How to Avoid Black Markets for Appointments With Online Booking Systems

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Online Appendix

A Main Proofs

In the following, we provide the proofs of the main results of the paper, namely propositions 1 and 2. We also state and prove Proposition 3, which describes an additional equilibrium of the batch system. Note that an equilibrium with any belief off the equilibrium path is a weakly perfect Bayesian equilibrium in this game. This is because the scalper sets the price before learning how many seekers are looking to buy from him, and thus his booking decision is unaffected by his belief about the seekers' types. We focus on symmetric equilibria in which every seeker uses a symmetric strategy. We say that a symmetric strategy has the cutoff $\hat{v}(p)$ if a type lower than the cutoff directly applies and a type higher than the cutoff buys from the scalper.

Lemma 1. *Consider the immediate system or the batch system. Suppose that the scalper enters the market with price p. We consider the decision of a seeker with type v regarding whether to apply directly or buy. Given that all other seekers follow the symmetric strategy β, let D and B be the probabilities of a seeker getting a slot from a direct application and from buying. Moreover, let $E(\mathrm{D}; v) = Dv$ an $E(\mathrm{B}; v) = B(v - p)$ be the expected payoff of a type v from a direct application and from buying. Then, the symmetric strategy β is optimal for a seeker if and only if it has the cutoff $\hat{v}(p)$. In particular, the cutoff $\hat{v}(p)$ satisfies*

1. *$\hat{v}(p) = \underline{v}$ when $D < B$ and $E(\mathrm{D}, \underline{v}) - E(\mathrm{B}, \underline{v}) \leq 0$;*
2. *$\hat{v}(p)$ satisfies $E(\mathrm{D}, \hat{v}(p)) = E(\mathrm{B}, \hat{v}(p))$ when $D < B$ and $E(\mathrm{D}, \underline{v}) - E(\mathrm{B}, \underline{v}) > 0 > E(\mathrm{D}, \overline{v}) - E(\mathrm{B}, \overline{v})$;*
3. *$\hat{v}(p) = \overline{v}$ when $D < B$ and $E(\mathrm{D}, \overline{v}) - E(\mathrm{B}, \overline{v}) \geq 0$;*
4. *$\hat{v}(p) = \overline{v}$ when $D \geq B$.*

Proof. Case 1: $D < B$ and $E(\mathrm{D}, \underline{v}) - E(\mathrm{B}, \underline{v}) = -(B - D)\underline{v} + Bp \leq 0$. Then, for any $v \in (\underline{v}, \overline{v}]$, $E(\mathrm{D}; v) - E(\mathrm{B}; v) = -(B - D)v + Bp < -(B - D)\underline{v} + Bp \leq 0$. Hence, the optimality of the symmetric strategy implies $\hat{v}(p) = \underline{v}$. Moreover, it follows from the above inequality that the symmetric strategy with the cutoff $\hat{v}(p) = \underline{v}$ is optimal.
Case 2: $D < B$ and $E(\mathrm{D}, \underline{v}) - E(\mathrm{B}, \underline{v}) > 0 > E(\mathrm{D}, \overline{v}) - E(\mathrm{B}, \overline{v})$. Then, the function $E(\mathrm{D}; v) - E(\mathrm{B}; v) = -(B - D)v + Bp$ is strictly decreasing, and is positive at $v = \underline{v}$ and negative at $v = \overline{v}$. Thus, there is a unique $v^* \in (\underline{v}, \overline{v})$ such that $E(\mathrm{D}; v^*) - E(\mathrm{B}; v^*) = 0$. Moreover, for $v \in [\underline{v}, v^*)$,

$E(\mathrm{D}; v) - E(\mathrm{B}; v) > 0$; for $v \in (v^*, \overline{v}]$, $E(\mathrm{D}; v) - E(\mathrm{B}; v) < 0$. Thus, we have the cutoff $\hat{v}(p) = v^*$. It is straightforward to see that the symmetric strategy with the cutoff v^* is optimal.

Case 3: $D < B$ and $E(\mathrm{D}, \overline{v}) - E(\mathrm{B}, \overline{v}) = -(B - D)\overline{v} + p \geq 0$. Then, for any $v \in [\underline{v}, \overline{v})$, $E(\mathrm{D}; v) - E(\mathrm{B}; v) = -(B - D)v + Bp > -(B - D)\overline{v} + Bp \geq 0$. Hence, the optimality of the symmetric strategy implies $\hat{v}(p) = \overline{v}$. Moreover, it follows from the above inequality that the symmetric strategy with the cutoff $\hat{v}(p) = \overline{v}$ is optimal.

Case 4: $D \geq B$. Then, for any v, $E(\mathrm{D}; v) - E(\mathrm{B}; v) = (D - B)v + Bp > 0$. Hence, the optimality of the symmetric strategy implies $\hat{v}(p) = \overline{v}$. Moreover, it follows from the above inequality that the symmetric strategy with the cutoff $\hat{v}(p) = \overline{v}$ is optimal. □

A.1 Proof of Proposition 1: Equilibrium of the immediate system

We show that the following strategy profile is an equilibrium in the immediate system: (i) Scalper's application: When the scalpers enters the market with a price p, and n_b seekers buy the service, the number of applications by the scalper is $n_s(p, n_b) = m$; (ii) Scalper's pricing: the scalper sets the price p^* to maximize the profit Π as in footnote 10; (iii) Scalper's entry: If $\Pi(p^*) \geq 0$, the scalper enters the market. Otherwise, he does not; (iv) Each seeker follows the symmetric strategy with the following cutoff $\hat{v}(p)$

$$\hat{v}(p) = \begin{cases} \underline{v} & \text{if } p < \underline{v}, \\ p & \text{if } \underline{v} < p < \overline{v}, \\ \overline{v} & \text{if } \overline{v} < p. \end{cases} \tag{1}$$

The first claim is that the scalper's application stated above is optimal.

Lemma 2. *Consider the immediate system. Suppose that the scalper enters the market with price p, and n_b seekers buy the service. Then, the number of applications by the scalper, $n_s(p, n_b) = m$, is optimal.*

Proof. Let n_s be a number of applications by the scalper. Note that by definition, $0 \leq n_s \leq m$. We calculate the profit $\pi(n_s; p, n_b)$ in the two cases. If $m \leq n_b$, then the profit is $\pi(n_s; p, n_b) = pn_s - c$. On the other hand, if $n_b < m$, then the profit is

$$\pi(n_s; p, n_b) = \begin{cases} pn_s - c & \text{if } 0 \leq n_s \leq n_b, \\ pn_b - c & \text{if } n_b < n_s \leq m. \end{cases}$$

Thus, in any case, $n_s = m$ is optimal. □

The next claim is that the seeker's behavior stated above is optimal.

Lemma 3. *Consider the immediate system. Suppose that the scalper enters the market with price p and makes m applications for each n_b. Suppose that all seekers follow a symmetric strategy β. The symmetric strategy β is optimal for a seeker if and only if the cutoff $\hat{v}(p)$ is given by (1).*

Proof. Suppose that the symmetric strategy β is optimal. Then, by Lemma 1, it has the cutoff $\hat{v}(p)$. Denote $\hat{v} = \hat{v}(p)$. We consider the decision of a seeker of any type v. She knows her own valuation v and faces $(n-1)$ other seekers following strategy β. Then, denote by $D(\hat{v})$ and $B(\hat{v})$ the probabilities of her getting a slot from a direct application and from buying when the cutoff is $\hat{v}$. Since the scalper's number of applications is $n(p, n_b) = m$ for each n_b, we have $D(\hat{v}) = 0$ and

$$B(\hat{v}) \quad = \quad \begin{cases} \sum_{k=0}^{m-1} \binom{n-1}{k} F^{n-1-k}(\hat{v})(1-F(\hat{v}))^k + \sum_{k=m}^{n-1} \binom{n-1}{k} \frac{m}{k+1} F^{n-1-k}(\hat{v})(1-F(\hat{v}))^k & \text{if } m < n \\ 1 & \text{if } m \geq n, \end{cases}$$

where $B(\underline{v}) = \frac{m}{n}$ for $m < n$; $B(\underline{v}) = 1$ for $m \geq n$; and $B(\overline{v}) = 1$. Note $D(\hat{v}) < B(\hat{v})$ for any $\hat{v}$.
Case 1: $p < \underline{v}$. Then $E(\mathrm{D}, \underline{v}) - E(\mathrm{B}, \underline{v}) = -B(\hat{v})(\underline{v} - p) < 0$. Then, by Lemma 1-(1), $\hat{v} = \underline{v}$.
Case 2: $\underline{v} < p < \overline{v}$. Then $E(\mathrm{D}, \underline{v}) - E(\mathrm{B}, \underline{v}) = -B(\hat{v})(\underline{v} - p) > 0$ and $E(\mathrm{D}, \overline{v}) - E(\mathrm{B}, \overline{v}) = -B(\hat{v})(\overline{v} - p) < 0$. Thus, by Lemma 1-(2), $\hat{v}$ satisfies $E(\mathrm{D}, \hat{v}) - E(\mathrm{B}, \hat{v}) = -B(\hat{v})(\hat{v} - p) = 0$, i.e., $\hat{v} = p$.
Case 3: $\overline{v} < p$. Then $E(\mathrm{D}, \overline{v}) - E(\mathrm{B}, \overline{v}) = -B(\hat{v})(\overline{v} - p) > 0$. Then, by Lemma 1-(3), $\hat{v} = \overline{v}$.

Therefore, Lemma 3 follows from Lemma 1. □

Proof of Proposition 1. Given the scalper's application and the seeker's behavior as stated above, we can calculate the profit in the following two cases.
Case 1: $m \geq n$. If $p \leq \underline{v}$, $\Pi(p) = pn - c$; if $p > \overline{v}$, $\Pi(p) = -c$; if $\underline{v} \leq p \leq \overline{v}$, then

$$\Pi(p) = \sum_{k=0}^{n} \binom{n}{k} F^{n-k}(p)(1-F(p))^k pk - c.$$

Case 2: $m < n$. If $p \leq \underline{v}$, $\Pi(p) = pm - c$; if $p > \overline{v}$, $\Pi(p) = -c$; if $\underline{v} \leq p \leq \overline{v}$, then

$$\Pi(p) = \sum_{k=0}^{m} \binom{n}{k} F^{n-k}(p)(1-F(p))^k pk + \sum_{k=m+1}^{n} \binom{n}{k} F^{n-k}(p)(1-F(p))^k pm - c.$$

Since Π is continuous, and $\Pi(p) \leq \Pi(\underline{v})$ for $p \leq \underline{v}$ and $\Pi(p) < 0$ for $p > \overline{v}$, we can formulate as in footnote 10 where the existence of price p^* is guaranteed as it maximizes the continuous function on the compact set $[\underline{v}, \overline{v}]$.

Therefore, together with lemmas 2 and 3, the strategy profile stated in the beginning is an equilibrium. □

A.2 Proof of Proposition 2: Equilibria of the batch system

Consider the batch system. We need several lemmas.

Lemma 4. *Consider the batch system. Suppose that the scalper enters the market with price p, and n_b seekers buy the service. Then, the number of applications by the scalper, $n_s(p, n_b)$, is optimal if* *and only if*

$$n_s(p, n_b) = \begin{cases} \textit{any integer in } [0, Q] & \textit{if } n_b = 0, \\ n_b & \textit{if } n_b > 0. \end{cases} \tag{2}$$

Proof. Suppose that $n_s(p, n_b)$ is optimal for the scalper.

Case 1: $n_b = 0$. Then the scalper cannot sell any slot and thus the profit is $\pi(n_s, p, n_b) = -c$ for any $n_s \geq 0$. Thus, $n_s(p, n_b) \in [0, Q]$.

Case 2-(a): $n_b > 0$ and $m < n$. Note that as $n_b + n_d = n < Q$, we have $m - n_d < n_b$, and $n_b < Q$. Then we can calculate the profit as follows.

$$\pi(n_s, p, n_b) = \begin{cases} pn_s - c & \text{if } 0 \leq n_s \leq m - n_d, \\ p\frac{m}{n_s + n_d} n_s - c & \text{if } m - n_d \leq n_s \leq n_b, \\ p\frac{m}{n_s + n_d} n_b - c & \text{if } n_b \leq n_s \leq Q. \end{cases}$$

Thus, the profit $\pi(n_s, p, n_b)$ is strictly increasing in $n_s \in [0, n_b]$ and strictly decreasing in $n_s \in [n_b, Q]$. Thus, $n_s(p, n_b) = n_b$.

Case 2-(b): $n_b > 0$ and $n \leq m$. Note that as $n = n_b + n_d$, we have $n_b \leq m - n_d$. If $0 \leq n_s \leq m - n_d$, then $n_s + n_d \leq m$ and thus the scalper is certain to get all of his slots. If $m - n_d \leq n_s$, the scalper gets a slot with probability $\frac{m}{n_s + n_d}$. Thus, we have the profit:

$$\pi(n_s, p, n_b) = \begin{cases} pn_s - c & \text{if } 0 \leq n_s \leq n_b, \\ pn_b - c & \text{if } n_b \leq n_s \leq m - n_d, \\ p\frac{m}{n_s + n_d} n_b - c & \text{if } m - n_d \leq n_s \leq Q. \end{cases}$$

Thus, the profit $\pi(n_s, p, n_b)$ is strictly increasing in $n_s \in [0, n_b]$ and strictly decreasing in $n_s \in [n_b, Q]$. Therefore, $n_s(p, n_b) = n_b$.

Conversely, it follows from the above analysis that the number of applications, $n_s(p, n_b)$, satisfying (2) is optimal for the scalper. □

Lemma 5. *Consider the batch system. Suppose that the scalper enters the market with price p and all seekers follow a symmetric strategy β. The symmetric strategy β is optimal for a seeker if and only if the cutoff $\hat{v}(p)$ satisfies the following.*

1. *When the number of applications by the scalper is $n_s(p, n_b) = n_b$ for each n_b, we have $\hat{v}(p) = \overline{v}$.*

2. *Suppose that the number of applications by the scalper is $n_s(p, n_b) = Q$ for $n_b = 0$; and $n_s(p, n_b) = n_b$ for each $n_b > 0$.*

   (a) *If $\min\left\{1, \frac{m}{n}\right\} p < \left(\min\{1, \frac{m}{n}\} - \frac{m}{n+Q}\right) \overline{v}$, then $\hat{v}(p)$ is a unique solution in $(\underline{v}, \overline{v})$ to the equation $-\left(\min\left\{1, \frac{m}{n}\right\} - \frac{m}{n+Q}\right) xF^{n-1}(x) + \min\left\{1, \frac{m}{n}\right\} p = 0$.*

(b) *If* $\min\left\{1, \frac{m}{n}\right\} p \geq \left(\min\{1, \frac{m}{n}\} - \frac{m}{n+Q}\right) \overline{v}$, *then* $\hat{v}(p) = \overline{v}$, *i.e., all types apply directly.*

Proof. Suppose that the symmetric strategy β is optimal. Then, by Lemma 1, it has the cutoff $\hat{v}(p)$. We consider the decision of a seeker of any type v. She knows her valuation v and faces $(n-1)$ other seekers. Denote $\hat{v} = \hat{v}(p)$. Let $B(\hat{v})$ and $D(\hat{v})$ be the probabilities of a seeker getting a slot from buying and from applying directly, respectively.

Part (1). Denote by $\hat{n}_b$ and $\hat{n}_d$ the number of buyers and direct applicants among the $(n-1)$ other seekers, respectively. Note that $\hat{n}_b + \hat{n}_d + 1 = n$.

Case 1: $n \leq m$. Then, suppose that the seeker applies directly. The number of direct applications is $(\hat{n}_d + 1) = n - \hat{n}_b$, while the scalper's number of applications is $\hat{n}_b$ by our assumption. Thus, the total number of applications is $(n - \hat{n}_b) + \hat{n}_b = n \leq m$. Hence, the seeker gets a slot for sure and gets the utility of v. On the other hand, suppose that the seeker buys the service from the scalper. In this case the number of direct applications is $\hat{n}_d$, while the scalper's number of applications is $\hat{n}_b + 1$ by our assumption. Thus, the total number of applications is $\hat{n}_d + (\hat{n}_b + 1) = n \leq m$. Hence, she gets a slot for certain and gets the utility of $(v - p)$. Therefore, the direct application is optimal for her, i.e., $\hat{v} = \overline{v}$.

Case 2: $n > m$. If the seeker makes the direct application, the probability of getting a slot is $\frac{m}{(\hat{n}_d+1)+\hat{n}_b} = \frac{m}{n}$, and thus her expected utility is $\frac{m}{n}v$. On the other hand, if she buys, the probability of getting a slot is $\frac{m}{\hat{n}_d+(\hat{n}_b+1)} = \frac{m}{n}$ and thus her expected utility is $\frac{m}{n}v - p$. Thus, the direct application is optimal for her, i.e., $\hat{v} = \overline{v}$.

Conversely, by Lemma 1, we can verify that the symmetric strategy with this cutoff is optimal.

Part (2). We first find $\hat{v}$. Suppose that she buys. There is then at least one buyer and thus the scalper makes n_b applications for any $n_b \geq 1$. Thus

$$B(\hat{v}) = \begin{cases} 1 & \text{if } n \leq m, \\ \frac{m}{n} & \text{if } n > m. \end{cases}$$

On the other hand, we have

$$D(\hat{v}) = \begin{cases} F^{n-1}(\hat{v})\frac{m}{n+Q} + (1 - F^{n-1}(\hat{v})) & \text{if } n \leq m, \\ F^{n-1}(\hat{v})\frac{m}{n+Q} + (1 - F^{n-1}(\hat{v}))\,\frac{m}{n} & \text{if } n > m. \end{cases}$$

Note that $B(\hat{v}) \geq D(\hat{v})$. Let

$$\begin{aligned} g(\hat{v}) &= E(\mathrm{D}, \hat{v}) - E(\mathrm{B}, \hat{v}) = D(\hat{v})\hat{v} - B(\hat{v})(\hat{v} - p) \\ &= -\left(\min\left\{1, \frac{m}{n}\right\} - \frac{m}{n+Q}\right) F^{n-1}(\hat{v})\hat{v} + \min\left\{1, \frac{m}{n}\right\} p. \end{aligned}$$

Then g is strictly decreasing, $g(\underline{v}) = \min\left\{1, \frac{m}{m}\right\} > 0$, and

$$g(\overline{v}) = -\left(\min\left\{1, \frac{m}{n}\right\} - \frac{m}{n + n_s(p,0)}\right)\overline{v} + \min\left\{1, \frac{m}{n}\right\} p.$$

<u>Case 1:</u> $\min\left\{1, \frac{m}{n}\right\} p < \left(\min\{1, \frac{m}{n}\} - \frac{m}{n+Q}\right)\overline{v}$. Then $g(\overline{v}) < 0$. Moreover, since $g(\underline{v}) > 0$ and g is strictly decreasing, there is a candidate type, $v^* \in (\underline{v}, \overline{v})$ such that to $g(v^*) = 0$. Moreover, $g(\underline{v}) > 0 > g(\overline{v})$, i.e., $E(\mathrm{D}, \underline{v}) - E(\mathrm{B}, \underline{v}) > 0 > E(\mathrm{D}, \overline{v}) - E(\mathrm{B}, \overline{v})$. Thus, by Lemma 1-(2), the cutoff is $\hat{v} = v^*$.
<u>Case 2:</u> $\min\left\{1, \frac{m}{n}\right\} p \geq \left(\min\{1, \frac{m}{n}\} - \frac{m}{n+Q}\right)\overline{v}$. Then $g(\overline{v}) \geq 0$. Consider $\overline{v}$ as a candidate of $\hat{v}$. Then $D(\overline{v}) = \frac{m}{n+Q} < B(\overline{v}) = \min\{1, \frac{m}{n}\}$, and $g(\overline{v}) = E(\mathrm{D}, \overline{v}) - E(\mathrm{B}, \overline{v}) \geq 0$. Thus, by Lemma 1-(3), the cutoff is $\hat{v} = \overline{v}$.

Conversely, by Lemma 1, the symmetric strategy with the cutoff in each of the above is optimal.

□

Proof of the first half of Proposition 2. It is sufficient to show that the following strategy profile is an equilibrium: (1) the scalper does not enter the market; (2) the scalper makes n_b applications for each price p and each number n_b of buyers; (3) every seeker applies directly.

By Lemma 4, the scalper's number of applications, n_b, is optimal for any p and n_b. Given this, by Lemma 5-(1), for any p and n_b, it is optimal for any seeker to apply directly. Then the scalper's profit is $-c$ if he enters the market, and 0 if he does not. If he enters, since no seeker buys the service from the scalper, any price is optimal. Therefore, it is an equilibrium.

Proof of the second half of Proposition 2. Suppose on the contrary that there is a symmetric equilibrium on whose outcome path the scalper enters the market with some price p. Then, since $\Pi(p) \geq 0$, there are some types who buy the service. Thus, by Lemma 5-2(a), there is a type $\hat{v} \in (\underline{v}, \overline{v})$ such that

$$-\left(\min\left\{1, \frac{m}{n}\right\} - \frac{m}{n+Q}\right)\hat{v}F^{n-1}(\hat{v}) + \min\left\{1, \frac{m}{n}\right\} p = 0. \tag{3}$$

This equation implies

$$p < \overline{v}F^{n-1}(\hat{v}). \tag{4}$$

Since $\hat{v} \in (\underline{v}, \overline{v})$, for each $n_b > 0$, there is a positive probability of the event in which n_b seekers buy on the equilibrium path. For each such event, the scalper makes n_b applications on the equilibrium path by Lemma 4. Thus, we can calculate the expected revenue, $R(p)$, as follows.

$$R(p) = \begin{cases} \sum_{k=1}^{n} \binom{n}{k} F^{n-k}(\hat{v})(1 - F(\hat{v}))^k pk & \text{if } n \leq m, \\ \sum_{k=1}^{m} \binom{n}{k} F^{n-k}(\hat{v})(1 - F(\hat{v}))^k \frac{m}{n} pk + \sum_{k=m+1}^{n} \binom{n}{k} F^{n-k}(\hat{v})(1 - F(\hat{v}))^k \frac{m}{n} pm & \text{if } n > m. \end{cases} \tag{5}$$

Thus

$$R(p) \quad \leq \quad \sum_{k=1}^{n} \binom{n}{k} F^{n-k}(\hat{v})(1-F(\hat{v}))^k \frac{m}{n} kp < \sum_{k=1}^{n} \binom{n}{k} F^{2n-k-1}(\hat{v})(1-F(\hat{v}))^k \frac{m}{n} k\overline{v} \tag{6}$$

where the first inequality follows from (5) and the second follows from (4). We use the next claim.

Claim 1. Let $S_k(x) = x^{2n-k-1}(1-x)^k$ be the real-valued function on $[0,1]$. There is a unique element $x^* = \frac{2n-k-1}{2n-1} \in (0,1)$ that maximizes S_k.

Proof of Claim 1. It is sufficient to show that x^* is a unique local maximizer of S_k, since the value of S_k is larger than that of S_k at the boundary points 0 and 1, i.e., $S_k(x^*) > 0 = S_k(0) = S_k(1)$. Thus, we need to show that $S'_k(x^*) = 0$ and $S''_k(x^*) < 0$.

First, we have $S'_k(x) = (2n-k-1)x^{2n-k-2}(1-x)^k - k(1-x)^{k-1}x^{2n-k-1} = 0$. Thus $(2n-k-1)(1-x) - kx = 0$. Thus,

$$x^* = \frac{2n-k-1}{2n-1}. \tag{7}$$

Note that since $1 \leq k \leq n$, we have $x^* \in (0,1)$ and is a unique element that satisfies $S'_k(x^*) = 0$. It remains to be shown that $S''(x^*) < 0$.

$$S''_k(x) = (2n-k-1)(2n-k-2)x^{2n-k-3}(1-x)^k - 2k(2n-k-1)(1-x)^{k-1}x^{2n-k-2} + k(k-1)(1-x)^{k-2}x^{2n-k-1}$$

Divide it by $x^{2n-k-3}(1-x)^{k-2} > 0$.

$$\begin{aligned} \frac{S''_k(x)}{x^{2n-k-3}(1-x)^{k-2}} &= (2n-k-1)(2n-k-2)(1-x)^2 - 2k(2n-k-1)(1-x)x + k(k-1)x^2 \\ &= (2n-k-1)(1-x)((2n-k-2)(1-x) - 2kx) + k(k-1)x^2 \end{aligned}$$

Then inserting $x^* = \frac{2n-k-1}{2n-1}$, we get

$$\frac{S''_k(x^*)}{(x^*)^{2n-k-3}(1-x^*)^{k-2}} = -\frac{k(2n-k-1)}{2n-1} < 0.$$

Since $(x^*)^{2n-k-3}(1-x^*)^{k-2} > 0$, we get $S''_k(x^*) < 0$. This completes the proof of Claim 1.

Thus, by (6) and Claim 1,

$$\begin{aligned} R(p) &\leq \sum_{k=1}^{n} \binom{n}{k} \left(\frac{2n-k-1}{2n-1}\right)^{2n-k-1} \left(\frac{k}{2n-1}\right)^k \frac{m}{n} k\overline{v} \qquad (\because (6) \text{ and Claim 1}) \\ &\leq \sum_{k=1}^{n} \binom{n}{k} \left(\frac{2n-k-1}{2n-1}\right)^{2n-k-1} \left(\frac{k}{2n-1}\right)^k \frac{\max\{n-1, m\}}{n} k\overline{v} \quad (\because m < n) \end{aligned}$$

Thus, the right-hand side of the last inequality is R in Proposition 2. Therefore, by our assumption, $\Pi(p) \leq R - c < 0$. Hence, on the equilibrium path, the scalper should not enter the market. This is a contradiction. $\square$

Proposition 3. *Under the batch booking system, the following is a symmetric equilibrium:*

1. *Scalper's application: When the scalper enters the market with a price* p *and* n_b *seekers buy the service, the scalper's number of applications is* $n_s(p, n_b) = Q$ *applications for* $n_b = 0$*, and* $n_s(p, n_b) = n_b$ *for* $n_b > 0$*.*

2. *Each seeker follows the symmetric strategy* β *according to Lemma 5-(2) for any* p*.*

3. *Scalper's pricing and entry: Let* p^* *be the price that maximizes the following profit* $\Pi(p)$ *in the compact set* $\{p \in \mathcal{P} \mid \min\left\{1, \frac{m}{n}\right\} p \leq \left(\min\{1, \frac{m}{n}\} - \frac{m}{n+Q}\right) \overline{v}\}$*.*

$$\Pi(p) = \begin{cases} \sum_{k=1}^{m} \binom{n}{k} F^{n-k}(\hat{v}(p))(1 - F(\hat{v}(p)))^k kp - c & \text{if } n \leq m, \\ \sum_{k=1}^{m} \binom{n}{k} F^{n-k}(\hat{v}(p))(1 - F(\hat{v}(p)))^k) \frac{m}{n} kp & \\ \quad + \sum_{k=m+1}^{n} \binom{n}{k} F^{n-k}(\hat{v}(p))(1 - F(\hat{v}(p)))^k) \frac{m}{n} mp - c & \text{if } n > m. \end{cases} \tag{8}$$

When $\Pi(p^*) \geq 0$*, the scalper enters the market with price* p^**. Otherwise, he does not enter the market.*

Proof of Proposition 3. We verify that the strategy profile specified in the proposition is an equilibrium. By Lemma 4, the scalper's application is optimal for the scalper. Moreover, by Lemma 5-(2), the seeker's behavior is optimal for a seeker. Finally, we consider the scalper's pricing and entry. When the scalper enters the market, given the behavior shown above, the profit $\Pi(p)$ of the scalper in setting price p is as follows: If $\min\left\{1, \frac{m}{n}\right\} p \geq \left(\min\{1, \frac{m}{n}\} - \frac{m}{n+Q}\right) \overline{v}$, then $\Pi(p) = -c$. If $\min\left\{1, \frac{m}{n}\right\} p < \left(\min\{1, \frac{m}{n}\} - \frac{m}{n+Q}\right) \overline{v}$, then $\Pi(p)$ is given as (8). Note that $\Pi(p)$ is continuous on the compact set $\{p \in \mathcal{P} \mid \min\left\{1, \frac{m}{n}\right\} p \leq \left(\min\{1, \frac{m}{n}\} - \frac{m}{n+Q}\right) \overline{v}\}$, there is a price, denoted by p^*, that maximizes Π in the compact set. If $\Pi(p^*) \geq 0$, the scalper enters the market. Otherwise, he does not. Therefore, the strategy profile stated above is an equilibrium. □

A.3 On the upper bound R of the scalper's revenue

Figure A.1 presents a graph of the upper bound of the revenues of the scalper for the case of excess demand and $\bar{v} = 1$. In the case of excess demand the upper bound R depends only on n. Note that the relationship between the revenues and the entry cost determines whether non-entry of the scalper is the unique equilibrium outcome of the batch system. It is evident from the graph that the function is bounded from above and cannot exceed $0.55\overline{v}$. Thus, whenever the cost of entry is higher than $0.55\overline{v}$, the batch system has a unique equilibrium outcome where the scalper does not enter the market.

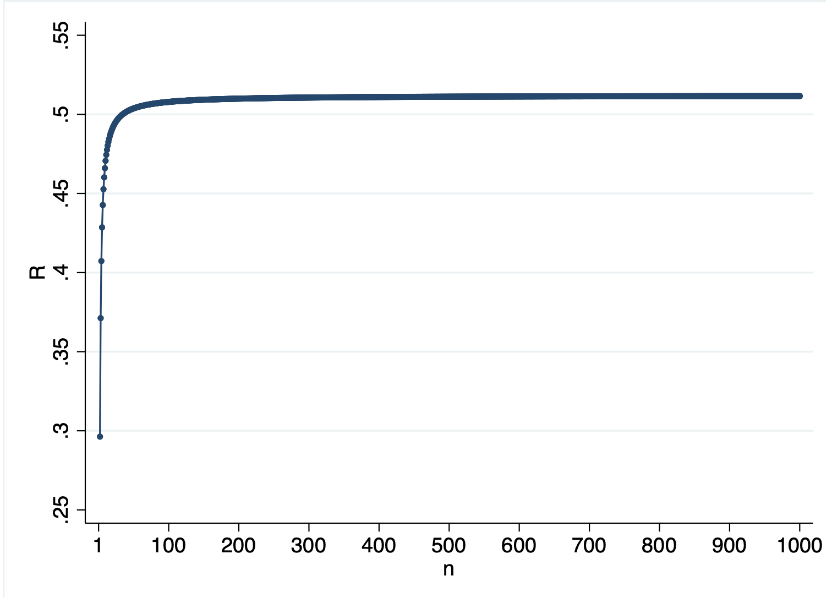

Figure A.1: Upper bound of revenues of scalper in the batch system given $\bar{v} = 1$, depending on n in the case of excess demand

B Robustness to Booking Deposits (for online appendix)

We study the robustness of the equilibria of the two booking systems to the introduction of a booking deposit. A booking deposit is often suggested as a remedy to fight scalping in the immediate system. It has to be paid when a slot is assigned and is reimbursed at the time of the appointment. We model the deposit as a cost $\epsilon > 0$ that the scalper has to pay for each slot that is assigned to a fake ID. The reason is that a deposit only creates a real cost when a booked slot is not used, e.g., when bookings are made under fake names. Thus, the slots sold by the scalper and the slots that are booked directly by the seekers do not come with a cost since the deposit is returned. We therefore ignore these payments in the model.[1]

B.1 The immediate system with a booking deposit

The model of the immediate system captures the possibility to cancel and re-book slots with the help of the simplifying assumption that appointment seekers buy from the scalper before he makes the bookings. This shortcut renders booking deposits toothless, since it is exactly the cancellations and re-bookings that are costly under the policy of booking deposits. In this section, we therefore consider the case where all slots are first booked under fake names and are then sold to the clients.[2] For each slot, the scalper has to pay the deposit $\epsilon > 0$, which will be lost once the slot is re-booked

[1]Strictly speaking, seekers pay ϵ at the moment of being assigned a slot directly, or pay $p + \epsilon$ when booking through the scalper, and they receive this deposit of ϵ back at the moment of the appointment. Importantly, ϵ is only requested from those who are assigned a slot, not from all applicants.

[2]Note that this represents the worst case for the scalper when a booking deposit has to be paid. We bias the modeling of the deposit against the scalper in the immediate system, to show that scalping can still be profitable.

on the name of a client. This case is extreme in the sense that deposits have the strongest possible effect on scalpers. While the payoff function of seekers remains unchanged, we have

$$\begin{array}{l}\text{the scalper's payoff}\\ \text{in the immediate system}\end{array} = \begin{cases} m'p - n_s\epsilon - c & \text{if he sells } m' \text{ slots to seekers} \\ & \qquad \text{and makes } n_s \text{ bookings,} \\ 0 & \text{if he is not active.} \end{cases}$$

As shown in Proposition 1, in the equilibrium without a booking deposit, the scalper can be active even when there is no excess demand, that is, when $m \geq n$. By booking all available slots independent of the number of buyers, the scalper can ensure that the probability for an appointment seeker to obtain a slot is zero if she does not buy from the scalper. With a booking deposit $\epsilon > 0$, blocking slots without having a buyer for them is no longer credible, as this cost has to be paid after customers had the opportunity to buy. Thus, the scalper will not block more slots than the number of buyers. Since the appointment seekers no longer believe that the scalper will block all slots and because there are more slots than appointment seekers, the appointment seekers expect to receive a slot with certainty, regardless of whether they buy or apply directly. Hence, they prefer to apply directly for any positive price charged by the scalper. Thus, the equilibrium of Proposition 1 is not robust to the introduction of a deposit in the absence of excess demand.

With excess demand, that is, $m < n$, the scalper can profitably offer his service. The reason is that he can increase a seeker's probability of receiving a slot compared to the seeker applying directly. The higher the excess demand, the higher the increase in the probability of a seeker receiving a slot through the scalper. If the price of the scalper's service is below the expected benefit from the increased probability of receiving the slot, the seekers are willing to pay for his service.

Proposition 4 (Immediate system with booking deposit)**.** *Consider the immediate booking system with booking deposit $\epsilon > 0$.*

1. *In the case of excess supply ($m \geq n$), there exists a unique equilibrium outcome where the scalper does not enter the market.*
2. *In the case of excess demand ($m < n$),*
   (a) *if $\underline{v} \leq \left(1 - \frac{m}{n}\right) \overline{v}$, there are market parameters, such that the scalper enters the market in a symmetric equilibrium;*
   (b) *otherwise, when $m\underline{v} < c$, there is a unique symmetric equilibrium outcome where the scalper does not enter the market.*

The conditions under which scalping is robust to the booking deposit are excess demand and a condition regarding the support of the valuations, $\underline{v} \leq \left(1 - \frac{m}{n}\right) \overline{v}$. This condition states that the larger the support of the distribution of valuations, the more likely the scalper can make positive profits. A seeker's application decision depends on the increase in the probability of receiving a slot by applying through the scalper and the difference between her valuation and the price. The larger the support of the valuations, the larger the expected difference between the valuation and the price. The larger the excess demand, the less demanding the condition regarding the support of the seekers' valuations. This is because even with a small difference between a seeker's valuation and the price, the scalper can offer a higher increase in the probability of receiving the slot when excess demand is higher.

Example. Again, consider a market with 20 seekers competing for 15 slots where the valuations of the seekers are uniformly distributed on the interval [10,100] and the scalper can enter the market for an entry cost of 100. Now the authority requests a booking deposit of 2, which is reimbursed at the time of the appointment. In equilibrium with excess demand, the scalper enters the market and sets the profit-maximizing price of 28. All seekers with valuations above 35.5 buy his service. (To see this, consider a seeker with a valuation of 36. Given the cutoff of 35.5, her chance of receiving a slot is 21% if she applies directly and 97% if she buys the scalper's service. This increase in probability makes it profitable to pay for the scalper's service, since $0.21 \cdot 36 < 0.97 \cdot 8$.) In equilibrium, the expected profit of the scalper is 259, and the expected number of slots sold is around 14. Thus, direct applicants only have a small chance of getting a slot. Overall, the booking deposit eliminates the deadweight loss of unassigned slots but does not preclude the scalper from being active in the market.

Just as in the model without a booking deposit, our setup regarding the order of moves is disadvantageous from the scalper's perspective. In particular, the relevance of excess demand depends on the order of moves. If the scalper books slots before the appointment seekers can buy his services, then blocking slots can be credible even with a deposit fee. Blocking slots before the seekers arrive allows the scalper to affect the seekers' behavior. Thus, even slots that will not be sold can increase the scalper's profit because they raise the appointment seekers' demand for slots. The necessity of excess demand to make the market entry of the scalper optimal in the presence of an arbitrarily small deposit of ϵ, is driven by the modeling choice of a one-period interaction and the order of moves. Thus, scalping in the immediate system is profitable in equilibrium even under conditions that are worse than those typically encountered in reality.

Overall, we find that the equilibrium where the scalper profitably enters the market is robust to the introduction of a booking deposit for a set of parameters.

B.2 The batch system with a booking deposit

In the batch system, the scalper has to pay the deposit for every successful fake application. Thus, he has to pay the deposit for every application in excess of the number of buyers, and he can no longer costlessly flood the market with fake applications.

While the payoff function of seekers remains unchanged, we have

$$\begin{array}{c}\text{the scalper's payoff}\\ \text{in the batch system}\end{array} = \begin{cases} m'p - (n'-m')\epsilon - c & \text{if he sells } m' \text{ slots to seekers} \\ & \quad \text{and receives in total } n' \text{ slots,} \\ 0 & \text{if he is not active.} \end{cases}$$

The consequence of a booking deposit in the batch system is that the strategy of flooding the market in the case of no buyers is no longer credible, since flooding the market is costly. Thus, the booking deposit for applications means that the scalper will no longer decide to flood the system on the equilibrium path. This is the intuition behind the following proposition:

Proposition 5 (Batch system with booking deposit)**.** *Under the batch booking system with a booking deposit $\epsilon > 0$, there is a unique equilibrium outcome where the scalper does not enter the market.*

Thus, when the deposit is introduced, the scalper does not enter the market in any equilibrium, i.e., the black market disappears. Moreover, the scalper not entering the market is robust against booking deposits.

Example. Again, consider a market with 20 seekers competing for 15 slots where the valuations of the seekers are uniformly distributed on the interval [10,100], and the scalper can enter the market for an entry cost of 100. The authority requests a booking deposit of 2, which is reimbursed at the time of the appointment. According to proposition 5, the scalper not entering the market is the unique equilibrium outcome. Consider, for example, that the scalper enters the market. Whenever he sets a positive price, seekers prefer to apply directly, as they have the same probability of getting the slot as when they apply through the scalper, because in all cases the scalper will submit as many applications as the number of buyers (even if it is zero), since submitting fake applications is costly for the scalper. Thus, the scalper will have zero buyers, and will incur a loss of 100.

B.3 Proofs

Note that Lemma 1 still holds when booking deposits are in place.

B.3.1 Proof of Proposition 4: Equilibrium of the immediate system

We extend Lemma 2 for the number of applications by the scalper as follows.

Lemma 6. *Consider the immediate system with a booking deposit $\epsilon > 0$. Suppose that the scalper enters the market with price $p > \epsilon$ and n_b seekers buy the service. Then, the number of applications* *by the scalper, $n_s(p, n_b)$, is optimal if and only if*

$$n_s(p, n_b) = \begin{cases} n_b & \textit{if } n_b < m, \\ m & \textit{if } m \leq n_b. \end{cases} \tag{9}$$

Proof. Suppose that $n_s(p, n_b)$ is optimal. Denote the number of applications by the scalper by n_s, and the profit from n_s applications by $\pi(n_s; p, n_b)$. Note that by definition, $0 \leq n_s \leq m$. We calculate the profit $\pi(n_s; p, n_b)$ in the two cases.
Case 1: $n_b < m$. Then the profit can be calculated as

$$\pi(n_s; p, n_b) = \begin{cases} pn_s - \epsilon n_s - c & \text{if } 0 \leq n_s \leq n_b, \\ pn_b - \epsilon n_s - c & \text{if } n_b < n_s \leq m. \end{cases}$$

Thus, we have $n_s(p, n_b) = n_b$.
Case 2: $m \leq n_b$. Then the profit can be calculated as $\pi(n_s; p, n_b) = p_{n_s} - \epsilon n_s - c$. Thus, $n_s(p, n_b) = m$.

Conversely, it follows from the above analysis that the number of applications, $n_s(p, n_b)$, satisfying (9) is optimal for the scalper. □

We extend Lemma 3 for seekers' behavior as follows.

Lemma 7. *Consider the immediate system with a booking deposit $\epsilon > 0$. Suppose that the scalper enters the market with price p, and all seekers follow the symmetric strategy β. The symmetric strategy β is optimal for a seeker if and only if the cutoff $\hat{v}(p)$ satisfies the following.*

1. With excess demand $(m < n)$ the number of applications by the scalper is $n(p, n_b) = n_b$ for each $n_b \leq m$, and $n_b(p, n_b) = m$ for $m > n_b$.

   (a) If $\underline{v} < \left(1 - \frac{m}{n}\right) \overline{v}$,

   $$\hat{v}(p) = \begin{cases} \underline{v} & \text{if } p < \underline{v}, \\ \text{some } \hat{v} \in (\underline{v}, \overline{v}) & \text{if } \underline{v} < p < \left(1 - \frac{m}{n}\right) \overline{v}, \\ \overline{v} & \text{if } \left(1 - \frac{m}{n}\right) \overline{v} < p. \end{cases}$$

   (b) If $\left(1 - \frac{m}{n}\right) \overline{v} < \underline{v}$

   $$\hat{v}(p) = \begin{cases} \underline{v} & \text{if } p \leq \left(1 - \frac{m}{n}\right) \overline{v}, \\ \overline{v} & \text{if } \left(1 - \frac{m}{n}\right) \overline{v} \leq p. \end{cases}$$

2. With excess supply $(n \leq m)$ the number of applications by the scalper is $n_s(p, n_b) = n_b$ for each n_b, and we have $\hat{v}(p) = \overline{v}$.

Proof. Suppose that the symmetric strategy β is optimal. Then, by Lemma 1, it has the cutoff $\hat{v}(p)$. Denote $\hat{v} = \hat{v}(p)$. We consider the decision of a seeker of any type v. She knows her own valuation v and faces $(n-1)$ other seekers following strategy β. Then, denote by $D(\hat{v})$ and $B(\hat{v})$ the probabilities of her getting a slot from a direct application and from buying when the cutoff is $\hat{v}(p)$.

Part (1). With excess demand $(m < n)$, the number of applications by the scalper is $n(p, n_b) = n_b$ for each $n_b \leq m$, and $n_b(p, n_b) = m$ for $m > n_b$. We have

$$B(\hat{v}) \quad = \quad \begin{cases} \sum_{k=0}^{m-1} \binom{n-1}{k} F^{n-1-k}(\hat{v})(1-F(\hat{v}))^k + \sum_{k=m}^{n-1} \binom{n-1}{k} \frac{m}{k+1} F^{n-1-k}(\hat{v})(1-F(\hat{v}))^k & \text{if } m < n \\ 1 & \text{if } m \geq n, \end{cases}$$

where $B(\underline{v}) = \frac{m}{n}$ for $m < n$; $B(\underline{v}) = 1$ for $m \geq n$; and $B(\overline{v}) = 1$.

$$D(\hat{v}) = \sum_{k=0}^{m-1} \binom{n-1}{k} \frac{m-k}{n-k} F^{n-1-k}(\hat{v})(1-F(\hat{v}))^k.$$

where $D(\underline{v}) = 0$ and $D(\overline{v}) = \frac{m}{n}$. Note that $D(\hat{v}) < B(\hat{v})$.

Part 1-(a). Suppose $\underline{v} < \left(1 - \frac{m}{n}\right)\overline{v}$, and suppose that β is optimal for a seeker.

<u>Case 1:</u> $p < \underline{v}$. Then $E(\mathrm{D}, \underline{v}) - E(\mathrm{B}, \underline{v}) = -B(\hat{v})(\underline{v} - p) < 0$. Thus, by Lemma 1-(1), $\hat{v} = \underline{v}$.

<u>Case 2:</u> $\underline{v} < p < \left(1 - \frac{m}{n}\right)\overline{v}$. We first find $\hat{v}$. Let $g(\hat{v}, p) = E(\mathrm{D}, \hat{v}) - E(\mathrm{B}, \hat{v}) = D(\hat{v})\hat{v} - B(\hat{v})(\hat{v} - p)$. Note that $g(\underline{v}, p) = -\frac{m}{n}(\underline{v} - p) > 0$ and $g(\overline{v}, p) = -\left(1 - \frac{m}{n}\right)\overline{v} + p < 0$. Since g is a continuous function, there is a candidate type $v^* \in (\underline{v}, \overline{v})$ such that $g(v^*, p) = 0$.

Then $E(\mathrm{D}, \underline{v}) - E(\mathrm{B}, \underline{v}) = D(v^*)\underline{v} - B(v^*)(\underline{v} - p) > 0$ and $E(\mathrm{D}, v^*) - E(\mathrm{B}, v^*) = g(v^*, p) = 0$. Thus, since the function $E(\mathrm{D}, v) - E(\mathrm{B}, v) = -(B(\hat{v}) - D(\hat{v}))v + B(\hat{v})p$ is strictly decreasing, we have $E(\mathrm{D}, \underline{v}) - E(\mathrm{B}, \underline{v}) < 0$. Therefore, by Lemma 1-(2), the cutoff is $\hat{v} = v^*$.

<u>Case 3:</u> $\left(1 - \frac{m}{n}\right)\overline{v} < p$. Consider $v^* = \overline{v}$ as a candidate of $\hat{v}$. Then $D(\hat{v}) = \frac{m}{n} < B(\hat{v}) = 1$. Thus $E(\mathrm{D}, \overline{v}) - E(\mathrm{B}, \overline{v}) = \frac{m}{n}\overline{v} - (\overline{v} - p) > 0$. Then, by Lemma 1-(3), the cutoff is $\hat{v} = v^* = \overline{v}$.

Conversely, by Lemma 1, the symmetric strategy with the cutoff in each of the above is optimal.

Part 1-(b). Suppose $\left(1 - \frac{m}{n}\right)\overline{v} \leq \underline{v}$. Suppose that β is optimal for a seeker.

<u>Case 1:</u> $p < \left(1 - \frac{m}{n}\right)\overline{v}$. Consider $\underline{v}$ as a candidate of $\hat{v}$. Then $E(\mathrm{D}, \underline{v}) - E(\mathrm{B}, \underline{v}) = D(\underline{v})\underline{v} - B(\underline{v})(\underline{v} - p) = -\frac{m}{n}(\underline{v} - p) < 0$. Thus, by Lemma 1-(1), the cutoff is $\hat{v} = \underline{v}$.

<u>Case 2:</u> $\left(1 - \frac{m}{n}\right)\overline{v} < p$. Consider $\overline{v}$ as a candidate of $\hat{v}$. Then $E(\mathrm{D}, \overline{v}) - E(\mathrm{B}, \overline{v}) = D(\overline{v})\overline{v} - B(\overline{v})(\overline{v} - p) = \frac{m}{n}\overline{v} - (\overline{v} - p) = p - (1 - \frac{m}{n})\overline{v} > 0$. Thus, by Lemma 1-(3), the cutoff is $\hat{v} = \overline{v}$.

Conversely, by Lemma 1, the symmetric strategy with the cutoff in each of the above is optimal.

Part (2). Under the excess supply with $n \leq m$, the scalper's number of applications is $n_s(p, n_b) = n_b$ for each n_b. Consider $\overline{v}$ as a candidate of $\hat{v}$. Then $D(\overline{v}) = B(\overline{v}) = 1$. Thus $E(\mathrm{D}, \overline{v}) - E(\mathrm{B}, \overline{v}) = \overline{v} - (\overline{v} - p) = p > 0$. Thus, by Lemma 1-(3), the cutoff is $\hat{v} = \overline{v}$. Conversely, by Lemma 1, the symmetric strategy with this cutoff is optimal. $\square$

Lemma 8. *In the immediate system, the following strategy profile is a symmetric equilibrium.*

1. *Scalper's application: When the scalper enters the market with a price p and n_b seekers buy the service, the scalper makes $n_s(p, n_b)$ applications according to Lemma 6.*

2. *Scalper's pricing: When the scalper enters the market, he sets the price p^* that maximizes the profit*
$$\Pi(p) = \sum_{k=0}^{n} \binom{n}{k} F^{n-k}(\hat{v}(p))(1 - F(\hat{v}(p)))^k (p - \epsilon) n_s(p, k) - c.$$

3. *Scalper's entry: If $\Pi(p^*) \geq 0$, the scalper enters the market. Otherwise, he does not.*

4. *Each seeker follows the symmetric strategy with the cutoff type $\hat{v}(p)$ according to Lemma 3.*

Proof. This is straightforward. □

Proof of Proposition 4-(1). Note that this proposition is for $m \geq n$. For the proof, we specify the strategies as required by Lemma 8. For the scalper's application, as in Lemma 6, since we always have $n_b \leq m$ (due to the assumption of $n \leq m$), the scalper makes n_b applications for any number of buyers, n_b. By Lemma 7-(2), all seekers apply directly for any price p. Thus the profit $\Pi(p) = -c$ for any price p and thus $\Pi(p^*) < 0$ where p^* is a maximizing price. Thus, the scalper does not enter the market. Hence, by Lemma 8, these strategies constitute an equilibrium.

To show the uniqueness of symmetric equilibrium outcomes, suppose that there is another symmetric equilibrium such that on its equilibrium path the scalper enters the market with some price p. Note that the scalper enters the market and a seeker uses a symmetric strategy. Thus, there is some set of types with a positive measure in which each type in the set buys the service. Then, since type v_i is independent, there is a positive probability for each event that any number of seekers will buy the service. Thus, by Lemma 6, since $n_b \leq m$ due to our assumption of $n \leq m$, the scalper makes n_b applications for any number of buyers, n_b. Then, on the equilibrium path, all seekers apply directly by Lemma 7-(2), and thus the profit of the scalper is negative, and he does not enter the market. This is a contradiction. □

Proof of Proposition 4-(2). Part (a) follows from the example right after Proposition 4. For Part (b), note that this proposition is for $m < n$, $\epsilon > 0$, and $\underline{v} > \left(1 - \frac{m}{n}\right) \overline{v}$. For the proof, we specify the strategies as in Lemma 8. For the scalper's application, as in Lemma 6, the scalper makes n_b applications when n_b ($\leq m$) seekers buy and makes m applications when n_b ($> m$) seekers buy. By Lemma 7-(1b), when $p \leq \left(1 - \frac{m}{n}\right) \overline{v}$, all seekers buy the service; when $\left(1 - \frac{m}{n}\right) \overline{v} < p$, all seekers make direct applications. To show that these strategies and the scalper being inactive constitute an equilibrium, by Lemma 8, it is sufficient to check whether the scalper makes a loss. Now the profit is

$$\Pi(p) = \begin{cases} (p-\epsilon)m - c & \text{if } p \leq \left(1-\frac{m}{n}\right)\overline{v}, \\ -c & \text{if } p > \left(1-\frac{m}{n}\right)\overline{v}. \end{cases}$$

Thus, $\Pi(p)$ is increasing up to $\left(1-\frac{m}{n}\right)\overline{v}$, and then becomes constant with $-c$. And

$$\begin{aligned} \Pi\left(\left(1-\frac{m}{n}\right)\overline{v}\right) &= \left(1-\frac{m}{n}\right)\overline{v}m - \epsilon m - c \\ &< m\underline{v} - \epsilon m - c \quad \left(\because \left(1-\frac{m}{n}\right)\overline{v} < \underline{v}\right) \\ &< 0 \quad (\because m\underline{v} < c) \end{aligned}$$

Hence, $\Pi(p^*) < 0$. Thus, the scalper does not enter the market.

To show the uniqueness of symmetric equilibrium outcomes, suppose that there is another equilibrium such that on its equilibrium path, the scalper enters the market with some price p. Thus, there is some set of types with a positive measure, and each type in the set buys the service. Then, since type v_i is drawn independently, there is a positive probability that any number of seekers buy the service. Thus, by Lemma 6, for any number of buyers, n_b, the scalper makes n_b applications for $n_b \leq m$, and makes m applications for $m \leq n_b$. Then, following the discussion in the previous paragraph, we can show that $\Pi(p) < 0$. Thus, the scalper should not enter the market in the equilibrium, a contradiction. □

B.3.2 Proof of Proposition 5: Equilibrium of the batch system

We extend Lemma 4 regarding the number of applications by the scalper as follows.

Lemma 9. *Consider the batch system with a booking deposit. Suppose that the scalper enters the market with price $p > \epsilon$ and n_b seekers buy the service. Then the number of applications by the scalper, $n_s(p, n_b)$, is optimal if and only if $n_s(p, n_b) = n_b$.*

Proof. Suppose that $n_s(p, n_b)$ is optimal for the scalper.
Case 1: $m < n$. Note that as $n_b + n_d = n < Q$, we have $m - n_d < n_b$, and $n_b < Q$. Then we can calculate the profit as follows:

$$\pi(n_s, p, n_b) = \begin{cases} pn_s - c & \text{if } 0 \leq n_s \leq m - n_d, \\ p\frac{m}{n_s+n_d}n_s - c & \text{if } m - n_d \leq n_s \leq n_b, \\ p\frac{m}{n_s+n_d}n_b - \epsilon\frac{m}{n_s+n_d}(n_s - n_b) - c & \text{if } n_b \leq n_s \leq Q. \end{cases}$$

Thus, the profit $\pi(n_s, p, n_b)$ is continuous in n_s, strictly increasing in $n_s \in [0, n_b]$, and strictly decreasing in $n_s \in [n_b, Q]$. Therefore, $n_s(p, n_b) = n_b$.

Case 2: $n \leq m$. Note that as $n = n_b + n_d$, we have $n_b \leq m - n_d$. If $0 \leq n_s \leq m - n_d$, then $n_s + n_d \leq m$ and thus the scalper gets all of his slots for sure. If $m - n_d \leq n_s$, the scalper gets a slot with probability $\frac{m}{n_s+n_d}$. Thus, the scalper's profit is

$$\pi(n_s, p, n_b) = \begin{cases} pn_s - c & \text{if } 0 \leq n_s \leq n_b, \\ pn_b - \epsilon(n_s - n_b) - c & \text{if } n_b \leq n_s \leq m - n_d, \\ p\frac{m}{n_s+n_d}n_b - \epsilon\frac{m}{n_s+n_d}(n_s - n_b) - c & \text{if } m - n_d \leq n_s \leq Q. \end{cases}$$

The profit $\pi(n_s, p, n_b)$ is continuous in n_s, strictly increasing in $n_s \in [0, n_b]$, and strictly decreasing in $n_s \in [n_b, Q]$. Thus, $n_s(p, n_b) = n_b$.

Conversely, it follows from the above analysis that the number of applications, $n_s(p, n_b) = n_b$ is optimal for the scalper. □

Note that Lemma 5 still holds when booking deposits are required.

Proof of Proposition 5. It is sufficient to show that the following strategy profile is an equilibrium: (i) the scalper does not enter the market; (ii) the scalper makes n_b applications for each price p and each number n_b of buyers; (iii) every seeker applies directly. By Lemma 9, the scalper's number of applications, n_b, is optimal for any p and n_b. Given this, by Lemma 5-(1), for any p and n_b, it is optimal for any seeker to apply directly. Then the scalper's profit is $-c$ if he enters the market and 0 if he does not. In case he enters, since no seeker buys the service from the scalper, any price is optimal. Therefore, it is an equilibrium.

To show the uniqueness of equilibrium outcomes, suppose on the contrary that there is a symmetric equilibrium where on the equilibrium path, the scalper enters the market. Note that the scalper makes a nonnegative profit, and seekers use a symmetric strategy. Thus, there exists a set of types with a positive measure in which each type in the set buys the service. Then, since types are drawn independently, there is a positive probability for each event with any number of seekers, including zero, buying the service. Then, by Lemma 9, the scalper makes n_b applications for any number n_b of buyers. Thus, by Lemma 5-(1), every seeker applies directly in any event. Thus, the scalper makes a negative profit of $-c$ and does not enter the market. This is a contradiction. □

C The Experiment

C.1 Instructions

Welcome to this experiment about decision-making. You and the other participants in the experiment will be asked to make a number of decisions. In this situation, you can earn money that will be paid out to you in cash at the end of the experiment. How much you will earn depends on the decisions that you and the other participants make. These instructions describe the situation in which you have to make a decision in detail. Note that the instructions are identical for all participants in the experiment. It is very important that you read the instructions carefully so that you understand the decision-making problem well. If you have any questions, please let us know by raising your hand. We will then answer your questions individually. Please do not, under any circumstances, ask your questions aloud. You are also not permitted to give information of any kind to other participants. Please do not speak to other participants at any time throughout this experiment. Whenever you have a question, please raise your hand and we will come to you in order to help you. If you break these rules, we may have to terminate the experiment. Once everyone has read the instructions and there are no further questions, we will conduct a short quiz where each of you will complete some tasks on your own. Afterwards, we will come to you and check your answers, and solve any remaining problems. The only purpose of the quiz is to ensure that you thoroughly understand the crucial details of the decision-making problem. Your anonymity and the anonymity of the other participants will be guaranteed throughout the entire experiment. You will neither learn about the identity of the other participants, nor will they learn about your identity.

General description

This experiment is about booking an appointment at a public office. The 24 participants in the room are grouped into four groups of six persons each. Each of these groups consists of five participants taking up the roles as appointment seekers and of one participant acting as a service firm. Three out of five appointment seekers need an appointment in every block whereas the other two only in every second block. This means that there will be blocks in which three participants seek an appointment and others with five appointment seekers. Your role is randomly determined at the beginning of the experiment and will be fixed for the entire experiment. Your group consisting of five appointment seekers and one firm will also be the same for the entire experiment. The experiment consists of 40 independent decisions, i.e., 40 rounds, each of which represents an appointment allocation process. To receive a time slot, an ID number has to be provided, and this cannot be changed after the slot has been assigned. Each appointment seeker obtains a unique ID. This ID is changed every round for the purpose of anonymity and is assigned automatically. There are two alternative booking systems–System A and System B. In each system four appointments are provided in total, and there will be either three or five appointment seekers. Prior to every block consisting of five rounds, the number of appointment-seekers and the booking system (A or B) will be announced. These two properties remain the same throughout the five rounds.

Each round consists of two steps. Step 1 is the same for both booking systems while step 2 differs between them.

In step 1, each appointment seeker's valuation for a slot is determined randomly and will be a natural number between (and including) 50 and 100. Thus, each number has an equal chance of being drawn. Each appointment seeker is only informed about his own valuation, and the service firm does not know any of the valuations drawn. The service firm decides whether it wants to enter the market. This means the service firm decides whether to participate in the appointment allocation process or not. Entering the market costs the service firm 150 points. The firm also determines the price for a slot that has to be paid by appointment seekers if the slot is provided. The service firm has a choice between the following prices: 15, 20, 25,. . . ,75, 80, or 85 points. Appointment seekers decide whether they want to pay for the firm's service at the price asked by the firm or whether they want to apply without the firm. Remember that if the firm does not provide a slot to the appointment seeker, the appointment seeker does not have to pay the price.

Step 2 differs between the two booking systems.

System A:

In step 2, if the firm is active in the market (that is, it entered the market in step 1 at a cost of 150 points), it can book as many slots as it wants for free. If the firm has sold a slot to an appointment seeker in step 1, it assigns a slot to the seeker by providing the ID. The firm can also book more slots than it has sold in step 1 by entering fictitious IDs. To book more slots, the firm only has to indicate the number of additional slots. The computer will then generate the number of fictitious IDs needed. Slots with fake IDs are blocked and cannot be used by appointment seekers. If the firm does not book all available slots, the remaining slots are randomly allocated to the appointment seekers who have applied without the firm in step 1. Appointment seekers do not have to take any decision in the second step, and only receive a slot or not.

System B:

In step 2, appointment seekers and the service firm do not book the slots themselves, but have to apply for the slots. Each appointment seeker can apply for one slot by using the ID. If the firm is active in the market (that is, it entered the market in step 1 at the cost of 150 points), it can submit as many applications as it wants for free. The firm enters the IDs of appointment seekers who decided to apply through the firm in step 1. Each ID can be entered in the system only once. Firms can also enter fake IDs. To apply for more slots, the firm needs to indicate the number of additional slots needed. The number of applications can be greater than the number of available slots. The computer will generate and insert the fictitious IDs according to the number of slots indicated by the firm. The allocation of slots is determined randomly in the following way: all applications of the firm and of appointment seekers who decided to apply directly are put into an urn. Then, one by one, applications are randomly drawn from the urn to fill all the slots. Note that if a firm receives a slot for a fake ID, the firm cannot sell it to the appointment seekers.

Payoffs

Each appointment seeker has an endowment of 220 points at the beginning of each five-round block. Within a block, points are added and deducted to this endowment in the course of the five rounds. Thus, the appointment seeker earns her valuation (the randomly drawn number between 50 and 100) minus the price asked by the firm if he receives a slot through the firm. If the appointment seeker receives a slot without the firm, the appointment seeker simply earns his valuation, without paying anything. If the appointment seeker does not receive a slot, either with or without the firm, he earns nothing in this round.

Example: An appointment seeker has 210 points at the beginning of the round and his valuation for a slot is 75. The service firm decides to participate and sets a price of 60 for a slot. If the appointment seeker decides to apply through the firm and the application is successful, his payoff at the end of the round is 210+75-60=225. If the appointment seeker decides to apply directly, without the help of service firm, and receives the slot, then his account at the end of the round is 210+75=285. If the appointment seeker applies through the firm or directly and does not receive a slot or decides not to apply for a slot, his account at the end of the round is 210.

The service firm has an endowment of 750 points at the beginning of each block of five rounds, and points are added and deducted to this endowment in the course of the five rounds. If the firm enters the market, it has to pay the cost of 150 points and it receives the price times the number of slots sold to applicants. If the firm decides not to enter the market, it does not earn anything in this round.

Example: A service firm has 750 points at the beginning of a round, and there are four available slots at the public office. The service firm decides to participate and offers a slot at the price of 70. If three appointment seekers decide to apply through the firm and the firm is able to provide slots to them, the firm's account at the end of the round is (750-150+3*70)=350. If one appointment seeker decides to apply through the firm and the firm is able to provide a slot, the firm's account at the end of the round is 750-150+1*70=670. If no appointment seeker decides to apply through the firm, the firm's account at the end of the round is 750-150=600.

In total there will be 40 rounds, consisting of eight blocks of five rounds each. At the end of the experiment, one randomly chosen block will be paid out. The exchange rate is 1 point = 1.5 Euro Cent.

C.2 Experimental implementation of booking systems

Immediate system. In step 2, when the scalper enters the market, he learns how many appointment seekers have bought his service. He can book as many slots as he wants for free. If the scalper sold a slot to a seeker in step 1, the system assigns him a slot for that seeker's ID. This is implemented automatically in the experiment, i.e., if a seeker buys the service of the scalper, her ID is automatically used for one of the slots booked by the scalper (if the scalper booked any slots). If there are more seekers who bought the service than the number of slots booked by the scalper, it is randomly determined who receives a slot. The scalper can also book more slots than he has sold in step 1 by entering fake IDs. The fake IDs are created by the computer if the scalper decides to book more slots than the number of buyers. The number of booked slots cannot exceed the total supply. In the experiment, slots with fake IDs are blocked and cannot be taken by appointment seekers. If the scalper does not book all available slots, the remaining slots are randomly distributed among appointment seekers who have applied for slots directly, without the scalper, in step 1. Appointment seekers do not have to take any decision in the second step, and only receive a slot or not.

Batch system. In step 2, if the scalper is active in the market (that is, he entered the market in step 1 at a cost of 150 points), the scalper learns how many seekers bought his service. He can then submit as many applications for slots as he wants for free. The scalper enters the IDs of the seekers who decided to apply through him in step 1. Similar to the immediate system, this is automatically implemented in the experiment, i.e., if a seeker buys the service of the scalper, her ID is automatically used for one of the applications if the scalper submitted applications for slots. If there are more seekers who bought the service than the number of applications submitted by the scalper, the system randomly determines whose IDs to use. Each ID can be entered into the system once. The scalper can also enter fake IDs. The maximum number of applications is 10,000,000. The allocation of slots is determined randomly in the following way: all applications of the scalper and the applications of the seekers who decided to apply directly are put into an (imaginary) urn. Then, one by one, four applications are randomly drawn from the urn to fill the slots. Note that if the scalper receives a slot for a fake ID, he cannot sell it to the seekers.

C.3 Sequence of experimental treatments

Round	Block	System	Demand (n)	Treatment
1-5	1	Immediate	High (5)	Im5
6-10	2		Low (3)	Im3
11-15	3	Batch	High (5)	Batch5
16-20	4		Low (3)	Batch3
21-25	5	Immediate	High (5)	Im5
26-30	6		Low(3)	Im3
31-35	7	Batch	High (5)	Batch5
36-40	8		Low (3)	Batch3

Table C.1: Characteristics and sequence of treatments in each session

C.4 Equilibrium predictions

Treatment	Im5	Im3	Batch5	Batch3
Entry by scalper	yes	indifferent	no	no
Price after entry (p)	60	50	40	45
# of slots booked by scalper (a)	4	4	# of buyers (n_b); indiff. if n_b=0	# of buyers (n_b); indiff. if n_b=0
Expected # of slots sold	3.67	0 [3.00]	0	0
Expected profit of scalper	70.34	0 [0]	0	0
Expected payoff of seekers	14.68 (18.35)	25.00	60 (75)	75

Notes: The predictions refer to one round. The numbers for the immediate system are calculated based on Proposition 1, while the numbers for the batch system are calculated based on Proposition 2. The numbers in square brackets denote the continuation equilibrium after the scalper enters the market, calculated based on Proposition 3. The equilibrium payoff of seekers in Im3 is calculated given entry of the scalper in case of indifference. The numbers in parentheses refer to the normalized payoffs of appointment seekers where payoffs in Im5 and Batch5 are divided by 0.8 to make them comparable to payoffs in Im3 and Batch3.

Table C.2: Equilibrium predictions

C.5 Experimental results: market entry by the scalper over time

As shown by Figure C.1, market entry in Im5 and Im3 is relatively stable over the rounds. In Batch5, market entry decreases between blocks (from 29 to six out of 40 scalpers in the first versus the last round), and the decrease sets in within the first block. Since the first block of Batch5 is preceded by Im5 and Im3 where entry is (weakly) profitable, the decline in entry within the first block of Batch5 reflects the adjustment of scalpers to the batch system where entry is unprofitable. In Batch3, which follows after Batch5, we do not observe a similar decrease due to the small proportion of scalpers entering the market in the initial rounds in the first place.

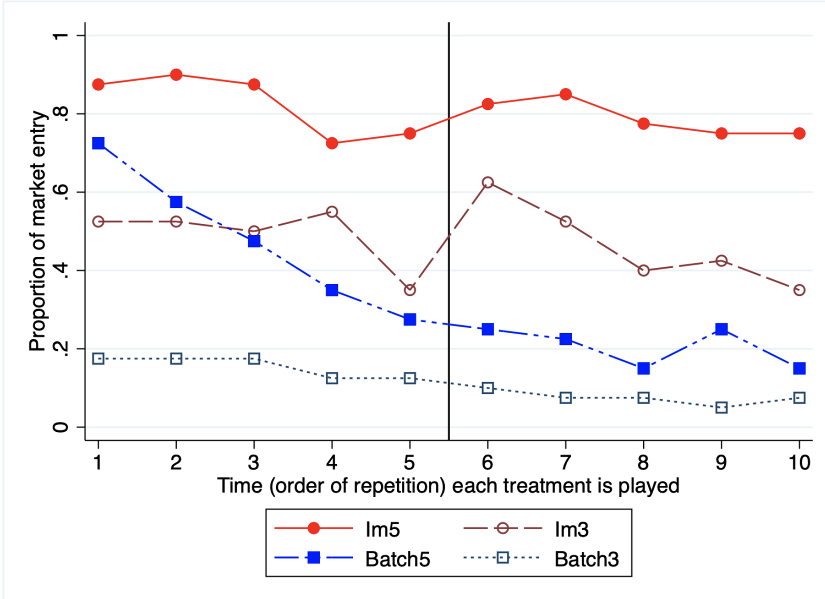

Figure C.1: Proportion of scalpers entering the market by treatments
Notes: Rounds 1-5 form the first block of a given treatment, while rounds 6-10 form the second block. The black vertical line separates the first and the second block.

C.6 Experimental results: appointment seekers' decisions to buy from the scalper

To analyze the purchase decisions of appointment seekers, we run regression analyses. Table C.3 presents probit regressions of the dummy for buying from the scalper. The sample is restricted to those rounds in which the scalper is active in the market. We find that over time appointment seekers become more likely to buy slots from the scalper in Im5 and less likely in Batch5. The higher an appointment seeker's valuation and the lower the price, the more likely it is the appointment seeker will buy from the scalper (except in Batch3 due to the small sample size caused by the infrequent entry of scalpers). If the scalper books all slots in the immediate system, seekers are more likely in the following round to buy the service from the scalper. This does not hold in the batch system: Blocking the system by booking at least 10 slots in the batch system is not correlated with more seekers buying the service from the scalper in the following round. Thus, the seekers understand that the scalpers' attempts to threaten them in the batch system are empty while they learn to buy from the scalper in the immediate system.

C.7 Experimental results: welfare

C.7.1 Allocation of slots

Figure C.2 presents the proportion of slots allocated to appointment seekers by treatments. It includes both the slots assigned through the scalper and the slots that the appointment seekers

	(1) Im5	(2) Im3	(3) Batch5	(4) Batch3
Time played	.02**	-.00	-.03*	-.02
	(.01)	(.01)	(.01)	(.02)
Valuation for a slot	.02***	.02***	.01***	.01*
	(.00)	(.00)	(.00)	(.00)
Price of service	-.02***	-.02***	-.01***	-.00
	(.00)	(.00)	(.00)	(.00)
Scalper booked all slots (Im)	.21***	.27***	-.06	.07
or blocked in previous round (Def)	(.05)	(.09)	(.07)	(.13)
Observations	1440	510	540	117
No. of clusters	39	31	28	10
log(likelihood)	-651.21	-249.29	-293.28	-59.37

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Notes: "Scalper books all slots" or "blocked in previous round" is a lagged dummy for booking all four slots in the immediate system and submitting more than 10 applications in the batch system.

Table C.3: Purchase decisions of appointment seekers

received directly. Note that if all three seekers receive a slot in Im3 and Batch3, we count this as 100% of slots being allocated.

In the treatments with excess demand, Im5 and Batch5, around 90% of slots are allocated to seekers. In Im5, the 10% unfilled slots are explained by some appointment seekers refusing to buy the service, while the unfilled slots in Batch5 are due to some scalpers entering the market and blocking the system.

In the treatments with an excess supply of slots, Batch3 leads to a higher proportion of slots allocated in the last five rounds than Im3 (p<0.01).[3] In Im3, around 15–20% of the slots are not filled (excluding slots that are in excess of demand) due to scalpers entering the market and setting a price higher than in equilibrium, together with the seekers' tendency to refuse to buy slots if the difference between price and valuation is low. In Batch3, we observe a loss of around 5% of slots in the last block of the treatment, which is explained by the irrational choice of some scalpers to enter the market despite losses and to block the system with fake applications.

C.7.2 Who gets a slot?

For the sake of completeness, we also show which seekers get a slot under the two booking systems. First, we define for each seeker in each round an ordinal rank based on her valuation of a slot compared to the other active seekers. Thus, the seeker with the highest valuation in a given round

[3]The p-values refer to the coefficient of the treatment dummy in a probit regression of the proportion of slots allocated to appointment seekers on this dummy. Standard errors are clustered at the level of matching groups, and the sample is restricted to the treatments of interest for the test.

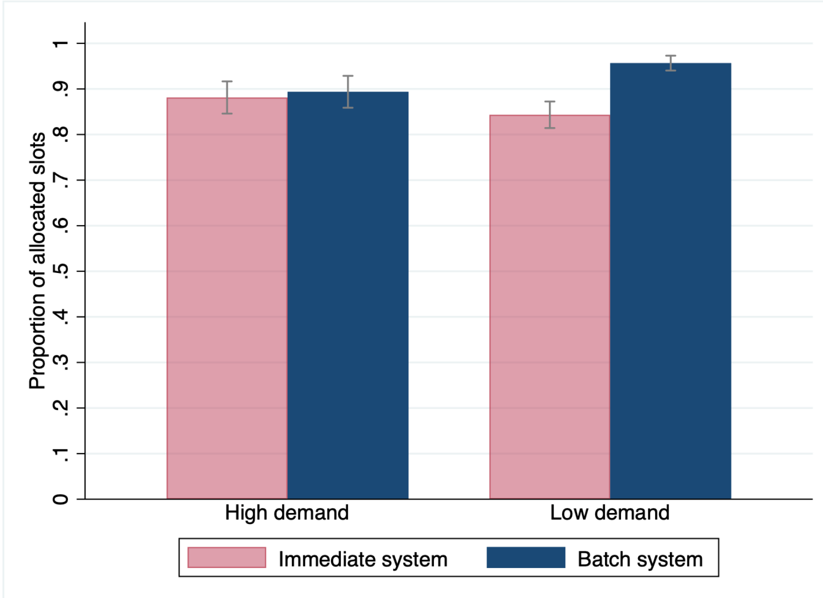

Figure C.2: Proportion of slots that were allocated

Notes: Gray bars represent 95% confidence intervals. High demand stands for five seekers (Im5 and Batch5) while low demand for three seekers (Im3 and Batch3). The figure is based on all decisions in the second block.

receives an ordinal rank of one, the seeker with the second highest valuation receives a rank of two, and so on.[4] Thus, in Im5 and Batch5 we have ranks from one to five, and in Im3 and Batch3 ranks from one to three.

Table C.4 presents the average ranks based on the valuation of a slot of all seekers receiving a slot by treatments. We partition the sample with respect to the total number of seekers assigned in a round. Comparing Im5 and Batch5, the average rank of seekers who are assigned a slot is lower in Im5 than in Batch5, that is, seekers who value the slots more highly in relative terms receive a slot in Im5 compared to Batch5. The difference is not significant in rounds with one seeker receiving a slot (p=0.17) but it is significant for rounds with two, three, and four seekers receiving a slot (p<0.01). As for the difference between Im3 and Batch3, the difference goes in the same direction, and is significant for the rounds where one seeker is assigned (p=0.01). Thus, the presence of scalpers and their pricing decisions have the expected effect: in the immediate system, seekers with higher evaluations receive slots more often than in the batch system.

[4]If seekers have equal valuations, they are assigned the average of two ranks. For instance, if two seekers have the highest valuation in a round, they are both assigned a rank of 1.5.

Rounds with	Im5	Im3	Batch5	Batch3
... one seeker receiving a slot				
Average rank of assigned seekers	1.68	1.37	2.37	1.88
Number of rounds	28	43	16	13
... two seekers receiving a slot				
Average rank of assigned seekers	2.10	1.70	3.07	1.84
Number of rounds	54	64	31	14
... three seekers receiving a slot				
Average rank of assigned seekers	2.41	2.00	3.03	2.00
Number of rounds	68	273	31	360
... four seekers receiving a slot				
Average rank of assigned seekers	2.88		3.01	
Number of rounds	224		292	

Note: The table displays the average rank based on the valuations of the seekers who obtain a slot. Only data from the second block are used.

Table C.4: Average ranks of seekers who received a slot