

Policy Experimentation in Committees: a Case against Veto Rights under Redistributive Constraints

ONLINE APPENDIX

Vincent Anesi T. Renee Bowen

A Proof of Lemma A1

We begin with part (i). Suppose first that $L \notin \mathcal{D}$. To see that there exists an equilibrium with the proposed outcome, consider the following (stationary Markov) strategy profile:

- Whenever the status quo is R , all proposers pass (i.e., propose R), and each voter i accepts proposal S if and only if $i \in L$;
- whenever the status quo is S , each proposer i proposes R if $i \notin L$ and passes otherwise, and each voter i accepts proposal R if and only if $i \notin L$.

It is easy to check that this strategy profile constitutes an equilibrium. (In particular, proposers who prefer S to R do not deviate and propose to amend status quo R because they anticipate that such a proposal would be rejected.)

Next we show that this is the unique equilibrium outcome. Our proof shares some of the intuitions of the Shaked and Sutton (1984) proof of equilibrium uniqueness for the Rubinstein (1982) model. Let the set of equilibria of $\Gamma(R, 1)$ be denoted by $\mathcal{E}(R, 1)$. In $\Gamma(R, 1)$, committee member i 's expected payoff in every period t is a convex combination of $\gamma\Delta r_i$ and Δs_i . Therefore, for every strategy profile σ her average discounted payoff is of the form $V_i(\sigma) \equiv \beta(\sigma)\gamma\Delta r_i + [1 - \beta(\sigma)]\Delta s_i$, with $\beta(\sigma) \in [0, 1]$. This implies that, for any two strategy profiles σ and σ' , and any committee member $i \in \{i \in N : \gamma r_i > s_i\}$, we have $V_i(\sigma) \geq V_i(\sigma')$ if and only if $\beta(\sigma) \geq \beta(\sigma')$.

Let $\{\sigma^m\}$ be a sequence in $\mathcal{E}(R, 1)$ that satisfies $\lim_{m \rightarrow \infty} \beta(\sigma^m) = \inf_{\sigma \in \mathcal{E}(R, 1)} \beta(\sigma)$, so that $\lim_{m \rightarrow \infty} V_i(\sigma^m) = \inf_{\sigma \in \mathcal{E}(R, 1)} V_i(\sigma)$ for all $i \in W \equiv \{i \in N : \gamma r_i \geq s_i\}$. Fix $m \in \mathbb{N}$. Every proposal that may successfully be made by the last proposer in the first period under σ^m (both on and off the path) must be accepted by some decisive player i in W . That is, i 's continuation payoff from accepting the proposal, say U_i^a , must be at least as large as her payoff from rejecting it; i.e., $U_i^a \geq (1 - \delta)\gamma\Delta r_i + \delta V_i(\sigma^r)$, where $\sigma^r \in \mathcal{E}(R, 1)$ is the equilibrium of $\Gamma(R, 1)$ that is played from the next period on if i rejects the proposal in the first period. From the argument in the previous paragraph, we thus have $U_j^a \geq (1 - \delta)\gamma\Delta r_j + \delta V_j(\sigma^r)$ for all $j \in W$. Similarly, every proposal that may successfully be made by the penultimate proposer in the first period under σ^m (both on and off the path) must also be accepted by some member i of W . Her payoff (and therefore the payoff of all members of W) from accepting must be at least as large as the payoff from rejecting which, as previously shown, must be at least $(1 - \delta)\gamma\Delta r_i + \delta \inf \{V_i(\sigma) : \sigma \in \mathcal{E}(R, 1)\}$. Applying the same argument recursively, we obtain that the acceptance of any proposal in the first period must give a payoff of at least $(1 - \delta)\gamma\Delta r_i + \delta \inf \{V_i(\sigma) : \sigma \in \mathcal{E}(R, 1)\}$ for all $i \in W$. Hence,

$$V_i(\sigma^m) \geq (1 - \delta)\gamma\Delta r_i + \delta \inf \{V_i(\sigma) : \sigma \in \mathcal{E}(R, 1)\},$$

for all $i \in W$. Taking the limit as $m \rightarrow \infty$ and recalling the definition of $\{\sigma^m\}$, we obtain $\gamma\Delta r_i = \inf \{V_i(\sigma) : \sigma \in \mathcal{E}(R, 1)\}$ (since $\gamma\Delta r_i$ is maximum feasible payoff for a player $i \in W$ when R is good with probability one). This in turn implies that R must be implemented with probability one in every period of every equilibrium of $\Gamma(R, 1)$.

The argument for the case where $L \in \mathcal{D}$ is analogous.

We now turn to part (ii). To prove the second part of the lemma, we proceed in three steps: first, we show that the infimum of every player i 's equilibrium payoff in $\Gamma(S, \alpha_k)$ converges to Δs_i as $k \rightarrow \infty$; then, we show that for sufficiently large k , alternative S is implemented in every period of $\Gamma(S, \alpha_k)$; finally, we use the previous result to complete the proof of the lemma.

Let $\alpha_k \in A \setminus \{1\}$; and let $\mathcal{E}(S, \alpha_k)$ be the set of equilibria of $\Gamma(S, \alpha_k)$. Every period of $\Gamma(S, \alpha_k)$ begins with a belief α that alternative R is good; then, either S is implemented,

in which case committee member i receives a payoff of Δs_i ; or R is implemented, in which case i 's expected payoff is $\alpha\gamma\Delta r_i$. Therefore, every strategy profile σ yields an expected payoff of the form

$$V_i^k(\sigma) \equiv \Delta \left[\beta_s^k(\sigma) s_i + \beta_1^k(\sigma) \gamma r_i + \sum_{\ell=k}^{\infty} \beta_\ell^k(\sigma) \alpha_\ell \gamma r_i \right]$$

to player i in $\Gamma(S, \alpha_k)$, where $\beta_s^k(\sigma) + \beta_1^k(\sigma) + \sum_{\ell=k}^{\infty} \beta_\ell^k(\sigma) = 1$ and $\beta_s^k(\sigma), \beta_1^k(\sigma), \beta_\ell^k(\sigma) \in [0, 1]$ for each $\ell = k, k+1, \dots$. Moreover, as R must have been successfully tried at least once to be known to be good, $\beta_1^k(\cdot)$ is bounded above by $\alpha_k \gamma$. Coupled with the fact that $\alpha_\ell \leq \alpha_k$ for all $\ell \geq k$, this implies that $\lim_{k \rightarrow \infty} M_i^k \equiv \sup_{\sigma \in \mathcal{E}(S, \alpha_k)} \left[\beta_1^k(\sigma) + \sum_{\ell=k}^{\infty} \beta_\ell^k(\sigma) \alpha_\ell \right] \gamma r_i$ for each $i \in N$. This in turn implies that there is a null sequence $\{\varepsilon^k\}$ in \mathbb{R}_+ such that, for all $\sigma \in \mathcal{E}(S, \alpha_k)$, we have

$$\max_{i \in N} |V_i^k(\sigma) - \beta_s^k(\sigma) s_i \Delta| \leq \Delta \max_{i \in N} M_i^k < \varepsilon^k,$$

for every $k \in \mathbb{N}$. Now for each $k \in \mathbb{N}$, let $\{\sigma^{k,m}\}$ be a sequence in $\mathcal{E}(S, \alpha_k)$ such that $\lim_{m \rightarrow \infty} \beta_s^k(\sigma^{k,m}) = \inf_{\sigma \in \mathcal{E}(S, \alpha_k)} \beta_s^k(\sigma)$. As $\sigma^{k,m}$ is an equilibrium of $\Gamma(S, \alpha_k)$, there must be at least one committee member, say i_k , such that

$$V_{i_k}^k(\sigma^{k,m}) \geq (1 - \delta) \Delta s_{i_k} + \delta \inf_{\sigma \in \mathcal{E}(S, \alpha_k)} V_{i_k}^k(\sigma);$$

otherwise some player would have a profitable deviation in the first period of $\Gamma(S, \alpha_k)$. It follows that

$$\beta_s^k(\sigma^{k,m}) \Delta s_{i_k} + \varepsilon^k \geq (1 - \delta) \Delta s_{i_k} + \delta \left[\inf_{\sigma \in \mathcal{E}(S, \alpha_k)} \beta_s^k(\sigma) \Delta s_{i_k} - \varepsilon^k \right].$$

Taking the limit as $m \rightarrow \infty$, we obtain $\inf_{\sigma \in \mathcal{E}(S, \alpha_k)} \beta_s^k(\sigma) \geq 1 - 2(\varepsilon^k / \Delta s_{i_k})$. This implies that $\inf_{\sigma \in \mathcal{E}(S, \alpha_k)} \beta_s^k(\sigma)$ converges to one as $k \rightarrow \infty$ and, therefore, that there exists a null sequence $\{\eta^k\}$ such that $\lim_{k \rightarrow \infty} \max_{i \in N} \left| \inf_{\sigma \in \mathcal{E}(S, \alpha_k)} V_i^k(\sigma) - \Delta s_i \right| < \eta^k$, for all $k \in \mathbb{N}$, thus completing the first step of the argument.

We now turn to the second step of the proof. Observe first that, as $(1 - \delta)(s_i - \alpha_k \gamma r_i) \Delta - \delta \eta^k$ converges to $(1 - \delta) \Delta s_i > 0$ as $k \rightarrow \infty$, there is a sufficiently large $K \in \mathbb{N}$ such that $(1 - \delta)(s_i - \alpha_k \gamma r_i) \Delta - \delta \eta^k > 0$, for all $k \geq K$. Let $k \geq K$, and suppose

that $\Gamma(S, \alpha_k)$ has an equilibrium in which alternative R is implemented with positive probability. Consider the first period of $\Gamma(S, \alpha_k)$ in which R may be implemented. Every decisive voter i 's benefit from rejecting any proposal to change S to R is bounded below by $(1 - \delta)(s_i - \alpha_k \gamma r_i) \Delta + \delta [(\Delta s_i - \eta^k) - \Delta s_i] > 0$, where the bracketed term represents the difference between the lower and upper bounds on i 's continuation payoffs from rejecting R and accepting it, respectively. (Recall that each committee member i 's maximum payoff is Δs_i when the belief is smaller than or equal to $\hat{\alpha}_n$.) Hence, every proposal to amend S to R is rejected in any equilibrium of $\Gamma(S, \alpha_k)$. We thus have $V_i^k(\sigma) = \Delta s_i$, for all $i \in N$ and all $\sigma \in \mathcal{E}(S, \alpha_k)$.

If $\alpha_K > \hat{\alpha}_n$, then Lemma 1(ii) follows immediately from the previous paragraph; so suppose that $\alpha_K \leq \hat{\alpha}_n$. To complete the proof of the result, consider the first period of $\Gamma(S, \alpha_K)$. If alternative R is implemented, then the expected payoff to each committee member i is $[1 - \delta(1 - \gamma\Delta)]\alpha_K \gamma \Delta r_i + \delta(1 - \alpha_K \gamma \Delta) \Delta s_i < \Delta s_i$, where the inequality follows from $\alpha_K \leq \hat{\alpha}_n$ and the definition of the committee members' optimal cutoffs in Subsection 3.2; if alternative S is instead implemented, then her expected payoff will be a convex combination of $[1 - \delta(1 - \gamma\Delta)]\alpha_K \gamma \Delta r_i + \delta(1 - \alpha_K \gamma \Delta) \Delta s_i$ (if R is implemented with positive probability in a future period) and Δs_i , with a positive coefficient on the latter. Therefore, all committee members are strictly better off implementing R : they all reject proposals to amend S to R (when decisive). Hence, $V_i^k(\sigma) = \Delta s_i$, for all $i \in N$, $k \geq K$ and $\sigma \in \mathcal{E}(S, \alpha_k)$. Applying the same argument recursively from belief α_{K-1} to belief $\hat{\alpha}_n$ we obtain that, for all $\alpha_k \leq \hat{\alpha}_n$, $\Gamma(S, \alpha_k)$ has a unique equilibrium outcome: Alternative S is implemented in every period. By the same logic, the same is also true in game $\Gamma(S, \alpha_k)$, $\alpha_k \leq \hat{\alpha}_n$. In such a game, every decisive voter receives her largest possible payoff Δs_i if she accepts a proposal to change the status quo R to S , since the latter will then never be amended. Any such a proposal must therefore be successful and, as S is the ideal policy of all players, some proposer must successfully propose it in equilibrium.

Finally, S being the ideal alternative of all the players, it is easy to construct an equilibrium in which all players always propose alternative S (conditional on being recognized to propose), accept any proposal to change status quo R to alternative S , and reject any

proposal to amend status quo S .

B Proof of Proposition 2: Collegial Rules

The main Appendix contains a proof of Proposition 2 for cases where the voting rule is noncollegial. This section covers all other cases.

(i) Unanimity rule. Suppose now that $\mathcal{D} = \{N\}$. As in the proof of Proposition 1, we denote by $\Gamma(p \mid \alpha)$ the continuation game that begins with status quo $p \in \{R, S\} \times [0, 1] \times X$ and belief $\alpha \in A$.

Lemma B1. *Suppose \mathcal{D} is unanimity rule and $\hat{\tau} = 1$. Then there exists $\bar{\Delta}_1 > 0$ such that, for all $\Delta < \bar{\Delta}_1$ and all $(\tau, x) \in [0, 1] \times X$:*

- (i) $\Gamma(R, \tau, x \mid 1)$ has a renegotiation-proof equilibrium and, in any such equilibrium, each committee member i 's payoff is $w_i(R, \tau, x \mid 1) \equiv \gamma\Delta[(1 - \tau)r_i + \tau x_i \bar{r}]$; and
- (ii) the set of renegotiation-proof equilibrium payoffs for $\Gamma(S, \tau, x \mid 1)$ is the simplex $\{(w_1, \dots, w_n) \in \mathbb{R}^n : \sum_{i=1}^n w_i = V^*(1) \text{ and } w_i \geq [(1 - \tau)s_i + \tau x_i \bar{s}]\Delta, \forall i \in N\}$.

Proof. Let $\bar{\Delta}_1 \equiv \sup \{\Delta \in \mathbb{R}_+ : (1 - e^{-\rho\Delta}) < e^{-\rho\Delta}(n - 2)\} > 0$. Henceforth, we assume that $\Delta < \bar{\Delta}_1$.

Let $(\tau, x) \in [0, 1] \times X$, let $w^0 \in \{(w_1, \dots, w_n) \in \mathbb{R}^n : \sum_{i=1}^n w_i = V^*(1) \text{ and } w_i \geq [(1 - \tau)s_i + \tau x_i \bar{s}]\Delta\}$. To prove the lemma, we will construct a renegotiation-proof equilibrium σ for $\Gamma(S, \tau, x \mid 1)$ in which every committee member i receives w_i^0 , thus establishing part (ii). In that equilibrium, the optimal stopping rule will be implemented in every period both on and off the path. As $\Gamma(R, \tau, x \mid 1)$ is itself a continuation game in $\Gamma(S, \tau, x \mid 1)$, this will also establish that $\Gamma(R, \tau, x \mid 1)$ has a renegotiation-proof equilibrium. If the status quo is (R, τ, x) and the belief is equal to 1, then each committee member i can obtain a payoff of $w_i(R, \tau, x \mid 1)$ by rejecting any future proposal to amend the status quo. As the payoff vector $(w_j(R, \tau, x \mid 1))_{j \in N}$ is in the Pareto frontier (and \mathcal{D} is unanimity rule), it follows that $w_i(R, \tau, x \mid 1)$ is i 's payoff in any (renegotiation-proof) equilibrium for $\Gamma(R, \tau, x \mid 1)$.

We begin with an intuitive description of the equilibrium σ . Alternative R is implemented in each period (both on and off the path). As the belief is equal to 1, this implies that payoff vectors are Pareto optimal in every continuation game. Once S has been implemented, all proposers pass in all future periods, irrespective of the tax rate and distribution of revenues. If R has not yet been implemented, then behavior is determined by a set of n “phases,” each corresponding to one committee member in N . In committee member i ’s phase, every proposer successfully offers a policy that gives a payoff of $\gamma\Delta\bar{r} - \sum_{j \neq i} [(1 - \tau')s_j + \tau y_j \bar{s}] \Delta$ to i and a payoff of $[(1 - \tau')s_j + \tau y_j \bar{s}] \Delta$ to each committee member $j \neq i$, where (S, τ', y) is the status quo policy. The idea is that i receives her “reward payoff” and the others their “punishment payoffs.” If a proposer, say i , deviates, then every committee member (other than i) rejects her proposal and the game transitions to the phase of the first committee member who rejected the proposal. If voter i rejects a proposal which she should have accepted, then the game moves to the another committee member’s phase.

We now turn to the formal definition of σ for the continuation game $\Gamma(S, \tau, x | 1)$. As in the case of noncollegial rules, we divide each period into n “parts,” each consisting of a proposal stage and the n voting stages that follow it. Changes of phases can only occur at the end of these parts. A phase is formally represented by a pair $(\ell, i) \in \{1, \dots, n\} \times \{0, 1, \dots, n\}$. In every period that begins with status quo $p = (a, \tau', y)$ and an order of proposers (π_1, \dots, π_n) , σ prescribes the following behavior in phase (ℓ, i) :

(a) If $a = R$, then proposer π_ℓ passes; and if $a = S$, then she offers policy $(R, 1, y^i)$,

where

$$y_j^i \equiv \begin{cases} w_j^0 / V^*(1) & \text{if } i = 0, \\ w_j(p | 1) \Delta / V^*(1) & \text{if } i \neq 0 \text{ \& } j \neq i, \\ [V^*(1) - \sum_{k \neq i} w_k(p | 1)] / V^*(1) & \text{if } i \neq 0 \text{ \& } j = i, \end{cases}$$

for all $j \in N$;

(b) if $a = S$ and π_ℓ offered $(R, 1, y^i)$, then every voter accepts it (irrespective of the previous voters’ actions);

(c) if $a = R$ and π_ℓ proposed some policy $p' = (a', \tau'', z) \neq p$, then voter $j \in N$ votes

to accept it if and only if

$$w_j(p | 1) < (1 - \delta)w_j(p' | 1) + \delta \begin{cases} w_j(p' | 1) & \text{if } a' = R, \\ y_j^i V^*(1) & \text{if } a' = S. \end{cases} ;$$

(d) if $a = S$ and π_ℓ proposed some policy $p' \notin \{(R, 1, y^i), p\}$, then every voter j acts according to the following rules:

(d1) if any previous voter has already rejected p' , then j also rejects it;

(d2) if she is the n th voter and p' has not yet been rejected by any voter, then she accepts p' if and only if the following holds

$$\begin{cases} (1 - \delta)w_j(p' | 1) + \delta y_j^{\pi_\ell} V^*(1) > (1 - \hat{\delta}_\ell)w_j(p | 1) + \hat{\delta}_\ell y_j^j V^*(1) & \text{if } j \neq \pi_\ell, \\ (1 - \delta)w_j(p' | 1) + \delta y_j^{\pi_\ell} V^*(1) > (1 - \hat{\delta}_\ell)w_j(p | 1) + \hat{\delta}_\ell y_j^{\hat{i}} V^*(1) & \text{if } j = \pi_\ell, \end{cases}$$

where $\hat{i} \equiv \min N \setminus \{\pi_\ell\}$ and

$$\hat{\delta}_\ell \equiv \begin{cases} 1 & \text{if } \ell \in \{1, \dots, n-1\}, \\ \delta & \text{if } \ell = n; \end{cases}$$

(d3) if she is the k th voter, $k < n$, and p' has not yet been rejected by any voter, then she accepts p' if and only if all the remaining voters will also accept it and the following holds

$$\begin{cases} (1 - \delta)w_j(p' | 1) + \delta y_j^{\pi_\ell} V^*(1) > (1 - \hat{\delta}_\ell)w_j(p | 1) + \hat{\delta}_\ell y_j^j V^*(1) & \text{if } j \neq \pi_\ell, \\ (1 - \delta)w_j(p' | 1) + \delta y_j^{\pi_\ell} V^*(1) > (1 - \hat{\delta}_\ell)w_j(p | 1) + \hat{\delta}_\ell y_j^{\hat{i}} V^*(1) & \text{if } j = \pi_\ell. \end{cases}$$

Observe that, from (a) and (b) above, every committee member j 's continuation value at the start of phase (ℓ, i) is $w_j(p | 1)$ if $a = R$, and $y_j^i V^*(1)$ if $a = S$.

In period 1, play begins in phase $(1, 0)$. Then in every period, at the end of any part that began with status quo $p = (a, \tau', y)$ and in some phase $(\ell, i) \in \{1, \dots, n\} \times \{0, 1, \dots, n\}$:

(t1) If $a = R$ and π_ℓ passed, then the game transitions to phase $(\ell + 1, i)$ (we set $\ell + 1 = 1$ whenever $\ell = n$);

(t2) if $a = S$ and π_ℓ proposed $(R, 1, y^i)$ which was accepted, then the game transitions to phase $(1, i)$;

(t3) if $a = S$ and π_ℓ proposed $(R, 1, y^i)$ which was rejected, then the game transitions to phase $(\ell + 1, j + 1)$ (set $j + 1 = 1$ whenever $j = n$), where j is the first voter who rejected it;

(t4) if $a = R$ and π_ℓ proposed some policy $p' \neq p$, then the game transitions to phase $(1, i)$ if p' was accepted, and to phase $(\ell + 1, i)$ otherwise;

(t5) if $a = S$ and π_ℓ proposed some policy $p' \notin \{(R, 1, y^i), p\}$ which was accepted, then the game transitions to phase $(\ell + 1, \pi_\ell)$;

(t6) if $a = S$ and π_ℓ proposed some policy $p' \notin \{(R, 1, y^i), p\}$ which was rejected by some voter in $N \setminus \{\pi_\ell\}$, then the game transitions to phase $(\ell + 1, j)$, where j is the first voter in $N \setminus \{\pi_\ell\}$ who rejected p' ; and

(t7) if $a = S$ and π_ℓ proposed some policy $p' \notin \{(R, 1, y^i), p\}$ which was only rejected by π_ℓ herself, then the game transitions to phase $(\ell + 1, \hat{i})$;

(t8) if $a = S$ and π_ℓ passed, then the game transitions to phase $(\ell + 1, \pi_\ell + 1)$.

We now verify that for $\Delta < \bar{\Delta}_1$, σ is an equilibrium. Take an arbitrary committee member $j \in N$, and consider a voting stage with status quo $p = (a, \tau', y)$ in some phase (ℓ, i) . Suppose first that $a = S$ and π_ℓ proposed $(R, 1, y^i)$, so that σ prescribes j to accept this proposal (see (b)). If some previous voter has already rejected the proposal, then j has trivially no profitable deviation: her decision will have no impact on her payoff. If she is the first voter, or if all previous voters have accepted the proposal, then her decision does impact her payoff. If she accepts $(R, 1, y^i)$ then, from (b) and (t2) above, she receives a payoff of $y_j^i V^*(1)$; if she rejects $(R, 1, y^i)$ then, from (t3), the game transitions to phase $(\ell + 1, j + 1)$ and she receives $(1 - \hat{\delta})w_j(p | 1) + \hat{\delta}y_j^{j+1}V^*(1)$. Since

$$y_j^i V^*(1) \geq w_j(p | 1) = (1 - \hat{\delta}_\ell)w_j(p | 1) + \hat{\delta}_\ell y_j^{j+1} V^*(1) ,$$

she is better off accepting.

Suppose now that $a = R$ and π_ℓ proposed some policy $p' \neq p$. If p' is accepted then, from (t4), the game transitions to phase $(1, i)$ and committee member j receives $(1 - \delta)w_j(p' | 1) + \delta \begin{cases} w_j(p' | 1) & \text{if } a' = R , \\ y_j^i V^*(1) & \text{if } a' = S; \end{cases}$ if p' is rejected then she receives $w_j(p | 1)$.

It follows from (c) that, under σ , her decision is optimal whenever she is decisive. Hence, she cannot profitably deviate from σ .

Finally, suppose that $a = S$ and π_ℓ proposed some policy $p' \notin \{(R, 1, y^j), p\}$:

- If some voter in $N \setminus \{\pi_\ell, j\}$ has already rejected the proposal then, from (t6), her decision will not have any impact on her payoff and is therefore optimal.

- If π_ℓ is the only voter who has already rejected p' , then the choice of voter $j \neq \pi_\ell$ has no impact on her stage-game payoff in this period but will impact the transition to the next phase. It follows from (t6) and (t7) that she cannot improve on rejecting, which is the action prescribed by σ (see (d1)).

- If p' has not yet been rejected and j is the n th voter, then it follows from (d2) and (t5)-(t7) that σ prescribes her to accept p' if and only if she is strictly better off doing so. The same is true if j is not the last voter and she anticipates that all the remaining voters will accept p' — see (d3).

- If p' has not yet been rejected, j is not the last voter and she anticipates the some of the remaining voters will reject p' , then her choice has no impact on the policy that will be implemented in the current period. If, in addition, $j = \pi_\ell$ then her choice does not have any impact on her continuation value either and, therefore, rejecting is optimal. If instead $j \neq \pi_\ell$, then her decision will impact the transition to the next phase. It follows from (t6) and (t7) that she is better off rejecting, as prescribed by σ . (She can only be indifferent if π_ℓ is the only other voter who will reject p' , and $j = \hat{i}$.)

This proves that deviations in voting stages are unprofitable. We now turn to proposal stages. Consider the proposal stage of any phase (ℓ, i) that begins with a status quo $p = (a, \tau', y)$. Suppose first that $a = R$. If $j = \pi_\ell$ passes, as prescribed by σ , then from (t1) she receives a payoff of $y_j^i V^*(1)$. If she deviates by proposing a policy $p' \neq p$ then, from (c), her proposal will be rejected: as the payoff vector $(w_k(p | 1))_{k \in N}$ belongs to the Pareto frontier, it is impossible to offer every committee member k a higher payoff than $w_k(p | 1)$. It then follows from (a) that proposer j gets a payoff of $w_j(p | 1) \leq y_j^i V^*(1)$ (with a strict inequality if and only if $i = j$). Hence, j cannot profitably deviate from passing.

Suppose now that $a = S$. If $j = \pi_\ell$ proposes $(R, 1, y^i)$, as prescribed by σ , then from (b), (t2) and (a), she receives a payoff of $y_j^i V^*(1)$. If she deviates by passing, then she receives $(1 - \delta)w_j(p | 1)$ in the current period and the game then transitions to phase $(\ell + 1, j + 1)$ (see (t8)). She thus receives

$$(1 - \hat{\delta}_\ell)w_j(p | 1) + \hat{\delta}_\ell y_j^{j+1} V^*(1) = w_j(p | 1) \leq y_j^i V^*(1)$$

(with a strict inequality if and only if $i = j$). Hence, passing is not a profitable deviation. Finally, if she deviates by proposing a policy $p' \neq (R, 1, y^i)$ then, from (c), her proposal will be rejected. To see this, observe that from (d), she would have to offer more than $(1 - \hat{\delta}_\ell)w_k(p | 1) + \hat{\delta}_\ell y_k^k V^*(1)$ to all the other committee members $k \neq j$, and more than $w_j(p | 1)$ to herself. Summing across the committee members and rearranging terms, a successful proposal would have to generate a total sum of payoffs that exceeds $(1 - \hat{\delta}_\ell)\bar{s}\Delta + \hat{\delta}_\ell[V^*(1) + (n - 2)[V^*(1) - \bar{s}\Delta]] = (1 - \hat{\delta}_\ell)\bar{s}\Delta + \hat{\delta}_\ell[\gamma\bar{r} + (n - 2)[\gamma\bar{r} - \bar{s}]]\Delta$. As aggregate payoffs are bounded above by $V^*(1) = \gamma\Delta\bar{r}$, this would require that

$$(1 - \hat{\delta}_\ell)\bar{s} + \hat{\delta}_\ell[\gamma\bar{r} + (n - 2)[\gamma\bar{r} - \bar{s}]] < \gamma\bar{r} \quad (\text{B2})$$

or, equivalently, $1 - \hat{\delta}_\ell < \hat{\delta}_\ell(n - 2)$. This is impossible since $\Delta < \bar{\Delta}_1$. Moreover, it follows from (t6)-(t7) that the game will then transition to phase $(\ell + 1, j)$. As a result, j obtains a payoff of

$$(1 - \hat{\delta}_\ell)w_j(p | 1) + \hat{\delta}_\ell y_j^k V^*(1) = w_j(p | 1) \leq y_j^i V^*(1)$$

(with $k \neq j$) if she deviates by making a proposal $p' \neq (R, 1, y^i)$. Hence, such a deviation is unprofitable. \square

Lemma B2. *Suppose \mathcal{D} is unanimity rule and $\hat{\tau} = 1$. Then there exists $\bar{\Delta}_2 > 0$ such that, for all $\Delta < \bar{\Delta}_2$, all beliefs $\alpha_k \leq \alpha^*$ and all $(\tau, x) \in [0, 1] \times X$:*

- (i) $\Gamma(S, \tau, x | \alpha_k)$ has a renegotiation-proof equilibrium and, in any such equilibrium, each committee member i 's payoff is $w_i(S, \tau, x | \alpha_k) \equiv \alpha_k \gamma \Delta [(1 - \tau)s_i + \tau x_i \bar{s}]$; and
- (ii) the set of renegotiation-proof equilibrium payoffs for $\Gamma(R, \tau, x | \alpha_k)$ is the simplex $\{(w_1, \dots, w_n) \in \mathbb{R}^n : \sum_{i=1}^n w_i = \bar{s}\Delta \text{ and } w_i \geq \alpha_k \gamma \Delta [(1 - \tau)r_i + \tau x_i \bar{r}], \forall i \in N\}$.

Proof. An application of l'Hôpital's rule gives

$$\lim_{\Delta \rightarrow 0} \alpha^* = \frac{\rho \bar{s}}{\gamma[(\rho + \gamma)\bar{r} - \bar{s}]},$$

so that

$$\bar{s} - \gamma \bar{r} \lim_{\Delta \rightarrow 0} \alpha^* = \frac{\bar{s}(\gamma \bar{r} - \bar{s})}{(\rho + \gamma)\bar{r} - \bar{s}} > 0.$$

It follows that

$$\tilde{\Delta}_2 \equiv \sup \{ \Delta > 0 : [1 - \delta(1 - \alpha^* \gamma \Delta)] \bar{s} - [1 - \delta(1 - \gamma \Delta)] \alpha^* \gamma \bar{r} < \delta(1 - \alpha^* \gamma \Delta)(n - 2)[\bar{s} - \alpha^* \gamma \bar{r}] \}$$

is well-defined and positive. Observe that if $\Delta < \tilde{\Delta}_2$, then the following inequality holds for all beliefs $\alpha_k \leq \alpha^*$:

$$[1 - \delta(1 - \alpha_k \gamma \Delta)] \bar{s} - [1 - \delta(1 - \gamma \Delta)] \alpha_k \gamma \bar{r} < \delta(1 - \alpha_k \gamma \Delta)(n - 2)[\bar{s} - \alpha_k \gamma \bar{r}].$$

Henceforth, we assume that $\Delta < \bar{\Delta}_2 \equiv \min\{\bar{\Delta}_1, \tilde{\Delta}_2\}$.

To prove the lemma, one can use an equilibrium construction that parallels that in the proof of Lemma B1. In this equilibrium, when the status quo is of the form $p = (a, \tau', y)$ and the belief is $\alpha \leq \alpha_k$, committee member i 's reward payoff is $V^*(\alpha) - \sum_{j \neq i} w_j(p \mid \alpha) = \bar{s} \Delta - \sum_{j \neq i} \alpha \gamma \Delta [(1 - \tau') r_j + \tau' y_j \bar{s}]$ and her punishment payoff is $w_i(p \mid \alpha) = \alpha \gamma \Delta [(1 - \tau') r_i + \tau' y_i \bar{s}]$. If the belief becomes equal to one, then the equilibrium described in Lemma B1 is played. The argument is then exactly the same. In particular, the key condition (B2), necessary for proposers to have profitable deviations, now becomes

$$[1 - \hat{\delta}_\ell(1 - \gamma \Delta)] \alpha \gamma \Delta \bar{r} + \hat{\delta}_\ell(1 - \alpha \gamma \Delta) [V^*(\alpha) + (n - 2)(V^*(\alpha) - \alpha \gamma \Delta \bar{r})] < V^*(\alpha)$$

or, equivalently,

$$\hat{\delta}_\ell(1 - \alpha_k \gamma \Delta)(n - 2)[\bar{s} - \alpha \gamma \bar{r}] < [1 - \hat{\delta}_\ell(1 - \alpha \gamma \Delta)] \bar{s} - [1 - \hat{\delta}_\ell(1 - \gamma \Delta)] \alpha \gamma \bar{r}.$$

This cannot hold since $\Delta < \bar{\Delta}_2$. □

Lemma B3. *Suppose \mathcal{D} is unanimity rule and $\hat{\tau} = 1$. Then there exists $\bar{\Delta}_2 > 0$ such that, for all $\Delta < \bar{\Delta}_2$, all beliefs $\alpha_k \in [\alpha_1, \alpha^*)$ and all $(\tau, x) \in [0, 1] \times X$:*

(i) $\Gamma(a, \tau, x \mid \alpha_k)$, $a \in \{R, S\}$, has a renegotiation-proof equilibrium that sustains the optimal stopping rule; and

(ii) $\Gamma(R, 1, x \mid \alpha_k)$ has a renegotiation-proof equilibrium in which each committee member i 's expected payoff is $x_i V^*(\alpha_k)$.

Proof. Let $\bar{\Delta}_2$ be defined as in Lemma B2. To prove Lemma B3, consider first the simple variant on the standard Baron-Ferejohn model, denoted $\mathbf{G}(a, \tau, x \mid \alpha_k)$, in which the policy space is not the unit simplex but $\{(w_1, \dots, w_n) \in \mathbb{R}^n : \sum_{i=1}^n w_i \leq V^*(\alpha_k) \text{ and } w_i \geq w_i(S, \tau, x \mid \alpha_k), \forall i \in N\}$, and the probability that committee member i is selected to propose is equal to the probability q_i that she is the last proposer in our model. It is well known that this game has a (pure strategy) stationary subgame perfect equilibrium, in which the selected proposer makes the same (successful) proposal, $w^i(S, \tau, x \mid \alpha_k)$, in every period.

Let $k^* \in \mathbb{N}$ be implicitly defined by $\alpha_{k^*} = \alpha^*$, and let $(\tau, x) \in [0, 1] \times X$. Consider a strategy profile σ^{k^*-1} for $\Gamma(R, \tau, x \mid \alpha_{k^*-1})$ that prescribes the following behavior in any period that begins with a status quo $p = (a, \tau', y)$, a belief $\alpha \in \{\alpha_k \in A : k \geq k^* - 1\} \cup \{1\}$, and an order of proposers (π_1, \dots, π_n) :

a) If $\alpha = \alpha_{k^*-1}$ and $a = R$, then all proposers pass (irrespective of the previous history of play);

b) if $\alpha = \alpha_{k^*-1}$, $a = R$, and some proposer has offered a policy $p' = (a', \tau'', z) \neq p$, then voter i votes to accept p' if and only if

$$\frac{w_i(p \mid \alpha_{k^*-1})}{\alpha_{k^*-1} \gamma \Delta \bar{r}} V^*(\alpha_{k^*-1}) < \begin{cases} \frac{w_i(p' \mid \alpha_{k^*-1})}{\alpha_{k^*-1} \gamma \Delta \bar{r}} V^*(\alpha_{k^*-1}) & \text{if } a' = R, \\ (1 - \delta) s_i \Delta + \delta V^*(\alpha_{k^*-1}) \sum_{j=1}^n q_j w_i^j(p' \mid \alpha_{k^*-1}) & \text{if } a' = S; \end{cases}$$

c) if $\alpha = \alpha_{k^*-1}$ and $a = S$, then each proposer π_ℓ , $\ell < n$, passes and proposer π_n offers policy $(R, 1, y^{\pi_n}(S, \tau', y \mid \alpha_{k^*-1}))$;

d) if $\alpha = \alpha_{k^*-1}$, $a = S$, and some proposer has offered a policy $p' = (a', \tau'', z) \neq p$, then voter i makes the same decision as when she is offered the following policy in the stationary subgame perfect equilibrium of $\mathbf{G}(S, \tau', y \mid \alpha_{k^*-1})$:

$$\begin{cases} \frac{w_i(p' \mid \alpha_{k^*-1})}{\alpha_{k^*-1} \gamma \Delta \bar{r}} V^*(\alpha_{k^*-1}) & \text{if } a' = R, \\ (1 - \delta) s_i \Delta + \delta V^*(\alpha_{k^*-1}) \sum_{j=1}^n q_j w_i^j(p' \mid \alpha_{k^*-1}) & \text{if } a' = S; \end{cases}$$

e) if $\alpha \leq \alpha^*$ and $a = R$, then the committee plays an equilibrium of $\Gamma(R, \tau', y | \alpha)$ in which each committee member i 's payoff is $\frac{w_i(p|\alpha)}{\alpha\gamma\Delta\bar{r}}V^*(\alpha)$ (see Lemma B2(ii)); if $\alpha \leq \alpha^*$ and $a = S$, then the committee plays an equilibrium of $\Gamma(R, \tau', y | \alpha)$ in which each committee member i 's payoff is $w_i(p | \alpha)$ (see Lemma B2(i));

d) if $\alpha = 1$ and $a = R$, then the committee plays an equilibrium of $\Gamma(R, \tau', y | 1)$ in which each committee member i 's payoff is $w_i(p | 1)$ (see Lemma B1(i)); if $\alpha = 1$ and $a = S$, then the committee plays an equilibrium of $\Gamma(S, \tau', y | 1)$ in which each committee member i 's payoff is $\frac{w_i(p|1)}{\bar{s}\Delta}$ (see Lemma B1(ii)).

It is readily checked that σ^{k^*-1} is an equilibrium for $\Gamma(R, \tau, x | \alpha_{k^*-1})$. In particular, the acceptance condition in case b) compares the voter's continuation value from rejecting the proposal (left side of the inequality) with her continuation value from accepting it (right side). As the optimal stopping rule is implemented in case of rejection (and the voting rule is unanimity), at least one voter must voter to reject the proposal. It follows that any proposal is unsuccessful in case a) and, therefore, passing is optimal for all proposers. In the other cases, any deviation is by construction unprofitable. Moreover, as the optimal stopping rule is implemented in every continuation game both on and off the equilibrium path, σ^{k^*-1} is a renegotiation-proof equilibrium. As $\Gamma(S, \tau, x | \alpha_{k^*-1})$ is a continuation game of $\Gamma(R, \tau, x | \alpha_{k^*-1})$, the restriction of σ^{k^*-1} to $\Gamma(S, \tau, x | \alpha_{k^*-1})$ is also a renegotiation-proof equilibrium. To complete the proof of the lemma for the case where $k = k^* - 1$, observe that if $\tau = 1$, then each committee member i 's payoff in equilibrium σ^{k^*-1} is $x_i V^*(\alpha_{k^*-1})$.

To obtain the result for any $k \in \{1, \dots, k^*-1\}$, one can then proceed recursively: having obtained an equilibrium σ^{k+1} for every continuation game of the form $\Gamma(a, \tau', y | \alpha_{k+1})$, one can apply the same construction as above at belief α_k to obtain a renegotiation-proof equilibrium σ^k for every game $\Gamma(a, \tau, x | \alpha_k)$. \square

Lemma B4. *Suppose \mathcal{D} is unanimity rule and $\hat{\tau} = 1$. Then there exists $\bar{\Delta} > 0$ such that, for all $\Delta < \bar{\Delta}$, the set of renegotiation-proof equilibrium payoffs for $\Gamma(S, 0, x^0 | \alpha_0)$ is the simplex $\{(w_1, \dots, w_n) \in \mathbb{R}^n : \sum_{i=1}^n w_i = V^*(\alpha_0) \text{ and } w_i \geq s_i \Delta, \forall i \in N\}$.*

Proof. It follows from Assumption A1 that, for sufficiently small Δ , $V^*(\alpha_0)/\Delta > \bar{s}$. Therefore, the threshold

$$\tilde{\Delta}_3 \equiv \{\Delta > 0: \delta(n-2)[(V^*(\alpha_0)/\Delta) - \bar{s}] < (1-\delta)[(V^*(\alpha_0)/\Delta) - \bar{s}]\}$$

is well-defined and positive. Henceforth, we assume that $\Delta < \bar{\Delta} \equiv \min\{\bar{\Delta}_2, \tilde{\Delta}_3\}$.

To prove the lemma, one can use again an equilibrium construction that parallels that in the proof of Lemma B1. In this equilibrium, when the status quo is of the form $p = (S, \tau', y)$ and the belief is α_0 , committee member i 's reward payoff is $V^*(\alpha_0) - \sum_{j \neq i} w_j(p | \alpha_0) = \bar{s}\Delta - \sum_{j \neq i} s_j\Delta$ and her punishment payoff is $w_i(p | \alpha_0) = s_i\Delta$. If the belief becomes equal to α_1 , then an equilibrium as described in Lemma B3 is played — in particular, the equilibrium described in Lemma B3(ii) if the status quo is of the form $(R, 1, x)$ for some $x \in X$ — and if the belief becomes equal to one, then the equilibrium described in Lemma B1 is played. The argument is then exactly the same. In particular, the key condition (B2), necessary for proposers to have profitable deviations, is now

$$(1 - \hat{\delta}_\ell)\bar{s}\Delta + \hat{\delta}_\ell[V^*(\alpha_0) + (n-2)(V^*(\alpha_0) - \bar{s}\Delta)] < V^*(\alpha_0)$$

or, equivalently,

$$\hat{\delta}_\ell(n-2) \left[\frac{V^*(\alpha_0)}{\Delta} - \bar{s} \right] < (1 - \hat{\delta}_\ell) \left[\frac{V^*(\alpha_0)}{\Delta} - \bar{s} \right].$$

As $\Delta < \bar{\Delta}$, this inequality cannot hold. \square

Let $\Delta < \bar{\Delta}$, where $\bar{\Delta} > 0$ is the threshold defined in Lemma B4. To complete the proof for the unanimity case, observe that in any equilibrium, each committee member i 's expected payoff must be greater than or equal to $s_i\Delta$; otherwise, i could profitably deviate by rejecting all proposals in every period. Hence, the set of equilibrium payoff vectors is a subset of $\{(w_1, \dots, w_n) \in \mathbb{R}^n: \sum_{i=1}^n w_i \leq V^*(\alpha_0) \text{ and } w_i \geq s_i\Delta, \forall i \in N\}$. It follows from Lemma B4 that any equilibrium that fails to support the optimal stopping rule is Pareto dominated by some renegotiation-proof equilibrium. Therefore, the set of renegotiation-proof equilibrium payoff vectors is $\{(w_1, \dots, w_n) \in \mathbb{R}^n: \sum_{i=1}^n w_i = V^*(\alpha_0) \text{ and } w_i \geq s_i\Delta, \forall i \in N\}$ (Lemma B4).

(ii) **Other collegial rules.** Suppose first that $\emptyset \neq V \equiv \bigcap \mathcal{D} \notin \mathcal{D}$. Let $m = |N \setminus V|$, and let $\bar{\Delta} \equiv \sup\{\Delta \in \mathbb{R}_+ : 2(1 - e^{-\rho\Delta}) < e^{-\rho\Delta}/m\} > 0$. To establish the result in this case, we will use an analogous argument to that used for noncollegial rules: we will show that every payoff vector in $W \equiv \{(w_1, \dots, w_n) \in \mathbb{R}^n : \sum_{i=1}^n w_i = V^*(\alpha_0), w_i \geq s_i\Delta \forall i \in V, \text{ and } w_i \geq 0 \forall i \notin V\}$ can be supported in a renegotiation-proof equilibrium. As the set of equilibrium payoff vectors must be contained in $\{(w_1, \dots, w_n) \in \mathbb{R}^n : \sum_{i=1}^n w_i \leq V^*(\alpha_0), w_i \geq s_i\Delta \forall i \in V, \text{ and } w_i \geq 0 \forall i \notin V\}$, this implies that the set of renegotiation-proof equilibrium vectors is W .

Take an arbitrary $w^0 \in W$, and let $y^0 \in X$ be defined by $y_i^0 \equiv w_i^0/V^*(\alpha_0)$ for all $i \in N$. Our objective is to construct an equilibrium σ , in which: (i) the committee implements the optimal stopping rule in every continuation game (so that σ is renegotiation-proof); (ii) on the path, aggregate revenues are distributed according to y^0 in every period. To this end, we first define revenue distributions $y^i(p | \alpha) \in X$, for all $i \in N$ and $p \in \{R, S\} \times [0, 1] \times X$, as follows: Let $S(p | \alpha) \equiv V^*(\alpha) - \sum_{k \in V} w_k(p | \alpha)$; and let

$$y_j^i(p | \alpha) \equiv \begin{cases} \frac{w_i(p|\alpha)}{V^*(\alpha)} & \text{if } j \in V, \\ 0 & \text{if } j = i \ \& \ j \notin V, \\ \frac{1}{m}S(p | \alpha) & \text{if } j \neq i \in V \ \& \ j \notin V, \\ \frac{1}{m-1}S(p | \alpha) & \text{if } j \neq i \notin V \ \& \ j \notin V, \end{cases}$$

for all $j \in N$. As the optimal stopping rule is implemented in every continuation game both on and off the path, such an equilibrium must be renegotiation-proof.

We define the strategy profile σ in terms of “phases,” formally represented by pairs in $\{1, \dots, n\} \times \{0, 1, \dots, n\}$. Every phase (ℓ, i) prescribes behavior in the ℓ th proposal stage of any given period and in the n voting stages that follow it: “ i ” indicates that σ prescribes policy $(a^*(\alpha), 1, y^i)$ to be implemented. Specifically, in any period in where the status quo is p , the belief is $\alpha \in A$ and the order of proposers is (π_1, \dots, π_n) , if the game is in phase $(\ell, i) \neq (n, \pi_n)$, then σ prescribes the following behavior:

(P1) proposer π_ℓ offers policy $(a^*(\alpha), 1, y^i)$;

(V1.a) if π_ℓ offered $(a^*(\alpha), 1, y^i)$ where $i \notin V$, then every voter $j \neq i$ accepts it, and voter i accepts it if and only if one of the following conditions hold: she is the first voter;

or all the previous voters accepted $(a^*(\alpha), 1, y^i)$; or some of the previous voters rejected it and

$$y_i^i(p | \alpha)V^*(\alpha) \geq \begin{cases} (1 - \delta)w_i(p | \alpha) + \delta\mathbb{E}[y_i^k(p | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] & \text{if } \ell = n, \\ & \text{or } \ell = n - 1 \text{ \& } i = \pi_n, \\ y_i^k(p | \alpha)V^*(\alpha) & \text{otherwise,} \end{cases}$$

where k is the last of the previous voters who rejected it.

(V1.b) if π_ℓ offered $(a^*(\alpha), 1, y^i)$ where $i \in V$, then all voters accept it.

(V2a) if π_ℓ offered any $p' \neq (a^*(\alpha), 1, y^i)$ in $P(p | \alpha) \equiv \{p'' \in \{R, S\} \times [0, 1] \times X : w_j(p'' | \alpha) \leq w_j(p | \alpha), \forall j \in V\}$, then each voter j acts as follows:

- If she is a vetoer and $w_j(p' | \alpha) \leq w_j(p | \alpha)$, then she rejects p' (irrespective of the previous voters' choices);
- otherwise, she accepts p' if and only if $(1 - \delta)w_j(p' | \alpha) + \delta\mathbb{E}[y_j^k(p' | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p]$ is greater than

$$\begin{cases} (1 - \delta)w_j(p' | \alpha) + \delta\mathbb{E}[y_j^k(p' | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] & \text{if } \ell = n - 1 \text{ \& } k = \pi_n, \text{ or } \ell = n, \\ y_j^k(p' | \alpha)V^*(\alpha) & \text{otherwise,} \end{cases}$$

where k is the last of the vetoers in $V^r \equiv \{i \in V : w_j(p' | \alpha) \leq w_j(p | \alpha)\}$ in the sequence of voters.

(V2b) if π_ℓ offered any $p' \neq (a^*(\alpha), 1, y^i)$ outside $P(p | \alpha)$, then each voter j acts as follows:

- If she is a vetoer, then she accepts p' if and only if $w_j(p' | \alpha) \geq w_j(p | \alpha)$;
- if she is not a vetoer, then she rejects p' .

If the game is in phase (n, π_n) , then σ prescribes the following behavior: (i) proposer π_n passes; and (ii) if π_n proposed some policy $p' \neq p$, then j behaves as in case (V2a) if $p' \in P(p | \alpha)$, and as in case (V2b) otherwise.

Phases evolve according to the following recursive rules. In period 1, play begins in phase $(1, 0)$. Then in every period, at the end of any sequence of votes that began in any phase $(\ell, i) \neq (n, \pi_n)$:¹

¹We set $\ell + 1 = 1$ whenever $\ell = n$.

(t1.a) If policy $(a^*(\alpha), 1, y^i)$, where $i \notin V$, was proposed and accepted by all voters but i , then the game transitions to phase $(1, i)$;

(t1.b) If policy $(a^*(\alpha), 1, y^i)$, where $i \in V$, was proposed and accepted by all voters, then the game transitions to phase $(1, i)$;

(t2.a) if policy $(a^*(\alpha), 1, y^i)$, where $i \notin V$, was accepted but some voters different from i rejected it, then the game transitions to phase $(1, k)$, where k is the last of those voters;

(t2.b) if policy $(a^*(\alpha), 1, y^i)$, where $i \in V$, was accepted but some voters rejected it, then the game transitions to phase $(1, k)$, where k is the last of those voters;

(t3.a) if policy $(a^*(\alpha), 1, y^i)$, where $i \notin V$, was proposed and rejected, then the game transitions to phase $(\ell + 1, k)$, where k is the last voter different from i who rejected $(a^*(\alpha), 1, y^i)$;

(t3.b) if policy $(a^*(\alpha), 1, y^i)$, where $i \in V$, was proposed and rejected, then the game transitions to phase $(\ell + 1, k)$, where k is the last voter who rejected $(a^*(\alpha), 1, y^i)$;

(t4) if the status quo differs from $(a^*(\alpha), 1, y^i)$ and proposer π_ℓ passes, then the game moves to phase $(\ell + 1, \pi_\ell)$;

(t5) if proposer π_ℓ offered a policy $p' \neq (a^*(\alpha), 1, y^i)$ in $P(p | \alpha)$ and her proposal was rejected by all vetoers j such that $w_j(p' | \alpha) \leq w_j(p | \alpha)$, then the game transitions to phase $(\ell + 1, \pi_\ell)$;

(t6) if proposer π_ℓ offered a policy $p' \neq (a^*(\alpha), 1, y^i)$ in $P(p | \alpha)$ and her proposal was rejected by the committee, but accepted by some vetoers j such that $w_j(p' | \alpha) \leq w_j(p | \alpha)$, then the game transitions to phase $(\ell + 1, k)$, where k is the last of those vetoers who accepted p' ;

(t7) if proposer π_ℓ offered a policy $p' \neq (a^*(\alpha), 1, y^i)$ in $P(p | \alpha)$ and her proposal was accepted by the committee, then the game transitions to phase $(1, k)$, where k is the last vetoer j such that $w_j(p' | \alpha) \leq w_j(p | \alpha)$;

(t8) if proposer π_ℓ offered a policy $p' \neq (a^*(\alpha), 1, y^i)$ outside $P(p | \alpha)$ and her proposal was rejected by all voters in $N \setminus V$, then the game transitions to phase $(\ell + 1, \pi_\ell)$;

(t9) if proposer π_ℓ offered a policy $p' \neq (a^*(\alpha), 1, y^i)$ outside $P(p | \alpha)$ and her proposal was rejected by the committee, but accepted by some voters in $N \setminus V$, then the game

transitions to phase $(\ell + 1, k)$, where k is the last of the voters in $N \setminus V$ who accepted p' ;

(t10) if proposer π_ℓ offered a policy $p' \neq (a^*(\alpha), 1, y^i)$ outside $P(p | \alpha)$ and her proposal was accepted by the committee, then the game transitions to phase $(1, k)$, where k is the last of the voters in $N \setminus V$ who accepted p' ;

At the end of any sequence of votes that began in phase (n, π_n) , we have the following transitions that parallel cases (t5)-(t10) above:

(t11) If the proposer π_n passed, then the game transitions to phase $(1, \pi_n)$;

(t12) if proposer π_ℓ offered a policy $p' \neq p$ in $P(p | \alpha)$ and her proposal was rejected by all vetoers j such that $w_j(p' | \alpha) \leq w_j(p | \alpha)$, then the game transitions to phase $(1, \pi_n)$;

(t13) if proposer π_ℓ offered a policy $p' \neq p$ in $P(p | \alpha)$ and her proposal was rejected by the committee, but accepted by some vetoers j such that $w_j(p' | \alpha) \leq w_j(p | \alpha)$, then the game transitions to phase $(1, k)$, where k is the last of those vetoers who accepted p' ;

(t14) if proposer π_ℓ offered a policy $p' \neq p$ in $P(p | \alpha)$ and her proposal was accepted by the committee, then the game transitions to phase $(1, k)$, where k is the last vetoer j such that $w_j(p' | \alpha) \leq w_j(p | \alpha)$;

(t15) if proposer π_ℓ offered a policy $p' \neq p$ outside $P(p | \alpha)$ and her proposal was rejected by all voters in $N \setminus V$, then the game transitions to phase $(1, \pi_\ell)$;

(t16) if proposer π_ℓ offered a policy $p' \neq p$ outside $P(p | \alpha)$ and her proposal was rejected by the committee, but accepted by some voters in $N \setminus V$, then the game transitions to phase $(1, k)$, where k is the last of the voters in $N \setminus V$ who accepted p' ;

(t17) if proposer π_ℓ offered a policy $p' \neq p$ outside $P(p | \alpha)$ and her proposal was accepted by the committee, then the game transitions to phase $(1, k)$, where k is the last of the voters in $N \setminus V$ who accepted p' ;

We now verify that for $\Delta < \bar{\Delta}$, this strategy profile is an equilibrium. We begin with committee member j 's voting behavior. Consider in any period in where the status quo is p , the belief is $\alpha \in A$ and the order of proposers is (π_1, \dots, π_n) . There are several cases:

- *Case 1: In phase $(\ell, i) \neq (n, \pi_n)$, π_ℓ has proposed $(a^*(\alpha), 1, y^i)$.* Observe first that it follows from the definition of σ that voter j 's continuation value at the start of any phase

$(\hat{\ell}, \hat{i})$ is

$$\begin{cases} y_j^{\hat{i}}(p | \alpha)V^*(\alpha) & \text{if } \ell < n , \\ (1 - \delta)w_j(p | \alpha) + \delta\mathbb{E}[y_j^{\hat{i}}(p | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] & \text{if } \ell = n . \end{cases}$$

We assume that $i \notin V$; the case where $i \in V$ is analogous — just replace “all the previous voters but i ” and “the previous voters different from i ” by “all the previous voters.” We consider several cases in turn:

(1.a) $i = 0$ (so that $\ell = 1$ and $\alpha = \alpha_0$).

(1.a.i) $j \neq i$. Voter j is better off accepting $(R, 1, y^0)$, as prescribed. Indeed, if she is the first voter, or if all the previous voters but i have accepted $(R, 1, y^0)$, then she receives $y_i^0 V^*(\alpha_0)$ if she accepts; while if she rejects, then the game moves to phase $(2, j)$ and she receives $y_j^j(p | \alpha_0)V^*(\alpha_0)$; and, by construction $y_i^0 \geq y_j^j(p | \alpha_0)$. If some of the previous voters different from i have rejected $(R, 1, y^0)$, then she receives $y_j^j(p | \alpha_0)V^*(\alpha_0)$ if she also rejects it, and $y_j^k(p | \alpha_0)V^*(\alpha_0) > y_j^j(p | \alpha_0)V^*(\alpha_0)$ if she instead accepts it, where k is the last of the previous voters different from i who rejected it.

(1.a.ii) $j = i$. If voter i is the first voter, or if all the previous voters have accepted $(R, 1, y^0)$, then her choice does not affect her payoff. If some of the previous voters rejected $(R, 1, y^0)$, then her decision can only affect her payoff if she is pivotal. In the latter case, her voting strategy prescribes her to accept if and only if her continuation value from accepting is greater than or equal to her continuation value from rejecting. Hence, she cannot profitably deviate from σ .

(1.b) $i \neq 0$.

(1.b.i) $j \neq i$. There are several cases:

- $\ell < n - 1$, or $\ell = n - 1$ and $j \neq \pi_n$. In this case, the same argument as in (1.a) shows that voter j is better off accepting $(a^*(\alpha), 1, y^i(p | \alpha))$, as prescribed by σ .
- $\ell = n - 1$ and $j = \pi_n$. If j is the first voter, or if all the previous voters different from i have accepted $(a^*(\alpha), 1, y^i(p | \alpha))$, then her payoff is $y_j^i V^*(\alpha)$ if she also accepts it, and $(1 - \delta)w_j(p | \alpha) + \delta\mathbb{E}[y_j^j(p | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p]$ if she rejects it. We have $y_j^i V^*(\alpha) =$

$(1 - \delta)w_j(p | \alpha) + \delta\mathbb{E}[y_j^j(p | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] = w_j(p | \alpha)$ when j is a vetoer, and

$$\begin{aligned} y_j^i V^*(\alpha) &\geq \frac{1}{m}S(p | \alpha) > (1 - \delta)S(p | \alpha) \geq (1 - \delta)w_j(p | \alpha) \\ &= (1 - \delta)w_j(p | \alpha) + \delta\mathbb{E}[y_j^j(p | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] \end{aligned}$$

when she is not a vetoer. (The strict inequality follows from $\Delta < \bar{\Delta}$.) Hence, j cannot profitably deviate from accepting the proposal (as prescribed by σ) in this case.

Now suppose that some of the previous voters different from i have rejected $(a^*(\alpha), 1, y^i(p | \alpha))$. If j 's choice is not pivotal, then she is better off accepting the proposal (as prescribed by σ) in order to ensure a transition to a phase where she will receive her largest continuation payoff. If j 's choice is pivotal and she is a vetoer, then she is indifferent between accepting and rejecting: in both cases, some committee member k (possibly equal to j) will be "punished" and she will receive $w_j(p | \alpha)$. If j 's choice is pivotal and she is not a vetoer, then she has two options:

◦ If she votes to accept $p^* \equiv (a^*(\alpha), 1, y^i(p | \alpha))$ (so that it is accepted by the committee), then p^* is implemented and the game transitions to phase $(1, k)$, where $k \neq j$ is the last voter different from i who voted to reject p^* . In this case, j receives a payoff of

$$(1 - \delta)w_j(p^* | \alpha) + \delta\mathbb{E}[y_j^k(p^* | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p^*] = \frac{1}{\tilde{m}}S(p^* | \alpha) = \frac{1}{\tilde{m}}S(p | \alpha) \geq \frac{1}{m}S(p | \alpha),$$

where the first equality follows from $y_j^i(p^* | \alpha) = y_j^k(p^* | \alpha)$ (since $j \notin \{i, k\}$), and the second from the fact that $w_\ell(p^* | \alpha) = w_\ell(p | \alpha)$ for all $\ell \in V$.

◦ If she votes to reject $p^* \equiv (a^*(\alpha), 1, y^i(p | \alpha))$ (so that it is rejected by the committee), then the game moves to phase (n, π_n) , in which she will first pass as a proposer and will then receive her "punishment payoff." That is, she obtains

$$(1 - \delta)w_j(p | \alpha) + \delta\mathbb{E}[y_j^j(p | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] \leq (1 - \delta)S(p | \alpha) < \frac{1}{m}S(p | \alpha),$$

where the first inequality follows from the fact that $y_j^j(p | \tilde{\alpha}) < 1$ for all $\tilde{\alpha} \in A$, and the second from $\Delta < \bar{\Delta}$.

We conclude that voter j is better off accepting the proposal, as prescribed from σ .

• $\ell = n$ (so that $i \neq \pi_n$). The argument is exactly the same as in the case where $\ell = n - 1$ and $j = \pi_n$. (In particular, as in that case, if the proposal is rejected both by j and

by the committee, then the status quo policy p is implemented and j receives her lowest continuation value from the next period on.)

(1.b.i) $j = i$. The argument is exactly the same as in case (1.a.ii).

• *Case 2: In phase $(\ell, i) \neq (n, \pi_n)$, π_ℓ has proposed a policy $p' \neq (a^*(\alpha), 1, y^i)$ in $P(p | \alpha)$.*

(2.a) j is a vetoer and $w_j(p' | \alpha) \leq w_j(p | \alpha)$. Suppose first that, given the previous voters' choices and the remaining voters' strategies, voter j 's decision is not pivotal. In this case, her choice only affects the transition to the next phase. It follows from the transition rules (t5)-(t7) that she is always (weakly) better off rejecting p' , as prescribed by σ . Now suppose that her decision is pivotal. If she rejects p' then, from (t5)-(t7), the game transitions to phase $(\ell + 1, k)$ for some $k \neq j$, and she receives a payoff of $(1 - \delta)w_j(p | \alpha) + \delta\mathbb{E}[y_j^k(p | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] = y_j^k(p | \alpha)V^*(\alpha) = w_j(p | \alpha)$. If instead she deviates, then the game transitions to phase $(\ell + 1, j)$ (see (7)), and she receives $(1 - \delta)w_j(p' | \alpha) + \delta\mathbb{E}[y_j^j(p' | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] = w_j(p' | \alpha)$. As $w_j(p' | \alpha) \leq w_j(p | \alpha)$, she is better off rejecting.

(2.b) *Either j is not a vetoer or $w_j(p' | \alpha) > w_j(p | \alpha)$.* If, given the previous voters' choices and the remaining voters' strategies, voter j 's decision is not pivotal then, as above, any deviation from σ is unprofitable. If j 's vote is pivotal given the previous voters' moves and the remaining voters' strategies, then it must be the case that all the vetoers in V^r have already voted and they all chose to accept p' . Let k be the last member of V^r who moved. If j chooses to accept p' , then her payoff will be $(1 - \delta)w_j(p' | \alpha) + \delta\mathbb{E}[y_j^k(p' | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p]$; if she chooses to reject p' , then her payoff will be

$$\begin{cases} (1 - \delta)w_j(p | \alpha) + \delta\mathbb{E}[y_j^k(p | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] & \text{if } \ell = n - 1 \text{ \& } k = \pi_n, \text{ or } \ell = n, \\ y_j^k(p | \alpha)V^*(\alpha) & \text{otherwise.} \end{cases}$$

It follows that j cannot profitably deviate from σ .

• *Case 3: In phase $(\ell, i) \neq (n, \pi_n)$, π_ℓ has proposed a policy $p' \neq (a^*(\alpha), 1, y^i)$ outside $P(p | \alpha)$.*

(3a) j is a vetoer. By the same logic as above, she cannot profitably deviate from σ if she is not pivotal (given the previous voters' choices and the remaining voters' strategies).

Suppose she is pivotal. If she chooses to accept p' , then she receives $(1 - \delta)w_j(p' | \alpha) + \delta\mathbb{E}[y_j^k(p' | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] = w_j(p' | \alpha)$, where k is the last of the previous voters who accepted p' — there must be such voter, otherwise j would not be pivotal. If she chooses to reject p' , then she receives $w_j(p | \alpha)$: if $\ell = n - 1$ and $k = \pi_n$, or if $\ell = n$, then she gets $(1 - \delta)w_j(p | \alpha) + \delta\mathbb{E}[y_j^k(p | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] = w_j(p | \alpha)$; otherwise, she gets $y_j^k(p | \alpha)V^*(\alpha) | \alpha, p] = w_j(p | \alpha)$. It follows that she cannot profitably deviate from σ .

(3b) j is not a vetoer. By the same logic as in case (2.a) above, voter j cannot profitably deviate from σ if she is not pivotal (given the previous voters' choices and the remaining voters' strategies). If she is pivotal, then there are several cases:

(3.b.i) $\ell < n - 1$, or $\ell = n - 1$ and $j \neq \pi_n$. If j accepts the proposal, then she receives $(1 - \delta)w_j(p' | \alpha) + \delta\mathbb{E}[y_j^j(p' | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] = (1 - \delta)w_j(p' | \alpha) \leq (1 - \delta)S(p' | \alpha) < (1 - \delta)S(p | \alpha)$, where the equality follows from the fact that $y_j^j(p' | \tilde{\alpha}) = 0$ for all $\tilde{\alpha} \in A$, and the second inequality follows from $p' \notin P(p | \alpha)$ (and, therefore, $\sum_{l \in V} w_l(p' | \alpha) > \sum_{l \in V} w_l(p | \alpha)$). If she rejects p' (as prescribed by σ), then she receives $y_j^k(p | \alpha)V^*(\alpha) = \frac{1}{m}S(p | \alpha) \geq \frac{1}{m}S(p | \alpha) > (1 - \delta)S(p | \alpha)$, where the last inequality follows from $\Delta < \bar{\Delta}$.

(3.b.ii) $\ell = n - 1$ and $j = \pi_n$, or $\ell = n$. If j accepts the proposal then, by the same logic as above, she receives a payoff that is smaller than $(1 - \delta)S(p | \alpha)$. If she rejects p' (as prescribed by σ), then she receives $(1 - \delta)w_j(p | \alpha) + \delta\mathbb{E}[y_j^k(p | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p] = (1 - \delta)w_j(p | \alpha) + \delta[\frac{1}{m}S(p | \alpha) - (1 - \delta)w_j(a^*(\alpha), 1, y_j^k(p | \alpha) | \alpha)] \geq \delta[\frac{1}{m}S(p | \alpha) - (1 - \delta)S(p | \alpha)] > (1 - \delta)S(p | \alpha)$, where the last inequality follows from $\Delta < \bar{\Delta}$.

• *Case 4: In phase (n, π_n) , π_n has proposed a policy $p' \neq p$.* One can show that voter j cannot profitably deviate from σ by using the same arguments as in Cases 2 and 3.

We now turn to committee member i 's proposal behavior. Consider in any period in which the status quo is p , the belief is $\alpha \in A$ and the order of proposers is (π_1, \dots, π_n) . In phase $(\ell, i) \neq (n, \pi_n)$, σ prescribes her to propose $(a^*(\alpha), 1, y^i)$, thus receiving a payoff of $y_j^i(p | \alpha)V^*(\alpha)$. Suppose that either $\ell < n - 1$, or $\ell = n - 1$ and $i \neq \pi_n$. If j deviates by making any proposal $p' \neq (a^*(\alpha), 1, y^i)$, then her proposal is rejected by the committee, the game transitions to phase $(\ell + 1, j)$, and she receives $y_j^j(p | \alpha)V^*(\alpha) \leq y_j^i(p | \alpha)V^*(\alpha)$. The deviation is therefore unprofitable. Now suppose that either $\ell = n - 1$ and $i = \pi_n$, or $\ell = n$

(so that $j \neq i$). In this case, if j deviates by making any proposal $p' \neq (a^*(\alpha), 1, y^i)$, then her proposal is rejected, the status quo policy p is implemented, and the game transitions to the next period in phase $(1, j)$. Hence, her payoff is equal to $w^d \equiv (1 - \delta)w_j(p | \alpha) + \delta \mathbb{E}[y_j^j(p | \tilde{\alpha})V^*(\tilde{\alpha}) | \alpha, p]$. If j is a vetoer, then $w^d = w_j(p | \alpha) \leq y_j^i(p | \alpha)V^*(\alpha)$ (which holds with equality whenever $i \neq 0$), and the deviation is therefore unprofitable. If j is not a vetoer, then

$$w^d = (1 - \delta)w_j(p | \alpha) \leq (1 - \delta)S(p | \alpha) \leq \frac{1}{\tilde{m}}S(p | \alpha) = y_j^i(p | \alpha)V^*(\alpha) ,$$

where the second inequality follows from $\Delta < \bar{\Delta}$, and the last equality from $i \neq j \notin V$.

Finally, in phase (n, π_n) , σ prescribes proposer $j = \pi_n$ to pass. If she does so, then the status quo policy p is implemented and the period starts in phase $(1, \pi_n)$ (see (t11)). If she deviates, proposing any policy $p' \neq p$, then her proposal is rejected by the committee: if $p' \in P(p | \alpha)$, then it is rejected by at least one vetoer (see (V2a)); if $p' \notin P(p | \alpha)$, then it is rejected by all the voters without a veto (see V2b). Therefore, the status quo policy is implemented and, from (t12) and (t15), the next period starts in phase $(1, \pi_n)$. It follows that the proposer receive the same payoff irrespective of her choice and, consequently, cannot profitably deviate from σ .

We now turn to the case where $\emptyset \neq V \equiv \bigcap \mathcal{D} \in \mathcal{D}$, i.e., the voting rule is oligarchic. If $|V| = 1$, then the result is trivial: in every continuation game, the dictator redistributes all revenues to herself and, therefore, has the same objective function as the social planner. If $|V| \geq 3$, then one can use the same equilibrium constructions as above — or as in the $V = N$ case (section B in the main text) — to establish that $\{(w_1, \dots, w_n) \in \mathbb{R}_+^n : \sum_{i=1}^n w_i = V^*(\alpha_0) \text{ and } w_i \geq s_i \Delta, \forall i \in V\}$ is a subset of renegotiation-proof equilibrium payoff vectors. As the set of equilibrium payoff vectors must be a subset of $\{(w_1, \dots, w_n) \in \mathbb{R}_+^n : \sum_{i=1}^n w_i \leq V^*(\alpha_0) \text{ and } w_i \geq s_i \Delta, \forall i \in V\}$, this proves that the set of renegotiation-proof equilibrium vectors is $\{(w_1, \dots, w_n) \in \mathbb{R}_+^n : \sum_{i=1}^n w_i = V^*(\alpha_0) \text{ and } w_i \geq s_i \Delta, \forall i \in V\}$.

To complete the proof of the result for oligarchic rules, it remains to show that the same is true if $|V| = 2$. In the previous cases, we could sustain equilibria in which a vetoer i would receive a payoff of $w_i(p | \alpha)$ in any continuation game $\Gamma(p | \alpha)$ because it was always

possible to ensure that every proposal giving her more than $w_i(p | \alpha)$ would be rejected by at least one decisive coalition. In those equilibria, it was always impossible for i to offer *all* of the decisive voters more than their rewards for rejecting i 's proposals. The only reason why the previous constructions do not apply in the $|V| = 2$ case is that i only needs *one* of the other players (the other vetoer) to accept her proposal. As $\delta < 1$, whenever i is the last proposer (an event that occurs with positive probability in every period), she can always make proposal that gives her more than $w_i(p | \alpha)$ and the other vetoer more than the maximum the latter would get by rejecting the proposal. This in turn implies that, in any period where she proposes last, vetoer i can guarantee herself some payoff greater than $w_i(p | \alpha)$ by rejecting the first $(n - 1)$ proposals. One must therefore change the lower bounds on the vetoers' continuation values to account for this possibility. As in the previous construction, if a vetoer i deviates then, from the next period on, we "punish" her by giving the other vetoer the total surplus minus i 's minimum continuation value. For each $i \in V$, let ρ_i be the probability is the last proposer in each period, and let $V_i^+(p | \alpha)$ and $V_i^-(p | \alpha)$ be the solutions to the following functional equations:

$$\begin{aligned} V_i^-(p | \alpha) &= (1 - \delta)w_i(p | \alpha) + \delta \mathbb{E} \left[\varpi_i(p | \tilde{\alpha}) | \alpha, p \right] \\ V_i^+(p | \alpha) &= V^*(\alpha) - (1 - \delta)w_j(p | \alpha) - \delta \mathbb{E} \left[V^*(\tilde{\alpha}) - \varpi_i(p | \tilde{\alpha}) | \alpha, p \right], \end{aligned}$$

where $\varpi_i(p | \alpha) \equiv \rho_i V_i^+(p | \alpha) + (1 - \rho_i) V_i^-(p | \alpha)$ and $j \in V \setminus \{i\}$, for all $p \in \{R, S\} \times [0, 1] \times X$. Intuitively, $V_i^+(p | \alpha)$ [resp. $V_i^-(p | \alpha)$] stands for vetoer i 's minimum payoff in continuation game $\Gamma(p | \alpha)$ conditional on her being the last proposer [resp. not being the last proposer], and $\varpi_i(p | \alpha)$ stands for her minimum payoff in continuation game $\Gamma(p | \alpha)$ (computed before the realization of the order of proposers). We then obtain the result with an equilibrium construction as above, but substituting $\varpi_i(p | \alpha)$ to $w_i(p | \alpha)$ for each $i \in V$.

C Proof of Lemma C1

Let $i, j \in N$ with $i \in C^j$, and all $k \in \mathbb{N}$. By definition of W_i , we have

$$\begin{aligned} \frac{W_i^j(1) - (1 - \delta)\gamma\Delta\bar{r} - \delta W_i^0(1)}{\Delta} &= (1 - \delta)\gamma(1 - \hat{\tau})(r_i - \bar{r}) \\ &\quad + \gamma\hat{\tau}\bar{r}[\delta(x_i^j - \bar{x}_i) - (1 - \delta)(1 - x_i^j)] . \end{aligned}$$

As $x_i^j - \bar{x}_i > 0$, there exists $\hat{\Delta}_{i,j}^1 > 0$ such that $W_i^j(1) - (1 - \delta)\gamma\bar{r} - \delta W_i^0(1) > 0$ whenever $\Delta < \hat{\Delta}_{i,j}^1$. By the same logic, if $k \geq k^*$, then there exists $\hat{\Delta}_{i,j}^2 > 0$ such that $W_i^j(\alpha_k) - (1 - \delta)\bar{s}\Delta - \delta W_i^0(\alpha_k) = \bar{s}\Delta[\delta(x_i^j - \bar{x}_i) - (1 - \delta)(1 - x_i^j)] > 0$ whenever $\Delta < \hat{\Delta}_{i,j}^2$. Now suppose that $k < k^*$. Observe that

$$\begin{aligned} \frac{W_i^j(\alpha_k) - W_i^0(\alpha_k)}{\Delta} &= \left\{ [1 - \delta^{k^* - k}(1 - \gamma\Delta)^{k^* - k}] \alpha_k \gamma \bar{r} + \delta^{k^* - k} [1 - \alpha_k + (1 - \gamma\Delta)^{k^* - k} \alpha_k] \bar{s} \right\} \\ &\quad \times (x_i^j - \bar{x}_i) , \end{aligned}$$

where the first bracketed term on the right-hand side represents the expected social welfare (divided by Δ) under the optimal stopping rule. As $\alpha_k > \alpha^*$, this term is greater than or equal to \bar{s} . Hence, $W_i^j(\alpha_k) - W_i^0(\alpha_k) \geq \bar{s}\Delta(x_i^j - \bar{x}_i) > 0$, and

$$\frac{W_i^j(\alpha_k) - (1 - \delta)\bar{s}\Delta - \delta W_i^0(\alpha_k)}{\Delta} \geq \frac{1 - \delta}{\Delta} W_i^j(\alpha_k) - (1 - \delta)\bar{s} + \frac{\delta}{\Delta} \bar{s} \hat{\tau} (x_i^j - \bar{x}_i) .$$

An application of l'Hôpital's rule shows that $(1 - \delta)/\Delta \rightarrow \rho$ as $\Delta \rightarrow 0$. As $W_i^j(\cdot)$ and $W_i^0(\cdot)$ are bounded, there exists $\hat{\Delta}_{i,j}^3 > 0$ such that $W_i^j(\alpha_k) > (1 - \delta)\bar{s}\Delta - \delta W_i^0(\alpha_k) > 0$ whenever $\Delta < \hat{\Delta}_{i,j}^3$.

Consider now the last inequality in the lemma. Let $\Psi(\alpha_k) \equiv W_i^j(\alpha_k) - (1 - \delta)\alpha_k\gamma\Delta\bar{r} - \delta\alpha_k\gamma\Delta W_i^0(1) - \delta(1 - \alpha_k\gamma\Delta)W_i^0(\alpha_{k+1})$. Suppose first that $k \geq k^*$. It is readily checked that

$$\lim_{\Delta \rightarrow 0} \frac{\Psi(\alpha_k)}{\Delta} = (1 - \hat{\tau})s_i + \hat{\tau}x_i^j\bar{s} - [(1 - \hat{\tau})s_i + \hat{\tau}\bar{x}_i\bar{s}] = \hat{\tau}(x_i^j - \bar{x}_i)\bar{s} > 0 .$$

Therefore, there exists $\hat{\Delta}_{i,j}^4 > 0$ such that $W_i^j(\alpha_k) > (1 - \delta)\alpha_k\gamma\bar{r} + \delta\alpha_k\gamma\Delta W_i^0(1) + \delta(1 - \alpha_k\gamma\Delta)W_i^0(\alpha_{k+1})$ whenever $\Delta < \hat{\Delta}_{i,j}^4$. Finally, suppose that $k < k^*$. By definition of W_i^j ,

we have

$$\begin{aligned}
\Psi(\alpha_k) &= -(1 - \delta)\alpha_k\gamma\Delta\bar{r}(1 - x_i^j) + \delta\alpha_k\gamma\Delta[W_i^j(1) - W_i^0(1)] \\
&\quad + \delta(1 - \alpha_k\gamma\Delta)[W_i^j(\alpha_{k+1}) - W_i^0(\alpha_{k+1})] \\
&\geq -(1 - \delta)\alpha_k\gamma\Delta\hat{\tau}(1 - x_i^j)\bar{r} + \delta(x_i^j - \bar{x}_i)[\alpha_k\gamma\Delta\hat{\tau}\bar{r} + (1 - \alpha_k\gamma\Delta)\hat{\tau}\bar{s}] \\
&> -(1 - \delta)\alpha_k\gamma\Delta\hat{\tau}(1 - x_i^j)\bar{r} + \delta\hat{\tau}(x_i^j - \bar{x}_i)\bar{s},
\end{aligned}$$

where the first inequality follows from $W_i^j(\alpha_{k+1}) - W_i^0(\alpha_{k+1}) \geq \bar{s}\hat{\tau}(x_i^j - \bar{x}_i)$ (as established above), and the second follows from our assumption that $\gamma\bar{r} > \bar{s}$. Therefore, there exists $\hat{\Delta}_{i,j}^5 > 0$ such that $W_i^j(\alpha_k) > (1 - \delta)\alpha_k\gamma\Delta\bar{r} + \delta\alpha_k\gamma\Delta W_i^0(1) + \delta(1 - \alpha_k\gamma\Delta)W_i^0(\alpha_{k+1})$ whenever $\Delta < \hat{\Delta}_{i,j}^5$. Setting $\bar{\Delta} \equiv \min\{\hat{\Delta}_{i,j}^\ell: i, j \in N \ \& \ \ell = 1, \dots, 5\}$, we obtain the lemma.