Online Appendix of "Sectoral effects of social distancing"

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Table 1—Model Equations

Description	Equation	Input-Output Parameter(s)
Intermediate good price	for $\varepsilon \neq 1$, $\widehat{P}_i^M = \left(\sum_j \Omega_{ij} \widehat{p}_j^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$ for $\varepsilon = 1$, $\widehat{P}_i^M = \prod_j \widehat{p}_j^{\Omega_{ij}}$	$\Omega_{ij} = \frac{\overline{p}_j \overline{x}_{ij}}{\sum_j \overline{p}_j \overline{x}_{ij}}$
Capital/Labor bundle	$\widehat{a}_i = \left(\widehat{l_i}\right)^{\gamma_i} \left(\widehat{k_i}\right)^{1-\gamma_i}$	$\gamma_i = \frac{\overline{w}_i \overline{l}_i}{\overline{v}_i \overline{a}_i}$
Capital/Labor bundle price	$\widehat{v}_i = \widehat{z}_i^{\frac{\theta-1}{\theta}} \left(\frac{\widehat{y}_i}{\widehat{a}_i}\right)^{\frac{1}{\theta}} \widehat{p}_i$	
Labor Income	$\widehat{w}_i \widehat{l}_i = \widehat{v}_i \widehat{a}_i$	
Capital Income	$\widehat{r}_i \widehat{k}_i = \widehat{v}_i \widehat{a}_i$	
Final demand	$\widehat{f}_i = \widehat{p}_i^{-\sigma} \widehat{P}^{\sigma} \widehat{Y}$	
Intermediate demand	$\widehat{x}_{ij} = \widehat{z}_i^{\theta-1} \widehat{p}_i^{\theta} \widehat{p}_j^{-\varepsilon} \left(\widehat{P}_i^M\right)^{\varepsilon-\theta} \widehat{y}_i$	
Prices = MC	for $\theta \neq 1$, $\widehat{p}_i^{1-\theta} = \widehat{z}_i^{\theta-1} \left(\eta_i \widehat{v}_i^{1-\theta} + (1-\eta_i) \left(\widehat{P}_i^M\right)^{1-\theta}\right)$ for $\theta = 1$, $\widehat{p}_i = \widehat{z}_i^{-1} \widehat{v}_i^{\eta_i} \left(\widehat{P}_i^M\right)^{1-\eta_i}$	$\eta_i = \frac{\overline{v}_i \overline{a}_i}{\overline{p}_i \overline{y}_i}$ and $1 - \eta_i = 1 - \frac{\sum_j \overline{p}_j \overline{x}_{ij}}{\overline{p}_i \overline{y}_i}$
Markets Clearing	$\widehat{y}_i = \varphi_i \widehat{f}_i + \sum_j \Delta_{ji} \widehat{x}_{ji}$	$\varphi_i = \frac{\overline{p}_i \overline{f}_i}{\overline{p}_i \overline{y}_i}$ and $\Delta_{ij} = \frac{\overline{p}_j \overline{x}_{ij}}{\overline{p}_j \overline{y}_j}$
GDP Deflator	for $\sigma \neq 1$, $1 = \widehat{P}^{1-\sigma} = \sum_i \psi_i \widehat{p}_i^{1-\sigma}$ for $\sigma = 1$, $1 = \widehat{P} = \prod_i \widehat{p}_i^{\psi_i}$	$\psi_i = \frac{\overline{p}_i \overline{f}_i}{\sum_i \overline{p}_i \overline{f}_i}$

Note: For an equilibrium variable X whose value at an initial equilibrium is $\overline{X}$, we denote $\widehat{X} = X/\overline{X}$, the change from the initial equilibrium. For sectors i and j, P_i^M is the intermediate input bundle price, p_j is the price of good j, a_i is the capital/labor input bundle, l_i is the labor input, k_i is the capital input, v_i is the capital/labor bundle price, thus $v_i a_i$ is the value-added, z_i is the productivity, y_i is the quantity of good i , r_i is the rental rate of capital, w_i is the wage, f_i is the final demand for good i, P is the price index of the aggregate good, Y is the quantity of aggregate good, and, x_{ij} is the intermediate input of good j from sector i. Given exogenous variables, z_i, k_i, l_i, elasticities $\sigma, \theta, \varepsilon$, input-output parameters, the solution of this system of equations gives the endogenous variables, $P_i^M, p_i, y_i, a_i, v_i, r_i, w_i, f_i, Y, P = 1, x_{ij}$.