Forward Guidance and Durable Goods Demand

By Alisdair McKay and Johannes F. Wieland*

We study the monetary transmission mechanism in a quantitative fixed-cost model of durable goods demand. We show that aggregate demand is substantially more sensitive to contemporaneous interest rates than to forward guidance about future interest rates. Reducing the real interest rate one year from now increases output by only 41% as much as reducing the real interest rate today. The power of forward guidance declines further at longer horizons. We show analytically and quantitatively that this result is driven by the sensitivity of the extensive margin of durable adjustment to the contemporaneous user cost.
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Forward guidance plays an increasingly important role in the conduct of monetary policy as it is one of the main tools of unconventional monetary policy (Bernanke, 2020). Despite the prominence of forward guidance in modern monetary policy, the theoretical underpinnings of how future interest rates affect aggregate demand are still a matter of debate within monetary economics. Workhorse New Keynesian models are viewed by many as being too forward looking and thereby attributing too much power to forward guidance policies (Carlstrom, Fuerst and Paustian, 2015; Del Negro, Giannoni and Patterson, 2015). Indeed, the predictions of the Euler equation at the heart of the three-equation New Keynesian model illustrate the issue starkly: changes in expected real interest rates at *any horizon* have an equally large effect on the current level of aggregate demand. This implausible prediction has come to be known as the "forward guidance puzzle."

A number of authors have offered modifications to the New Keynesian framework that can reduce the power of forward guidance. These include market incompleteness (McKay, Nakamura and Steinsson, 2016; Werning, 2015; Acharya and Dogra, 2020), behavioral or informational frictions (Farhi and Werning, 2019;

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Gabaix, 2020; Angeletos and Lian, 2018), and including wealth in the utility function (Campbell et al., 2017; Michaillat and Saez, 2019). In these approaches, aggregate demand is solely determined by *nondurable* consumption. However, monetary policy is generally viewed as having a particularly strong influence on durable demand and investment spending (Erceg and Levin, 2006; Barsky, House and Kimball, 2007; Sterk and Tenreyro, 2018).

In this paper, we quantify the power of forward guidance in an incomplete markets model of durable goods demand subject to fixed adjustment costs. The model is based on McKay and Wieland (2020) where we show that it can match micro-data on durable adjustment hazards and the response of durable and non-durable expenditure to monetary policy shocks. We find that an announcement of an interest rate cut one year from now increases current output by only 41% as much as a contemporaneous interest rate cut. Interest rate cuts further in the future are even less effective. The power of forward guidance declines to 25% of the power of contemporaneous policy at a horizon of two years and settles around 20% at a horizon of four years. These patterns are due to a weaker response of durable expenditure to forward guidance. In a version of the model without durables, forward guidance is essentially as powerful as contemporaneous interest rate changes.

What explains these results? The demand for durables is particularly sensitive to the contemporaneous user cost of durables. A contemporaneous real interest rate cut stimulates durable demand by directly reducing the contemporaneous user cost. Forward guidance has a weaker, indirect effect on the contemporaneous user cost through expected capital gains, and is therefore less effective at stimulating durable demand.

The importance of the contemporaneous user cost comes from the extensive margin decision—the choice of when to make an adjustment to the durable stock. Optimality requires that a household at an adjustment threshold is indifferent between adjusting now versus waiting a short time (the smooth-pasting condition). Consider a household that wants to increase its durable position. Upgrading the durable position immediately brings a higher utility. But postponing the adjustment avoids paying the contemporaneous user cost on the addition to the durable stock. Because the choice of when to adjust is a short-term decision (now versus a short time later), the contemporaneous user cost plays a special role.

We verify that this logic drives our results in several ways. We decompose our main result and show that the extensive margin accounts for most of the result. The intensive margin—the choice of how many durables to purchase when an adjustment occurs—also contributes, but to a much lesser extent than the extensive margin. Second, we compare our results to a model without fixed costs. In such a frictionless model, households continuously adjust their durable positions to equate the marginal rate of substitution between durables and nondurables with the contemporaneous user cost. This is an extreme case in which durable demand is highly sensitive to the contemporaneous user cost. The results from the frictionless model are similar to what we obtain from the fixed-cost model. Finally, we quantitatively evaluate the terms in the smooth-pasting condition and show that the change in the contemporaneous user cost is the main driver of the extensive margin response to monetary policy.

It is often argued that forward guidance is powerful because it affects the interest rates on financing for durable goods purchases such as mortgage rates, which are long-term rates. While our model abstracts from long-term financing, we present an extension with a long-duration financial asset. We show the partial-equilibrium household decision problem is unchanged by the duration of financing. Therefore the importance of the contemporaneous user cost to the extensive margin decision remains the same with long-term financing.

I. Model

A. Households

Households consume nondurable goods, c, and a service flow from durable goods, s. Household $i \in [0,1]$ has preferences given by

$$E_0 \int_{t=0}^{\infty} e^{-\rho t} u\left(c_{it}, s_{it}\right) \mathrm{d}t.$$

The service flow from durables is generated from the household's stock of durable goods as we describe below. The felicity function is CES,

$$u(c,s) = \frac{\left[(1-\psi)^{\frac{1}{\xi}} c^{\frac{\xi-1}{\xi}} + \psi^{\frac{1}{\xi}} s^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi(1-1/\sigma)}{\xi-1}} - 1}{1-1/\sigma},$$

where ξ is the elasticity of substitution between nondurables and durables and σ is the intertemporal elasticity of substitution.

Households hold a portfolio of durables denoted d_{it} and liquid assets denoted a_{it}. When we calibrate the model, we will interpret durables broadly to include consumer durables and housing. When a household with pre-existing portfolio (a_{it}, d_{it}) adjusts its durable stock, it chooses a new portfolio (a'_{it}, d'_{it}) subject to the payment of a fixed cost $f p_t d_{it}$ such that

$$a'_{it} + p_t d'_{it} = a_{it} + (1-f) p_t d_{it}, \tag{1}$$

where p_t is the relative price of durable goods in terms of nondurable goods. We use d^*_{it} to denote the optimal post-adjustment durable stock.

The stock of durables depreciates at rate δ. A fraction χ of depreciation must be paid immediately in the form of maintenance expenditures so we have

$$\dot{d}_{it} = -(1-\chi)\delta d_{it}, \tag{2}$$

where a dot over a variable indicates a time derivative. Maintenance expenditures reduce the drift of the durable stock, which reduces the mass of households near an adjustment threshold and dampens the sensitivity of the extensive margin of durable demand (see Bachmann, Caballero and Engel, 2013).

The household pays a flow cost of operating the durable stock equal to $\nu p_t d_{it}$. These operating costs reflect expenditures such as fuel, utilities, and taxes. Operating costs raise the user cost of owning durables and therefore reduce the elasticity of user costs with respect to interest rates (see McKay and Wieland, 2020).

Liquid savings pay a safe real interest rate r_t. Borrowers pay a real interest rate $r_t + r^s$, where r^s is an exogenous borrowing spread. We include the borrowing spread for the sake of our quantitative analysis and for our analytical results we will assume $r^s = 0$ to simplify the expressions. The household is able to borrow against the value of the durable stock up to a loan-to-value (LTV) limit λ

$$a_{it} \geq -\lambda(1-f)p_t d_{it}. \tag{3}$$

When a household does not adjust its durable stock, its liquid assets evolve according to

$$\dot{a}_{it} = r_t a_{it} + r^s a_{it} I_{\{a_{it}<0\}} - c_{it} + y_{it} - (\chi\delta + \nu)p_t d_{it}. \tag{4}$$

Household after-tax income is given by $y_{it} = (1-\tau_t)z_{it}Y_t$, where Y_t is aggregate income, z_{it} is the household's idiosyncratic income share, and τ_t is a time-varying income tax rate. The log income share $\ln z_{it}$ follows the Ornstein-Uhlenbeck process

$$\mathrm{d}\ln z_{it} = \rho_z \ln z_{it}\,\mathrm{d}t + \sigma_z\,\mathrm{d}\mathcal{W}_{it} + (1-\rho_z)\ln\bar{z}\,\mathrm{d}t, \tag{5}$$

where $\mathrm{d}\mathcal{W}_{it}$ is a Brownian motion, $\rho_z < 0$ controls the persistence of the income process, σ_z determines the variance of the income process, and $\bar{z}$ is a constant such that $\int z_{it}\,\mathrm{d}i = 1$.

The service flow from durables is given by $s_{it} = \eta_{it}d_{it}$ where η_{it} represents the quality of the match between the household and its durable stock. η_{it} equals one when a durable adjustment takes place but subsequently drops to zero with Poisson intensity θ. These match-quality shocks stand in for unmodeled life events that cause households to adjust their durable positions such as a new job in a distant city. Match-quality shocks are a source of inframarginal adjustments of the household durable stock, which help the model match the sensitivity of durable demand to monetary policy shocks (see McKay and Wieland, 2020).

B. Firms

Nondurable goods are produced with a technology that is linear in labor, $Y_t = L_t$. Durable goods are produced by a representative firm that combines nondurables with a fixed factor

$$X_t = M_t^{1-\zeta} \bar{K}^{\zeta},$$

where X_t is aggregate durable production, M_t is nondurable input, and $\bar{K}$ is the supply of the fixed factor. The fixed factor could be interpreted as capturing the role of new land in the production of residential housing or as capturing the capital stock in the durable goods sector, which is approximately constant in the short run. We normalize $\bar{K}$ so the steady state relative price of durables is one. We then have

$$p_t = \left(\frac{X_t}{\bar{X}} \right)^{\frac{\zeta}{1-\zeta}}, \tag{6}$$

where $\bar{X}$ is steady state durable production. The payments to the fixed factor, which equal $\zeta p_t X_t$, are paid to the government.[1]

C. Government

We assume that the central bank directly chooses a path for the real interest rate, $\{r_s\}_{s \geq 0}$. Implicitly we assume nominal rigidities allow the central bank to implement this real rate path through an appropriate choice of the nominal interest rate.[2] Following the announcement of a real interest rate path, the economy follows a perfect foresight transition path. This is a common way of analyzing forward guidance (e.g. Alisdair McKay, Emi Nakamura and Jón Steinsson, 2016; Iván Werning, 2015).

Financial assets are in positive net supply due to a fixed supply of real government bonds $A_t = \bar{A}$. The tax rate τ_t adjusts to finance debt payments net of revenue from the fixed factor,

$$r_t \bar{A} - \zeta p_t X_t = \int_0^1 \tau_t z_{it} Y_t \, \mathrm{d}i = \tau_t Y_t.$$

[1] This assumption has little bearing on our quantitative results because these payments are small under our calibration. The purpose of the assumption is to avoid complicating the asset portfolio of the household sector. Similar assumptions appear in Favilukis, Ludvigson and Van Nieuwerburgh (2017) and Kaplan, Mitman and Violante (2020).

[2] We select the equilibrium in which the economy returns to steady state. This can be implemented by assuming that the central bank reverts to a standard interest rate rule at some arbitrarily far away date.

D. Market Clearing

By integrating over all households we obtain aggregate quantities,

$$C_t = \int_0^1 c_{it}\,\mathrm{d}i,$$
$$D_t = \int_0^1 d_{it}\,\mathrm{d}i.$$

Total durable expenditure, X_t, includes maintenance $\chi\delta D_t$, the durable expenditure by households making durable adjustments $(d_{it}^* - d_{it})$, and the fixed costs paid $f d_{it}$,

$$X_t = \chi\delta D_t + \int_0^1 \lim_{\mathrm{d}t\to 0} \frac{prob_{i,[t,t+\mathrm{d}t]}}{\mathrm{d}t}\left[(d_{it}^* - d_{it}) + f d_{it}\right]\mathrm{d}i \tag{7}$$

where $prob_{i,[t,t+\mathrm{d}t]}$ is the probability that household i makes an adjustment between t and $t+\mathrm{d}t$. The market for nondurable goods clears when

$$Y_t = C_t + \nu p_t D_t + M_t + r^s \int_0^1 a_{it} I_{\{a_{it}<0\}}\,\mathrm{d}i, \tag{8}$$

where the last term is an intermediation cost that gives rise to the borrowing spread r^s.

Total output (GDP) is given by $GDP_t = Y_t + \zeta p_t X_t = C_t + \nu p_t D_t + p_t X_t$.

As we analyze the demand response to a given path for the real interest rate, it is not necessary to calculate inflation so we do not need to specify all aspects of the supply side. In equilibrium, Y_t is determined by (8) and then divided among households according to $y_{it} = (1-\tau_t) z_{it} Y_t$. This approach to equilibrium income determination follows Werning (2015). In McKay and Wieland (2020) we provide a complete supply side that yields these equilibrium relationships. In that formulation, z_{it} is idiosyncratic labor productivity and wages are sticky.

II. Quantitative Results

We now quantitatively assess the power of forward guidance.

A. Calibration

The calibration largely follows McKay and Wieland (2020) where we show the model accurately captures the transmission of monetary shocks to the intensive and extensive margins of durable demand as well as nondurable consumption.[3]

[3] The model in McKay and Wieland (2020) also includes sticky information with respect to aggregate variables in the style of Carroll et al. (2020). These information rigidities do not meaningfully change the relative strength of contemporaneous interest rate changes and forward guidance so we omit them for simplicity.

We choose an elasticity of substitution between durables and nondurables of $\xi = 0.5$, which is at the lower end of the range of values estimated empirically (Ogaki and Reinhart, 1998; Davis and Ortalo-Magné, 2011; Pakoš, 2011; Albouy, Ehrlich and Liu, 2016). Higher values imply that durable demand is overly sensitive to monetary policy (McKay and Wieland, 2020). We have verified numerically that choosing a higher value for the elasticity of substitution ξ reduces the power of forward guidance relative to contemporaneous interest rates and in this sense our choice is conservative. We set the elasticity of intertemporal substitution to $\sigma = 0.25$, which allows the model to match the small response of nondurable consumption to monetary policy shocks (McKay and Wieland, 2020). This value is at the lower end of the range typical in calibrations, but on the higher end of traditional time-series estimates (Hall, 1988; Campbell and Mankiw, 1989; Yogo, 2004) as well as recent cross-sectional estimates (Best et al., 2020).

We calibrate the taste for durables, ψ, to match the value of the stock of durables relative to nondurable consumption from 1970-2019. Durables include housing and consumer durables. The depreciation rate is set to match durable stock depreciation in the BEA fixed asset table. We measure maintenance costs as the sum of intermediate goods and services consumed in the housing output table, the PCE on household maintenance, and the PCE on motor vehicle maintenance and repair. Operating costs include taxes on the housing sector, PCE on household utilities, and motor vehicle fuels and fluids.

We calibrate the discount rate, ρ, to match aggregate holdings of financial assets net of mortgage and auto loans. We set the steady state real interest rate to 1.5%, which is the average real federal funds rate between 1991 and 2007. We set the borrowing spread to 1.7%, which is the average spread between the 30-year mortgage and 10-year Treasury rates.

The fixed adjustment cost is set to match the frequency of durable adjustments. Our calibration target is a weighted average of the frequency of moving residence or making a housing addition or substantial repair and the frequency of buying a car. These frequencies are weighted in proportion to the values of the respective durable stocks. In McKay and Wieland (2020), we estimate the arrival intensity of match-quality shocks, θ, from PSID data on durable adjustments using the method of Berger and Vavra (2015). θ is identified by the frequency with which households adjust their durable position despite having a small gap between their existing durable position and their target position. The LTV limit, λ, is set to 80% and we take the parameters of the idiosyncratic risk process from Floden and Lindé (2001).

We calibrate the supply elasticity of durable goods based on land's share in the production of durables. This leads us to an inverse durable supply elasticity of $\zeta = 0.047$. This value reflects the share of residential investment in durable expenditure (36%), the share of new permanent-site structures in residential investment (58%), and the cost of land in new permanent-site structures (approx.

Table 1—Calibration of the Model.

	Name	Value	Source
ξ	Elasticity of substitution	0.5	See text
σ	Intertemporal elasticity of subs.	0.25	See text
ψ	Durable exponent	0.582	D/C ratio = 2.64
δ	Depreciation rate	0.068	BEA fixed asset table
χ	Required maintenance share	0.35	See text
ν	Operating cost	0.048	See text
ρ	Discount rate	0.094	Net assets/private GDP = 1.1
$\bar{r}$	Real interest rate	0.015	Annual real fed. funds rate
$\bar{r}^s$	Borrowing spread	0.017	Mortgage-Treasury spread
f	Fixed cost	0.199	Ann. adjustment prob = 0.19
θ	Match-quality shock freq.	0.158	McKay and Wieland (2020)
λ	Borrowing limit	0.8	20% Down payment
ρ_z	Income persistence	-0.090	Floden and Lindé (2001)
σ_z	Income standard deviation	0.216	Floden and Lindé (2001)
ζ	Inverse durable supply elasticity	0.047	See text

23%).[4] An elastic supply is consistent with the muted response of the relative price of durables to monetary shocks estimated by McKay and Wieland (2020) and with House and Shapiro's (2008) finding that capital goods production responds significantly to investment stimulus but prices do not.[5]

Table 1 summarizes the calibration. We solve the model using continuous-time methods from Achdou et al. (2017) and the sequence-space methods from Auclert et al. (2019).

B. A Frictionless Model

To gain intuition for the role that durable goods play in resolving the forward guidance puzzle, we first solve a special case of the model without adjustment costs ($f = 0$) and with fully collateralizable durables ($\lambda = 1$). Figure 1 shows the change in contemporaneous output in response to interest rate cuts at different horizons. We assume the central bank announces a 1% (annualized) reduction in the real interest rate that lasts for one quarter and we vary the horizon at which the interest rate cut occurs.

The model predicts that the effect of forward guidance on current output is

[4] The first two values are from NIPA Table 1.1.5 and NIPA Table 5.4.5, 1969-2007. We calibrate the cost of land in housing prices using the midpoint of new and existing houses in Davis and Heathcote (2007). See McKay and Wieland (2020) for further details.

[5] Price stickiness in durables would also manifest as a weak durable price response and elastic durable supply. Goolsbee (1998) also finds little price response for consumer durables (autos, computers, and furniture).

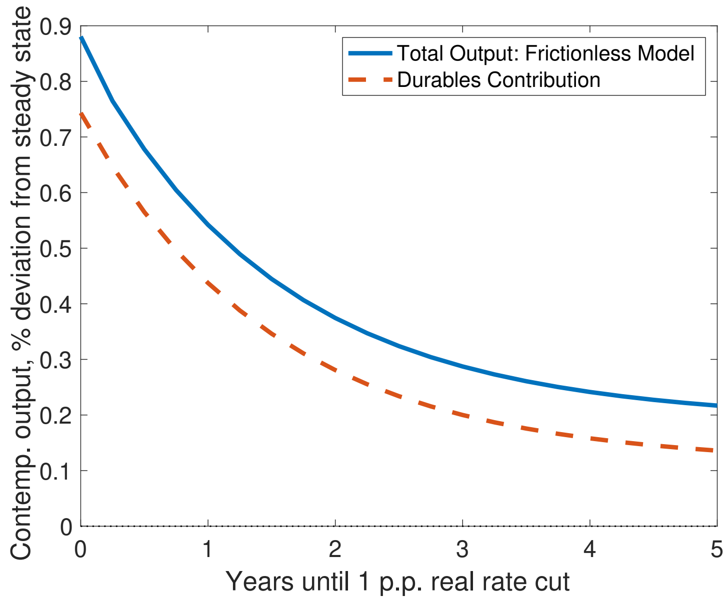

FIGURE 1. CONTEMPORANEOUS OUTPUT RESPONSE TO FORWARD GUIDANCE IN THE FRICTIONLESS MODEL.

Note: The real interest rate falls by 1 percentage point (annualized) for one quarter starting at the horizon indicated on the horizontal axis. The frictionless model has $f = 0$ and $\lambda = 1$.

substantially weaker than contemporaneous interest rates. The solid line in Figure 1 shows that a contemporaneous 1% cut in the real rate increases output by 0.88%. If the same interest rate change occurs, for example, one year from now, then today's output increases by only 0.55%. The power of forward guidance steadily falls with the horizon of the guidance. The dashed line in Figure 1 shows that this drop in effectiveness is entirely accounted for by a weaker response of durable expenditure.

To understand this behavior of durable demand note that in this frictionless durable model, households continuously adjust their durable positions to equate the marginal rate of substitution between durables and nondurables to the contemporaneous user cost,

$$\left(\frac{\psi}{1-\psi}\frac{c_{it}}{d_{it}}\right)^{\frac{1}{\xi}} = p_t(r_t + \nu + \delta) - \dot{p}_t \equiv r_t^d. \tag{9}$$

The user cost r_t^d captures the marginal cost of holding durables for an instant. A unit of durables acquired at p_t costs forgone interest $p_t r_t$, operating costs $p_t \nu$, depreciation $p_t \delta$, and potential capital losses $-\dot{p}_t$.

Durable demand is particularly sensitive to the contemporaneous interest rate because that is the interest rate that appears in the contemporaneous user cost.

In contrast, future interest rates do not directly appear in equation (9). To understand why the power of forward guidance declines smoothly with the horizon in Figure 1, the distinction between interest rates and user costs is key. A reduction in future interest rates lowers the future user cost and leads to an increase in future durable demand, which bids up future durables prices. The anticipated increase in relative prices raises the future user cost and lowers the current user cost through anticipated capital gains. In this way, equilibrium relative price movements smooth out the relationship between durable demand and real interest rates. However, the effect on the contemporaneous user cost declines as the horizon of the interest rate change increases.

A reduction in the user cost prompts households to increase their consumption of durables and movements in the user cost are the key reason durable demand behaves differently from nondurable demand. This mechanism is particularly strong for durables with a low depreciation rate, since a one percentage point change in interest rates leads to a very large percentage change in user costs. In contrast, when δ is very large, the percentage change in the user cost is small and durable demand behaves similarly to nondurable demand.

In the frictionless model, households only consider the contemporaneous user cost because they plan to adjust again the next instant. As a result, durable demand is extremely sensitive to the contemporaneous user costs and much less sensitive to future user costs. Of course this logic is inconsistent with the observation that durables purchases are lumpy and households can go years without adjusting. In contrast, the fixed-cost model is consistent with long periods of inaction. As we show next, the contemporaneous user cost remains a key determinant of durable demand in the fixed-cost model due to the extensive margin decision. It is this common emphasis on the contemporaneous user cost that makes contemporaneous interest rates more powerful than forward guidance in both models.[6]

C. Main Results

Panel A of Figure 2 shows the change in contemporaneous output in response to interest rate cuts at different horizons in the fixed-cost model. A contemporaneous interest rate cut increases output by 0.74%. Promises of interest rate cuts in the future are less powerful and substantially so at more distant horizons. If the same interest rate change occurs one year from now, then today's output increases by 0.30%, only 41% as much compared to contemporaneous stimulus. An interest rate cut at a horizon of two years is about 25% as effective as a contemporaneous one. For promises more than four years out, the power of forward guidance settles

[6]Models with durable goods subject to smooth adjustment cost, particularly higher order adjustment costs, place more weight on future user costs as households gradually build up their desired durable stocks in small increments. Durable demand reacts more strongly to forward guidance in these models. The gradual adjustment behavior is inconsistent with the infrequent, lumpy adjustment in the micro data.

around 20% of the effectiveness of a contemporaneous cut. In short, forward guidance is considerably less powerful than contemporaneous interest rate cuts.

We plot results from two other models for comparison. First, as is well known, the three-equation New Keynesian model predicts that real rate changes at any horizon have the same effect on output today. This prediction of the model is widely regarded as implausible and at the heart of the forward guidance puzzle (see Carlstrom, Fuerst and Paustian, 2015; Del Negro, Giannoni and Patterson, 2015). Our model differs from the three-equation model in several ways, but the addition of durables is particularly important. We plot the effect of forward guidance in a version of our model with only nondurables.[7] In that model, forward guidance effects are only slightly attenuated relative to the three-equation model. For example, an interest rate cut two years from now is 92% as effective as a contemporaneous interest rate cut. Panel B of Figure 2 shows the contribution of durable demand to the total output response. The weaker output response of forward guidance is almost entirely accounted for by a weaker response of durable spending, which parallels the total output response.

The increase in durable expenditure to monetary stimulus can be decomposed into two margins. First, the extensive margin: holding fixed the desired durable stock d^*_{it}, monetary stimulus increases durable expenditure by increasing the probability of a durable adjustment ($\frac{prob_{i,[t,t+dt]}}{\mathrm{d}t}$ in equation (7)). Second, the intensive margin: holding fixed the frequency of adjustment, a lower real interest rate increases the desired durable stock conditional on an adjustment, d^*_{it}.[8]

Panel B of Figure 2 shows the decomposition of durable expenditure into the contributions of the extensive and intensive margins. Both lines slope down meaning both margins are less responsive to forward guidance than to contemporaneous interest rate changes, but the effect is much stronger for the extensive margin. For example, the extensive margin accounts for 73% of the weaker response of output to forward guidance at a horizon of one year.

III. The Special Role of the Contemporaneous User Cost

Why is forward guidance weaker in the fixed-cost model? And, in particular, why does the extensive margin of adjustment account for the majority of this effect? We now show analytically and quantitatively that the contemporaneous user cost plays a special role in the extensive margin decision. In this manner, the extensive margin plays a similar role in reducing the power of forward guidance as equation (9) in the frictionless model.

[7]In this model, the durable share in utility is set to zero, $\psi = 0$, rendering $\delta, \xi, f, \theta, \nu$ irrelevant. The intertemporal elasticity of substitution, σ, is calibrated to match the impact response of output to a contemporaneous 1% real rate reduction in our full model. The borrowing limit is set to $-\lambda$ times the 25^{th} percentile of durable holdings in our full model. The parameter ρ is set to match the same net asset to GDP ratio as in the full model. Other parameters are unchanged.

[8]Because the durable stock is pre-determined, the initial impact of a monetary shock has no effect on maintenance expenditures.

Define $V_t(a, d, z)$ as the value function of a household with liquid assets a, durable stock d, and productivity z. The space of individual state variables is divided into an inaction region and an adjustment region. When an adjustment takes place, the household picks the optimal durable stock given its cash-on-hand $m_t \equiv a + (1-f)p_t d$ to maximize the post-adjustment value subject to the LTV constraint

$$V_t^{adj}(m, z) = \max_{d'} V_t(m - p_t d', d', z)$$
$$\text{s.t. } p_t(1 - \lambda(1-f))d' \leq m.$$

The solution to this problem is the optimal durable stock $d_t^*(m, z)$. We use the notation d_t^* when the state variables are clear from the context.

When no adjustment takes place, the value function follows the standard Hamilton-Jacobi-Bellman equation,

$$\rho V_t(a, d, z) = \max_{c_t} \left\{ u(c_t, d) + \mathbb{E}_t \frac{\mathrm{d}}{\mathrm{d}t} V_t(a, d, z) \right\}, \tag{10}$$

subject to the laws of motion for the individual states and the LTV constraint (2)-(5).

A. *Extensive Margin*

The optimal adjustment thresholds are characterized by the value-matching and smooth-pasting conditions. For a point (a, d, z) on an optimal adjustment threshold at time t, the value-matching condition simply states that the value function is continuous at an adjustment point,

$$V_t\left(a - p_t(d_t^* - (1-f)d), d_t^*, z\right) = V_t(a, d, z).$$

The smooth-pasting condition requires that the household is indifferent between adjusting and waiting another instant,

$$\mathbb{E}_t \frac{\mathrm{d}}{\mathrm{d}t} V_t(a - p_t(d_t^* - (1-f)d), d_t^*, z) = \mathbb{E}_t \frac{\mathrm{d}}{\mathrm{d}t} V_t(a, d, z).$$

In Appendix A, we show that the smooth-pasting condition can be expressed as[9]

(11)

$$
\begin{aligned}
&\frac{1}{V_{a,t}(a_t^*, d_t^*, z)} \left[u(c_t^*, d_t^*) - u(c_t, d)\right] = \\
&\quad r_t^d \left(d_t^* - d\right) + \left[r_t^d - (\nu + \delta\chi)p_t\right] fd + (c_t^* - c_t) \\
&\quad + \frac{\frac{V_{d,t}(a_t^*, d_t^*, z)}{p_t V_{a,t}(a_t^*, d_t^*, z)} - 1}{1 - \lambda(1-f)} \left\{ \frac{a}{p_t} \left[r_t^d - (\nu + \delta\chi)p_t\right] + z(1-\tau_t)Y_t - c_t - (\nu + \delta\chi)p_t d \right\},
\end{aligned}
$$

where c_t^* and a_t^* are post-adjustment consumption and assets and c_t is pre-adjustment consumption. This equation characterizes the indifference between adjusting now versus waiting for another instant by equating the benefit of adjusting now (first line) with the benefit of waiting for another instant (second and third lines).

To understand the individual components of the smooth-pasting condition, begin with the benefit of a durable adjustment this instant given by the first line of (11). For concreteness, consider an upward adjustment, $d_t^* > d$. The term $u(c_t^*, d_t^*) - u(c_t, d)$ captures the increased flow utility from upgrading durables, which is converted into nondurable goods units via $V_{a,t}(a_t^*, d_t^*, z)^{-1} = u_c(c_t^*, d_t^*)^{-1}$.

The second line of (11) represents the benefit of delaying the adjustment for a household that is not LTV-constrained. For this household, delaying the purchase $d_t^* - d$ incurs a flow benefit given by the contemporaneous user cost: the household earns additional interest, pays lower operating and maintenance costs, and does not incur any capital losses on the purchase. In addition, the household delays the payment of the fixed cost, which is valued at the contemporaneous user cost less the operating and maintenance costs.[10] Finally, complementarities in the choices of nondurable consumption and durable consumption through, for example, the utility function or borrowing constraints, yield an additional benefit of delaying equal to $c_t^* - c_t$.

At an adjustment point, an unconstrained household sets $V_{d,t} = p_t V_{a,t}$ so the third line of (11) drops out. However, with a binding LTV constraint, $V_{d,t} > p_t V_{a,t}$ leading to an additional benefit from waiting. Accumulating more assets can relax the LTV constraint, with $\frac{1}{1-\lambda(1-f)}$ leveraging up these savings. The value of savings grows faster with a higher user cost both because interest accumulates and also because the relative price of durables may decline.

Equation (11) implies that the contemporaneous user cost r_t^d plays a central role in determining durable demand. If the user cost is low, for example because r_t is low, then the benefit of waiting shrinks. We would then expect households

[9] For simplicity, this derivation assumes $\theta = 0$. Appendix A shows that $\theta > 0$ introduces an additional term that captures the costs of an inframarginal adjustment. We include this cost in Figure 3.

[10] Subtracting the operating and maintenance costs leaves the interest expense $r_t p_t$ and reduction in resale value $p_t \delta(1-\chi) - \dot{p}_t$.

to accelerate their durable purchases and a corresponding increase in aggregate durable demand. The contemporaneous interest rate is more powerful in stimulating durable demand than are future interest rates because it directly affects the contemporaneous user cost.

Figure 3 demonstrates the special role of the contemporaneous user cost in the quantitative model. The figure shows the net benefit from adjusting now rather than waiting an instant—the left hand side minus the right hand side of (11). The figure is drawn for households with the average level of liquid assets and income and with the existing durable position shown on the horizontal axis.[11] The adjustment threshold is the point at which the net benefits are zero. The net benefit curve slopes down primarily because the utility gain from acquiring more durables is smaller with a larger existing stock.

Panel A shows that the net benefit of adjusting increases after a surprise contemporaneous interest rate cut. We use a very large change in interest rates to make the comparison more visible but the same patterns occur for smaller changes. The immediate extensive margin response is given by the mass of households for whom the net benefits become positive after the interest rate cut.

The dashed line in panel A isolates the contribution of the contemporaneous user cost terms in equation (11). Specifically, we fix all other variables at their steady state value and only change r_t^d in equation (11). This change alone accounts for the majority of the increase in net benefits in panel A. The remaining increase in the net benefit of adjusting primarily reflects an increase in the desired durable stock (intensive margin) and thus a larger utility gain from adjusting.

Panel B of the figure shows the net benefit of adjusting after an announced real interest rate cut in a year's time. In this case, the net benefit line shifts up by much less, because future real interest rate cuts have a much weaker effect on the contemporaneous user cost. The contrast between panels A and B demonstrates the importance of the contemporaneous user cost to the extensive margin decision, which is the main reason why forward guidance is so much less powerful in the fixed-cost model.

B. *Intensive Margin*

The intensive margin also contributes to the weaker power of forward guidance. To see why, define the cumulative user cost from t to $t+\tau$ as

(12)
$$r^d_{t,t+\tau} = p_t e^{\int_0^\tau r_{t+u}\,\mathrm{d}u} - p_{t+\tau} e^{-\delta(1-\chi)\tau} + (\nu + \delta\chi)\int_0^\tau e^{\int_k^\tau r_{t+u}\,\mathrm{d}u - \delta(1-\chi)k} p_{t+k}\,\mathrm{d}k.$$

[11] Appendix Figures A.1-A.3 show the same patterns hold for different levels of liquid assets and income.

This is the cost of holding a unit of durables from t to $t+\tau$. The first two terms accumulate lost interest, depreciation, and capital losses over the holding period. The third term accumulates (with interest) the flow payments for operating and maintenance costs over the holding period.

The intensive margin first order condition can be expressed as (see Appendix B)

$$\mathbb{E}_t \int_0^\tau e^{-(\rho+\delta(1-\chi))s} u_d(c_{t+s}, e^{-\delta(1-\chi)s} d)\, \mathrm{d}s = \mathbb{E}_t e^{-\rho\tau} V^{adj}_{m,t+\tau} \left[r^d_{t,t+\tau} + e^{-\delta(1-\chi)\tau} p_{t+\tau} f \right] + \mathbb{E}_t \int_0^\tau e^{-\rho s} \Psi_{t+s} \left[r^d_{t,t+s} + (1-\lambda(1-f)) e^{-\delta(1-\chi)s} p_{t+s} \right] \mathrm{d}s \tag{13}$$

where $t+\tau$ is the optimal (stochastic) stopping time when the next durable adjustment takes place, $V^{adj}_{m,t+\tau}$ is the marginal value of cash-on-hand at the next adjustment, and Ψ_t is the Lagrange multiplier on the borrowing constraint at date t. An unconstrained household ($\Psi = 0$) equates the expected discounted marginal utility of durables over the holding period to the expected discounted cumulative user cost over the holding period plus the losses from the fixed cost. When borrowing constraints bind, the household also considers how liquid assets are affected by increasing the durable position, which is given by $r^d_{t,t+s}$ plus the required equity in the durable stock.

The crucial thing to note about (13) is that the planning horizon stops at the next adjustment date $t+\tau$. Since τ is stochastic it is integrated out by the expectation operator. This integration weighs the user cost at $t+s$ by the probability the durable position has not been adjusted before that date. At longer horizons, it is quite likely that the household has already adjusted its durable position so user costs at these horizons receive less weight relative to those in the more immediate future. This explains why the intensive margin also accounts for some weaker forward guidance effects in Figure 2, though to a much lesser extent than the extensive margin.

IV. Long-Term Financing

Forward guidance is often thought to affect household purchasing decisions by moving long-term financing rates such as mortgage rates. Our model abstracts from this mechanism as households use short-term assets for financing. We now describe an extension of the model in which households borrow through a long-duration bond.

The long term bond trades at a price q_t and promises an arbitrary sequence of coupon payments. We redefine a_{it} as the total value of liquid assets including holdings of short- and long-term bonds. No-arbitrage implies that all assets must pay the same return along a perfect foresight path, so the return on the long-term bond r^b_t is the same as the short-term interest rate, $r^b_{t+k} = r_{t+k}$ for $k \geq 0$.

Therefore, the return on total liquid assets is equal to the short-rate r_t irrespective of the portfolio weights on short-term and long-term bonds, and the law of motion of total liquid wealth is identical to the model with short-term bond only (equation (4)).[12] It follows that all of the household constraints in Section I (equations (1)-(5)) are unchanged by the long-term bond. Therefore, conditional on the initial states (a_{i0}, d_{i0}, z_{i0}) and the paths for aggregate variables, the household's decision problem is identical to the model with short-term debt only. We show this formally in Appendix D.

As the decision problem is unchanged, we obtain *exactly* the same smooth-pasting condition (11) with and without long-term debt. Thus, the contemporaneous user cost plays the same important role as in our baseline model. Households are still making a short-term decision at the extensive margin—to adjust now or a little bit later. The cost of issuing long-term debt now rather than the next instant is determined by the instantaneous return on the long-term bond. Therefore, the contemporaneous user cost under long-term financing is $p_t \left(r_t^b + \delta + v \right) - \dot{p}_t$. But under no-arbitrage $r_t^b = r_t$ and this is the same user cost as in (9). Intuitively, what matters over the next instant is not so much the level of the long-term interest rate but the change in the interest rate, which is closely related to the return on the bond. If short-term rates are currently low, households have an incentive to lock-in an interest rate as long-term rates are expected to increase.

V. Conclusion

Forward guidance policies have received considerable attention not only because of their relevance to unconventional monetary policy strategies but also because they raise questions about the plausibility of the strongly forward-looking behavior in workhorse macroeconomic models. We show that incorporating durable goods demand subject to fixed adjustment costs substantially reduces the power of forward guidance. Forward guidance at a one year horizon is only 41% as powerful as contemporaneous stimulus and guidance at longer horizons is even less powerful. We view the fixed-cost model as an attractive approach for modeling forward guidance because durable goods are particularly sensitive to monetary policy and because fixed adjustment costs are supported by the microeconomic lumpiness of durable adjustments.

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[12] We assume that borrowing through either the short-term or long-term bond incurs the borrowing spread r^s.

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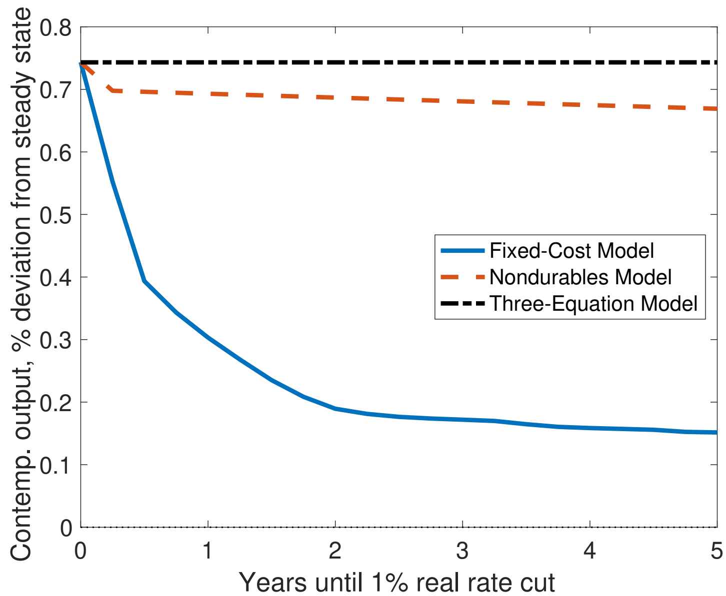
A. Power of Forward Guidance in the Fixed-Cost Model and Alternative Models.



B. Fixed-Cost Model: Contributions From the Extensive and Intensive Margins.

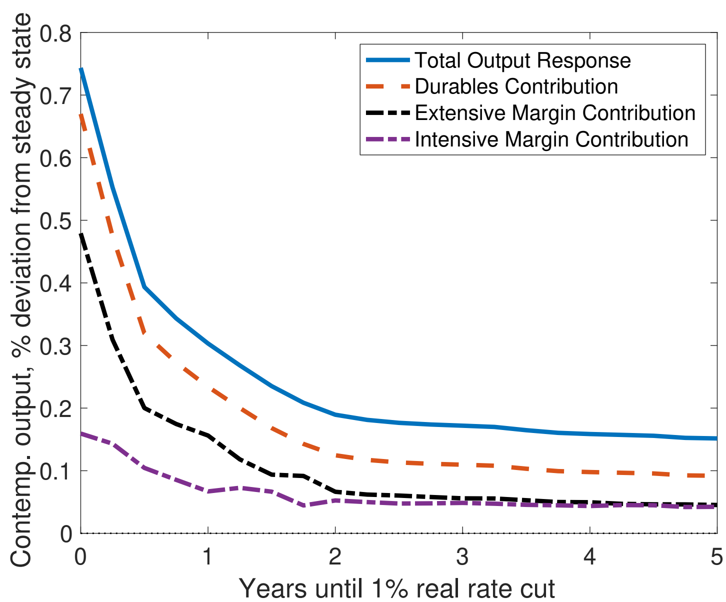
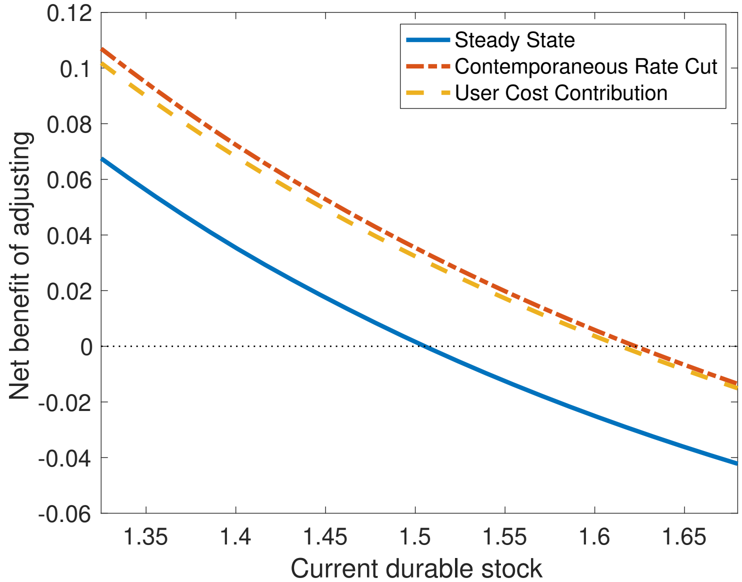


FIGURE 2. CONTEMPORANEOUS OUTPUT RESPONSE TO FORWARD GUIDANCE IN THE FIXED-COST MODEL.

Note: The real interest rate falls by 1 percentage point (annualized) for one quarter starting at the horizon indicated on the horizontal axis. Both panels: The solid blue line shows the output response in the fixed-cost model. Panel A: The dashed red line is a version of our baseline model without durables and the dashed-dotted gray line is the standard three-equation model. The alternative models are calibrated to yield the same output effects for a contemporaneous real interest rate cut as our main model. Panel B: The dashed red line shows the contribution from total durable expenditure. The dash-dot black and purple lines show the contributions from the extensive and intensive margins, respectively.

A. Net Benefit of Adjusting Before and After Contemporaneous Real Rate Cut



B. Net Benefit of Adjusting Before and After Announcement of Real Rate Cut in One Year

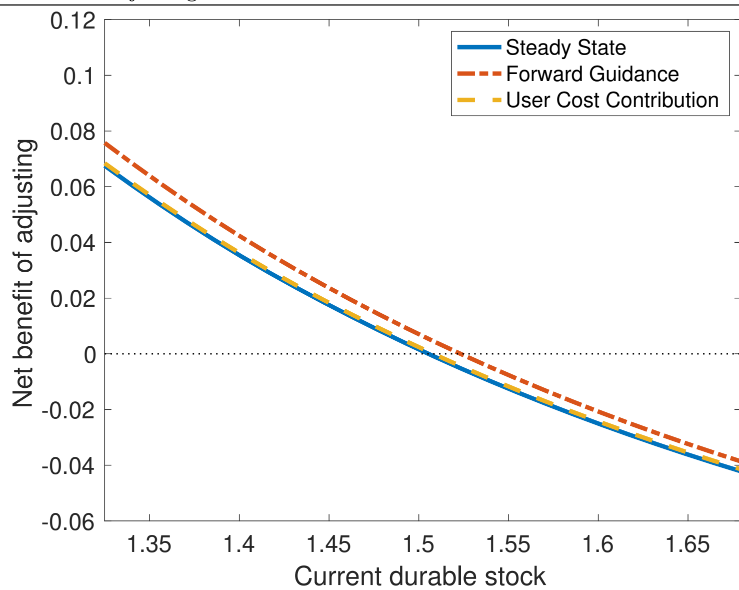


FIGURE 3. THE NET BENEFIT OF MAKING A DURABLE ADJUSTMENT.

Note: The figure shows the net benefit of adjusting now rather than waiting for an instant for different levels of the durable stock, which is given by the left-hand side minus the right-hand side of the smooth-pasting equation (11). Liquid assets and income are fixed at their average values. Panel A: The solid blue line shows the steady state net benefit. The dash-dotted red line shows the net benefit after a contemporaneous 10% real interest rate cut that lasts for one quarter. The dashed yellow line shows the net benefit after a contemporaneous 10% real interest rate cut but fixing all terms except the contemporaneous user cost r_t^d at their steady state values. Panel B: Same as Panel A, except that the shock is an announced real interest rate of 10% in one year.