Online Appendix to ”News Uncertainty in Brexit U.K.”

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1 Method of Simulated Moments

The Method of Simulated Moments (MSM) retrieves the parameters that minimize the sum of squared residuals between the moments of the data and those of the model. The minimization problem reads as follows:

$$\Theta = \arg\min_{\Theta} \mathbf{d}(\Theta)' W \mathbf{d}(\Theta), \tag{1}$$

where Θ is a $N \times 1$ vector of parameters, $\mathbf{d}(\Theta)$ is a $M \times 1$ vector of residuals, and W is a $M \times M$ weighting matrix. It is required that there should be at least as many parameters (N) as moments (M), that is $N \geq M$. In the case of $N = M$ the model is just-identified, whereas if $N > M$ the model is over-identified. Note that setting W as a matrix with the reciprocal of the squared data moments on the diagonal and zero elsewhere implies that solving eq. 1 is equivalent to minimizing the sum of squared residuals between the moments of the data and those of the model.

In order to solve the MSM we rely on the root-finding method of Nelder and Mead (1965). Since we use a local root-finding method, we conduct robustness checks by altering both the initial starting values and the step factor. We find that the results are not sensitive to such modifications.

2 Computational Strategy

This section describes the numerical methods used to compute the model's steady state, as well as the transitional dynamics under policy uncertainty. We first describe how the state space is discretized.

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2.1 State Space Discretization

The model contains four states: idiosyncratic productivity (a), idiosyncratic policy state (z), idiosyncratic capital (k), aggregate state (ζ). The discretization of the four states is as follows:

- Idiosyncratic productivity (a) is discretized into a grid $\mathbf{a} \in \{\overline{a}_1, ..., \overline{a}_{N_a}\}$ comprising of $N_a = 15$ log-linearly spaced points.
- The idiosyncratic policy states (z) is discretized into a grid containing $N_z = 3$ idiosyncratic states represented by $\mathbf{z} \in \{\overline{z}_+, \overline{z}_-, \overline{z}_0\}$.
- The idiosyncratic capital state (k) is discretized into a grid $\mathbf{k} \in \{\overline{k}_1, ..., \overline{k}_{N_k}\}$ containing of $N_k = 100$ points spaced log-linearly between 1×10^0 and 1×10^2.
- The aggregate states (ζ) are four ($N_\zeta = 4$): the pre-referendum state (ζ^P), the negotiations state (ζ^N), the Soft Brexit state (ζ^S) and the Hard Brexit state (ζ^H). These aggregate states can be represented into the following grid $\mathbf{Z} \in \{\overline{\zeta^P}, \overline{\zeta^N}, \overline{\zeta^S}, \overline{\zeta^H}\}$.

The state space is $N_a \times N_k \times N_z \times N_\zeta$, or more specifically $15 \times 100 \times 3 \times 4$.

We also discretize the following three exogenous stochastic processes:

- The stochastic process of the aggregate states can be represented by the transition matrix Γ_ζ of size $N_\zeta \times N_\zeta$ where $\sum_{l=1}^{N_\zeta} \pi^\zeta_{j,l} = 1$ for all $j \in \{1, ..., N_\zeta\}$. The transition matrix for the aggregate policy state of the economy is given by:

$$\Gamma_\zeta(\zeta_{t+1} = \zeta^i | \zeta_t = \zeta^q) =$$
$$\begin{array}{cc} \downarrow \zeta^q_t, \zeta^i_{t+1} \rightarrow & \begin{array}{cccc} \zeta^N & \zeta^S & \zeta^H & \zeta^P \end{array} \\ \begin{array}{c} \zeta^N \\ \zeta^S \\ \zeta^H \\ \zeta^P \end{array} & \begin{pmatrix} 1-\gamma^R-\theta & \theta(1-\gamma^H) & \theta\gamma^H & \gamma^R \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}. \tag{2}$$

- The stochastic process of idiosyncratic productivity can be represented by the transition matrix Γ_a of size $N_a \times N_a$ discretized using Tauchen's method.

- The stochastic process of the idiosyncratic policy states can be represented by the transition matrix Γ_z of size $N_a \times N_a \times N_z \times N_z$. Where $\Gamma_z(\zeta' = \zeta^j, z'|\zeta = \zeta^i, z) = \mathbb{I}$ if $i = \{P, S, H\}$ and $j = \{P, N, S, H\}$, as these states do not entail the draw of z. Moreover, if $i = \{N\}$, $j = \{N, P\}$, and $z_n = \{z_+, z_0, z_-\}$: $\Gamma_z(\zeta' = \zeta^j, z'|\zeta = \zeta^i, z) = \mathbb{I}$. However, if $i = \{N\}$, $j = \{S, H\}$, and $z_n = \{z_+, z_0, z_-\}$:

$$\Gamma_z(\zeta' = \zeta^j, z'|\zeta = \zeta^i, z) = \begin{array}{cc} \downarrow z, z' \rightarrow & \begin{array}{ccc} z_+ & z_0 & z_- \end{array} \\ \begin{array}{c} z_+ \\ z_0 \\ z_- \end{array} & \begin{pmatrix} q_+ & (1 - q_+ - q_-) & q_- \\ q_+ & (1 - q_+ - q_-) & q_- \\ q_+ & (1 - q_+ - q_-) & q_- \end{pmatrix} \end{array}. \tag{3}$$

2.2 Steady State

We compute the stochastic steady states of the pre-referendum economy abstracting from the possibility that Brexit may happen, i.e. we do not calculate the *'risky'* steady state. In what follows we describe the solution algorithm based on value function iteration.

2.3 Steady State Solution Algorithm:

1. Solve the problem of the firms using value function iteration, given the prices β and w:

   (a) guess an initial value function $V_g(a, z, k)$, for instance $V_g(e, z, k) = 0$;

   (b) solve for $V^{NA}(a, z, k)$ and $V^A(a, z, k)$ by taking expectations over the exogenous processes of a and z and using $V'(a, z, k) = V_g(a, z, k)$, and obtain the policy functions $K(a, z, k)$ and $L(a, z, k)$;

   (c) using $V^{NA}(a, z, k)$ and $V^A(a, z, k)$ find $\tilde{V}(a, z, k, \phi)$;

   (d) then find the policy function for the fixed capital adjustment cost threshold $\phi^T(a, z, k)$;

   (e) calculate $V(a, z, k)$ by taking expectations of $\tilde{V}(a, z, k, \phi)$ over ϕ using the threshold $\phi^T(a, z, k)$;

   (f) check whether the absolute percentage deviation between the guessed value function $V_g(a, z, k)$ and the obtained value function $V(a, z, k)$ is within a pre-set tolerance. If the absolute deviation is smaller than the tolerance then exit the algorithm and save the optimal policy functions $(K(a, z, k)$,$L(a, z, k)$,$\phi^T(a, z, k))$, otherwise update the guess $V_g(a, z, k) = V(a, z, k)$ and repeat steps (a)-(e) until convergence.

2. Using the policy functions $K(a,z,k)$ and $\phi^T(a,z,k)$ solve for the stationary distribution as a fixed point, defined as $\mu'(a',z',k') = \mu(a,z,k)$, by iterating on the distribution of firms over idiosyncratic productivity, idiosyncratic policy, and idiosyncratic capital holdings. In doing so, the transitional probability matrices, Γ_a and Γ_z, for the exogenous processes for a and z, respectively, are used for the evolution of the distribution:

$$\begin{aligned}&\mu'(a',z',k')\\&=\sum_{a\in\mathbf{a}}^{N_a}\sum_{z\in\mathbf{z}}^{N_z}\mu(a,z,k)\Gamma_a(a'=a_l|a=a_q)\Gamma_z(z'=z_i|z=z_j)\mathbb{I}(k',a,z,k),\end{aligned} \tag{4}$$

where $\mathbb{I}(k',a,z,k)=1$ if $k'=K(a,z,k)$ and 0 otherwise.

3. Once the stationary distribution is obtained, it is possible to multiply it by the relevant policy decision to obtain the aggregates K, L, Y, I.

2.4 Transitional Dynamics under Policy Uncertainty

We solve for the transitional dynamics under policy uncertainty. The model features policy uncertainty, coming through the stochastic processes represented by the transition matrices Γ_ζ and Γ_z.

We set $T = 100$ and $N^* = 16$, where T is the total number of periods in the simulation and N^* denotes the first period in which uncertainty is resolved. For the transition from ζ^P to ζ^j where $j \in \{S, H, P\}$:

1. Solve the model for the initial steady state (ζ^P) using value function iteration and obtain the initial distribution $\mu_0(a,z,k)$ by solving the fixed point of the stationary distribution.

2. Solve the model for all the aggregate states with the aggregate policy stochastic process (Γ_ζ) and the idiosyncratic policy stochastic process (Γ_z) and obtain the optimal policy functions $K_t(a,z,k;\zeta^i)$, $L_t(a,z,k;\zeta^i)$, and $\phi_t^T(a,z,k;\zeta^i)$ where $i \in \{P, N, S, H\}$ using value function iteration.

3. Using the optimal policy functions and $\mu_{t-1}(a,z,k)$, obtain aggregates and solve for the next period distribution $\mu_t(a,z,k)$ for $t = 1, ..., N^*$ under the aggregate state ζ^N.

4. Again, using the optimal policy functions and $\mu_{t-1}(a,z,k)$, obtain aggregates and solve for the next period distribution $\mu_t(a,z,k)$ for $t = N^* + 1, ..., T$ under the aggregate state ζ^j.

We have used alternative maximum time periods for the algorithm, namely, $T = 200, 300$ and checked that the results do not change.

References

Nelder, J. A. and Mead, R. (1965). A Simplex Method for Function Minimization. *The Computer Journal*, 7(4):308–313.