Online Appendix for "Endogenous Education and Long-Run Factor Shares"

by

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September 2020

Proofs from Section I

Capital-skill complementarity

Let $G(K, S, U)$ be a three-factor production function that is homogeneous of degree one in (K, S, U), twice continuously differentiable and has strictly positive first and second derivatives in all its arguments. Let $\sigma_{KJ} \equiv \frac{G_K G_J}{G G_{KJ}}$ for $J = S, U$. We show that Assumption 1 implies $\sigma_{KU} > \sigma_{KS}$. For the nested constant elasticity of substitution production function used by Krusell et al. (2000), $\sigma_{KU} > \sigma_{KS}$ if and only if (equipment) capital is more substitutable with unskilled labor than with skilled labor.

Assumption 1 requires $\varphi > 0$ where

$$\varphi \equiv \frac{d \ln (F_h/F_L)}{d \ln K} = \frac{K F_{Kh}}{F_h} - \frac{K F_{KL}}{F_L}. \tag{A.1}$$

Let $L = S + U$ and $h = S/L$. Then we can write $F(K, L, h) = G(K, hL, (1-h)L)$, which is equivalent to $F(K, S+U, S/(S+U)) = G(K, S, U)$. Differentiating yields

$$\begin{aligned}\sigma_{KU} - \sigma_{KS} &= \frac{F_K}{F} \frac{F_L - hF_h/L}{F_{KL} - hF_{Kh}/L} - \frac{F_K}{F} \frac{F_L + (1-h)F_h/L}{F_{KL} + (1-h)F_{Kh}/L}, \\ &= \frac{F_K F_L F_h}{LF} \frac{1}{G_{KU} G_{KS}} \left(\frac{F_{Kh}}{F_h} - \frac{F_{KL}}{F_L} \right),\end{aligned}$$

which is positive if and only if $\varphi > 0$.

Derivation of equation (1)

Let $k = K/L$. Output is homogeneous of degree one in K and L by Assumption 1, meaning that the optimal capital use equation can be written as $R = F_K(k, 1, h)$. Differentiating yields

$$dk = \frac{F_K}{F_{KK}} d \ln R - \frac{F_{Kh}}{F_{KK}} dh. \tag{A.2}$$

Likewise, the capital share is given by $\theta = Rk/F(k, 1, h)$ and differentiating implies

$$d\theta = \theta d \ln R - \theta \frac{F_h}{F} dh + \frac{\theta(1-\theta)}{k} dk.$$

.
Using (A.2) to substitute for dk then gives

$$d\theta = \theta\left[1 + (1-\theta)\frac{F_K}{kF_{KK}}\right] d\ln R - \theta\left[\frac{F_h}{F} + (1-\theta)\frac{F_{Kh}}{kF_{KK}}\right] dh.$$

Noting that the homogeneity of F implies $kF_{KK} = -F_{KL}$ and using equation (A.1) for φ, we can rearrange this expression to obtain equation (1).

Proofs from Section II

Optimal education: derivation of equation (6)

Let $\hat{F}(k,h) = F(k,1,h)$ denote the production function in intensive form where $k = K/L$. Let $\kappa_t(h)$ be the units of physical capital that are combined with a unit of labor bearing human capital h at time t. Optimal capital use requires

$$\hat{F}_k(\kappa_t(h), h) = R, \tag{A.3}$$

and since competitive producers make zero profits, the wage schedule is given by

$$w_t(h) = \hat{F}(\kappa_t(h), h) - R\kappa_t(h). \tag{A.4}$$

Differentiating these expressions and suppressing the arguments of $\hat{F}(\kappa_t(h), h)$ yields

$$\begin{aligned} \kappa_t'(h) &= -\frac{\hat{F}_{kh}}{\hat{F}_{kk}}, & \frac{\partial \kappa_t(h)}{\partial t} &= g_R\frac{\hat{F}_k}{\hat{F}_{kk}}, \\ w_t'(h) &= \hat{F}_h, & \frac{\partial w_t(h)}{\partial t} &= \frac{\partial \hat{F}}{\partial t} - g_R\kappa_t(h)\hat{F}_k, \end{aligned} \tag{A.5}$$

where g_R denotes the growth rate of R. Note also that using the intensive form production function we can write: $\theta = \kappa_t(h)\hat{F}_k/\hat{F}$; $\sigma = -\hat{F}_k(1-\theta)/(\kappa_t(h)\hat{F}_{kk})$; $\varphi = \kappa_t(h)\hat{F}_{kh}/\hat{F}_h - \theta/\sigma$.

Each individual chooses her labor supply path to maximize the expected present value of lifetime earnings. Consider an individual with human capital h_t at time t and labor supply path ℓ_τ for $\tau \geq t$. Let $\tilde{\ell}_\tau$ be an alternative labor supply path defined by

$$\tilde{\ell}_\tau = \begin{cases} \ell_\tau + \epsilon, & \tau \in [t, t+\Delta], \\ \ell_\tau - \epsilon, & \tau \in (t+\Delta, t+2\Delta], \\ \ell_\tau, & \tau > t + 2\Delta. \end{cases}$$

where $\epsilon \in \mathbb{R}$ and $\Delta > 0$. The individual's human capital under labor supply path $\tilde{\ell}_\tau$ is given by

$$\tilde{h}_\tau = \begin{cases} h_\tau - \epsilon(\tau - t), & \tau \in [t, t+\Delta], \\ h_\tau - \epsilon\left(t + 2\Delta - \tau\right), & \tau \in [t+\Delta, t+2\Delta], \\ h_\tau, & \tau \geq t + 2\Delta. \end{cases}$$

Note that this labor supply perturbation does not affect the individual's human capital outside the interval $(t, t+2\Delta)$.

Let S be the difference between the individual's expected present value of earnings under $\tilde{\ell}_\tau$ and under ℓ_τ. We have

$$\begin{aligned}
S &= \int_t^{t+2\Delta} e^{-\int_t^\tau (r_s+\nu)ds}\left[\tilde{\ell}_\tau w_\tau(\tilde{h}_\tau) - \ell_\tau w_\tau\left(h_\tau\right)\right] d\tau, \\
&= \int_t^{t+\Delta} e^{-\int_t^\tau (r_s+\nu)ds}\left\{\ell_\tau\left(w_\tau\left[h_\tau - \epsilon(\tau-t)\right] - w_\tau\left[h_\tau\right]\right) + \epsilon w_\tau\left[h_\tau - \epsilon(\tau - t)\right]\right\} d\tau \\
&+ \int_{t+\Delta}^{t+2\Delta} e^{-\int_t^\tau (r_s+\nu)ds}\left\{\ell_\tau\left(w_\tau\left[h_\tau - \epsilon\left(t+2\Delta-\tau\right)\right] - w_\tau\left[h_\tau\right]\right) - \epsilon w_\tau\left[h_\tau - \epsilon\left(t+2\Delta-\tau\right)\right]\right\} d\tau,
\end{aligned}$$

where the second equality uses the expressions for $\tilde{\ell}_\tau$ and $\tilde{h}_\tau$ above. Expressing the functions in the integrands as Taylor series around t, computing the integrals and dropping terms that are $o(\Delta^2)$ implies that for Δ close to zero

$$S \approx \epsilon\Delta^2\left[(r_t+\nu)w_t(h_t) - w_t'(h_t) - \frac{\partial w_t(h_t)}{\partial t}\right]. \tag{A.6}$$

The intuition for this expression is as follows. When $\epsilon > 0$, switching from labor supply path ℓ_τ to $\tilde{\ell}_\tau$ means working more today and less tomorrow. The benefit of this switch is $(r_t+\nu)w_t(h_t)$, which equals the increase in the expected present value of earnings from bringing forward the time at which labor income is received. The costs of delaying schooling are: $w_t'(h_t)$, which gives the decline in earnings from having lower human capital tomorrow, and; $\frac{\partial w_t(h_t)}{\partial t}$, which is positive when wages are increasing over time. Since human capital accumulation and labor supply are both linear in ℓ_t, agents for whom the benefits of delaying schooling exceed the costs will choose to work full-time, while agents for whom the costs are greater will devote all their time to schooling.

Agents are indifferent between working and learning if and only if the right hand side of (A.6) equals zero for all ϵ, which requires

$$\tilde{S}_t(h_t) \equiv (r_t+\nu)w_t(h_t) - w_t'(h_t) - \frac{\partial w_t(h_t)}{\partial t} = 0. \tag{A.7}$$

We now make the following assumption

ASSUMPTION A.1: *The production function and parameters of the economy are such that for all t*
(i) There exists $h_t^* > 0$ such that $\tilde{S}_t(h_t^*) = 0$;
(ii) $\Gamma_t(h_t^*) > 0$ for all k where

$$\Gamma_t \equiv \frac{1}{\hat{F}}\left[\left(\hat{F}_h + \frac{\partial \hat{F}}{\partial t} - g_R\kappa_t(h_t^*)\hat{F}_k\right)\frac{\hat{F}_h}{\hat{F} - \kappa_t(h_t^*)\hat{F}_k} - \hat{F}_{hh} + \frac{\hat{F}_{kh}^2}{\hat{F}_{kk}} - \frac{\partial \hat{F}_h}{\partial t} - g_R\frac{\hat{F}_k\hat{F}_{kh}}{\hat{F}_{kk}}\right].$$

Assumption A.1.i imposes that a solution to equation (A.7) exists. This is a relatively weak restriction. To see why, note that $\tilde{S}_t(h_t)$ is continuous in h_t whenever the production function is continuously differentiable in k, h and t. Then if a solution does not exist, either all individuals work full-time with $\ell_t = 1$ or all individuals are in full-time education with $\ell_t = 0$. It is straightforward to impose sufficient conditions to rule out such equilibria. For example, if individuals with no human capital produce no output then $w_t(0) = 0$, meaning that working full-time cannot be optimal for newborn agents. In addition, if the economy has a positive capital stock and the marginal product of capital is unbounded as the capital input approaches zero, then it cannot be optimal for all agents to be in full-time education.

Assumption A.1.ii is a second order condition for educational choice that ensures the solution to equation (A.7) is unique. To show this we differentiate $\tilde{S}_t(h_t)$ given by (A.7). Using equations (A.3)–(A.5) and setting $h_t = h_t^*$ yields $\tilde{S}_t'(h_t^*) = \hat{F}\Gamma_t(h_t^*)$. Thus, the gradient of $\tilde{S}_t(h_t)$ is positive if $\tilde{S}_t(h_t) = 0$.

This single-crossing property guarantees that equation (A.7) has a unique solution $h_t = h_t^*$. It also implies that $\tilde{S}_t(h_t) < 0$ for all $h_t < h_t^*$ and $\tilde{S}_t(h_t) > 0$ for all $h_t > h_t^*$. Consequently, individuals with human capital below the threshold h_t^* prefer to study today and work tomorrow, while the opposite is true for individuals with human capital above h_t^*. Since labor supply is bounded on the interval $[0, 1]$ it follows that optimal labor supply is given by $\ell_t = 0$ if $h_t < h_t^*$ and $\ell_t = 1$ if $h_t > h_t^*$.

Setting $h_t = h_t^*$ and rearranging equation (A.7) gives equation (6) in the paper. Taking the total derivative of this expression for given t and using equations (A.3)–(A.5) together with the definitions of φ, σ and θ yields

$$dh_t^* = -\frac{1}{\Gamma_t}\frac{\sigma\varphi}{1-\theta}\frac{F_h}{F}d\ln R + \frac{1-\theta}{\Gamma_t}\left(dg_{w|h_t^*,t} - dr_t - d\nu\right), \tag{A.8}$$

where $dg_{w|h_t^*,t}$ denotes the change in the growth rate of wages evaluated at h_t^*. Equation (A.8) shows that whenever there is capital-skill complementarity as defined in Assumption 1 (meaning $\varphi > 0$) and the technical conditions in Assumption A.1 hold, an increase in the rental rate of capital R reduces the optimal human capital threshold h_t^*. Moreover, even in the absence of capital-skill complementarity, the human capital threshold is increasing in the growth rate of wages, but decreasing in the real interest rate and the risk of death.

Optimal human capital in a model of occupational choice

Suppose there are two types of labor – skilled and unskilled – and h denotes the fraction of the labor force that is skilled. Formally, let S denote the skilled labor force and U the unskilled labor force. Then $L = S + U$ and human capital $h = S/L$. Let w^U denote the unskilled wage and $w^S = \psi w^U$ the skilled wage, where ψ denotes the skill premium. For this economy the wage schedule $w_t(h)$ satisfies

$$w_t(h) = \frac{w^U U + w^S S}{L} = w^U\left[1 + h\left(\psi - 1\right)\right], \tag{A.9}$$

implying that $w_t'(h) = w^U(\psi - 1)$.

Competitive firms hire capital and labor taking the rental rate and the wage schedule as given, implying that equations (A.3)-(A.5) hold. Using the wage schedule in (A.9) to differentiate (A.3), (A.4) together with the expression for $w_t'(h)$ in (A.5) we obtain

$$dh = -\frac{1}{\tilde{\Gamma}_t}\frac{\sigma\varphi}{1-\theta}d\ln R - \frac{1}{\tilde{\Gamma}_t}\frac{1}{1+h(\psi-1)}\frac{d\psi}{\psi-1}, \tag{A.10}$$

where

$$\tilde{\Gamma}_t \equiv \frac{1}{\hat{F}_h}\left(\frac{\hat{F}_{kh}^2}{\hat{F}_{kk}} - \hat{F}_{hh}\right),$$

and we assume $\tilde{\Gamma}_t > 0$ for all t to ensure that the second order condition for profit maximization holds. Thus, the relative demand for skilled labor is declining in the skill premium ψ and also decreasing in the capital rental rate R whenever there is capital-skill complementarity.

Equation (A.10) gives demand for human capital conditional on the skill premium. However, when individuals choose whether or not to invest in becoming skilled, the skill premium also affects occupational choice. Suppose all newborns are unskilled, but have the opportunity to become skilled workers by attending school for ζ periods. Apart from this change to the education technology, the economy is as specified in Section II.

To maximize dynastic utility, each individual chooses the occupation that offers the highest expected present value of lifetime earnings. We restrict attention to equilibria where at each instant some, but not all, unskilled individuals choose to become skilled. This requires that unskilled individuals are indifferent over whether or not to attend school. Skilled agents earn nothing for ζ periods and then receive the skilled wage, while unskilled agents always earn the unskilled wage. Therefore, the indifference condition at time τ is

$$\int_\tau^\infty e^{-\int_\tau^t (r_z+\nu)dz} w_t^U\, dt = \int_{\tau+\zeta}^\infty e^{-\int_\tau^t (r_z+\nu)dz} \psi_t w_t^U\, dt,$$

where the left hand side is the expected present value of earnings of an unskilled worker and the right hand side is the expected present value of earnings of an individual that chooses to become skilled. Differentiating the indifference condition with respect to τ yields

$$w_\tau^U = e^{-\int_\tau^{\tau+\zeta}(r_z+\nu)dz}\psi_{\tau+\zeta} w_{\tau+\zeta}^U. \tag{A.11}$$

Thus, the unskilled wage at time τ equals the expected present value of the skilled wage at time $\tau+\zeta$, which is when skilled agents who start schooling at τ join the labor force.

Let $g_w^U(t,\zeta) = w_t^U/w_{t-\zeta}^U$ denote growth in the unskilled wage between $t-\zeta$ and t and $r(t,\zeta) = e^{\int_{t-\zeta}^t (r_z+\nu)dz}$ be the inverse of the discount factor used to value time t earnings at time $t-\zeta$. Then differentiating (A.11) with $\tau = t-\zeta$ gives

$$\frac{d\psi_t}{\psi_t} = \frac{dr(t,\zeta)}{r(t,\zeta)} - \frac{dg_W^U(t,\zeta)}{g_W^U(t,\zeta)},$$

and using this expression to substitute for $d\psi_t$ in (A.10) yields

$$dh_t = -\frac{1}{\tilde{\Gamma}_t}\frac{\sigma\varphi}{1-\theta}d\ln R + \frac{1}{\tilde{\Gamma}_t}\frac{1}{1+h_t(\psi_t-1)}\frac{\psi_t}{\psi_t-1}\left[\frac{dg_w^U(t,\zeta)}{g_w^U(t,\zeta)} - \frac{dr(t,\zeta)}{r(t,\zeta)}\right]. \tag{A.12}$$

Equation (A.12) is analogous to equation (A.8) from the baseline model. As in the baseline model, an increase in the capital rental rate reduces equilibrium human capital h whenever there is capital-skill complementarity. In addition, h is increasing in the growth rate of unskilled wages, but decreasing in the compound interest rate during the period when individuals attend school. This shows that the qualitative results concerning the determinants of optimal human capital derived in Section II continue to hold in a model of occupational choice with endogenous supplies of skilled and unskilled labor.

Proofs from Section III

Proof of Lemma 1 and Proposition 1

Imposing the functional form in Assumption 2 and noting that optimal capital use satisfies equation (8), a firm that hires labor with human capital h_t at time t has capital share $\theta\left[z_t(h_t)\right]$ where $\theta(z) \equiv zf'(z)/f(z)$ and $z_t(h) \equiv e^{-(a+b)h}\frac{A_t\kappa_t(h)}{B_t}$. Moreover, equation (8) implies z_t is strictly decreasing in h_t and Grossman et al. (2017a) show that $\theta(z)$ is strictly decreasing in z. It follows that $\theta\left[z_t\left(h_t\right)\right]$ is strictly increasing in h_t.

Differentiating the wage schedule in (9) yields

$$\frac{1}{w_t(h)}\frac{\partial w_t(h)}{\partial t} = \gamma_L + (g_A - g_R)\frac{\theta\left[z_t\left(h\right)\right]}{1-\theta\left[z_t\left(h\right)\right]},$$

and substituting this expression together with equation (14) into equation (A.7) gives

$$\tilde{S}_t(h_t) = \left(r_t + \nu - b - \gamma_L + (a + g_R - g_A)\frac{\theta\left[z_t\left(h_t\right)\right]}{1-\theta\left[z_t\left(h_t\right)\right]}\right)w_t(h_t).$$

Now, assume that for all t there exists $h_t^* > 0$ that solves $\tilde{S}_t(h_t^*) = 0$ and that $a+g_R-g_A > 0$, which ensures $\tilde{S}_t'(h_t^*) > 0$ because $\theta\left[z_t\left(h_t\right)\right]$ is strictly increasing in h_t. We prove below that these assumptions hold on a balanced growth path (BGP). Then Assumption A.1 is satisfied. It follows that h_t^* defines a human capital threshold such that at time t all individuals with human capital below h_t^* are in full-time education and all individuals with human capital above h_t^* work full-time.

Next, suppose the economy is on a BGP. The no arbitrage condition for capital accumulation implies that on a BGP where the interest rate is constant $g_R = -g_q$. Therefore, on a BGP $a+g_R-g_A = a-\gamma_K$, which is strictly positive by Assumption 3.i. It follows that $a+g_R-g_A > 0$ on a BGP as assumed above.

Setting $\tilde{S}_t(h_t^*) = 0$ implies the human capital threshold on a BGP satisfies

$$\frac{\theta\left[z_t\left(h_t^*\right)\right]}{1-\theta\left[z_t\left(h_t^*\right)\right]} = \frac{b+\gamma_L-(r+\nu)}{a-\gamma_K}, \tag{A.13}$$

showing that $z_t(h_t^*) = z^*$ must be constant on a BGP which proves equation (11) in Lemma 1. Differentiating (8) with respect to time while holding $z_t(h_t^*)$ constant then yields

$$\dot{h}_t^* = \frac{\gamma_K}{a}.$$

Therefore, in order to keep their human capital rising at the same rate as h_t^*, individuals that are in the labor force must choose labor supply $\ell = 1 - \gamma_K/a$ as claimed in equation (10) of

Lemma 1.

At time t any individuals with human capital above h_t^* work full-time. Consequently, on a BGP it is not possible for individuals to have human capital above h_t^* since h_t^* is growing over time. Given this observation, the remaining properties of the unique BGP can be derived as in the discussion following Lemma 1 in the paper. In particular, equation (16) gives the real interest rate on the BGP and substituting (16) into (A.13) gives (18), which determines the BGP value of θ. Assumption 3.iii ensures the discount rate is sufficiently large that dynastic utility is finite on the BGP. Finally, since $g_R = -g_q$ and the real interest rate r satisfies (16), Assumption 3.ii guarantees that, as assumed above, for all t there exists $h_t^* > 0$ that solves $\tilde{S}_t(h_t^*) = 0$.

This completes the proof that there exists a unique BGP. In our working paper Grossman et al. (2017b) we analyze the stability of the BGP and show that the BGP is locally saddle-path stable in a calibrated version of the model.

Proof of Proposition 2

Differentiating equation (18) with respect to γ_K yields

$$\frac{1}{(1-\theta)^2}\frac{\partial\theta}{\partial\gamma_K} = -\frac{\eta-1}{a-\gamma_K}\frac{b-\lambda}{a} - \frac{(\eta-1)(\gamma_L + \frac{b-\lambda}{a}\gamma_K) - \lambda + \nu + \rho}{(a-\gamma_K)^2}.$$

The first term on the right hand side is negative when $\eta > 1$ since Assumption 2 imposes $b > \lambda$. The second term on the right hand side is negative by Assumption 3.iii which guarantees finite utility on the BGP. It follows that an increase in γ_K reduces θ or, equivalently, that a reduction in γ_K reduces labor's share of income.

Differentiating equation (18) with respect to γ_L yields

$$\frac{1}{(1-\theta)^2}\frac{\partial\theta}{\partial\gamma_L} = -\frac{\eta-1}{a-\gamma_K},$$

which is negative if and only if $\eta > 1$. Thus, a reduction in γ_L increases θ and lowers labor's share of income.

Quantitative Exploration

We present the results of calibrating the model and quantifying the impact on factor shares of a one percentage point reduction in trend growth of labor productivity. For a complete description of the calibration and quantitative analysis see our working paper Grossman et al. (2017b).

We rely on the empirical literature to set some of our parameters and choose others to match moments from the U.S. historical experience as detailed in Table A1. Conveniently, steady state factor shares can be calculated without assuming a functional form for $\tilde{F}(\cdot)$. However, we have no firm basis for specifying the magnitude of the capital-skill complementarity that is reflected in the parameter a in $\tilde{F}\left(e^{-ah}A_tK, e^{bh}B_tL\right)$. Given our other moments, this parameter would be pinned down if we knew the bias of technical progress in the pre-slowdown period. However, the Diamond-McFadden "Impossibility Theorem" tells us that we cannot identify this from time series data. Consequently, we pursue two different approaches to calibrating a. First, we introduce plausible but *ad hoc* assumptions about the bias in technical progress along the initial BGP. Then, we employ cross-sectional data for U.S. regions and industries in a crude attempt to estimate a directly.

Table A1—Targeted Moments and Parameters

Parameter/Moment		**Value**
Birth rate	λ	2.16%
Death rate	ν	0.95%
Internal Rate of Return on schooling	ι	10%
Capital share	θ	0.35
Growth in labor productivity	$\gamma_L + \frac{b}{a}\gamma_K$	0.024
Increase in schooling	$\dot{s}_T = \frac{\gamma_K}{a-\gamma_K}$	0.088
Intertemporal elasticity of substitution	$\frac{1}{\eta}$	0.5

Table A2—Response of Capital Share to Productivity Slowdown: Ad Hoc Examples

$\gamma_K = \gamma_L = 1.1\% \Rightarrow a = 0.132, b = 0.164$						
	γ_K	γ_L	Growth in per capita Income	Annual Increase in Schooling	Interest Rate	Capital Share
Baseline	1.1%	1.1%	2.2%	0.09	10.0%	0.35
$\gamma_L \downarrow$	1.1%	0.1%	1.2%	0.09	8.0%	0.383
$\gamma_K \downarrow$	0.3%	1.1%	1.4%	0.02	8.3%	0.39

$g_q = 2.0\%$, $g_A = \gamma_L = 0.4\% \Rightarrow a = 0.293, b = 0.251$						
	γ_K	γ_L	Growth in per capita Income	Annual Increase in Schooling	Interest Rate	Capital Share
Baseline	2.4%	0.4%	2.2%	0.09	10.0%	0.35
$\gamma_L \downarrow$	2.4%	-0.6%	1.2%	0.09	8.0%	0.365
$\gamma_K \downarrow$	1.2%	0.4%	1.3%	0.04	8.2%	0.368

Table A2 shows the quantitative results when we make ad-hoc assumptions about the bias of technical progress. For the top panel, we assume that technical change in the pre-slowdown period was factor neutral, so that $\gamma_K = \gamma_L$. For the lower panel, we assume that the observed average decline in investment goods prices of 2% per year represents the full extent of investment-specific technical change, and that the disembodied technological progress was factor neutral ($g_A = \gamma_L$). In both cases we report the new steady state following a permanent one percentage point slowdown in labor productivity growth caused by a decline in either labor-augmenting or capital-augmenting technical progress. We see that the increase in capital's share of national income following a productivity slowdown varies between 1.5 and 4 percentage points.

Our second approach to calibrating the model estimates a from the association between labor shares and wage growth across states and industries in the US, details are given in Grossman et al. (2017b). We find an inverse relationship between the average labor share in the state-industry and the average rate of wage growth, as would be predicted by our model assuming that the US has an integrated national capital market. Our preferred estimate implies $a = 0.19$.

In Table A3, we repeat the exercise of simulating the effects of a one percentage point slowdown in annual labor-productivity growth. In this case, the values of γ_K and γ_L in the baseline

Table A3—Response of Capital Share to Productivity Slowdown: Estimates of Capital-Schooling Complementarity using Cross-Sectional Data

Central Estimate of a: $a = 0.19, b = 0.195$						
	γ_K	γ_L	Growth in per capita Income	Annual Increase in Schooling	Interest Rate	Capital Share
Baseline	1.5%	0.8%	2.2%	0.09	10.0%	0.35
$\gamma_L \downarrow$	1.5%	-0.2%	1.2%	0.09	8.0%	0.373
$\gamma_K \downarrow$	0.6%	0.8%	1.3%	0.03	8.2%	0.378

calibration are those needed for the model to match the annual increase in schooling, the capital share, the rate of return on education, and the growth rate of labor productivity in the pre-slowdown period. Again, we simulate the slowdown in labor-productivity growth as being the result of either a deceleration of capital-augmenting technological progress or of labor-augmenting technological progress.

We find that a one percentage point slowdown in trend productivity growth can account for a sizeable shift in income from labor to capital. With the parameters reflected in the table, the capital share rises between two and three percentage points. In our working paper we also analyze the sensitivity of the quantitative results and argue that once we admit a reasonable amount of capital-skill complementarity (as captured by the parameter a), a productivity slowdown can account for a substantial redistribution of income from labor to capital for all plausible values of the other parameters.