat{z}_i^{\alpha}$, and let $\hat{z}^{\alpha} = (\hat{z}_1^{\alpha_1}, \hat{z}_2^{\alpha})$.

I first develop some preliminary results about mediation. I assume throughout that rational agents always compromise, $\rho_i^{\alpha} = 1$. These generalize the findings about noisy mediation from Proposition 3. I then characterize unmediated bargaining as the probability of commitment vanishes, and show this allows the preliminary results on mediation to be applied.

Preliminaries: mediation systems

Consider the continuation game with an arbitrary demand pair $\boldsymbol{\alpha}$ and reputations after demand announcements $\hat{z}^{\alpha}$ consistent with unmediated bargaining. Suppose the mediator announcement probability is $b^{\alpha} \in [0, 1)$, and continuation payoffs are $\boldsymbol{m}^{\alpha} = (m_1^{\alpha}, m_2^{\alpha})$ after an announcement. Let $U_i^{c,\alpha}(b^{\alpha}, \boldsymbol{m}^{\alpha})$ be agent i's expected payoff when she messages the mediator and $U_i^{n,\alpha}(b^{\alpha})$ when she doesn't. These expressions are developed more fully in various different forms in the main text. They are continuous in $\boldsymbol{m}^{\alpha}$ and b^{α}. Furthermore, let $\underline{\hat{z}}_i^{\alpha}(b^{\alpha}) = \hat{z}_i^{\alpha}/(1-(1-\hat{z}_i^{\alpha})b^{\alpha})$, be agent i's reputation (rational agent j's belief that i is committed) when there is no announcement from the mediator.

Let $\overline{C}_1 = \{\alpha_1 \in C_1 : \mu_1(\alpha_1) > 0\}$ be the set of demands made by rational agent 1 in unmediated bargaining, and given α_1 let $\overline{C}_2^{\alpha_1} = \{\alpha_2 \in C_2 : \mu_2^{\alpha_1}(\alpha_2) > 0\}$ be the set of demands made by rational agent 2. Given any $\alpha_1 \in \overline{C}_1$ I define a *conditional mediation system* to be a continuous function on some nondegenerate interval domain $[0, \overline{\beta}^{\alpha_1}]$, specifying for each incompatible demand pair $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$ and $\beta \in [0, \overline{\beta}^{\alpha_1}]$, mediation probabilities $\hat{b}^{\alpha}(\beta)$ and mediation shares $\hat{\boldsymbol{m}}^{\alpha}(\beta)$ such that: $\hat{b}^{\alpha}(0) = 0$; $\hat{b}^{\alpha}(\beta) = 0$ if $\alpha_2 \notin \overline{C}_2^{\alpha_1}$; each agent wants to message the mediator, $U_i^{c,\alpha}(\hat{b}^{\alpha}(\beta), \hat{\boldsymbol{m}}^{\alpha}(\beta)) \geq U_i^{n,\alpha}(\hat{b}^{\alpha}(\beta), \hat{\boldsymbol{m}}^{\alpha}(\beta))$; each agent's payoff $U_i^{c,\alpha}(\hat{b}^{\alpha}(\beta), \hat{\boldsymbol{m}}^{\alpha}(\beta))$ is strictly increasing in β for all $\alpha_2 \in \overline{C}_2^{\alpha_1}$; there is no incentive for agent 2 to change her counterdemand choices from unmediated bargaining, $U_2^{c,\alpha}(\hat{b}^{\alpha}(\beta), \hat{\boldsymbol{m}}^{\alpha}(\beta)) = U_2^{c,\alpha'}(\hat{b}^{\alpha'}(\beta), \hat{\boldsymbol{m}}^{\alpha'}(\beta))$ for all $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$,

$\boldsymbol{\alpha}' = (\alpha_1, \alpha_2')$ such that $\alpha_2, \alpha_2' \in \overline{C}_2^{\alpha_1}$. Given the continuity of $\hat{\boldsymbol{m}}^{\alpha}$ and $\hat{b}^{\alpha}$ we must have that $U_i^{c,\boldsymbol{\alpha}}(\hat{b}^{\alpha}(\beta), \hat{\boldsymbol{m}}^{\alpha}(\beta))$ is continuous in β.

Consider some demand $\alpha_1 \in \overline{C}_1$, and a conditional mediation system on domain $[0, \overline{\beta}^{\alpha_1}]$. Agent 1's expected payoff from demanding α_1 is:

$$(1) \quad U_1^{\alpha_1}(\beta) = \sum_{\alpha_2 > 1-\alpha_1} (z_2\pi_2(\alpha_2) + \mu_2^{\alpha_1}(\alpha_2)) U_1^{\alpha}(\hat{b}^{\alpha}(\beta), \boldsymbol{m}^{\alpha}(\beta))) + \sum_{\alpha_2 < 1-\alpha_1} z_2\pi_2(\alpha_2)\alpha_1,$$

where $\mu_2^{\alpha_1}(\alpha_2)$ is determined by unmediated bargaining. It is clear given the properties of a conditional mediation system that $U_1^{\alpha_1}(0)$ is agent 1's payoff in unmediated bargaining and that $U_1^{\alpha_1}$ is strictly increasing in β.

If we can establish a collection of conditional mediation systems, one for each $\alpha_1 \in \overline{C}_1$, then the proof will be complete. To see this, let $\hat{\alpha}_1 \in \arg\min_{\alpha_1 \in \overline{C}_1} U_1^{\alpha_1}(\overline{\beta}^{\alpha_1})$ then for each $\alpha_1 \in \overline{C}_1$ we must have a unique $\beta^{\alpha_1} \in (0, \overline{\beta}^{\alpha_1}]$, such that $U_1^{\alpha_1}(\beta^{\alpha_1}) = U_1^{\hat{\alpha}_1}(\overline{\beta}^{\hat{\alpha}_1}) > U_1^{\alpha_1}(0)$. The implied mediation probabilities and shares $(\hat{b}^{\alpha}(\beta^{\alpha_1}), \hat{\boldsymbol{m}}^{\alpha}(\beta^{\alpha_1}))$ ensure that neither agent will adjust her demand choices from unmediated bargaining and both agents strictly benefit from mediation. For $\alpha_1 \notin \overline{C}_1$, we can clearly ignore mediation (i.e. impose $b^{\alpha} = 0$).

Preliminaries: residual claimant mediation

I next show how to construct a conditional mediation system for $\alpha_1 \in \overline{C}_1$ in which for each $\alpha_2 \in \overline{C}_2^{\alpha_1}$, continuation payoffs after the mediator's announcement imply that one agent is indifferent between messaging the mediator and not, while her opponent receives the residual share. I call this a *residual claimant* conditional mediation system. The analysis closely follows Proposition 3.

Consider a continuation game after an arbitrary demand pair $\boldsymbol{\alpha}$ and updated reputations $\hat{z}^{\alpha}$, such that agent i would immediately concede with strictly positive probability in unmediated bargaining. For some sufficiently small $\overline{b} > 0$, if the probability of mediator's announcement is $b \in [0, \overline{b}]$, then agent i must still immediately concede with strictly positive probability after no mediator announcement by continuity. For $b \in [0, \overline{b}]$ I define the function, $\check{\boldsymbol{m}}^{i,\alpha}(b)$ to be the compromise shares that make agent i indifferent to compromising (i.e. it ensures $(U_i^{c,\alpha}(b, \check{\boldsymbol{m}}^{i,\alpha}(b)) - U_i^{n,\alpha}(b)))/b = 0$). As established in Proposition 3, i's share must satisfy:

$$(2) \qquad \check{m}_i^{i,\alpha}(b) = (1 - \alpha_j)\Big(1 + \frac{\underline{\hat{z}}_j^{\alpha}(b)(1 - (\underline{\hat{z}}_j^{\alpha}(b))^{\frac{r_i}{\lambda_j}})}{1 - \underline{\hat{z}}_j^{\alpha}(b)}\Big),$$

which is clearly continuous in b. It is also strictly increasing in b, because a larger b speeds up concession by rational agent j if there is no mediator announcement, and $\check{m}_i^{i,\alpha}(b)$ is exactly the integral of discounted payoffs from such concession (again, see Proposition 3). Clearly $U_i^{\alpha}(b, \check{\boldsymbol{m}}^{i,\alpha}(b))$ and $U_j^{\alpha}(b, \check{\boldsymbol{m}}^{i,\alpha}(b))$

are also continuous in b, with $U_i^{\alpha}(b, \check{\boldsymbol{m}}^{i,\alpha}(b))$ strictly increasing in b. To establish when agent j also benefits from mediation, let $\underline{U}_j^{\alpha}$ be her unmediated bargaining payoff. We can then define a continuous function D_j^{α} on $[0, \bar{b}]$, as a normalization of the difference between agent j's payoff under mediation and in unmediated bargaining, following the logic of Proposition 3:

$$
\begin{aligned}
(3) \qquad D_j^{\alpha}(b) =& \frac{U_j^{c,\alpha}(b, \check{\boldsymbol{m}}^{i,\alpha}(b)) - \underline{U}_j^{\alpha}}{b\hat{z}_i^{\alpha}(\hat{z}_j^{\alpha})^{-\frac{\lambda_i^{\alpha}}{\lambda_j^{\alpha}}}(\alpha_i + \alpha_j - 1)} \\
=& \frac{1 - (1 - (1 - \hat{z}_j^{\alpha})b)^{\frac{\lambda_i^{\alpha}}{\lambda_j^{\alpha}}}}{b} - \frac{(\hat{z}_j^{\alpha})^{1+\frac{\lambda_i^{\alpha}}{\lambda_j^{\alpha}}}(1 - \hat{z}_i^{\alpha})(1 - (\underline{\hat{z}}_j^{\alpha}(b))^{\frac{r_i}{\lambda_j}})\lambda_j^{\alpha}}{\hat{z}_i^{\alpha}(1 - \hat{z}_j^{\alpha})(1 - b)r_i}
\end{aligned}
$$

where $D_j^{\alpha}(0) = \lim_{b \to 0^+} D_j^{\alpha}(b)$, is easy evaluated given $\lim_{b \to 0^+}(1 - (1 - (1 - \hat{z}_j^{\alpha})b)^{\lambda_i^{\alpha}/\lambda_j^{\alpha}})/b = (1 - \hat{z}_j^{\alpha})\lambda_i^{\alpha}/\lambda_j^{\alpha}$ by l'Hopital's rule. Agent j benefits from mediation conditional on demands $\boldsymbol{\alpha}$, if $b > 0$ and $D_j^{\alpha}(b) > 0$. Given that $U_j^{c,\alpha}(0, m) = \underline{U}_j^{\alpha}$ for any m, if $D_j^{\alpha}(0) > 0$ we must have that $U_j^{c,\alpha}(b, \check{m}_i^{i,\alpha}(b))$ is strictly increasing in b on some non-degerate interval $[0, \underline{b}]$.

We next define the continuous function Q_j^{α} on $[0, \underline{b}]$ as a normalization of the difference between agent j's payoff from messaging the mediator and not:

$$
\begin{aligned}
(4) \qquad Q_j^{\alpha}(b) =& \frac{(U_j^{c,\alpha}(b, \check{\boldsymbol{m}}^{i,\alpha}(b)) - U_j^{n,\alpha}(b))(1 - b)}{b\hat{z}_i^{\alpha}(\hat{z}_j^{\alpha})^{-\frac{\lambda_k^{\alpha}}{\lambda_l^{\alpha}}}(\alpha_i + \alpha_j - 1)} \\
=& (1 - (1 - \hat{z}_j^{\alpha})b)^{\frac{\lambda_i^{\alpha}}{\lambda_j^{\alpha}}} - (\hat{z}_j^{\alpha})^{\frac{\lambda_i^{\alpha}}{\lambda_j^{\alpha}}} - \frac{(\hat{z}_j^{\alpha})^{1+\frac{\lambda_i^{\alpha}}{\lambda_j^{\alpha}}}(1 - \hat{z}_i^{\alpha})(1 - (\underline{\hat{z}}_j^{\alpha}(b))^{\frac{r_i}{\lambda_j^{\alpha}}})\lambda_j^{\alpha}}{\hat{z}_i^{\alpha}(1 - \hat{z}_j^{\alpha})r_i} \\
&- \frac{(\hat{z}_j^{\alpha})^{\frac{\lambda_i^{\alpha}}{\lambda_j^{\alpha}}}(1 - (\underline{\hat{z}}_j^{\alpha}(b))^{\frac{r_j}{\lambda_j^{\alpha}}})\lambda_i^{\alpha}}{r_j}
\end{aligned}
$$

Agent j is willing to compromise given $\boldsymbol{\alpha}$, if this is positive.

If $(\hat{z}_j^{\alpha})^{1+\lambda_1^{\alpha}/\lambda_2^{\alpha}}/\hat{z}_i^{\alpha} \to 0$ as $z_i^n \to 0$, then it is clear that for all sufficiently small z_i^n we have $Q_j^{\alpha}(0) > 0$ and $D_j^{\alpha}(0) > 0$. A necessary condition for that convergence is that $\hat{z}_j^{\alpha} \to 0$ as $z_i^n \to 0$. In addition to that necessary condition: if $\hat{z}_i^{\alpha}/\hat{z}_j^{\alpha} \in [1/L, L]$ for some constant $L \geq 1$, then I say that updated reputations are *similar*; if $\hat{z}_i^{\alpha}(\hat{z}_j^{\alpha})^{-\lambda_i^{\alpha}/\lambda_j^{\alpha}} \geq L$ for some constant $L > 0$ then I say that updated reputations are *slightly different*; if $\hat{z}_j^{\alpha} < \hat{z}_i^{\alpha}$, then I say that there is a *smaller winner reputation*. Notice that our assumption that agent i immediately concedes with positive probability in unmediated bargaining ensures that $\hat{z}_i^{\alpha}(\hat{z}_j^{\alpha})^{-\lambda_i^{\alpha}/\lambda_j^{\alpha}} < 1$. Similar reputations, a smaller winnner reputation, or slightly different reputations, all imply $(\hat{z}_j^\alpha)^{1+\lambda_1^\alpha/\lambda_2^\alpha}/\hat{z}_i^\alpha \to 0$ as $z_i^n \to 0$ and, therefore, ensure $Q_j^\alpha(0) > 0$ and $D_j^\alpha(0) > 0$, for all sufficiently small z_i^n.

Given $\alpha_1 \in \overline{C}_1$, we want to identify a conditional mediation system. Suppose that in unmediated bargaining agent i immediately concedes with strictly positive probability after every counterdemand $\alpha_2 > 1 - \alpha_1$ made by agent 2. In this case, I say that the *conditions for residual claimant mediation are met* if $Q_j^\alpha(0) > 0$, $D_j^\alpha(0) > 0$, for each $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$ with $\alpha_2 \in \overline{C}_2^{\alpha_1}$. For such $\boldsymbol{\alpha}$ and b in some non-degenerate interval $[0, \overline{b}^{\alpha}]$ we must have that agent i immediately concedes with positive probability if there is no mediator announcement, $Q_j^\alpha(b) > 0$, while $U_i^\alpha(b, \check{\boldsymbol{m}}^{i,\alpha}(b))$ and $U_j^\alpha(b, \check{\boldsymbol{m}}^{i,\alpha}(b))$ are strictly increasing in b. Given this, choose some $\check{\boldsymbol{\alpha}} \in \arg\min_{\alpha \in (\alpha_1, \overline{C}_2^{\alpha_1})} U_2^\alpha(\overline{b}^{\alpha}, \check{\boldsymbol{m}}^{i,\alpha}(\overline{b}^{\alpha}))$. We can then define the continuous functions $\hat{b}^\alpha$ and $\hat{\boldsymbol{m}}^\alpha$ on the non-degenerate interval domain $[0, \overline{b}^{\check{\alpha}}] = [0, \overline{\beta}^{\alpha_1}]$ by $\hat{\boldsymbol{m}}^\alpha(\beta) = \check{\boldsymbol{m}}^{i,\alpha}(\hat{b}^\alpha(\beta))$ and the equality $U_2^\alpha(\hat{b}^\alpha(\beta), \hat{\boldsymbol{m}}^\alpha(\beta)) = U_2^{\check{\alpha}}(\beta, \check{\boldsymbol{m}}^{i,\check{\alpha}}(\beta))$ for $\alpha_2 \in \overline{C}_2^{\alpha_1}$, and $\hat{b}^\alpha(\beta) = 0$ for $\alpha_2 \notin \overline{C}_2^{\alpha_1}$. Clearly, we always have $\hat{b}^\alpha(0) = 0$.

Characterization of unmediated bargaining

We are now ready to start proving that collections of conditional mediation systems must exist as $z_i^n \to 0$. To do this, I will first partition the parameter space, before carefully characterizing the limit of unmediated bargaining outcomes in each partition element as commitment vanishes. These characterizations will show that conditions for residual claimant mediation are met for each demand $\alpha_1 \in C_1$ for all sufficiently small z_i^n, completing the proof.

If $r_2(1 - \min C_1) > r_1(1 - \max C_2)$, then define $\underline{\alpha}_2^{\alpha_1} = \min\{\alpha_2 \in C_2 : \alpha_2 > 1 - \alpha_1\}$ and $\underline{\alpha}_1 = \max\{\alpha_1 \in C_1 : r_2(1 - \alpha_1) > r_1(1 - \underline{\alpha}_2^{\alpha_1})\}$. And so, $\underline{\alpha}_2^{\alpha_1}$ is agent 2's smallest counterdemand that is incompatible with α_1 (this is always well defined given $\max C_2 + \min C_1 > 1$), while $\underline{\alpha}_1$ is agent 1's maximum demand such that she will concede at a faster rate than agent 2 for any incompatible counterdemand. If $\underline{\alpha}_1 < \max C_1$ we can further define $\overline{\alpha}_1 = \min\{\alpha_1 \in C_1 : \alpha_1 > \underline{\alpha}_1\}$ and $\overline{\alpha}_2 = \max\{\alpha_2 \in C_2 : r_1(1 - \alpha_2) > r_2(1 - \overline{\alpha}_1)\}$. And so, $\overline{\alpha}_1$ is agent 1's smallest demand that is greater than $\underline{\alpha}_1$, while $\overline{\alpha}_2$ is agent 2's largest counterdemand against $\overline{\alpha}_1$ such that agent 2 concedes at a faster rate than 1. I say agent 2 is *uniquely strong* if $r_2(1 - \min C_1) < r_1(1 - \max C_2)$. I say that C_1 is *richer* than C_2 if agent 2 is not uniquely strong and either (i) $\underline{\alpha}_1 = \max C_1$ or (ii) $\underline{\alpha}_1 > 1 - \overline{\alpha}_2$. Finally, I say that C_2 is *richer* than C_1 if agent 2 is not uniquely strong, and C_1 is not richer than C_2.

Agent 2 is uniquely strong

This is the easiest set of parameters to characterize unmediated bargaining outcomes as $z_i^n \to 0$.

A first simple observation about unmediated bargaining from AG , which I will use repeatedly, is that given a pair of demands $\boldsymbol{\alpha}$, if agent i concedes at a slower rate than j in the concession game ($\lambda_i^\alpha < \lambda_j^\alpha$) and her updated reputation vanishes at a weakly faster rate than j's ($\hat{z}_i^\alpha \to 0$ and $\hat{z}_i^\alpha/\hat{z}_j^\alpha \leq L$ for some constant $L > 0$) then she must concede at time 0 with probability approaching one. To see this, notice that $1 - F_i^{\alpha,c}(0) \leq \hat{z}_i^\alpha(\hat{z}_j^\alpha)^{-\lambda_i^\alpha/\lambda_j^\alpha} \leq (\hat{z}_i^\alpha)^{1-\lambda_i^\alpha/\lambda_j^\alpha} L^{\lambda_i^\alpha/\lambda_j^\alpha} \to 0$ where $F_i^{\alpha,c} = (1-\hat{z}_i^\alpha)G_i^{\alpha,c}$.

A second simple observation from AG, is that if agent i makes a demand with strictly positive limit probability as $z_i^n \to 0$ ($\lim \mu_1(\alpha_1) > 0$ or $\lim \mu_2^{\alpha_1}(\alpha_2) > 0$ along some subsequence if necessary for convergence), then her reputation does vanish at a weakly faster rate than agent j's ($\hat{z}_i^\alpha \to 0$ and $\hat{z}_i^\alpha/\hat{z}_j^\alpha \leq L$ for some constant $L > 0$). To see this notice that $\hat{z}_1^{\alpha_1}/\hat{z}_2^\alpha \approx \mu_2^{\alpha_1}(\alpha_2)/\mu_1(\alpha_1)$ for large n.

Given that agent 2 is uniquely strong, she must always concede at a faster rate than agent 1 in any concession game, $\lambda_1^\alpha/\lambda_2^\alpha = r_2(1-\alpha_1)/(r_1(1-\alpha_2)) < 1$. If agent 1 makes some demand with positive limit probability $\lim \mu_1(\alpha_1) > 0$ as $z_i^n \to 0$, therefore, she must subsequently immediately concede with probability approaching one against any of agent 2's counterdemands. This in turn implies $\overline{C}^{\alpha_1} = \{\max C_2\}$ and $C_1 = \overline{C}_1$ for all sufficiently large n, and that payoffs are $1 - \max C_2$ for agent 1, and approach $\max C_2$ for agent 2 in the limit. Agent 1's demand choice is slightly indeterminate, in that it doesn't affect outcomes whether she immediately accepts agent 1's demand (action Q) or first makes a counterdemand and then concedes at time 0 in the concession game. We may assume, therefore, that she never plays Q and for sufficiently large n demands each $\alpha_1 \in C_1$ with probability $1/|C_1|$, and so $\hat{z}_1^{\alpha_1}/\hat{z}_2^{\max C_2} \in [1/L, L]$ for some constant $L > 0$.

The characterization above, means that updated reputations are similar (according to our definition) for each demand pair $\boldsymbol{\alpha} = (\alpha_1, \max C_2)$ with agent 1 immediately conceding with positive probability in the limit. And so, the conditions for residual claimant mediation are met for each demand $\alpha_1 \in C_1$ for all sufficiently small z_i^n.

C_1 is richer than C_2

We now consider characterizing unmediated bargaining demands as $z_i^n \to 0$ in the more complicated case in which C_1 is richer than C_2. By the definition of $\underline{\alpha}_1$, if agent 1 demands $\alpha_1 \leq \underline{\alpha}_1$ then she concedes at a faster rate than agent 2 after any counterdemand. And so if $\alpha_1 \leq \underline{\alpha}_1$ and agent 2 makes some counterdemand with positive limit probability, agent 2 must immediately concede with probability approaching one (by AG's simple observations). For large n therefore, player 1 would get approximately α_1, and player 2 would get exactly $(1-\alpha_1)$. If agent 1 demands $\alpha_1 > \underline{\alpha}_1$ with positive limit probability, however, then agent 2 could counterdemand $\overline{\alpha}_2$ and concede at a faster rate than 1, so that 1 would subsequently have to immediately concede with probability approaching one. For large n, this would imply payoffs of approximately $\overline{\alpha}_2$ to player 2 and $(1 - \overline{\alpha}_2) < \underline{\alpha}_1$ to player 1 (where the inequality holds because C_1 is richer than C_2). Hence, agent 1 must demand $\underline{\alpha}_1$ with probability one in the limit, giving limit equilibrium payoffs of $\underline{\alpha}_1$ and $(1 - \underline{\alpha}_1)$ respectively.

There is again some indeterminacy in rational agent 2's counterdemand to $\underline{\alpha}_1$ (although again, not to outcomes), but we shall assume that she makes each counterdemand with probability $1/|\{\alpha_2 \in C_2 : \alpha_2 > 1 - \underline{\alpha}_1\}|$ for all large n. This implies that after $\underline{\alpha}_1$, reputations are similar for all counterdemands. Given this, conditions for residual claimant mediation are met for the demand $\underline{\alpha}_1$ and all sufficiently small z_i^n.

Of course if $\underline{\alpha}_1 < \max C_1$, then agent 1 must demand $\alpha_1 > \overline{\alpha}_1$ with positive probability, albeit vanishing small as $z_i^n \to 0$. Indeed, we must have $\hat{z}_1^{\alpha_1} \to 0$, otherwise for any counterdemand made with positive limit probability, the probability that agent 2 immediately concedes would converge one, giving agent 1 a continuation payoff greater than $\alpha_1 > \overline{\alpha}_1$, a contradiction. To see this, notice that if $\hat{z}_1^{\alpha_1} \to K > 0$, then for any counterdemand α_2 made with positive limit probability we would have $\hat{z}_2^{\alpha} \to 0$ and so the probability that 2 must immediately concede in the concession game would be $1 - \hat{z}_2^{\alpha}(\hat{z}_1^{\alpha_1})^{-\lambda_2^\alpha/\lambda_1^\alpha} \to 1$.

Suppose that agent 1 demands $\alpha_1 > \underline{\alpha}_1$, and that $\underline{\alpha}_2^{\alpha_1} < \overline{\alpha}_2^{\alpha_1}$. I claim that agent 2 must then counterdemand $\tilde{\alpha}_2 = \max\{\alpha_2 \in C_2 : \alpha_2 < \overline{\alpha}_2^{\alpha_1}\}$ with probability approaching $(\underline{\alpha}_1 + \overline{\alpha}_2^{\alpha_1} - 1)/(\overline{\alpha}_2^{\alpha_1} - \tilde{\alpha}_2)$ in the limit and $\overline{\alpha}_2^{\alpha_1}$ with the remaining probability. Agent 1 then immediately concedes with strictly positive limit probability against any counterdemand. To establish this claim, suppose instead that agent 1 does not concede with positive probability along some sequence of bargaining games, then agent 2 must imitate every incompatible counterdemand with positive probability. We can allow any concession by 2 to occur immediately after 1's demand announcement (action Q), so that she never concedes at time 0 in the concession stage. In order for neither player to concede at time 0, we need $(1 - F_i^{\alpha,c}(0)) = 1 = \hat{z}_2^{\alpha}(\hat{z}_1^{\alpha})^{-\lambda_2^\alpha/\lambda_1^\alpha}$ and so $\hat{z}_2^{\alpha} = (\hat{z}_1^{\alpha_1})^{\lambda_2^\alpha/\lambda_1^\alpha}$. Given $\lambda_2^\alpha/\lambda_1^\alpha > \lambda_2^{\alpha'}/\lambda_1^{\alpha'}$ for $\alpha_2' > \alpha_2$, $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$ and $\boldsymbol{\alpha}' = (\alpha_1, \alpha_2')$ we must have that $\hat{z}_2^{\alpha}/\hat{z}_2^{\alpha'} = (\hat{z}_1^{\alpha_1})^{\lambda_2^\alpha/\lambda_1^\alpha - \lambda_2^{\alpha'}/\lambda_1^{\alpha'}} \to 0$. And so, the probability that agent 2 counterdemands $\alpha_2 > \underline{\alpha}_2^{\alpha_1}$ converges to zero, giving agent 1 a limit payoff greater than $(1 - \underline{\alpha}_2^{\alpha_1}) > \overline{\alpha}_1$, a contradiction. Hence agent 1 must immediately concede with positive probability for sufficiently large n. The probability that agent 1 does not concede after counterdemand α_2 is $\hat{z}_1^{\alpha_1}(\hat{z}_2^{\alpha})^{-\lambda_1^\alpha/\lambda_2^\alpha} \in [0, 1]$. Agent 2 must clearly be indifferent between each $\alpha_2 \in \overline{C}_2^{\alpha_1}$. The indifference condition for two demands $\alpha_2' > \alpha_2$, is:

$$\alpha_2' - \alpha_2 = \hat{z}_1^{\alpha_1}\Big((\hat{z}_2^{\alpha'})^{-\frac{\lambda_1^{\alpha'}}{\lambda_2^{\alpha'}}}(\alpha_1 + \alpha_2' - 1) - (\hat{z}_2^{\alpha})^{-\frac{\lambda_1^{\alpha}}{\lambda_2^{\alpha}}}(\alpha_1 + \alpha_2 - 1)\Big). \tag{5}$$

If $\hat{z}_2^{\alpha'} \leq L\hat{z}_2^{\alpha}$ for some $L > 0$, then the above condition cannot hold unless $\hat{z}_1^{\alpha_1}(\hat{z}_2^{\alpha})^{-\lambda_1^\alpha/\lambda_2^\alpha} \to 0$ (or else the right hand side will become infinitely large given

$\lambda_1^{\alpha'}/\lambda_2^{\alpha'} > \lambda_1^{\alpha}/\lambda_2^{\alpha}$). This implies that agent 2 can imitate at most two consecutive counterdemands with positive probability in the limit (i.e. α_2 and α_2', such that there is no $\alpha_2'' \in (\alpha_2, \alpha_2')$). These must be $\tilde{\alpha}_2$ and $\underline{\alpha}_2^{\alpha_1}$, with the former demand made with probability approaching $(\underline{\alpha}_1 + \underline{\alpha}_2^{\alpha_1} - 1)/(\underline{\alpha}_2^{\alpha_1} - \tilde{\alpha}_2)$ so that agent 1's payoff from α_1 approaches her limit equilibrium payoff $\underline{\alpha}_1$.

For counterdemands $\tilde{\alpha}_2$ and $\underline{\alpha}_2^{\alpha_1}$, we must have a smaller winner reputation (according to our definition), because agent 1 imitates demand α_1 with probability approaching zero, while agent 2 makes these counterdemands with strictly positive probability in the limit (and so $\hat{z}_2^{\alpha}/\hat{z}_1^{\alpha_1} \to 0$). To ensure that agent 2 is indifferent between all counterdemands $\alpha_2 > \tilde{\alpha}_2$ and $\tilde{\alpha}_2$ we must have that the probability that agent 1 does not immediately concede, $\hat{z}_1^{\alpha_1}(\hat{z}_2^{\alpha})^{-\lambda_1^{\alpha}/\lambda_2^{\alpha}}$, converges to $(\alpha_2 - \tilde{\alpha}_2)/(\alpha_1 + \alpha_2 - 1) \in (0, 1)$. For such counterdemands, therefore, reputations are only slightly different (according to our definition). Given this characterization of unmediated bargaining, for all small enough z_i^n, conditions for residual claimant mediation are met for the demand α_1.

Next, suppose that agent 1 demands $\alpha_1 > \underline{\alpha}_1$, but now $\underline{\alpha}_2^{\alpha_1} = \underline{\alpha}_2^{\alpha_1}$. We must eventually have that agent 2 concedes immediately with positive probability for small z_i^n, otherwise agent 2 counterdemands $\alpha_2 \geq \underline{\alpha}_2^{\alpha_1} > 1 - \overline{\alpha}_1$, giving agent 1 a continuation payoff of at most, $(1 - \underline{\alpha}_2^{\alpha_1}) < \underline{\alpha}_1$, a contradiction. There is again some indeterminacy regarding whether agent 2 concedes (with positive probability) immediately after seeing α_1 (action Q), or first counterdemands $\alpha_2 \geq \alpha_2^{\alpha_1}$ and then concedes at time 0 in the concession stage. If we assumed no time 0 concession in the concession stage, then we would have $\hat{z}_2^{\alpha} = (\hat{z}_1^{\alpha_1})^{\lambda_2^{\alpha}/\lambda_1^{\alpha}}$ and so $\hat{z}_2^{\alpha}/\hat{z}_2^{\alpha'} = (\hat{z}_1^{\alpha_1})^{\lambda_2^{\alpha}/\lambda_1^{\alpha} - \lambda_2^{\alpha'}/\lambda_1^{\alpha'}} \to 0$ when $\alpha_2' > \alpha_2$. In the limit, therefore, agent 2 must immediately concede or counterdemand $\underline{\alpha}_2^{\alpha_1}$. To ensure that agent 1 receives a limit payoff of $\underline{\alpha}_1$, agent 2 must therefore immediately concede with probability $(\underline{\alpha}_1 + \underline{\alpha}_2^{\alpha_1} - 1)/(\alpha_1 + \underline{\alpha}_2^{\alpha_1} - 1)$ in the limit.

Assuming that any immediate concession by agent 2 occurred before making a counterdemand was useful for establishing the distribution of limit outcomes, however, it is now useful to make the reverse assumption that rational agent 2 never plays action Q and instead always first makes a counterdemand but then sometimes concedes at time 0 in the concession stage. In fact, we can choose the probabilities of agent 2's counterdemands $\mu_2^{\alpha_1}(\alpha_2)$ so that she subsequently concedes at with the same conditional probability after each counterdemand. In other words, we have $\hat{z}_2^{\alpha}(\hat{z}_1^{\alpha_1})^{-\lambda_2^{\alpha}/\lambda_1^{\alpha}} = \hat{z}_2^{\alpha'}(\hat{z}_1^{\alpha_1})^{-\lambda_2^{\alpha'}/\lambda_1^{\alpha'}}$ for all incompatible counterdemands, which we know must approach $(\underline{\alpha}_2^{\alpha_1} + \overline{\alpha}_1 - 1)/(\alpha_1 + \underline{\alpha}_2^{\alpha_1} - 1)$ as $z_i^n \to 0$. This implies that after α_1, agents' reputations are only slightly different for every counterdemand, and so for all small enough z_i^n, conditions for residual claimant mediation are met for the demand α_1.

For all sufficiently small z_i^n we have established the existence of a residual claimant conditional mediation system, for each $\alpha_1 \in \overline{C}_1 = \{\alpha_1 \in C_1 : \alpha_1 \geq \underline{\alpha}_1\}$.

C_2 is richer than C_1

We finally turn to the most complicated case, when C_2 is richer than C_1. In unmediated bargaining, suppose that agent 1 demands $\overline{\alpha}_1$ with positive limit probability as $z_i^n \to 0$. If agent 2 counterdemands $\alpha_2 \leq \overline{\alpha}_2$ with positive limit probability then because she concedes at a faster rate than agent 1, agent 1 must immediately concede with probability approaching one. On the other hand, if agent 2 counterdemands $\alpha_2 > \overline{\alpha}_2$ with positive limit probability, then because she concedes at a slower rate than agent 1, agent 2 herself must immediately concede with probability approaching one. Hence, agent 2 must counterdemand $\overline{\alpha}_2$ with probability approaching one, giving a payoff of $1 - \overline{\alpha}_2$ to agent 1 and approximately $\overline{\alpha}_2$ to agent 2. Given that C_2 is richer than C_1 we have $1-\overline{\alpha}_2 > \underline{\alpha}_1$ and so agent 1, must certainly never imitate $\alpha_1 < \overline{\alpha}_1$ (equivalently $\alpha_1 \leq \underline{\alpha}_1$) for large n .

I claim that agent 1 does not demand $\alpha_1 > \overline{\alpha}_1$ with positive limit probability. Suppose otherwise, so agent 1 does make some demand $\alpha_1 > \overline{\alpha}_1$ with positive limit probability. Agent 1 must concede with probability one in the limit against any counterdemand with $r_1(1 - \alpha_2) > r_2(1 - \alpha_1)$, and so in particular against $\overline{\alpha}_2$. If this is also true for some counterdemand $\alpha_2 > \overline{\alpha}_2$, then agent 1 would receive a limit continuation payoff of less than $(1 - \alpha_2) < (1 - \overline{\alpha}_2)$, a contradiction. Suppose, therefore, that there is no such counterdemand. For any counterdemand $\alpha_2 > \overline{\alpha}_2$ we must certainly have $\hat{z}_2^{\alpha_1,\alpha_2} \to 0$, otherwise agent 1 would concede with probability approaching one, giving agent 2 a limit payoff greater than $\alpha_2 > \overline{\alpha}_2$ and agent 1 a payoff strictly below $1 - \overline{\alpha}_2$, a contradiction. For large enough n, agent 1's expected payoff from imitating α_1 is then:

$$
\begin{aligned}
(6) \qquad \underline{U}^{\alpha_1} = & \sum_{\alpha_2 < 1-\alpha_1} z_2\pi_2(\alpha_2)\alpha_1 + \sum_{\alpha_2 > 1-\alpha_1} ((1 - z_2)\mu_2^{\alpha_1}(\alpha_2) + z_2\pi_2(\alpha_2))(1 - \alpha_2) \\
= & (1 - \overline{\alpha}_2) + \sum_{\alpha_2 < 1-\overline{\alpha}_1} z_2\pi_2(\alpha_2)(\alpha_1 + \overline{\alpha}_2 - 1) \\
& + \sum_{\alpha_2 \in (1-\alpha_1, \overline{\alpha}_2)} z_2\pi_2(\alpha_2)(\overline{\alpha}_2 - \alpha_2) - \sum_{\alpha_2 > \overline{\alpha}_2} \frac{z_2\pi_2(\alpha_2)}{\hat{z}_2^{\alpha_1,\alpha_2}}(\alpha_2 - \overline{\alpha}_2),
\end{aligned}
$$

where I use the fact that $(1 - z_2)\mu_2^{\alpha_1}(\alpha_2) + z_2\pi_2(\alpha_2) = z_2\pi_2(\alpha_2)/\hat{z}_2^{\alpha_1,\alpha_2}$. Notice that $1 - \overline{\alpha}_2 > \underline{\alpha}_1 > 1 - \max C_2$, implies that $\max C_2 > \overline{\alpha}_2$. Equation (6) with $\alpha_1 = \overline{\alpha}_1$ also provides a lower bound on agent 1's payoff when demanding $\overline{\alpha}_1$. Her actual payoff might, for instance, be $\mu_2^{\overline{\alpha}_1}(Q)(1 - z_2)(\overline{\alpha}_1 + \overline{\alpha}_2 - 1) \geq 0$ larger as we haven't assumed she demands $\overline{\alpha}_1$ with positive limit probability, although it must clearly still converge to $(1 - \overline{\alpha}_2)$. For large n, the normalized difference between this lower bound on agent 1's payoff when she demands $\overline{\alpha}_1$ and her payoff when demanding $\alpha_1 > \overline{\alpha}_1$ with positive limit probability is then:

$$(7)\quad \frac{\underline{U}^{\overline{\alpha}_1} - \underline{U}^{\alpha_1}}{z_2} = \sum_{\alpha_2 \in (1-\alpha_1, 1-\overline{\alpha}_1)} \pi_2(\alpha_2)(\overline{\alpha}_1 + \alpha_2 - 1) + \sum_{\alpha_2 > \overline{\alpha}_2} \pi_2(\alpha_2)(\alpha_2 - \overline{\alpha}_2) \frac{1 - \frac{\hat{z}_2^{\alpha_1,\alpha_2}}{\hat{z}_2^{\overline{\alpha}_1,\alpha_2}}}{\hat{z}_2^{\alpha_1,\alpha_2}}.$$

Next consider the subclaim that $\hat{z}_2^{\alpha_1,\alpha_2}/\hat{z}_2^{\overline{\alpha}_1,\alpha_2} \to 0$ for all $\alpha_2 > \overline{\alpha}_2$. If this subclaim is established then as commitment vanishes, (7) must explode because we know $\hat{z}_2^{\alpha_1,\alpha_2} \to 0$. The payoff to the demand $\overline{\alpha}_1$, therefore, must eventually strictly exceed that of α_1, a contradiction which establishes the original claim: that agent 1 cannot imitate $\alpha_1 > \overline{\alpha}_1$ with positive limit probability.

To establish the subclaim, suppose that after the demand $\alpha_1 \geq \overline{\alpha}_1$, agent 2 does not immediately concede with positive probability for large n. Rational agent 2 must then make all counterdemands $\alpha_2 \geq \overline{\alpha}_2$ with positive probability, or else agent 1's payoff from demanding α_1 would be strictly below $1 - \overline{\alpha}_2$ in the limit, a contradiction. Rearranging (5), the condition for indifference between these demands, we must then have:

$$(8)\qquad \hat{z}_2^{\alpha_1,\alpha_2} = \left(\frac{\hat{z}_1^{\alpha_1}(\alpha_1 + \alpha_2 - 1)}{(\alpha_2 - \overline{\alpha}_2) + \hat{z}_1^{\alpha_1} (\hat{z}_2^{\alpha_1,\overline{\alpha}_2})^{-\frac{\lambda_1^{\alpha_1,\overline{\alpha}_2}}{\lambda_2^{\alpha_1,\overline{\alpha}_2}}} (\alpha_1 + \overline{\alpha}_2 - 1)} \right)^{\frac{\lambda_2^{\alpha_1,\alpha_2}}{\lambda_1^{\alpha_1,\alpha_2}}},$$

On the other hand, suppose that agent 2 immediately concedes with positive probability after the demand α_1, and assume whenever 2 does this she chooses action Q rather than first making a counterdemand. We must then have $\hat{z}_2^{\alpha_1,\alpha_2} = (\hat{z}_1^{\alpha_1})^{\lambda_2^{\alpha_1,\alpha_2}/\lambda_1^{\alpha_1,\alpha_2}}$. Either way, for some constant $K^{\alpha_1,\alpha_2} > 1$, we must have $\hat{z}_2^{\alpha_1,\alpha_2} \in [(\hat{z}_1^{\alpha_1,\alpha_2})^{\lambda_2^{\alpha_1,\alpha_2}/\lambda_1^{\alpha_1,\alpha_2}}, K^{\alpha_1,\alpha_2}(\hat{z}_1^{\alpha_1,\alpha_2})^{\lambda_2^{\alpha_1,\alpha_2}/\lambda_1^{\alpha_1,\alpha_2}}]$. This implies:

$$(9)\qquad \frac{\hat{z}_2^{\alpha_1,\alpha_2}}{\hat{z}_2^{\overline{\alpha}_1,\alpha_2}} \leq \frac{K^{\alpha_1,\alpha_2}(\hat{z}_1^{\alpha_1})^{\frac{\lambda_2^{\alpha_1,\alpha_2}}{\lambda_1^{\alpha_1,\alpha_2}}}}{(\hat{z}_1^{\overline{\alpha}_1})^{\frac{\lambda_2^{\overline{\alpha}_1,\alpha_2}}{\lambda_1^{\overline{\alpha}_1,\alpha_2}}}}.$$

Given $\alpha_1 > \overline{\alpha}_1$, we have $\lambda_2^{\alpha_1,\alpha_2}/\lambda_1^{\alpha_1,\alpha_2} > \lambda_2^{\overline{\alpha}_1,\alpha_2}/\lambda_1^{\overline{\alpha}_1,\alpha_2}$ and given the assumption that agent 1 counterdemands α_1 with positive limit probability, we have $\hat{z}_1^{\alpha_1} \leq L\hat{z}_1^{\overline{\alpha}_1}$ for some constant $L > 0$. Together, these imply that the right hand side of (9) vanishes as $\hat{z}_1^{\alpha_1} \to 0$. This establishes the subclaim, and so the original claim: the probability that agent 1 demands $\alpha_1 > \overline{\alpha}_1$ must vanish.

We have established that in equilibrium: agent 1 demands $\overline{\alpha}_1$ with probability approaching one, after observing $\overline{\alpha}_1$ agent 2 must counterdemand $\overline{\alpha}_2$ with probability approaching one, and after observing counterdemand $\overline{\alpha}_2$, agent 1 must concede with probability approaching one. Given the demand $\overline{\alpha}_1$ and counterdemand $\overline{\alpha}_2$, it is clear that updated reputations are similar. The probability that agent 1 does not immediately concede after the demand $\overline{\alpha}_1$ and counterdemand $\alpha_2 > \overline{\alpha}_2$ is $\hat{z}_1^{\alpha_1}(\hat{z}_2^{\alpha})^{\lambda_1^\alpha/\lambda_2^\alpha}$, which must converge to $(\alpha_2 - \overline{\alpha}_2)/(\overline{\alpha}_1 + \alpha_2 - 1) > 0$, to ensure that agent 2 is indifferent between this counterdemand and $\overline{\alpha}_2$. This implies that updated reputations are only slightly different. And so, for all small enough z_i^n, conditions for residual claimant mediation are met for the demand $\overline{\alpha}_1$.

Next, I claim that if agent 2 observes the demand $\alpha_1 > \overline{\alpha}_1$ she counterdemands $\overline{\alpha}_2$ with probability approaching one, and after observing the counterdemand $\overline{\alpha}_2$ agent 1 immediately concedes with probability approaching one.

Clearly, for $\alpha_1 > \overline{\alpha}_1$ we must have $\hat{z}_1^{\alpha_1} \to 0$, or agent 2 would immediately concede with probability approaching one, giving agent 1 a continuation payoff of $\alpha_1 > (1 - \overline{\alpha}_2)$, a contradiction. Suppose that given $\alpha_1 > \overline{\alpha}_1$, agent 2 concedes with positive probability along some sequence of bargaining games. Assuming that any concession by agent 2 occurs immediately after 1's demand (action Q) rather than in the concession stage, we must have $\hat{z}_2^{\alpha} = (\hat{z}_1^{\alpha_1})^{\lambda_2^\alpha/\lambda_1^\alpha}$. This implies $\hat{z}_2^{\alpha}/\hat{z}_2^{\alpha'} = (\hat{z}_1^{\alpha_1})^{\lambda_2^\alpha/\lambda_1^\alpha - \lambda_2^{\alpha'}/\lambda_1^{\alpha'}} \to 0$ where $\boldsymbol{\alpha} = (\alpha_1, \underline{\alpha}_2^{\alpha_1})$ and $\boldsymbol{\alpha}' = (\alpha_1, \alpha_2')$ for any $\alpha_2' > \underline{\alpha}_2^{\alpha_1}$. And so, agent 2 must counterdemand $\underline{\alpha}_2^{\alpha_1}$ with probability approaching one. Clearly, in this case we must have $\underline{\alpha}_2^{\alpha_1} = \overline{\alpha}_2$, or else agent 1's limit payoff would be greater than $(1 - \overline{\alpha}_2)$, a contradiction. Also notice that agent 2 cannot concede with positive probability in the limit or else agent 1 would get a payoff strictly greater than $(1 - \underline{\alpha}_2)$.

On the other hand, if agent 2 does not concede along some sequence of bargaining games, in order for her to be indifferent between two arbitrary demands $\alpha_2' > \alpha_2$, (5) must be satisfied. As argued previously, if $z_2^{\alpha'} \leq L z_2^{\alpha}$ for some constant $L > 0$, then agent 1 concedes against α_2 with probability approaching one (i.e. $\hat{z}_1^{\alpha_1}(z_2^{\alpha})^{-\lambda_1^\alpha/\lambda_2^\alpha} \to 0$). And so, agent 2 can imitate at most two consecutive counterdemands with positive limit probability. Given this, agent 2 must counterdemand $\overline{\alpha}_2$ with probability approaching one so that agent 1 can receive a limit payoff of $(1 - \overline{\alpha}_2)$.

The part of the claim to still establish is that after observing counterdemand $\overline{\alpha}_2$, agent 1 concedes with probability one in the limit. Suppose not. I first establish a subclaim: assuming agent 1 concedes against $\overline{\alpha}_2$ with probability less than one in the limit, then for some constant $K^{\alpha_1,\alpha_2} > 0$ we must have $\hat{z}_2^{\alpha_1,\alpha_2} \geq K^{\alpha_1,\alpha_2}(\hat{z}_2^{\alpha_1,\overline{\alpha}_2})^{(1-\alpha_2)/(1-\overline{\alpha}_2)}$ for $\alpha_2 \geq \overline{\alpha}_2$ for all large n. This subclaim is certainly true if agent 2 concedes with positive (but vanishing) probability along some sequence of bargaining games, because (assuming any such concession occur before making a counterdemand) we must have $(\hat{z}_2^{\alpha})^{\lambda_1^\alpha/\lambda_2^\alpha} = \hat{z}_1^{\alpha_1}$, which implies $\hat{z}_2^{\alpha_1,\alpha_2} = (\hat{z}_2^{\alpha_1,\overline{\alpha}_2})^{(1-\alpha_2)/(1-\overline{\alpha}_2)}$. If agent 1 concedes against $\overline{\alpha}_2$ along the sequence of bargaining games, but with limit probability less than one, then $\hat{z}_1^{\alpha_1}(\hat{z}_2^{\alpha_1,\overline{\alpha}_2})^{-\lambda_1^{\alpha_1,\overline{\alpha}_2}/\lambda_2^{\alpha_1,\overline{\alpha}_2}} \to L > 0$ for some constant L. For the indifference condition (5) to be satisfied, we must then have $\hat{z}_1^{\alpha_1}(\hat{z}_2^{\alpha_1,\alpha_2})^{-\lambda_1^{\alpha_1,\alpha_2}/\lambda_2^{\alpha_1,\alpha_2}} \to ((\alpha_2 - \underline{\alpha}_2) + L(\alpha_1 + \overline{\alpha}_2 - 1))/(\alpha_1 + \overline{\alpha}_2 - 1)$. And so for some $K^{\alpha_1,\alpha_2} > 0$ we must have $(\hat{z}_2^{\alpha_1,\alpha_2}) \geq K^{\alpha_1,\alpha_2}(\hat{z}_2^{\alpha_1,\overline{\alpha}_2})^{(1-\alpha_2)/(1-\overline{\alpha}_2)}$, as desired.

For large n, agent 1's payoff when she demands $\overline{\alpha}_1$ must be defined by (6). This equation also defines a lower bound on agent 1's payoff when she demands $\alpha_1 > \overline{\alpha}_1$. The normalized difference between agent 1's payoff from demanding $\overline{\alpha}_1$ and the lower bound on her payoff after $\alpha_1 > \overline{\alpha}_1$ is then given by (7). If $\hat{z}_2^{\alpha_1,\alpha_2}/\hat{z}_2^{\overline{\alpha}_1,\alpha_2} \to \infty$ for all $\alpha_2 > \overline{\alpha}_2$, therefore, then 1's continuation payoff after demanding $\alpha_1 > \overline{\alpha}_1$ must eventually be strictly greater than after $\overline{\alpha}_1$, a contradiction. To show that $\hat{z}_2^{\alpha_1,\alpha_2}/\hat{z}_2^{\overline{\alpha}_1,\alpha_2} \to \infty$ for all $\alpha_2 > \overline{\alpha}_2$, first notice that after the demand $\overline{\alpha}_1$, for agent 2 to be indifferent between counterdemand $\overline{\alpha}_2$ and $\alpha_2 > \overline{\alpha}_2$ requires $\hat{z}_1^{\overline{\alpha}_1}(\hat{z}_2^{\overline{\alpha}_1,\alpha_2})^{-\lambda_1^{\overline{\alpha}_1,\alpha_2}/\lambda_2^{\overline{\alpha}_1,\alpha_2}} \to (\alpha_2 - \underline{\alpha}_2)/(\alpha_1 + \overline{\alpha}_2 - 1)$ and so $\hat{z}_2^{\overline{\alpha}_1,\alpha_2} \leq N^{\overline{\alpha}_1,\alpha_2}(\hat{z}_1^{\overline{\alpha}_1})^{\lambda_2^{\overline{\alpha}_1,\alpha_2}/\lambda_1^{\overline{\alpha}_1,\alpha_2}}$ for some constant $N^{\overline{\alpha}_1,\alpha_2} > 0$. And so for $\alpha_2 > \overline{\alpha}_2$ we have:

$$\frac{\hat{z}_2^{\alpha_1,\alpha_2}}{\hat{z}_2^{\overline{\alpha}_1,\alpha_2}} \geq \frac{K^{\alpha_1,\alpha_2}(\hat{z}_2^{\alpha_1,\overline{\alpha}_2})^{\frac{1-\alpha_2}{1-\overline{\alpha}_2}}}{N^{\overline{\alpha}_1,\alpha_2}(\hat{z}_1^{\overline{\alpha}_1})^{\frac{\lambda_2^{\overline{\alpha}_1,\alpha_2}}{\lambda_1^{\overline{\alpha}_1,\alpha_2}}}} \tag{10}$$

Because agent 1 demands $\overline{\alpha}_1$ with probability approaching one, for some constant $M > 0$ we must have $\hat{z}_2^{\alpha_1,\overline{\alpha}_2} \geq M\hat{z}_1^{\overline{\alpha}_1} \to 0$. We also have $(1-\alpha_2)/(1-\overline{\alpha}_2) < \lambda_2^{\overline{\alpha}_1,\alpha_2}/\lambda_1^{\overline{\alpha}_1,\alpha_2} = r_1(1-\alpha_2)/(r_2(1-\overline{\alpha}_1))$ given that by definition $r_2(1-\overline{\alpha}_1) < r_1(1-\overline{\alpha}_2)$. Hence, the right hand side of (10) must explode, delivering a contradiction and establishing the claim: after observing demand $\alpha_1 > \overline{\alpha}_1$ agent 2 counterdemands $\overline{\alpha}_2$ with probability approaching one, and after observing that counterdemand agent 1 immediately concedes with probability approaching one.

After the demand $\alpha_1 > \overline{\alpha}_1$, agent 2 cannot demand $\alpha_2 < \overline{\alpha}_2$ for all sufficiently large n, as agent 1 would concede against the larger demand $\overline{\alpha}_2$ with probability approaching one. After the counterdemand $\overline{\alpha}_2$ there is a smaller winner reputation because the demand $\alpha_1 > \overline{\alpha}_1$ is made with probability zero in the limit, while the counterdemand $\overline{\alpha}_2$ is made with probability one in the limit. After the counterdemand $\alpha_2 > \overline{\alpha}_2$, however, reputations are only slightly different, because $\hat{z}_1^{\alpha_1}(\hat{z}_2^{\alpha_1,\alpha_2})^{-\lambda_1^{\alpha_1,\overline{\alpha}_2}/\lambda_2^{\alpha_1,\alpha_2}} \to (\alpha_2 - \underline{\alpha}_2)/(\alpha_1 + \overline{\alpha}_2 - 1)$ in order to make agent 2 indifferent between this counterdemand and $\overline{\alpha}_2$. Hence, for sufficiently small z_i^n conditions for a residual claimant conditional mediation system are met.

For all sufficiently small z_i^n we have established that there is a residual claimant conditional mediation system associated with each $\alpha_1 \in \overline{C}_1 = \{\alpha_1 \in C_1 : \alpha_1 \geq \overline{\alpha}_1\}$, and so we are done. □

Ongoing mediation

I next show that the negative result for simple mediation is robust to allowing agents to privately compromise at any time over the infinite horizon. The mediator immediately suggest a deal when both parties have compromised. I call this the *ongoing mediation protocol*.

I have to slightly modify the game's extensive form to allow for this protocol. First agent 1 announces a demand, $a_1 \in [0,1]$; then agent 2 can immediately accept (giving payoffs a_1 and $1-a_1$ respectively) or make a counterdemand $a_2 \in [0,1]$. After observing both demands, each agent i chooses two stopping times, $t_i^c \in [0,\infty]$ and $t_i^y \in [0,\infty]$, where t_i^c is the time at which she compromises $(= c)$ sending a private message to the mediator, and t_i^y is the time at which she yields $(= y)$ to her opponent's demand (or concedes) if the mediator has not yet stopped the game. The mediator stops the game and publicly suggests an agreement at time $\max\{t_1^c, t_2^c\}$ when both agents have privately compromised, so long as bargaining has not already concluded, $\max\{t_1^c, t_2^c\} < \min\{t_1^y, t_2^y\}$. If $\max\{t_1^c, t_2^c\} = \min\{t_1^y, t_2^y\}$, the mediator makes her announcement before either agent has a chance to concede. If the mediator stops the game at time t (suggesting an agreement), first each agent i can simultaneously revise her demand to $a_i' \in [0,1]$, and then after observing demand changes, choose a new time $t_i^{y\prime} \in [t,\infty]$ to concede to her opponent's revised demand.

I focus on what I call *ongoing equilibria* in which rational agents always receive continuation payoffs (m_1, m_2) after the mediator's announcement. Such equilibria entail some loss of generality because those continuation payoffs could potentially vary over time. Given this restriction, however, it is then without loss of generality to assume $m_i \in (1-\alpha_j, \alpha_i)$, because if $m_i \geq \alpha_i$ then rational i would compromise with probability one at time 0 if this had any chance of affecting the outcome. It is also without loss of generality to then assume that each agent compromises before conceding $t_i^c \leq t_i^y$ because she would certainly do so if there was any possibility of it affecting the outcome.

Agent i's strategy can then be described by two c.d.f.s, $F_i^c \in [0,1]^{[0,\infty]}$ and $F_i^y \in [0,1]^{[0,\infty]}$, where $F_i^c(t)$ is the probability that agent i has compromised before time t, and $F_i^y(t)$ is the probability that agent i has conceded before time t, where $1 - z_i \geq F_i^c(t) \geq F_i^y(t)$ for $t < \infty$. Given agent j's equilibrium strategy, rational agent i's expected utility from compromising at time s and conceding at time $t \geq s$ is:

$$
\begin{aligned}
(11) \qquad U_i(s,t) = & \int_{v<s} e^{-r_i v}\alpha_i dF_j^y(v) + \int_{v\in(s,t]} e^{-r_i v} m_i dF_j^c(v) \\
& + (1 - F_j^c(t))e^{-r_i t}(1-\alpha_j) + (F_j^c(s) - \sup_{v<s} F_j^y(v))e^{-r_i s} m_i
\end{aligned}
$$

Of course, the unmediated bargaining equilibrium is still an ongoing equilibrium, where $F_i^c(t) = F_i^y(t)$ for all t. In fact, it is the only ongoing equilibrium.

Proposition 1. *The distribution of outcomes in any ongoing equilibrium is identical to unmediated bargaining.*

The idea of the proof is similar to that of Proposition 2, in that unless behavior matches unmediated bargaining with $F_i^c(t) = F_i^y(t)$, then indifference conditions for compromising and non-compromising agents would imply a contradiction to the fact that rational agents must concede within finite time. However, it is somewhat more involved.

Proof of Proposition 1

Consider some particular equilibrium. We clearly must have $A_i = \{(s,t) : U_i(s,t) = \max_{v,w} U_i(v,w)\} \neq \emptyset$. Define $T_i^y = \inf\{t : F_i^y(t) = 1 - z_i\}$ and $T^* = \max\{T_1^y, T_2^y\}$. As in the proof of Proposition 2, I will first prove a series of claims, which ultimately help establish the result.

(a) *We must have $T_i^y = T^* < \infty$.* This follows immediately from the proof of Proposition 2, claim (a). A rational agent would immediately concede if she knew she faced a committed opponent, and will not wait forever to concede if her opponent might be committed.

(b) *If F_i^y jumps at $t \in [0, T^*]$ then F_j^c is constant on $[t - \varepsilon, t]$ for some $\varepsilon > 0$.* This follows because if agent j has not compromised before $t - \varepsilon$, she would strictly increase her payoff by compromising an instant after t compared to slightly before as this would give her α_j rather than m_j with positive probability (at least $F_i^y(t) - \sup_{s<t} F_i^y(s) > 0$).

(c) *If F_i^c jumps at $t \in (0, T^*]$ then F_j^y is constant on $[t - \varepsilon, t)$ for some $\varepsilon > 0$.* This follows because agent j would prefer to concede an instant after t rather than slightly before as this would give her m_j rather than $1 - \alpha_i$ with positive probability (at least $F_i^c(t) - \sup_{s<t} F_i^c(t) > 0$).

(d) *Let $t' \leq t'' < t''' \leq T^*$. If $F_i^c(t''') = F_i^c(t')$ and $F_j^c(t'') > F_j^y(t'')$ then $F_j^y(t'') = F_j^y(t''')$.* If this is not true so that $F_j^y(t'') < F_j^y(t''')$, then there must exist some $s \leq t''$ and some $t \in (t'', t''']$ such that $(s,t) \in A_j$. However, given that $F_i^c(t'') = F_i^c(t''')$ the alternative strategy of conceding slightly earlier (e.g. at $(t'' + t)/2$) while still compromising at s is strictly more profitable as it moves the concession payoff $1 - \alpha_i$ forward in time (with probability greater than $z_i > 0$).

(e) *Let $t' < t''' \leq T^*$. If $F_i^c(t''') = F_i^c(t')$ then either $F_j^y(t') = F_j^y(t''')$ or for all $t \in [t', t''')$ we have $F_j^c(t) = F_j^y(t)$.* Suppose not, then for some $t'' \in [t', t''')$ we have $F_j^y(t'') < F_j^c(t'')$ and $F_j^y(t') < F_j^y(t''')$. Define $\check{t}_i = \sup\{t : F_i^c(t) = F_i^c(t')\}$. By claim (d) we have $F_j^y(t'') = \sup_{s<\hat{t}_i} F_j^y(s)$ and $F_j^c(t'') > F_j^y(t'')$.

This implies that F_i^c must be continuous at $\check{t}_i$, i.e. $F_i^c(\check{t}_i) = \sup_{s<\check{t}_i} F_i^c(s)$. To see this, notice that compromising at $\check{t}_i$ and conceding at some later date t must give i a strictly lower payoff than compromising slightly earlier (e.g. at $(\check{t}_i + t'')/2$ and still conceding at t (with probability $F_j^c(t'') - F_j^y(t'') > 0$ she receives the payoff m_i earlier). By claim (d), therefore, we must have $F_j^y(t''') = F_j^y(\check{t}_i)$. But in that case any strategy in which agent i compromises an instant after $\check{t}_i$ cannot be optimal either, contradicting the definition of the supremum $\check{t}_i$.

(f) *Let $T^* \geq t'' > t'$. If $F_i^y(t'') = F_i^y(t')$ and $F_i^c(t') > F_i^y(t')$ then $F_j^c(t') = F_j^c(t'')$* Suppose not so that $F_j^c(t') < F_j^c(t'')$. Then there exists $(s,t) \in A_j$ such that $s \in (t', t'']$. However, given $F_i^y(t'') = F_i^y(t')$, the alternative plan of compromising slightly earlier (e.g. at $\hat{s} = (t'+s)/2$) while still conceding at t would be strictly better for j as this gives her the payoff m_j with positive probability (at least $(F_i^c(t') - F_i^y(t')) > 0$) at an earlier time.

(g) *There is no jump in F_i^y at $t \in (0, T^*]$.* Suppose not, then by claim (b) F_j^c is constant on $[t-\varepsilon, t]$ for some $\varepsilon > 0$. Hence, by claim (e) either $F_i^y(t) = F_i^y(t-\varepsilon)$ (a direct contradiction) or $F_i^c(s) = F_i^y(s)$ for $s \in [t-\varepsilon, t)$. It must then be that F_i^c also jumps at t, because we must have $\sup_{s<t} F_i^c(s) = \sup_{s<t} F_i^y(s) < F_i^y(t) \leq F_i^c(t)$. Hence by claim (c), F_j^y is constant on $[t-\varepsilon, t)$ for some $\varepsilon > 0$ (assume the same ε without loss of generality). Given that F_i^c and F_i^y jump at t, we must have $(t,t) \in A_i$. However, the alternative strategy for i of both compromising and immediately conceding slightly earlier (e.g. at $t - \varepsilon/2$) delivers strictly higher expected profits as she gets the payoffs $(F_j^c(t-\varepsilon) - F_j^y(t-\varepsilon))m_i$ and $(1 - F_j^c(t-\varepsilon))(1-\alpha_j) > 0$ at an earlier date, without affecting other payoffs.

(h) *If F_i^y is continuous at $s \leq t$ then $U_i(s,t)$ is continuous at s, and if F_i^c is continuous at t then $U_i(s,t)$ is continuous at t.* This follows from how $U_i(s,t)$ is defined.

For claims (i)-(m) suppose that $F_1^c(t') > F_1^y(t')$ for some $t' \in [0,\infty)$ (symmetric arguments apply if $F_2^c(t') > F_2^y(t')$). Define $\bar{t}_1 = \inf\{t \geq t' : F_1^c(t) = F_1^y(t)\}$ and $\underline{t}_1 = \inf\{t : F_1^c(s) > F_1^y(s)\ \forall s \in [t,t']\}$. Notice that by claim (g), the continuity of F_1^y, we have $F_1^c(\bar{t}_1) = F_1^y(\bar{t}_1)$. Also note that $\bar{t}_1 > t' \geq \underline{t}_1$ and $F_1^c(t) > F_1^y(t)$ for all $t \in (\underline{t}_1, \bar{t}_1)$. Let $\bar{t}_1 \geq t''' > t'' > \underline{t}_1$.

(i) *We must have $F_2^c(t''') > F_2^c(t'')$.* Suppose not, and so let $\check{t}_2 = \sup\{t : F_2^c(t) = F_2^c(t'')\} \geq t'''$. I first establish the subclaim (i') that this must imply either $F_1^y(t'') = F_1^y(\check{t}_2)$ or $F_1^c(t) = F_1^y(t)$ for $t \in [t'', \check{t}_2)$. Suppose not (again), then $F_1^y(t'') < F_1^y(\check{t}_2)$ and there is some $t \in [t'', \check{t}_2)$ such that $F_1^y(t) < F_1^c(t)$. By claim (g), the continuity of F_1^y, we must have $F_1^y(t'') < F_1^y(\check{t}_2 - \varepsilon)$ for all $\varepsilon > 0$ sufficiently small. Choose such an appropriately small $\varepsilon < \check{t}_2 - t$, then we have $F_2^c(t'') = F_2^c(\check{t}_2 - \varepsilon)$, $F_1^y(t) < F_1^c(t)$ for some $t \in [t'', \check{t}_2 - \varepsilon)$ and $F_1^y(t'') < F_1^y(\check{t}_2 - \varepsilon)$, which contradicts claim (e).

By assumption we have $F_1^c(t) > F_1^y(t)$ for all $t \in [t'', \bar{t}_1)$ so that subclaim (i') in fact implies $F_1^y(t'') = F_1^y(\check{t}_2)$. This in turn ensures $\bar{t}_1 > \check{t}_2$ because $F_1^y(\check{t}_2) = F_1^y(t'') < F_1^c(t'') \leq F_1^c(\check{t}_2)$ whereas $F_1^c(\bar{t}_1) = F_1^y(\bar{t}_1)$. I next claim that it can't be optimal for agent 2 to compromise at $\check{t}_2$ while conceding at some $t \geq \check{t}_2$. To see this, notice that agent 2 would do strictly better compromising slightly earlier (e.g. at $(\check{t}_2 + t'')/2$) while still conceding at t as this would bring forward the payoff m_2 with positive probability (at least $F_1^c(t'') - F_1^y(t'') > 0$), without affecting other payoffs. Given claim (g), the continuity of F_i^y, this argument similarly also implies that compromising an instant after $\check{t}_2$ is strictly worse than compromising at $(\check{t}_2 + t'')/2$. This contradicts the definition of the supremum $\check{t}_2$.

(j) *We must have* $F_1^y(t''') > F_1^y(t'')$. Suppose not, then let $\check{t}_1 = \sup\{t : F_1^y(t) = F_1^y(t'')\} \geq t'''$. Given claim (g), the continuity of F_i^y, we have $F_1^y(\check{t}_1) = F_1^y(t'')$. Given $F_1^y(\check{t}_1) = F_1^y(t'') < F_1^c(t'') \leq F_1^c(\check{t}_1)$ we must have $\check{t}_1 < \bar{t}_1$. By claim (f) we must then have $F_2^c(\check{t}_1) = F_2^c(t'')$ which contradicts claim (i), that F_2^c is increasing on $(\underline{t}_1, \bar{t}_1]$.

(k) *We must have* $F_2^y(t''') > F_2^y(t'')$. Suppose not so that $F_2^y(t''') = F_2^y(t'')$. Given that F_2^c is increasing on the interval $[t'', t''']$ by claim (i), we must have $F_2^c(t) > F_2^y(t)$ for $t \in (t'', t''']$. Define $\bar{t}_2 = \inf\{t \geq t''' : F_2^c(t) = F_2^y(t)\}$ and $\underline{t}_2 = \inf\{t : F_2^c(s) > F_2^y(s) \forall s \in [t, t''']\}$, then switching the labelling for 1 and 2, claim (i) implies $F_1^c(t''') > F_1^c(t'')$ and claim (j) implies $F_2^y(t''') > F_2^y(t'')$, a contradiction.

(l) *We must have* $F_1^c(t''') > F_1^c(t'')$. Suppose not, and so $F_1^c(t''') = F_1^c(t'')$. Let $\check{t}_1 = \inf\{t : F_1^c(t) = F_1^c(t'')\}$. The right continuity of F_1^c ensures that $F_1^c(\check{t}_1) = F_1^c(t'')$. Clearly, we have $\check{t}_1 \geq \underline{t}_1$ (if $\check{t}_1 < \underline{t}_1$ then certainly at some $t \in (\check{t}_1, t'']$ we must have $F_1^c(t) = F_1^y(t) = F_1^c(t'') \geq F_1^y(t'') \geq F_1^y(t)$, which contradicts $F_1^c(t'') > F_1^y(t'')$). By claim (e), we then have either $F_2^y(t''') = F_2^y(\check{t}_1)$, which contradicts claim (k), or $F_2^c(t) = F_2^y(t)$ for all $t \in [\check{t}_1, t''')$. Notice that because F_1^y is strictly increasing on $[\check{t}_1, t''')$ by claim (j) while F_1^c is by assumption constant, for some $s \leq \check{t}_1$ and some $t \in (\check{t}_1, t''')$ we must have $(s, t) \in A_1$. Furthermore, if $(s', t') \in A_1$ where $s' \in [s, \check{t}_1]$ then $(s', t) \in A_1$. This is simply because at time s' an agent who compromised at s and another who previously compromised at s' have the same incentives to concede thereafter. I claim, however, that $(\check{t}_1, t) \notin A_1$. To see this, notice that such a strategy is strictly worse than both compromising and conceding at t, which gives agent 1 the higher payoff of α_1 instead of m_1 from the positive concession of agent 2 on the interval $[\check{t}_1, t)$. That is:

$$(12)\qquad U_1(t,t) - U_1(\check{t}_1, t) \geq \int_{\check{t}_1 \leq v \leq t} (\alpha_1 - m_1) e^{-r_1 v} dF_2^c(v)$$
$$\geq e^{-r_1 t}(\alpha_1 - m_1)(\sup_{v<t} F_2^c(v) - F_2^c(\check{t}_1)) > 0$$

where the first inequality follows from $F_2^c(t) = F_2^y(t)$ on $[\check{t}_1, t''')$, the second from $t \geq v \in [\check{t}_1, t]$ and the third from claim (i). For the same reason, compromising an instant before $\check{t}_1$ and conceding at t cannot be optimal either. This either contradicts the definition of $\check{t}_1$ as an infimum or implies $\check{t}_1 = 0$ and $F_2^c(0) = 0$. The latter possibility, however, clearly contradicts $F_1^c(v) > F_1^y(v)$ for all $v \in (\check{t}_1, t''')$.

(m) *F_i^c is continuous on $(\underline{t}_1, \bar{t}_1]$.* If F_i^c did jump at $t \in (\underline{t}_1, \bar{t}_1]$ then by (c), F_j^y is constant on $(t - \varepsilon, t)$ for some $\varepsilon > 0$, contradicting either claim (j) or (k).

We are almost done. Because F_1^c, F_1^y are increasing on $(\underline{t}_1, \bar{t}_1)$, established in claims (j) and (l), while by assumption $F_1^y(t) < F_1^c(t)$ on this interval, it follows that there is some $s' \in (\underline{t}_1, \bar{t}_1)$ such that A_1 is dense in the set $\{(s', t) : t \in [s', \bar{t}_1]\}$. Notice that regardless of whether agent 1 compromises at s' or $s \in (s', \bar{t}_1)$, she faces the same incentives to concede after s if she has not already done so. Notice also, that there is always a positive probability that agent 1 has compromised before s but has not conceded. From the continuity of F_2^c on $(\underline{t}_1, \bar{t}_1]$ it follows that $U_1(s', t)$ is constant on $[s', \bar{t}_1]$, and hence $\partial U_1(s', t)/\partial t = 0$. This implies:

$$(13)\qquad \frac{f_2^c(t)}{1 - F_2^c(t)} = \lambda_2^c = \frac{r_1(1-\alpha_2)}{m_1 - (1-\alpha_2)}$$

for $t \in [\underline{t}_1, \bar{t}_1]$. Solving this linear ODE gives $(1 - F_2^c(s)) = (1 - F_j^c(\underline{t}_1))e^{-\lambda_2^c(s - \underline{t}_1)}$.

By the same reasoning there must be some $s'' \in (\underline{t}_1, \bar{t}_1)$ such that A_1 is dense in the set $\{(s, s'') : s, \in [\underline{t}_1, s'']\}$. The continuity of F_2^y on $(\underline{t}_1, \bar{t}_1]$ then implies that $U_1(s, s'')$ is constant on $(\underline{t}_1, s'']$, and hence $\partial U_1(s, s'')/\partial s = 0$. This implies:

$$(14)\qquad \frac{f_2^y(s)}{F_2^c(s) - F_2^y(s)} = \lambda_2^y = \frac{r_1 m_1}{\alpha_1 - m_1}$$

This should already suggest a problem. When $F_2^c(s) - F_2^y(s)$ becomes arbitrarily small $f_2^y(s)$ must be similarly small. However, $f_2^c(t) \geq \lambda_2^c(1 - F_2^c(t)) \geq \lambda_2^c z_2$ is bounded above zero, implying $F_2^c(t) - F_2^y(t) > 0$ on $(\underline{t}_1, \bar{t}_1]$. To be more precise, the above linear ODE is solved to give:

$$(1 - F_2^y(s)) = \begin{cases} \phi_2^y e^{-\lambda_2^y(s-\underline{t}_1)} + \phi_2^c\psi_2(e^{-\lambda_2^c(s-\underline{t}_1)} - e^{-\lambda_2^y(s-\underline{t}_1)}) \text{ if } \lambda_2^y \neq \lambda_2^c \\ (\phi_2^y + \lambda_2^y\phi_2^c(s - \underline{t}_1))e^{-\lambda_2^y(s-\underline{t}_1)} \text{ if } \lambda_2^y = \lambda_2^c \end{cases} \tag{15}$$

where, $\psi_2 = \lambda_2^y/(\lambda_2^y - \lambda_2^c)$ and $\phi_2^y = (1 - F_2^y(\underline{t}_1)) \geq (1 - F_2^c(\underline{t}_1)) = \phi_2^c$. Define the gap between F_2^c and F_2^y as $d_2(s) = F_2^c(s) - F_2^y(s)$, and consider the following transformations of this gap:

$$\begin{aligned} d_2(s)\frac{e^{\lambda_2^y(s-\underline{t}_1)}}{\psi_2 - 1} &= \frac{\phi_2^y - \psi_2\phi_2^c}{\psi_2 - 1} + e^{(\lambda_2^y-\lambda_2^c)(s-\underline{t}_1)} \quad \text{if} \quad \lambda_2^y > \lambda_2^c \\ d_2(s)\frac{e^{\lambda_2^c(s-\underline{t}_1)}}{\phi_2^y - \psi_2\phi_2^c} &= e^{(\lambda_2^c-\lambda_2^y)(s-\underline{t}_1)} + \frac{\psi_2 - 1}{\phi_2^y - \psi_2\phi_2^c} \quad \text{if} \quad \lambda_2^y < \lambda_2^c \\ d_2(s)e^{\lambda_2^y(s-\underline{t}_1)} &= \phi_2^y + \lambda_2^y\phi_2^c(s - \underline{t}_1) - \phi_2^c \quad \text{if} \quad \lambda_2^y = \lambda_2^c \end{aligned} \tag{16}$$

Each of these transformations is positive. To see this, notice that $\psi_2 - 1 = \lambda_2^c/(\lambda_2^y - \lambda_2^c) > 0$ when $\lambda_2^y > \lambda_2^c$. Similarly $\phi_2^y - \psi_2\phi_2^c \geq -\phi_2^c\lambda_2^c/(\lambda_2^y - \lambda_2^c) > 0$ when $\lambda_2^y < \lambda_2^c$, where the first inequality follows from $\phi_2^y \geq \phi_2^c$. Each of the transformed gaps is strictly increasing in s, implying that $d_2(s) > 0$ for $s \in (\underline{t}_1, \bar{t}_1]$. Recall that we must have $\bar{t}_1 \leq T^* < \infty$, and $F_1^c(\bar{t}_1) = F_1^y(\bar{t}_1)$. Now define $\bar{t}_2 = \inf\{t > \underline{t}_1 : F_2^c(t) = F_2^y(t)\} \leq T^* < \infty$, where this is consistent with the definition of $\bar{t}_2$ in the proof of claim (k). We can now repeat the above arguments with the roles of agent 1 and 2 reversed to find that $d_1(s) > 0$ for $s \in (\underline{t}_1, \bar{t}_2]$ and $F_2^c(\bar{t}_2) = F_2^y(\bar{t}_2)$. Let $\bar{t} = \min\{\bar{t}_1, \bar{t}_2\}$. For some i we must have $\bar{t} = \bar{t}_i$, but that implies both $F_i^c(\bar{t}_i) = F_i^y(\bar{t}_i)$ and $d_i(\bar{t}_i) = F_i^c(\bar{t}_i) - F_i^y(\bar{t}_i) > 0$, a contradiction. We must, therefore, have $F_i^c(t) = F_i^y(t)$ for $t \in [0, \infty)$. Given this, the unique equilibrium must match unmediated bargaining by standard arguments (see AG). □