Crime Chains

By Mehmet Bac*

How should law enforcement resources be allocated to minimize the harms from flexible, chain-form trafficking organizations? I show that optimal interventions focus on one target alone, the feeding source (decapitation) *or the revenue-generating tail* (amputation). *Decapitation dismantles the crime chain under large budgets but induces maximal expansion otherwise, whereas amputation generates a rich set of detection outcomes and limits the chain's size response. A rule of thumb emerges for authorities to target tail segments under small budgets and high detection contiguity, qualified by chain profitability and enforcement parameters. Real-world interventions fail to coordinate on such efficient targeting.*
JEL: H50, K42
Keywords: organized crime, illicit trafficking, enforcement policy, budget allocation, intervention, connections, detection contagion, chain size

Illicit trafficking activities are organized in chain form. Human trafficking chains, for example, extend from a source unit that recruits victims to multiple units that transport, control and exploit them by forcing into crimes or working in hazardous jobs. Drugs, ivory, rhino horns and cultural artefacts are smuggled via chain-form connections.

Enormous budgets are branched out to combat tail units of these crime chains. In 2017, 50.4 percent of the U.S. federal drug control budget, excluding thus state-level resources, is allotted to disturbing trafficking activity through the borders and reducing availability of illicit drugs inside the country while 5.5 percent is spent in assistance to international partners.[1] It is unclear what results can be expected from such resource allocations predominantly targeting the tail units of trafficking chains while source units operate in low-risk environments. Occasional intervention campaigns in upstream source countries have also failed to deliver tangible results in the long run.[2] This paper scrutinizes a crime chain's expansion/contraction response to intervention policies and how, given this response,

* Faculty of Arts and Social Sciences, Sabanci University, Tuzla, Istanbul 34956, Turkey (e-mail: bac@sabanciuniv.edu). I thank Parimal Bag for bringing some of the literature to my attention and three referees for very useful comments and suggestions. TUBA member annual research grant is gratefully acknowledged.

[1]The main components of the domestic drug control budget are interdictions (16.6 percent) and law enforcement (33.8 percent). The remaining 44.1 percent of the total budget of $27.476 billion is spent on prevention and treatment (Executive Office of the President of the United States, National drug control budget, 2017).

[2]The number of people using drugs has increased by thirty percent between 2009 and 2017 (WDR, 2019). An ILO (2018) report estimates nine percent increase in the number of migrant workers worldwide since 2015.

the enforcement authority should allocate its resources between chain targets to minimize the harm.

There are a number of aspects to emphasize about crime chains and their interaction with law enforcement. The chain-form activity organization is a dictum of illicit trafficking. Without supplies from the source, tail units have no operation to cash in, and without tail units the source is dysfunctional. Removing a unit inflicts a damage proportional to the size of the disconnected segment. So, while the chain's Phoenix unit remains connected to the source in Mexico if law enforcement dismantles the Denver unit and cuts off the supply to Kansas downstream, detection of the Phoenix unit will disrupt all three. *Contiguity* of detections amplifies the risk. Another important aspect and contributing factor to the failure of enforcement policies is the capacity of crime chains to respond by suppressing or adding units. Such contractions/expansions entail changes in territorial control, hence the harm caused. Finally, for lack of sufficient intelligence and extreme mobility of the chain's clandestine operations, it is often impossible to control independently the detection probability of each tail unit. In light of this observation it is more appropriate to view the chain's revenue-generating tail group as one whole target and its source, typically discernible by location and activity, another target. The main question, then, is how to allocate the intervention budget between "fighting the mosquitos" and "drying up the marsh."[3]

The chain model incorporates the aspects listed above. It consists of a source unit which commits crime S and provides an input to successively ordered tail agents executing crimes A. The agents can be *directly detected* and traced back through a detected neighbor, that is, *indirectly detected*, by the State. Each agent's harm to society and contribution to chain profits are contingent on his connectivity to the source, which he loses if the source or any other agent along the path to the source is detected. In this setup, returns to chain growth are (eventually) diminishing because connectivity to source falls and indirect detection possibilities increase as the chain grows. The crime chain maximizes expected profits, incorporating thus a security-production tradeoff, whereas the State seeks to minimize the chain's expected harm by allocating its fixed budget between crimes S and A. An allocation, called *intervention strategy*, produces a pair of direct detection probabilities, one for the source, one for each agent on the operational tail segment.

The analysis starts by characterizing detection risks along the crime chain and presents comparative statics results, some of which are of independent interest. An increase in the (common) direct detection probability of the agents will skew

[3]Interventions targeting the chain's source would include policies that aim at, depending on the context, reducing recruitment of victims or destroying illicit crops as well as shipment interdictions. As for the chain's tail segment, besides counter-trafficking interdiction efforts by destination countries at their borders, enforcement would initially rely on methods such as finding informants, inspections in select locations, undercover policing and electronic surveillance to gather intelligence information. It takes a long period of time for these efforts to culminate in arrests, but resources must be committed in advance.

the harm distribution towards source-proximal units. So, to illustrate, if the domestic component of the U.S. drug control budget is raised to uniformly increase the probability of detection of tail units operating domestically, expect a lower benefit in areas served by the chain's ports of entry along the Mexican border or the Pacific coast relative to areas served by its distant tail units. On the other hand, expect higher benefits in source-proximal areas from an increase in the budget for disrupting the chain' source unit abroad.

A crime chain's expected harm is minimized by targeting exclusively the source unit *(decapitation)*[4] or the tail group *(amputation)* under any budget, detection contiguity and chain size (number of agents). This result stems from the serial interdependence between chain units. In comparison, decapitation is essentially an 'all-or-nothing' strategy: if it succeeds to capture the source the harm is eliminated, but if it fails the chain inflicts its full potential harm. Amputation in contrast generates a rich set of detection outcomes and its effectiveness depends on several parameters. Higher detection contiguity and larger chain size favor amputation. In this environment, it may not be optimal to target the most harmful chain segment. Full priority may be assigned to detecting the harmless dealers in cultural treasures (amputation) if their harmful partners looting archeological sites can be traced and detected with high probability. Nor does a more harmful tail segment imply a stronger case for amputation, for the same reason.

These results bear on fixed-size crime chains. Size is endogenized as the chain's best response to any intervention strategy, including thus simultaneous enforcement activities at the source and the tail. Whenever the tail is under positive scrutiny, the crime chain limits its size. The response to decapitation is strikingly different, however. By generating a fixed cost for the chain but incentivizing its growth, decapitation leaves the chain with two options, to be of maximal size and not to be. Factors that favor maximal expansion include (i) highly productive crime units, (ii) low detection contiguity, and (iii) small intervention budgets implying small detection probabilities for the source. The first condition bears on the crime chain's profitability, market opportunities in its trafficking business and efficiency of its operational units; the second condition increases incremental growth profits by lowering the agents' detection probability via the source. The third condition reduces what is almost like a fixed cost for the chain under decapitation.

This optimal response is incorporated into a dynamic game where the State moves first by setting its intervention strategy. In equilibrium, the State opts for decapitation if its intervention budget exceeds a critical level, or if the budget is small but amputation is not so effective in directly detecting the agents. A rule of thumb emerges for the State to target the tail part of the crime chain if its budget is small and detection contiguity is high, qualified by relative effectiveness of enforcement and profitability of the chain's activities.

[4] "*Kingpin* strategy" (the targeting of high-ranked members of criminal organizations) is an alternative for the term *decapitation* strategy, though the overlap between the two terms may not be perfect.

The analysis highlights elements of successful interventions under single-hand control. An extension to the transnational context indicates which country should be assisted given the harm distribution, enforcement resources and technologies, to minimize the global expected harm. Actual interventions typically fail to achieve such effective coordination, except the few instances confined to North-South cooperation under Northern pressure.[5] The chain structure of transnational trafficking activity lies at the root of cooperation failures. International crime links produce powerful enforcement externalities and incentives to free-ride, of magnitudes depending primarily on national budgets and relative harms from transnational and local crimes. Joint interventions to be effective should combine resources and focus on one side of the border, supported by intelligence sharing agreements that maximize trace-back information from the other side.

• **Related Literature.** Research on anti-drug policies focus predominantly on impacts in downstream markets controlled by tail units of drug chains.[6] Another strand of literature examines cost-effectiveness of expenditures on prevention, treatment, interdictions, domestic law enforcement and source-country interventions.[7] While these contributions highlight specific aspects of the drug trafficking problem, they offer little guidance as to how the drug control budget should be allocated between source- and tail-interventions.

Network models view organizations as a set of connections shaped by the trade-offs between internal trust building, value of connectivity and cooperation, security, contagion risks and defensive resource allocation between units. Baccara and Bar-Isaac (2008) study emergence of criminal networks in a repeated game between an enforcement authority and crime units. Except for units linked in pairs, however, their equilibria do not display the connection structure that is prevalent in illicit trafficking. Other models emphasize different network structures. In Goyal and Vigier (2014) the organization's structure is part of its defense response to an attacker whose objective is to destabilize the organization. A 'star' network is optimal in most situations. Cerdeiro, Dziubinski and Goyal (2017) explore the tradeoff between connectivity and protection from contagious attacks, showing that the designer may disconnect a subset of nodes to protect the network.[8] More generally, Galeotti, Golub and Goyal (2020) study welfare-maximizing in-

[5]An example of joint decapitation intervention is Plan Colombia, which costed \$7.3 billion to the Colombian government and \$4.3 billion to the U.S. between 2000 and 2008. See Mejia and Restrepo (2016) for an evaluation of the Plan's costs and benefits. Annual reports published by the United Nations draw attention to the need for international coordination in every issue; see for example WDR (2020).

[6]This line of literature includes Anderson (2010) on prevention through graphic advertising campaigns, Swensen (2015) on mortality impact of substance-abuse treatment availability, Dobkin and Nicosia (2009), Cunningham and Finlay (2017) on harm-reducing effects of limiting methamphetamine availability.

[7]MacCoun and Reuter (2001) find that treatment is more cost-effective than prevention efforts. Studies of source-country interventions are relatively few and campaign-specific (Dell, 2015; Mejia and Restrepo, 2016; Lindo and Padilla-Romo, 2018).

[8]The emphasis in social network settings is on inter-connectivity of nodes because individuals derive benefits from their connections, whereas trafficking networks rely primarily on source-connectivity of crime units.

terventions in networks where individual actions generate spillovers, given an intervention budget. Borrowing ideas from this literature (security-production tradeoff, contiguity of detections, resource constraint for the external authority) the present model incorporates a number of missing crime chain features discussed at the beginning of this section.

Network analysis is applied to several crime contexts, notably by (Dell, 2015), who finds that Mexican drug cartels change their route choices in response to shifts in interdiction resources.[9] Blattman et al. (2018) study street level interventions on gangs in an experiment carried out in Bogota. They find mixed impacts on crime dispersion and deterrence.[10] This paper abstracts from routing problems and gang-level interactions between law enforcement and organized crime.

A model related for its analysis of the resource allocation problem is Bac and Bag (2020), where continua of potential criminals match to execute an 'input crime' and an 'output crime' in dyads. Theirs is a diffuse, competitive criminal environment. While they find a tendency for enforcement to prioritize upstream crimes to prevent downstream crimes by intercepting upstream criminals prior to matching, this paper highlights the structure and response of the crime chain as prime determinants of the optimal intervention.

The next two sections present the model and preliminary results on detection risks and source-connection probabilities. Section III characterizes the optimal intervention for a crime chain of fixed size, followed in Section IV by the chain's response. Incorporating this response, Section V derives the optimal intervention strategy in an interactive setting. Section VI adapts the model to transnational crime context and Section VII discusses the results, the assumptions and other extensions, including a trafficking network example.

I. Model

A crime chain C_k consists of a source unit $\mathcal{S}$ and $k \geq 1$ successively linked agents as follows:

$$\mathcal{S} \longleftrightarrow A_{1,k} \longleftrightarrow A_{2,k} \;\cdot\; \cdots\cdots \;\cdot\; A_{k-1,k} \longleftrightarrow A_{k,k}.$$

$\mathcal{S}$, indexed 0, will commit crime S, which supplies an input for each agent to commit crimes of type A. Crime A is impossible without crime S and crime S

[9]The analysis in Ballester, Calvo-Armengol and Zenou (2006) to identify the key players in a network is applied by Konig et al. (2014) to insurgent groups in the Congo War. Brown et al. (2006) examine defensive network strategies under the threat of terrorist attacks. Interdiction strategies in networks with multiple routes between a source and a destination node are studied by many, such as Wollmer (1963) and Israeli and Wood (2002). Recent work on optimal interventions can also illuminate the effective control of downstream chain crimes. Eeckhout, Persico and Todd (2010) highlight random crackdowns and Banerjee et al. (2021) apply a randomized police deployment model with learning by potential criminals to an experiment carried out in Rajasthan.

[10]Empirical studies of small scale gang units that execute downstream crimes can produce insights on organizational aspects of large-scale trafficking activity. See also earlier work by Levitt and Venkatesh (2000).

generates no benefit without crime A.

This structure operates as a transmission line for criminal inputs and proceeds, therefore it is of prime relevance to macro-level intervention decisions. Each agent on the tail part of the chain represents a unit typically comprising a manager that coordinates distribution, collection of revenues, punishing defections, etc. The model abstracts from details of the chain's peripheral network which may include logistic and administrative support cells.

• **Detection possibilities**. All crime units, the source and the agents, can be detected each *directly* by an external authority, the State. They can also be detected *indirectly*, traced from detected neighbors. A unit is indirectly detected with probability $\delta \in [0, 1]$ if one of his immediate neighbors is detected. Thus, a successful shot by the State disrupts the chain and triggers a contamination in both directions, the source and the tail. The probability δ, capturing chain *contiguity*, is exogenously given; it would depend on factors such as the quality of information of each unit about his neighbor(s), enforcement technology, frequency of interactions and nature of the criminal activity. A chain with $\delta = 0$ perfectly shields its units from indirect detections. If $\delta = 1$, one detection suffices to (indirectly) detect all other chain units, so, either all will be detected or none. For intermediate values of $\delta \in (0, 1)$ indirect detection probabilities will be unit-specific, vary by location, or index, as shown in the sequel.[11]

• **Enforcement**. The analysis adopts an early-stage perspective for the State regarding which units or chain parts it can scrutinize. I assume that the State has little or no information about the identities and whereabouts of the extremely mobile agents. So, an agent-based allocation of enforcement resources to separately control the direct detection probability of each agent is impossible. The agents can be targeted as a group, not individually.[12]

ASSUMPTION 1: *Each A crime (agent) is directly detected with the same probability, μ_A.*

This assumption is relaxed in Section VII. It excludes the source unit which is often discernible by activity (recruiting victims, producing drugs) and location, possibly operating in a different country or in isolation from the agents. Also, different types of expertise and resources are needed to combat S crimes and A crimes. Accordingly, I assume that the State can separately target $\mathcal{S}$ and control its direct detection probability, denoted μ_S.

[11] One can introduce link detections as well, at the cost of additional complexity. The expressions chain profits and expected harms will be modified accordingly, but the qualitative results on optimal interventions and crime chain's size response obtained under node detections will go through.

[12] Enforcement activities in that stage are "general" in that they equally target the agents. In later stages these activities may become focused as information flows in. Signals about the presence of a human trafficking chain segment may be received, upon which a large amount of enforcement budget would be allotted to gathering information about working conditions in industries and identifying worksites and areas with high concentrations of temporary workers. Patrolling activities and random inspections would support intelligence development about chain members' transactions, their connections and whereabouts, etc.

The probabilities μ_S and μ_A are induced by allocating a fixed budget between crimes A and S, through a linear enforcement technology.

ASSUMPTION 2: *Let $\mu_S \equiv \bar{\mu} < 1$ if the entire budget is allotted to crime S.*

A fraction $x \in [0,1]$ of the budget induces $\mu_S = x \cdot \bar{\mu}$ if spent on crime S, $\mu_A = x \cdot \alpha\bar{\mu}$ if spent on crime A, where $\alpha > 0$ and $\alpha\bar{\mu} < 1$.

The parameter α captures the fact that resources invested in detecting $\mathcal{S}$ and those invested in detecting the agents are not equally effective. A feasible budget allocation (μ_S, μ), also called an *intervention strategy*, induces the direct detection probabilities μ_S and $\mu_A = \alpha\mu$ and satisfies the budget constraint $\mu + \mu_S = \bar{\mu}$.[13]

• **S-connection outcomes**. The chain generates its revenues from the agents' criminal activities. Each agent's potential revenue contribution is $v > 0$. This contribution is realized only if the path to $\mathcal{S}$ is open. So, if agent $A_{j,k}$ is detected the tail segment $\{A_{j,k}, \cdots, A_{k,k}\}$ is disconnected from $\mathcal{S}$ and becomes dysfunctional. Note that an agent need not be detected himself to lose his connection to $\mathcal{S}$. Also, the closer a detected agent's location to $\mathcal{S}$, the larger is the chain's loss. These notions, formalized below, capture the chain's vital role as transmission/exploitation line as well as its fundamental vulnerability.

DEFINITION 1: *Agent $A_{i,k}$ is S-connected if $\mathcal{S}$, $A_{1,k}, ..., A_{i-1,k}, A_{i,k}$ are not detected.*

The S-connection outcome with i agents occurs if $A_{i,k}$ is S-connected but $A_{i+1,k}$ is not. The $\emptyset$-connection outcome occurs if $A_{1,k}$ is not S-connected.

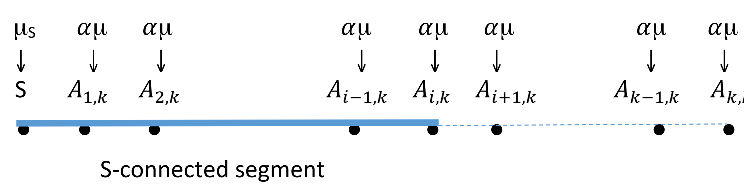

FIGURE 1. S-CONNECTION OUTCOME WITH i AGENTS IN C_k.

C_k has k potential S-connection outcomes. In the S-connection outcome with i agents shown in Figure 1, $\mathcal{S}$ commits crime S and the first i agents commit crime A each, generating the revenue $i.v$. Detection of $\mathcal{S}$ or $A_{1,k}$ produces the $\emptyset$-connection outcome.

[13]The linear enforcement technology in Assumption 2 is not absolutely necessary. Partly motivated by simplicity, it serves to highlight a structural nonconvexity in the State's objective function, which stems from serial interdependence of crime units. Section VII contains a discussion.

• **Objectives**. The net social harm from crime $j = S, A$ is $h_j \geq 0$. Though these harms are typically strictly positive both, the analysis admits the possibility that one $h_j = 0$; activity A *or* S may be almost harmless, yet classified a crime.[14]

Denote the probability of the S-connection outcome with i agents by $p_{i,k}$. In this outcome the chain inflicts the harm $h_S + i.h_A$. Thus, the expected harm by C_k is $SH = \sum_{i=1}^{k} p_{i,k}(h_S + i.h_A)$.[15]

The objective of the State is to minimize the expected harm from crimes S and A, given its intervention budget:

$$\begin{aligned} &\min_{\{\mu_S,\mu\}} \; SH = \textstyle\sum_{i=1}^{k} p_{i,k}(h_S + i.h_A), \\ &\text{subject to} \quad \mu_S + \mu = \bar{\mu}. \end{aligned} \tag{1}$$

The sanctions on crime A and crime S are denoted by s_A and s_S.[16] Given these sanctions and the intervention strategy (μ_S, μ), the crime chain sets itself a size $k \in \{0, 1, 2, \cdots, \bar{k}\}$ that maximizes profits,

$$\Pi(k) = R(k) - Es_S(k) - Es_A(k),$$

which consists of expected revenues $R(k)$ net of expected sanctions, $Es_S(k)$ for $\mathcal{S}$ and $Es_A(k)$ for the agents. That is,

$$\max_{k\in\{0,1,\cdots,\bar{k}\}} \Pi(k) = \max_{k\in\{0,1,\cdots,\bar{k}\}} \left[\sum_{i=1}^{k} p_{i,k}[i.v] - \mu_{0,k}.s_S - \sum_{i=1}^{k} \mu_{i,k} s_A \right]. \tag{2}$$

where $\mu_{0,k}$ and $\mu_{i,k}$ denote the detection probabilities of $\mathcal{S}$ and $A_{i,k}$, defined in the next section.

• **Timeline**. The analysis of the interaction between the State and the crime chain in sections IV and V adopts the following timeline.

(i) The State allocates its budget $\bar{\mu}$. The allocation (μ_S, μ) is publicly observed.

(ii) The chain determines k, forming C_k.

(iii) Detection and S-connection outcomes are realized. Detected members are sanctioned. S-connected agents commit crime A each while $\mathcal{S}$ commits crime S. The chain obtains its profit and inflicts the harms.

Two remarks: The chain is assumed to modify its size at no physical cost, for simplicity. Introducing size adjustment costs can of course create inertia and

[14]The magnitudes of these harms would depend on activity, of course: Production of opium and its transportation may not cause much harm as long as the drugs do not reach the consumers, whereas the bulk of the harm from wildlife crimes or smuggling of cultural artefacts are borne at the source.

[15]For simplicity the harm from crime S is assumed to be independent of the number of agents committing crime A. Alternative formulations under which the harm from crime S is a function of the S-connection outcome are of course admissible, but do not qualitatively affect the main results.

[16]These sanctions are exogenous, determined by the legislative branch of government. The State's cost or benefit from administering the sanctions would depend on the form the sanctions take. I shall assume that the sanctions are administered at zero cost, zero benefit.

limit the chain's size response. If adjustment costs are prohibitive, the analysis in Section III applies where chain size is taken to be fixed. Second, the assumption that the chain determines its final size after observing the intervention strategy is more appropriate than the chain-moves-first alternative if criminal organizations are more flexible in modifying their size and/or routes than state bureaucracies are in shifting large enforcement resources between targets when needed.[17]

II. Detections and S-connection outcomes

The risk distribution in the chain will not be uniform because detections are contiguous. To show this fact, it is useful to distinguish between indirect detections that originate from the source segment and those that originate from the tail segment.

DEFINITION 2: *The tail-detection probability* $\mu^+_{i,k}$ *of* $A_{i,k}$ *is his probability of* *indirect* *detection through direct detection of some* $A_{j,k} \in \{A_{i+1,k}, \cdots, A_{k,k}\}$*.*

The source-detection probability $\mu^-_{i,k}$ *of* $A_{i,k}$ *is his probability of* *indirect* *detection through direct detection of some* $A_{j,k} \in \{A_{1,k}, \cdots, A_{i-1,k}\}$ *or* $\mathcal{S}$*.*

The detection probability of $A_{i,k}$ can now be written as

$$\mu_{i,k} = \alpha\mu + (1-\alpha\mu)[\mu^-_{i,k} + (1-\mu^-_{i,k})\mu^+_{i,k}]. \tag{3}$$

In (3), the term in the squared brackets represents $A_{i,k}$'s indirect detection probability, which varies by agent location and produces a tendency for $\mu_{i,k}$ to rise as we move to the chain's central part.[18]

I turn now to the chain's S-connection outcomes (Definition 1), distinct by number of crimes committed, harms inflicted and profits earned. In the S-connection outcome with i agents, $\mathcal{S}$ and the agent group up to $A_{i,k}$ escape detection (which occurs with probability $(1-\mu_S)(1-\alpha\mu)^i$) while $A_{i,k}$'s right neighbor $A_{i+1,k}$ is detected. Thus, $p_{i,k} = (1-\mu_S)(1-\alpha\mu)^i(1-\delta)[\alpha\mu + (1-\alpha\mu)\mu^+_{i+1,k}]$, or,

$$p_{i,k} = \begin{cases} (1-\mu_S)(1-\alpha\mu)^i(1-\delta)\Big[\alpha\mu + (1-\alpha\mu)\delta\alpha\mu + \cdots \\ \qquad\qquad + (1-\alpha\mu)^{k-i-1}\delta^{k-i-1}\alpha\mu\Big] & \text{for } i < k, \\ (1-\mu_S)(1-\alpha\mu)^k & \text{for } i = k. \end{cases} \tag{4}$$

[17] See also Baccara and Bar-Isaac (2008). Section VII includes a discussion of this assumption.

[18] Appendix A (Lemma 3) contains formal expressions and some properties of $\mu^+_{i,k}$ and $\mu^-_{i,k}$. First, tail-detection probabilities fall and source-detection probabilities rise by distance to $\mathcal{S}$. Second, raising $\mu_A = \alpha\mu$ will increase both source- and tail-detection probabilities of all agents whereas raising μ_S will increase only the source-detection probabilities and leave tail-detection probabilities intact. Also, the source-detection probability at a given agent position is not affected by recruitments (adding new agents) to the tail end. In other words, recruitment affects the risk distribution in the chain solely through tail-detection probabilities.

As for the $\emptyset$-connection outcome, it occurs with probability $p_{\emptyset,k} = \mu_S + (1 - \mu_S)(\alpha\mu + (1-\alpha\mu)\mu^{+}_{1,k})$, or,

$$p_{\emptyset,k} = \mu_S + (1-\mu_S)\alpha\mu\big[1 + \sum_{i=1}^{k-1} \delta^i (1-\alpha\mu)^i\big]. \tag{5}$$

The following Lemma states two facts about the pattern of $p_{i,k}$.

LEMMA 1: *Let $\mu > 0$ and $\delta < 1$. (i) $p_{i,k} < p_{i+1,k}$; (ii) $p_{k,k} > p_{k,k+1}$ and, for $i < k$, $p_{i,k} < p_{i,k+1}$.*

Part (i) implies that high-harm outcomes occur less frequently than low-harm outcomes. Part (ii) states that recruitment increases each $p_{i,k}$ by raising the tail-detection probability of the $i+1$'th agent. The only exception is $p_{k,k}$, which falls (becoming $p_{k,k+1}$) because the new recruit introduces a tail-detection possibility for $A_{k,k}$ (becoming $A_{k,k+1}$).[19]

Consider now an increase in the domestic law enforcement (μ) or the source detection component (μ_S) of the drug control budget. How are the $p_{i,k}$ affected? Define $M_{i,k} = \{(\mu,\delta) \mid \frac{\partial p_{i,k}}{\partial \mu} \leq 0\}$ as the set of (μ,δ) combinations such that the probability of S-connection outcome with i agents is nondecreasing in μ (and α).

PROPOSITION 1: *Assume $\delta < 1$. The following properties hold.*

(i). $\dfrac{\partial p_{i,k}}{\partial \mu_S} < \dfrac{\partial p_{i+1,k}}{\partial \mu_S} < 0$ *for all* $1 \leq i \leq k-1$, *thus* $\displaystyle\sum_{i=1}^{k} \frac{\partial p_{i,k}}{\partial \mu_S}(h_S + i.h_A) < 0.$

(ii). $M_{1,k} \subset M_{2,k} \subset \cdots \subset M_{k,k} = [0,1)^2.$

Equivalently, $\dfrac{\partial p_{i,k}}{\partial \mu}|_{(\mu,\delta)} \leq 0 \Rightarrow \dfrac{\partial p_{i+1,k}}{\partial \mu}|_{(\mu,\delta)} < 0.$

(iii). $\dfrac{\partial p_{\emptyset,k}}{\partial \mu} > 0$ *and* $\displaystyle\sum_{i=1}^{k} \frac{\partial p_{i,k}}{\partial [\alpha\mu]}(h_S + i.h_A) < 0.$

An extra budget allotted to detect $\mathcal{S}$ will reduce all $p_{i,k}$, hence the expected harm, with stronger effects in source-proximal segments. However, an extra budget to uniformly raise the detection probability of the agents triggers two conflicting effects: a *source effect* reduces $p_{i,k}$ by raising the detection probabilities of agents $A_{1,k}$ to $A_{i,k}$ whereas a *tail effect* increases $p_{i,k}$ by raising the detection probability of $A_{i+1,k}$. Absence of a tail effect thus explains why $p_{k,k}$ always falls if μ is increased. The source effect dominates for large i, but the tail effect can

[19] Also note that if $\mu_S > 0$ but $\mu = 0$ or $\delta = 1$, the chain has but two potential outcomes: either all agents are S-connected or none.

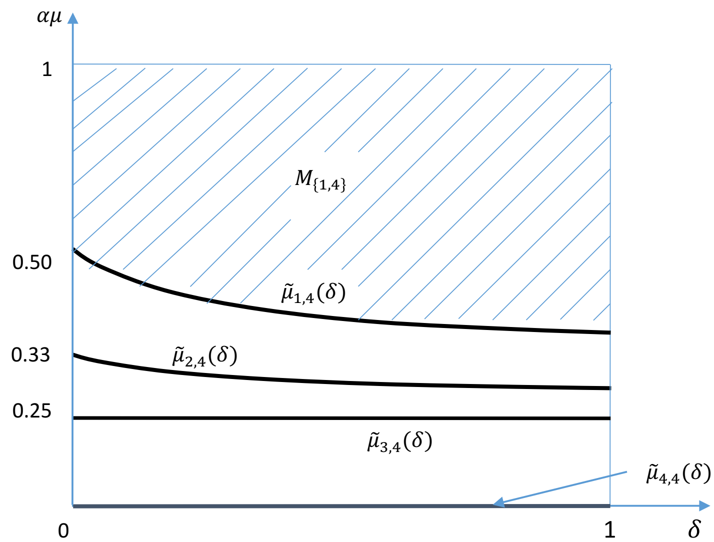

FIGURE 2. IMPACT OF μ ON $p_{i,k}$ IN THE CHAIN C_4.

Note: The set $M_{1,4}$ illustrates combinations of $(\alpha\mu, \delta)$ above the schedule $\tilde{\mu}_{1,4}(\delta)$, such that an increase in μ reduces $p_{1,4}$, the probability that $A_{1,4}$ only is connected to the source.

dominate in S-connection outcomes with small i (where a small segment of the chain survives).[20] Accordingly, the set $M_{i,k}$ becomes smaller as i approaches $\mathcal{S}$.

An increase in the budget to fight the tail segments of the crime chain can thus have unexpected effects on the geographic distribution of harms and chain activities. Figure 2 illustrates these effects in the chain C_4. For example, if $\alpha\mu < 0.25$ and $\alpha\mu$ is increased, $p_{1,4}$, $p_{2,4}$ and $p_{3,4}$ will rise but $p_{4,4}$ will fall regardless of detection contiguity δ. Starting from low levels to uniformly increase the intensity of enforcement on the tail, then, one should observe greater benefits in source-distant segments. However, if $\alpha\mu \geq 0.5$ or slightly below 0.5 but detections are contiguous ($\delta > 0$), the source effect will dominate and an increase in $\alpha\mu$ will reduce the probabilities of all S-connection outcomes.

The signs of some $\frac{\partial p_{i,k}}{\partial \mu}$ may be ambiguous, but $\sum_{i=1}^{k} \frac{\partial p_{i,k}}{\partial \mu}$ is negative, which implies that the probability of the $\emptyset$-connection outcome increases in μ. It follows that an increase in $\alpha\mu$ given μ_S, or an increase in μ_S given $\alpha\mu$, reduces the expected harm.

[20]These effects are intermediated by the detection contiguity parameter δ. A larger δ intensifies both the source and the tail effects. If the source effect dominates at a given (μ, δ) combination so that $\frac{\partial p_{i,k}}{\partial \mu} < 0$, it continues to dominate at larger δ.

III. Optimal interventions

This section characterizes the State's optimal intervention strategy, denoted (μ_S^F, μ^F), for a fixed-size, established crime chain.

The following intervention strategies will be of special interest.

• *Decapitation:* $(\mu_S, \mu) = (\bar{\mu}, 0)$. The State allocates its resources entirely to the capture of $\mathcal{S}$. These resources could be used in disrupting the production of crime inputs, intercepting their transportation, and possibly capture the management of the source unit.

• *Amputation:* $(\mu_S, \mu) = (0, \bar{\mu})$. This strategy deploys all resources to detect maximal number of agents. Amputation includes coordinated crackdowns on chain operations in various locations such as drug markets and houses, brothels, airports, random checkpoints, as well as intelligence activities.

If the expected harms are identical so that the State is indifferent between amputation and decapitation, assume that the State breaks the tie in favor of decapitation.

PROPOSITION 2: *Fix a crime chain C_k. The optimal intervention strategy is pure* decapitation *or pure* amputation*.*

The choice between decapitation and amputation depends on chain structure and enforcement parameters, as follows.

(i) (Relative enforcement effectiveness) *There exists a critical α_k such that amputation is optimal if $\alpha > \alpha_k$ and decapitation is optimal if $\alpha \leq \alpha_k$. This critical α_k depends on chain size, detection contiguity and relative crime harms.*

(ii) (Chain size) *Recruiting an agent to C_k raises expected harm more under decapitation than amputation, implying $\alpha_{k+1} < \alpha_k$, if*

$$(1 - \bar{\mu})h_A \; > \; (1 - \alpha\bar{\mu})^k \left[(-\alpha\bar{\mu}\delta^k)h_S + \left[1 - \alpha\bar{\mu}(1 + \sum_{i=1}^{k} \delta^i)\right] h_A \right]. \tag{6}$$

(iii) (Detection contiguity) *An increase in δ reduces the expected harm under amputation and leaves the expected harm unchanged under decapitation, implying α_k is decreasing in δ.*

(iv) (Relative crime harms) *Fix $h_A \geq 0$ and increase h_S. Expected harm will increase more under decapitation than amputation if $\alpha > \alpha'_k$, where $\alpha'_k < 1$ is defined through*

$$\alpha'_k \left[1 + \delta(1 - \alpha'_k\bar{\mu}) + \cdots + \delta^{k-1}(1 - \alpha'_k\bar{\mu})^{k-1}\right] = 1. \tag{7}$$

All resources are channelled to the source to capture $\mathcal{S}$ or to the tail and detect as many agents as possible. Discussed further in Section VII, this result stems from serial interdependence of chain units, best illustrated in the one source - one agent chain, C_1, assuming $\alpha = 1$. Investing the entire budget $\bar{\mu}$ to capture

$\mathcal{S}$ or investing it all to capture the agent eliminates the harm with the same probability, $\bar{\mu}$, whereas splitting the budget induces a pair of direct detection probabilities $\mu_S > 0$, $\mu = \bar{\mu} - \mu_S > 0$ and eliminates the harm with probability $\mu_S + (1 - \mu_S)\mu < \bar{\mu}$. It is therefore more effective to target $\mathcal{S}$ or the agent than targeting both. This holds under all parameter constellations and chain sizes with multiple harm (S-connection) outcomes.

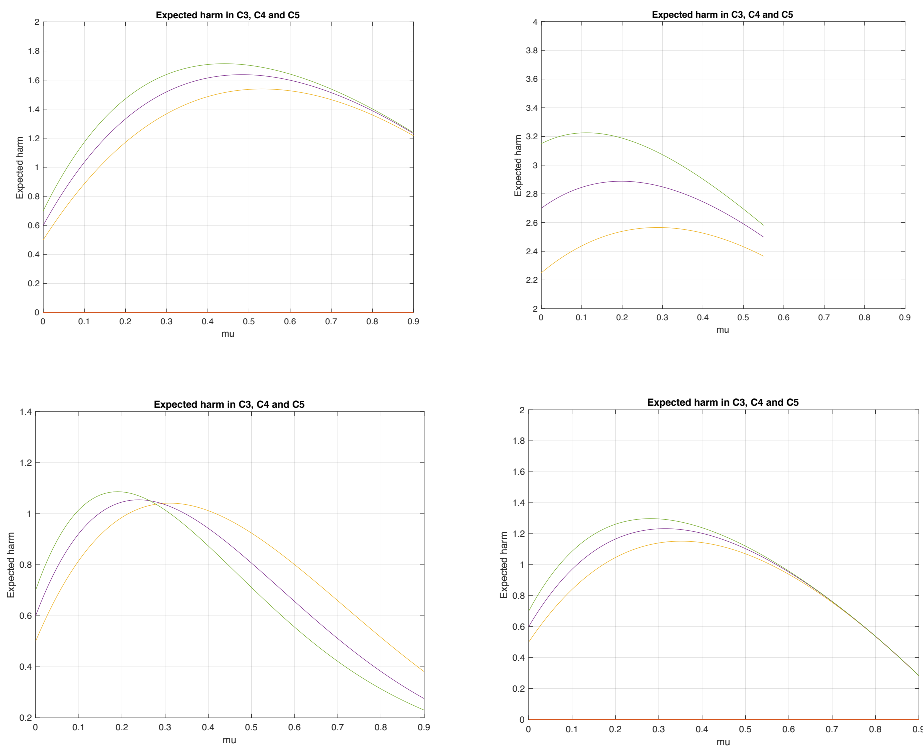

Figure 3. Expected harm as a function of μ, in C_3, C_4 and C_5.

Note: Decapitation corresponds to $\mu = 0$. The top left panel under (default) parameter values $h_A = 1$; $h_S = 2$; $\delta = 0.1$; $\bar{\mu} = 0.9$; $\alpha = 0.7$, where decapitation is optimal. The top right panel reduces budget to $\bar{\mu} = 0.55$, showing that decapitation is optimal in C_3 (lower curve) whereas amputation is optimal in C_4 and C_5. The bottom left panel raises contiguity to $\delta = 0.9$ and the bottom right panel raises α to 1, showing amputation becomes optimal in C_3, C_4 and C_5.

Whereas amputation sets up k independent Bernouilli draws for the agents, under decapitation the chain dismantles or survives and inflicts full harm. The choice between the two strategies depends on the relative enforcement effectiveness parameter α, chain size, detection contiguity and crime harms.

• **Role of chain size**. Figure 3 panels display the expected harm from C_3, C_4 and C_5 as μ is varied in the interval $[0, \bar{\mu}]$ on the x-axis, with the understanding that

$\mu_S = \bar{\mu} - \mu$. In all panels the curve with lowest intercept at $\mu = 0$ corresponds to C_3, followed by C_4 and C_5. The top left panel shows that decapitation is optimal if $\alpha = 0.7$ whereas the bottom right panel shows amputation is optimal if $\alpha = 1$, in all three chains, confirming part (i) of Proposition 2 that large α values favor amputation.

The impacts of chain growth on expected harm under decapitation and amputation are compared in (6). If this condition holds at chain size k, adding one more agent to it works in favor of amputation. Under decapitation, the new agent raises expected harm by $(1 - \bar{\mu})h_A$ regardless of chain size, which appears at the left hand side of (6). Under amputation, each new recruit actually reduces the h_S-component of harm. As for the expected harm from the tail part, it cannot fall as the number of agents grow provided the chain expects a nonnegative revenue from recruitment.[21] For sufficiently large chain size k amputation will dominate decapitation. Observe, in particular, that the right hand side of (6) is decreasing in δ, implying that under conditions of high detection contiguity further chain growth favors amputation. The top right panel illustrates the switch from decapitation to amputation occurring at $k = 4$, where decapitation is optimal for C_3 and amputation is optimal for C_4 and C_5, at $\delta = 0.1$, under the budget $\bar{\mu} = 0.55$.

• **Role of detection contiguity.** The State should attack the crime chain from the tail part if δ is high, the source if δ is low. This is fairly intuitive because the expected harm under decapitation is independent of δ whereas under amputation the number of detections and hence reductions in expected harm are proportional to it. In Figure 3, amputation is optimal in the bottom left panel where $\delta = 0.9$ whereas decapitation is optimal in the top left panel where $\delta = 0.1$, all else the same.

• **Role of crime harms**. When crime S becomes more harmful, say, as the supply of horns reduces the rhino stock to critical levels, the case for amputation may become stronger if sufficiently effective, that is, if α exceeds a threshold α'_k defined in (7). In this case it may become more effective to target the dealers downstream and not the poachers upstream. This holds, in addition to the large α requirement, if δ and k are large (high detection contiguity and chain size increase the indirect detection probability of $\mathcal{S}$).

IV. The chain's response

The analysis now probes into the crime chain's expansion/contraction decision. The timeline introduced in Section I defines a dynamic State-chain game in which given $\bar{\mu}$, the State's moves first with an intervention strategy, picking a pair (μ_S, μ) such that $\mu_S + \mu = \bar{\mu}$. The chain's strategy k^* maps such feasible pairs of

[21] For the harm from A-crimes to fall as k increases, the coefficient of h_A in (6) must be negative. This requires $\alpha\bar{\mu} > \frac{1}{1+\sum_{i=1}^{k}\delta^i}$ which in turn violates the condition for a nonnegative expected revenue from recruitment (in (11), see Proposition 3). The bottom left panel illustrates the possibility that expected harm falls when the chain grows under the amputation strategy ($\mu = \bar{\mu}$), for $\bar{\mu} = 0.9$, $\delta = 0.9$: C_3 is more harmful than C_4, which in turn inflicts a larger harm than C_5.

(μ_S, μ) into $\{0, 1, 2, \cdots, \bar{k}\}$. I focus on pure-strategy Subgame-Perfect equilibria (hitherto, *equilibria*) of this game, defined in the usual way: $k^*(\mu_S, \mu)$ maximizes (2) given any (μ_S, μ), and (μ_S^*, μ^*) solves (1) given the chain's response function $k^*(\mu_S, \mu)$.[22]

A. Recruitment

To characterize the chain's optimal size I study the incremental profits $\Delta\Pi(k)$ from recruitment, i.e., the change in profits from a chain extension $C_k \longrightarrow C_{k+1}$. Proposition 3 states the corresponding impacts on expected revenues $\Delta R(k)$ and expected sanctions $\Delta Es_S(k)$ and $\Delta Es_A(k)$ as a function of the intervention strategy, the chain's actual size, its detection contiguity and other model parameters.

PROPOSITION 3: *Fix a feasible intervention strategy (μ_S, μ) and consider a chain extension $C_k \to C_{k+1}$.*

- *Expected sanctions will increase by*

$$\Delta Es_S(k) = (1-\mu_S)\alpha\mu\delta^{k+1}(1-\alpha\mu)^k s_S \geq 0, \tag{8}$$

$$\Delta Es_A(k) = \left[\alpha\mu\sum_{i=1}^{k}(1-\mu_{i,k}^{-})\delta^{k-i+1}(1-\alpha\mu)^{k-i+1} + \mu_{k+1,k+1}\right]s_A \geq 0. \tag{9}$$

- *The impact on expected revenues is,*

$$\Delta R(k) = (1-\mu_S)(1-\alpha\mu)^k\left[1-\alpha\mu-\alpha\mu\delta\frac{1-\delta^k}{1-\delta}\right]v, \tag{10}$$

which is positive if and only if

$$\alpha\mu < \frac{1}{1+\delta\frac{1-\delta^k}{1-\delta}} \equiv m(\delta, k), \tag{11}$$

where $\frac{1}{1+\delta} = m(\delta,1) > \cdots > m(\delta,k) > m(\delta,k+1) > \cdots > m(\delta,\infty) = 1-\delta$.

If $\alpha\mu \leq 1-\delta$, $\Delta R(k) > 0$ for all k, whereas if $\alpha\mu > 1-\delta$ there exists a finite chain size $k_G > 0$ such that expected revenues fall beyond size k_G, that is, $\Delta R(k_G - 1) > 0 > \Delta R(k_G)$.

I discuss the crime chain's costs from growth, followed by the benefits.

• **Impact on expected sanctions**. Recruitment raises expected sanctions in two ways. First, there is the indirect tail-detection impact on the expected sanctions

[22] A tie-breaking assumption will be imposed, namely, if $\Pi(k') = \Pi(k'')$ given (μ_S, μ), then the chain chooses the size $min\{k', k''\}$; and if $SH(\mu_S', \mu') = SH(\mu_S'', \mu'')$ given $k^*(\mu_S', \mu')$ and $k^*(\mu_S'', \mu'')$, the State picks the pair involving $max\{\mu_S', \mu_S''\}$. If indifferent, the chain chooses the smaller size and the State, the larger μ_S.

of $\mathcal{S}$ and the agents, given by $\Delta Es_S(k)$ in (8) and the summation term in (9). The second impact, given by the last term in (9), is through $A_{k+1,k+1}$'s own expected sanction $\mu_{k+1,k+1}.s_A = [\alpha\mu + (1-\alpha\mu)\mu^{-}_{k+1,k+1}].s_A$, which can be written as (see end of Appendix A),

$$\mu_{k+1,k+1}.s_A = \left[\mu_S \delta^{k+1}(1-\alpha\mu)^{k+1} + \alpha\mu \frac{1-\delta^{k+1}(1-\alpha\mu)^{k+1}}{1-\delta(1-\alpha\mu)}\right] s_A. \tag{12}$$

The term $\delta^{k+1}(1-\alpha\mu)^{k+1}\mu_S$ is $A_{k+1,k+1}$'s source-detection probability through $\mathcal{S}$, exclusively, and the second term represents the combined effects from his own direct detection and indirect detection through the agent group.

Whenever $\mu > 0$ and the tail segment is under positive scrutiny, the expression in (12) is bounded below by $\alpha\mu s_A/[1-\delta(1-\alpha\mu)]$, its limit as k becomes extremely large.[23] On the other hand, if $\mu = 0$, $\Delta Es_S(k) = 0$ and the summation term in $\Delta Es_A(k)$ in (9) is also zero. The rise in expected sanctions is then confined to the source-detection part in (12), $\mu_S\delta^{k+1}(1-\alpha\mu)^{k+1}s_A$. Observe that this part also vanishes as k becomes large. It follows that when enforcement focuses on capturing the source unit alone, an active crime chain will see little risk to expanding from its tail.

• **Impact on expected revenues**. In contrast with expected sanctions, the effect of recruitment on expected revenues is ambiguous. There is a positive effect from the new recruit's expected contribution represented in (11) by the term $(1-\mu_S)(1-\alpha\mu)^k[1-\alpha\mu]v \equiv p_{k+1,k+1}v$. The negative effect stems from a detection externality on other agents' contributions, of magnitude $(1-\mu_S)(1-\alpha\mu)^k[-\alpha\mu\delta\frac{1-\delta^k}{1-\delta}]v$. These two effects are embodied in the schedules $m(\delta, k)$ defined in (11). We observe that the set of $(\alpha\mu, \delta)$ combinations under which expected revenues increase becomes smaller as k increases: The first recruit raises the expected revenue for all $\alpha\mu < 1$, implying $m(\delta, 0) = 1$, the second recruit raises it only if $\alpha\mu < m(\delta, 1) = \frac{1}{1+\delta}$, whereas the third recruit will raise it in a lower range of $\alpha\mu$ for each δ, implying $m(\delta, 2) < m(\delta, 1)$, and so forth.

Thus, if the agents' direct detection probability is not too low and detection contiguity not too small so that $\alpha\mu > 1-\delta$, the negative detection externality from the new recruit will eventually dominate and $\Delta R(k)$ will become negative at a size k_G. If $\alpha\mu \leq 1-\delta$, however, each recruitment will raise $R(k)$, but with smaller increments $\Delta R(k)$. In the limit case of $\mu = 0$ (under decapitation) each recruit's revenue contribution $\Delta R(k) = (1-\mu_S)v$ is constant.

[23]Lemma 4 in Appendix C shows that the rises in the expected sanction of $\mathcal{S}$ keep contracting at the rate $\delta(1-\alpha\mu) < 1$. On the other hand, the agent part of expected sanctions increases at a higher rate than $\delta(1-\alpha\mu)$, due to the expected sanction of the new recruit and the indirect detection possibilities he introduces for others.

B. The best response of the crime chain

The optimal chain size $k^*(\mu_S, \mu)$ can now be characterized in light of Proposition 3 and its discussion above. Recall, $\Delta\Pi(k) = \Delta R(k) - \Delta E s_S(k) - \Delta E s_A(k)$.

PROPOSITION 4: *Given an intervention strategy (μ_S, μ), the crime chain's optimal size response satisfies*

$$(13) \qquad k^*(\mu_S, \mu) \begin{cases} \in \{0, \cdots, \bar{k}-1\}, \ \text{if} \ \ \Delta\Pi(k^*-1) > 0 \geq \Delta\Pi(k^*) \\ \qquad\qquad\qquad\qquad \text{and} \ \ \Delta\Pi(\bar{k}) \leq 0; \\ = \bar{k}, \qquad\qquad \text{if} \ \ \Delta\Pi(\bar{k}) > 0, \end{cases}$$

where $0 < k^*(\mu_S, \mu) < \bar{k}$ only if $\mu > 0$. In particular, under decapitation

$$(14) \qquad k^*(\bar{\mu}, 0) = \begin{cases} 0, & \text{if} \ \bar{\mu} \geq \frac{\bar{k}v}{\bar{k}v + s_S + \sum_{i=1}^{k} \delta^i s_A} \equiv \bar{\mu}^c, \\ \bar{k}, & \text{otherwise.} \end{cases}$$

The chain's profit $\Pi(k^*)$ is decreasing, and its optimal size k^* is nonincreasing, in δ.

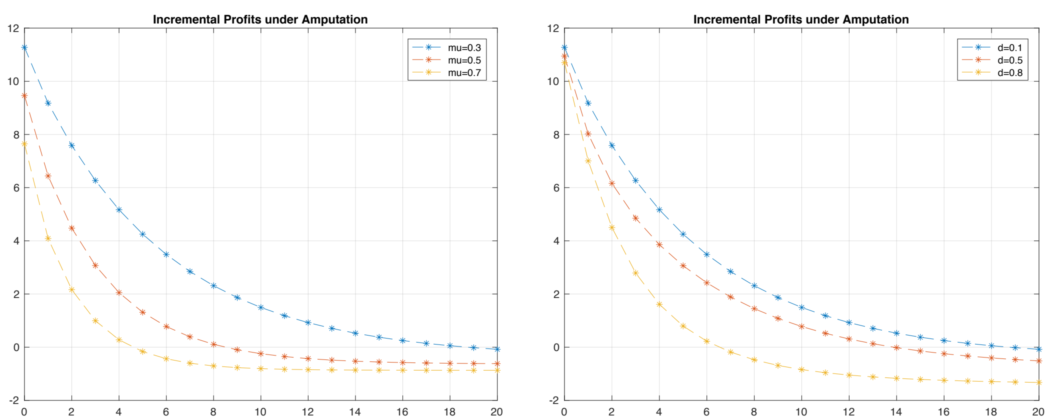

FIGURE 4. INCREMENTAL PROFITS $\Delta\Pi(k)$ UNDER AMPUTATION AS A FUNCTION OF CHAIN SIZE k.

Note: Parameter values: $v = 14$, $s_A = 2$, $s_S = 5$, $\alpha = 0.55$. Left panel displays $\Delta\Pi(k)$ for $\delta = 0.1$, raising $\bar{\mu} = 0.3$ to 0.5, then to 0.7, resulting in optimal sizes $k^* = 18$, 8 and 4, respectively. Right panel increases δ from 0.1 to 0.5, then to 0.8 at fixed $\bar{\mu} = 0.3$, and the size response falls from $k^* = 18$ to 14, and to 6.

Figure 4 displays incremental profits under amputation. The chain's best size response k^* is the largest k (on the x-axis) such that incremental profits are positive. By shifting down the $\Delta\Pi(k)$ schedule, higher budgets (left panel) and higher detection contiguity (right panel) lead the chain to reduce its size.

Under decapitation the size response takes a simple form. Expected sanctions are $\bar{\mu}s_S$ for $\mathcal{S}$ and $\delta^i\bar{\mu}s_A$ for agent $A_{i,k}$, so, the chain's expected profits are

$$\Pi(k) = (1-\bar{\mu})k.v - \bar{\mu}\big[s_S + \sum_{i=1}^{k}\delta^i s_A\big]. \tag{15}$$

Thus, $\Delta R(k) = (1-\bar{\mu})v > 0$ and $\Delta s_S(k) = 0$, whereas $\Delta s_A(k) = \bar{\mu}\delta^{k+1}s_A$ declines and converges to zero as k becomes very large. It follows that the chain will recruit up to $\bar{k}$ agents if $\bar{\mu}$ is small and/or $\bar{k}$ is sufficiently large so that $\Pi(\bar{k}) > 0$, and set $k^* = 0$ otherwise. This leads to the condition in (14) defining the critical budget size $\bar{\mu}^c$, such that under decapitation the chain expands maximally if $\bar{\mu} < \bar{\mu}^c$ and ceases to operate if $\bar{\mu} \geq \bar{\mu}^c$.

V. Equilibrium intervention strategy

The equilibrium intervention strategy minimizes the expected harm in (1) incorporating the crime chain's best size response $k^* = k^*(\mu_S, \mu)$. The analysis first delineates the environment in which the chain expands maximally under decapitation.[24]

LEMMA 2: *There exists a unique $\alpha = \alpha^c$ such that the State is indifferent between the following paths of the game:*

(a) decapitation followed by the maximal size response $\bar{k}$, producing the expected harm $SH(\{\bar{\mu}, 0\})$,

(b) amputation followed by the best size response $k^*(0, \bar{\mu})$, producing the expected harm $SH(\{0, \bar{\mu}\}, \alpha^c)$.

Further, $SH(\{\bar{\mu}, 0\}) > SH(\{0, \bar{\mu}\}, \alpha)$ if $\alpha > \alpha^c$ and $SH(\{\bar{\mu}, 0\}) < SH(\{0, \bar{\mu}\}, \alpha)$ if $\alpha < \alpha^c$.

Given the intervention budget $\bar{\mu}$, at the critical $\alpha = \alpha^c$ the expected harms from decapitation and amputation are identical,

$$SH(\{\bar{\mu}, 0\}) = SH(\{0, \bar{\mu}\}, \alpha^c). \tag{16}$$

If $\alpha < \alpha^c$, it is optimal to switch to decapitation despite the fact that the chain will respond by expanding maximally.

PROPOSITION 5: *The equilibrium intervention strategy and the crime chain's best size response are as follows.*

[24] The proof of Lemma 2 is integrated into the proof of Proposition 5.

• *If* $\bar{\mu} \geq \bar{\mu}^c$*, the State sets* $(\mu_S^*, \mu^*) = (\bar{\mu}, 0)$ *and the chain responds with* $k^* = 0$*: Decapitation eliminates the crime chain if the intervention budget is sufficiently large.*

• *If* $\bar{\mu} < \bar{\mu}^c$*, the chain is active and the equilibrium intervention depends on* α*.*

(i) *If* $\alpha \leq \alpha^c$*, the State continues with the decapitation strategy despite the chain's response to expand maximally:* $(\mu_S^*, \mu^*) = (\bar{\mu}, 0)$ *and the chain sets* $k^* = \bar{k}$*.*

(ii) *If* $\alpha > \alpha^c$*, the State opts for amputation and the chain responds with a moderate size:* $(\mu_S^*, \mu^*) = (0, \bar{\mu})$ *followed by* $k^* = k^*(0, \bar{\mu})$ *satisfying* (13).

The critical intervention budget $\bar{\mu}^c$ and amputation effectiveness α^c characterizing the choice between decapitation and amputation depend on model parameters as discussed below.

The roles of chain profitability and detection contiguity are apparent in the expression of $\bar{\mu}^c$ in (14). The larger the revenue contribution v per agent, the larger is the intervention budget required to eliminate the crime chain via decapitation. This means that demand side policies that aim at reducing the chain's drug profits, if sufficiently effective, will reduce the budget needed for a successful decapitation. Note also that $\bar{\mu}^c$ is continuously decreasing in δ, from $\bar{k}v/[\bar{k}v + s_S]$ at $\delta = 0$ to $\bar{k}v/[\bar{k}v + s_S + \bar{k}s_A]$ at $\delta = 1$. Thus, if the intervention budget is large enough to eliminate a crime chain with contiguity δ_x, it can also eliminate a crime chain with $\delta_y > \delta_x$.

For low intervention budgets $\bar{\mu} < \bar{\mu}^c$ the equilibrium choice between amputation and decapitation depends on the position of α relative to α^c defined in (16). This part of the characterization in Proposition 5 (second and third lines of (13)) is better illustrated in the case of zero detection contiguity.

• **The case** $\delta = 0$. Assume $\bar{\mu} < \bar{\mu}^c = \bar{k}v/[\bar{k}v + s_S]$, so that if $\delta = 0$ the chain's best size response to decapitation is $\bar{k}$. Under the amputation strategy, $\Delta Es_S(k) = 0$, $\Delta Es_A(k) = \alpha\bar{\mu}s_A$, and $\Delta R(k) = (1 - \alpha\bar{\mu})^{k+1}v$, so, incremental profits are

$$\Delta\Pi(k) = (1 - \alpha\bar{\mu})^{k+1}v - \alpha\bar{\mu}s_A. \tag{17}$$

Note that $\Delta\Pi(k)$ is decreasing in k and α. Using (17) it is easy to verify that there exist critical enforcement effectiveness parameters $\tilde{\alpha}_{\bar{k}-1} < \tilde{\alpha}_{\bar{k}-2} < \cdots < \tilde{\alpha}_1$ such that $\Delta\Pi(k) = 0$ at $\alpha = \tilde{\alpha}_k$ and $\Delta\Pi(k) > 0$, if $\alpha < \tilde{\alpha}_k$.[25] Intervals of α defined this way determine the chain's size response to amputation as $k^*(0, \bar{\mu}) = k$ if $\alpha \in [\tilde{\alpha}_k, \tilde{\alpha}_{k-1})$, and $k^*(0, \bar{\mu}) = \bar{k}$ if $\alpha < \tilde{\alpha}_{\bar{k}-1}$.

Turning to the State's objective, the expected harms from the two paths of play

[25]Observe that $\tilde{\alpha}_k$ will satisfy $0 < \tilde{\alpha}_k\bar{\mu} < 1$ for $k = 1, \cdots, \bar{k} - 1$.

defined in Lemma 2 are:

$$\begin{aligned} \text{Path (a):} \quad SH(\{\bar{\mu}, 0\}) &= (1-\bar{\mu})^{\bar{k}}(h_S + \bar{k}h_A), \\ \text{Path (b):} \quad SH(\{0, \bar{\mu}\}, \alpha) &= \alpha\bar{\mu}\sum_{j=1}^{k^*-1}(1-\alpha\bar{\mu})^j(h_S + jh_A) + (1-\alpha\bar{\mu})^{k^*}(h_S + k^*h_A). \end{aligned}$$

The critical α^c defined in (16) is determined as follows. Suppose, to begin with, that α is lower than $\tilde{\alpha}_{\bar{k}-1}$. When amputation is so ineffective that the chain's incremental profits remain positive at size $\bar{k}-1$ ($\Delta\Pi(\bar{k}-1) > 0$), the chain's optimal response to amputation is $\bar{k}$, same as its response to decapitation. Given this fact, the choice between amputation and decapitation is determined by comparing the expected harms, which depend on α. Note that $\lim_{\alpha\to 0} SH(\{0, \bar{\mu}\}, \alpha) = h_S + \bar{k}h_A > SH(\{\bar{\mu}, 0\})$. For such low α values, then, $SH(\{\bar{\mu}, 0\}) < SH(\{0, \bar{\mu}\}, \alpha)$, and decapitation is the optimal intervention. If $SH(\{\bar{\mu}, 0\}) \geq SH(\{0, \bar{\mu}\}, \tilde{\alpha}_{\bar{k}-1})$, then a critical $\alpha^c \leq \tilde{\alpha}_{\bar{k}-1}$ exists such that decapitation is used if $\alpha \leq \alpha^c$ and amputation is used otherwise. Now suppose $SH(\{\bar{\mu}, 0\}) < SH(\{0, \bar{\mu}\}, \tilde{\alpha}_{\bar{k}-1})$. At $\alpha = \tilde{\alpha}_{\bar{k}-1}$, $\Delta\Pi(\bar{k}-1) = 0$ whereas for α slightly above $\tilde{\alpha}_{\bar{k}-1}$, $\Delta\Pi(\bar{k}-1) < 0$, implying that the chain will reduce its size to $\bar{k}-1$. Raising α within the range in which the chain' size response remains $\bar{k}-1$ will keep reducing $SH(\{0, \bar{\mu}\}, \alpha)$. If α is raised further by taking into account the chain's contractions at $\alpha = \tilde{\alpha}_k$, eventually the critical α^c defined in (16) will be reached. For $\alpha > \alpha^c$ the expected harm is lower under amputation, $SH(\{\bar{\mu}, 0\}), > SH(\{0, \bar{\mu}\}, \alpha)$, and the State will optimally switch the focus from the chain's source unit to the tail units.

VI. Transnational crime chains

The results thus far characterize optimal interventions by a central authority. An important class of chain-form trafficking activity is transnational, where multiple enforcement authorities interact with objectives confined to minimizing harms within national borders. Practical cooperation is chronically in short supply.[26] This section adapts the model to the transnational context. The analysis highlights strong bilateral enforcement externalities that stem from the chain structure of trafficking activity, suggesting that the costs of failures to cooperate in joint interventions and intelligence sharing can be extremely large.

A crime chain operates in two countries. The source in upstream country U supplies its tail agents in downstream country D as illustrated in Figure 5. Country $\mathcal{U}$'s enforcement budget to control the transnational crime is μ_U^t and country $\mathcal{D}$'s budget is μ_D. Let μ_b be the "base level" direct detection probability

[26] According to World Drug Report WDR (2020, p. 21), joint drug operations have declined between 2010 and 2018, from 68 countries in 2010/11 to 57 in 2017/18. As causes for the decline, the Report highlights budgetary issues, "lack of agreements to enable operational cooperation and more practical issues such as an inability to identify appropriate counterparts."

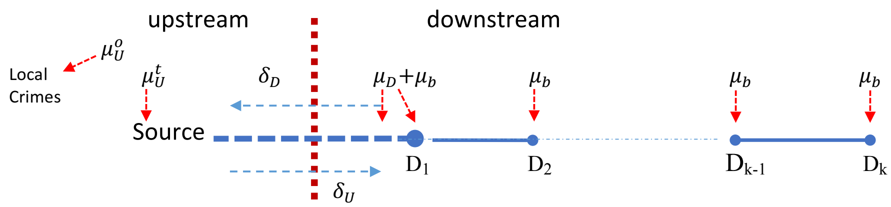

FIGURE 5. TRANSNATIONAL CRIME CHAIN AND LAW ENFORCEMENTS.

of crime units $D_1, \cdots, D_k$, induced by other domestic enforcement agencies in $\mathcal{D}$. Assume that μ_b is small, and $\mu_D + \mu_b < 1$.

From D's national perspective its border and port of entry D_1 constitute the source of a crime chain with tail units $D_2, \cdots, D_k$. The chain's border activity is therefore a natural target, distinct from its other clandestine units dispersed inside the country. Whereas coordinated action by the two countries can target individually the source $\mathcal{S}$, the border, and the tail group $D_2, \cdots, D_k$, in the absence of coordination $\mathcal{D}$'s intervention options are confined to domestic decapitation (targeting the border), domestic amputation (targeting internal tail agents), or combinations of the two.

If decapitation of this domestic chain is the optimal strategy (combination of small δ, small α, tail not too large−see Proposition 5), D will channel its transnational enforcement budget to the border force and interdictions. Under domestic decapitation, the direct detection probability of D_1 will be $\mu_D + \mu_b$ and hence expected crime harms in country $\mathcal{D}$ are

$SH_D = \sum_{i=1}^k p_{i,k}(i.h_A)$, where

$$p_{i,k} = \begin{cases} (1-\mu_D-\mu_b)(1-\mu_b)^{i-1}(1-\delta)\Big[\mu_b + (1-\mu_b)\delta\mu_b + \cdots \\ \qquad\qquad + (1-\mu_b)^{k-i-1}\delta^{k-i-1}\mu_b\Big] & \text{for } i < k, \\ (1-\mu_D-\mu_b)(1-\mu_b)^{k-1} & \text{for } i = k. \end{cases}$$

If the source is under positive scrutiny and detected with probability μ_U^t, the expressions of $p_{i,k}$ above are multiplied by $1-\mu_U^t$. Then, conditional on that the source is active, the probability that agent D_i is S-connected is

$$\rho_1 = (1-(\mu_D+\mu_b))(1-\mu_{1,k}^+) \text{ and}$$
$$\rho_i = (1-(\mu_D+\mu_b))(1-\mu_b)^{i-1}(1-\mu_{i,k}^+) \text{ for } i \geq 2,$$

where $\mu_{i,k}^+$ denotes the tail-detection probability of D_i. It can be shown (using (A3), see Lemma 3 in Appendix A), that source-connection probabilities fall,

$\rho_{i+1} < \rho_i$, and thus the probability of supply interruption, $1 - \rho_i$, increases along the tail. If drug prices are proportional to the risk of supply interruption, the domestic decapitation policy will produce at the border a sharp increase in the price, gradually rising by number of chain connections inside the downstream country.[27]

The analysis above takes transnational enforcement budgets as given. To focus on the interaction between national enforcements, assume for clarity of exposition one downstream tail unit D_1 and set $\mu_b = 0$. Upstream, country U allocates its budget μ_U as (μ_U^t, μ_U^o) between transnational and local crimes producing respectively the harms h_U^t and h_U^o.[28] Thus, $\mu_U = \mu_U^o + \mu_U^t$. If the countries do not coordinate, country U minimizes expected crime harms in its territory,

$$SH_U = (1 - \mu_U^o)h_U^o + (1 - \mu_U^t)(1 - \mu_D)h_U^t. \tag{18}$$

Replacing h_U^t by h_D^t in the second term yields the transnational harm component in country D's objective: $(1 - \mu_U^t)(1 - \mu_D)h_D^t$. From (18), the impact of a marginal balanced budget shift from local to transnational crime on expected harms is $h_U^o - (1 - \mu_D)h_U^t$.

Local crime harms h_U^o constitute the opportunity cost of funds allotted to transnational crime. Further, if $\mathcal{D}$ increases μ_D, the effectiveness of upstream enforcement μ_U^t decreases. This is a pure consequence of the chain structure of transnational crime and holds symmetrically, implying that *national enforcement efforts are strategic substitutes.*

The resulting free riding on efforts across the border is the prime obstacle to efficiency. The net impact of a marginal resource shift to transnational crime on expected harms, $h_U^o - (1 - \mu_D)h_U^t$, indicates that incentives to divert resources to local crimes will be stronger at the side with smaller transnational crime harms, relatively large local crime harms and lower budget. These features are typical of crime chains with source in the South feeding harmful tail units that span in the North.[29]

Turning now to the cooperative outcome, global expected harms from the crime chain are $(1 - \mu_U^t)(1 - \mu_D)[h_U^t + h_D^t]$. In light of Proposition 2 this expression is minimized by maximizing μ_U^t (decapitation) or μ_D (amputation): One country should avail its budget to the other and the total intervention budget should be determined in view of total harms at both sides. In practice such budget transfers are quite limited and mostly in kind (training, sending experts, personnel and equipment), and the few joint interventions in the South such as plans Colombia

[27]Shifting the border enforcement budget to detection of tail units operating inside (amputation) will reduce, but not eliminate, the price jump at border crossing.

[28]Normalize the measure of potential local criminals and set the sanction on the local crime equal to one. Assuming that crime benefits are distributed uniformly in the unit interval, the measure of deterred local crimes is μ_U^o.

[29]Another case in point where harms and resources are concentrated in opposite ends is supply of cultural artefacts looted from the South, where the harm h_U^t is large, to markets in the resourceful North where h_D^t is low.

and Merida are executed under Northern initiative. If the North expects limited impact from transferring resources to the South for reasons of corruption in law enforcement and lack of political will, its intervention options are by and large limited to its own borders.

At a more basic level, national governments should implement *intelligence sharing* agreements to raise δ_U and δ_D and maximize the exchange information about identities, whereabouts of chain members operating at the other side. This kind of cooperation will increase the probability of detection of the source from μ_U^t to $\mu_U^t + (1-\mu_U^t)\delta_D\mu_D$, that of the tail from μ_D to $\mu_D + (1-\mu_D)\delta_U\mu_U^t$. Of the two supplies of intelligence, the one that flows to the country where joint interventions take place, or to the country that predominantly enforces the transnational crime, is crucial: intelligence sharing agreements should maximize δ_D in decapitation to capture the source, δ_U in amputation to maximize tail disruption.

VII. Discussion

This section presents a number of extensions and special cases of the model, beginning with a discussion of preliminaries and assumptions.

A. Prioritized intervention strategy

The prescription to focus on one chain target at a time (Proposition 2) is the product of interdependence between the source and the tail units. If some real-life interventions appear to split forces between the source and the tail, the prime reason is the change in the authority's information set over time. Shifting resources from one part of the organization to another can be optimal upon new intelligence or tip-offs from captured members. Relatedly, the need to collect intelligence about one part of the chain may call for channelling some resources to other parts. Second, obtaining convictions for isolated chain members may be more difficult than convicting a group of chain members, which alone may favor simultaneous attacks to capture the source and as many agents as possible. Third, the enforcement technology, linear by Assumption 2, could play a role. Under a sufficiently concave technology satisfying the Inada conditions, marginal productivities of the first resources employed at the tail or the source would be extremely high and the State would allocate positive resources to each crime.[30] These factors must be powerful enough to offset the structural feature favoring prioritized interventions.

B. Chain formation

This paper has abstracted from the crime chain's formation process and its governance. In a simple centralized contractual procedure of chain formation $\mathcal{S}$

[30] Bac and Bag (2020) illustrate this fact in chains formed by two units.

assumes leadership and maximizes the chain's expected profits in (2) by recruiting agents for chain positions through contracts that specify transfers $\{T_{i,k}\}$ (or shares from realized revenues) depending on node location i in C_k and contingent on the detection outcome. If not detected himself, $\mathcal{S}$ collects the revenues net of transfers $\{T_{i,k}\}$ to agents who happen to be S-connected. Assuming risk-neutral chain members with zero outside options, $\mathcal{S}$ would set $T_{i,k}$ so as to bind $A_{i,k}$'s participation constraint, incorporating the expected sanction $\mu_{i,k}s_A$. The objective function of $\mathcal{S}$ will be that of the crime chain, hence the number of agents that $\mathcal{S}$ recruits will coincide with the optimal size response characterized in Section IV.[31]

C. *Changing the order of play*

The analysis has assumed that the State moves first with an intervention strategy and the chain responds by determining its size. If the crime chain moves first, its size could have commitment value if observable, which is clearly unrealistic. A flexible clandestine organization cannot be expected to maintain a size that is not optimal given the intervention strategy.

The chain-moves-first game will have a different equilibrium outcome only if the chain can beneficially set itself a different size that induces the State to switch from decapitation to amputation. To see this, let $\bar{\mu} \geq \bar{\mu}^c$ so that in the equilibrium of the State-moves-first game the State opts for decapitation and the chain responds with $k^* = 0$, making zero profits $\Pi(k^*(\bar{\mu}, 0)) = 0$. When the order of moves is reversed, the chain would set itself a positive k only if (i) the optimal intervention for C_k is amputation, that is, only if $\alpha > \alpha_k$ (Proposition 2-(i)) and (ii) $\Pi(k^*(0, \bar{\mu})) > 0$. Such differences in equilibrium outcomes from changing the order of moves can happen only in a subset of the model's parameter space.

D. *Tail agents can be targeted separately*

Suppose that the State can control each agent's direct detection probability, contrary to Assumption 1. An agent-based budget allocation $\{\mu^d_{S,k}, (\mu^d_{1,k}, \cdots, \mu^d_{k,k})\}$ is now possible. I maintain the linear technology assumption and, to adapt Assumption 2, assume that under identical enforcement budgets the agents' direct detection probabilities are identical.

[31]Recent contributions (e.g., Ostrovsky (2008) on chain-stable matchings, Antras and Chor (2013) on contractual relationships between final good producers and their multi-layered suppliers) have improved our understanding as to the governance and formation of complex supply chains. Though the crime chain context is quite different, the governance spectrum is just as rich: We may observe tail units of a human trafficking chain partly under under Balkan-origin partly under Russian or Middle-Eastern management, independent in many of their decisions from the upstream source in China, along with chains that operate under single-hand control. The formation and governance of a crime chain would also depend in certain ways, yet to be clarified, on law enforcement strategies. Because the chain structure is a building block for clandestine architectures, the insights gained will be useful in assessing policies aimed at deterring other forms of organized crime.

The State's objective is as stated in (1) except that the budget constraint becomes $\mu^d_{S,k} + \sum_{i=1}^k \mu^d_{i,k} = \bar{\mu}$. The following result obtains.

PROPOSITION 6: *Assume $\delta < 1$ and that the State can target each agent separately. Faced with crime chain C_k, the optimal intervention strategy of the State is*

$$\{\mu^{dF}_{S,k}, (\mu^{dF}_{1,k}, \cdots, \mu^{dF}_{k,k})\} = \begin{cases} (\bar{\mu}, \{0, 0, \cdots, 0\}) & \text{if } \alpha \leq 1, \\ (0, \{\bar{\mu}, 0, \cdots, 0\}) & \text{if } \alpha > 1. \end{cases} \tag{19}$$

It cannot be optimal to target any two agents simultaneously because by shifting resources to the agent with shorter path to $\mathcal{S}$ the State can lower the expected harm. Thus, if the State ever decides to target the tail segment, it should target the first agent. The alternative is of course to target $\mathcal{S}$, but the outcome set is identical: either the chain is dismantled or survives intact. It follows that the choice between $\mu^d_{1,k} = \bar{\mu}$ and $\mu^d_{S,k} = \bar{\mu}$ depends solely on the enforcement effectiveness parameter α.

E. Chain with pure transit agents

The analysis assumed that all tail units commit crime A. Let us drop this assumption and consider a chain where crime inputs are transported from $\mathcal{S}$ to one final destination $A_{k,k}$, through $A_{1,k}$ to $A_{k-1,k}$ "transit agents" who produce no value, no harm. Any detection now fully disrupts the chain.

The chain's potential harm is $h_S + h_A$, which it inflicts only if $A_{k,k}$ is S-connected, with probability $p_{k,k}$. The expected harm to be minimized is $SH = (1-\mu_S)(1-\alpha\mu)^k(h_S+h_A)$ subject to $\mu_S + \mu = \bar{\mu}$ if the agents cannot be targeted individually, $SH = (1-\mu_S)\prod_{i=1}^k(1-\alpha\mu^d_i)(h_S+h_A)$ subject to $\mu_S + \sum_{i=1}^k \mu^d_i = \bar{\mu}$ if the agents can be targeted individually.

PROPOSITION 7: *Under Assumption 1 and 2, the optimal intervention when the crime chain C_k consists of $k-1$ transit agents linking $\mathcal{S}$ to $A_{k,k}$ is:*

$$(\mu^F_S, \mu^F) = \begin{cases} (0, \bar{\mu}) & \text{if } \alpha > \alpha^T_k, \\ (\bar{\mu}, 0) & \text{if } \alpha \leq \alpha^T_k, \end{cases} \tag{20}$$

where $\alpha^T_k = \left[1 - (1-\bar{\mu})^{\frac{1}{k}}\right]/\bar{\mu}$.

The optimal enforcement strategy in (20) is similar to the one in Proposition 2 for a fixed-size crime chain, except the difference in the cutoff α level: target the tail group if resources are sufficiently effective in crime A enforcement (if $\alpha > \alpha^T_k$) and target $\mathcal{S}$ otherwise. The longer the transit line (the larger k), the lower is α^T_k and hence the stronger the case for amputation.[32]

[32] If the State *could* target individual agents separately, the optimal intervention would be independent

F. Chains vs Trees

A crime chain can branch out from its main transmission line to reach remote destinations. The resulting structure will be an arborescence in which every agent (node) has a single path to the source.

The new aspects that arise in such networks can be illustrated in the three-agent, one source organization, which is rich enough to capture various degrees and forms of centralization. In Figure 6, moving clockwise, linking A_3 to A_1 transforms C into TA. In TS, A_3 is directly linked to $\mathcal{S}$. Finally, source-centrality is maximal in the star network with direct links from $\mathcal{S}$ to all agents.

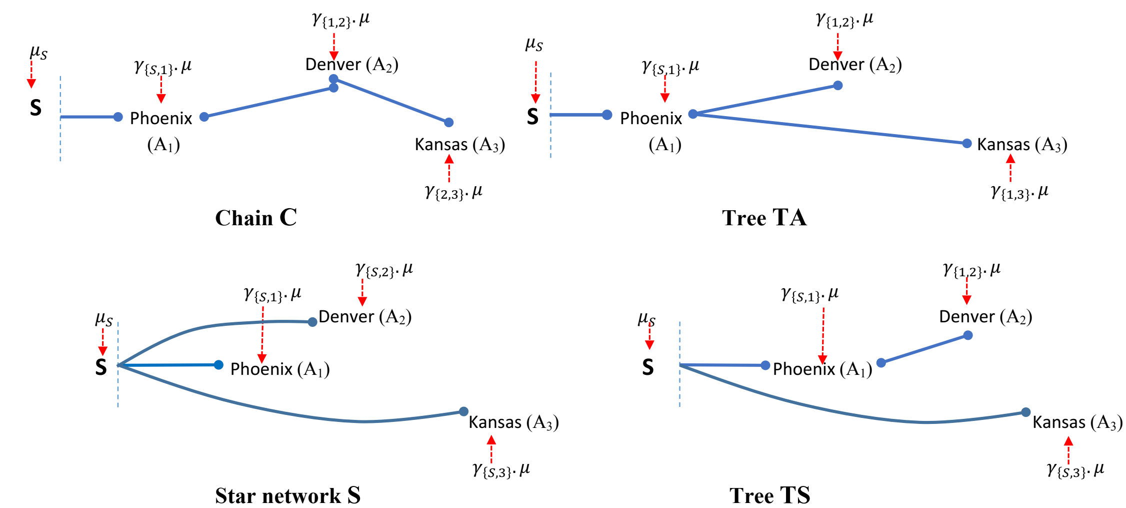

FIGURE 6. TRAFFICKING NETWORKS WITH THREE AGENTS.

In a geographically dispersed trafficking network, detection probabilities will increase by distance to supply connection.[33] This is captured by the coefficients $\gamma_{\{i,j\}}$: Linked to agent i, the direct detection probability of agent j is $\gamma_{\{i,j\}}\mu$. To illustrate, the direct detection probability of A_3, $\gamma_{\{2,3\}}\mu$ in C, becomes $\gamma_{\{1,3\}}\mu$ if linked to A_1, in TA. If A_3's distance to A_1 is larger than its distance to A_2, then $\gamma_{\{1,3\}} \geq \gamma_{\{2,3\}}$.

from the budget and the chain's size. In this case, capturing any single agent breaks the source connection of the final unit, so, it is optimal to target any one agent with all the resources. The target need not be chosen at random. Capturing the last agent $A_{k,k}$ specialized in crime A may be more important than the interim logistic support units for reasons not captured by the present model.

[33]Interdiction risks increase with node-to-node distance, to which traffickers respond with various tactics such as switching drivers and vehicles. Highway patrol officers trained in interceptions seize bulk shipments of illicit drugs and produce invaluable information for investigations aimed at major traffickers rooted in Colombia and Mexico. See Reuter (2014).

Normalize the distances between successive agents in C to one, so that $\gamma_{\{S,1\}} = \gamma_{\{1,2\}} = \gamma_{\{2,3\}} = 1$. If $\gamma_{\{1,3\}} > 1 = \gamma_{\{2,3\}}$, C has an advantage over TA. TA in turn has an advantage over TS if A_3's distance to $\mathcal{S}$ is larger than its distance to A_1 so that $\gamma_{\{S,3\}} > \gamma_{\{1,3\}}$. Finally, $\gamma_{\{S,2\}} > 1 = \gamma_{\{1,2\}}$ will favor TS over S. In the example in Figure 6, link size effects operate against network centralization.

How does the optimal intervention strategy vary across these networks? Let us maintain Assumption 1 (tail agents cannot be targeted individually) and focus on the pure network effect.

PROPOSITION 8: *Consider a trafficking network with $k = 3$ and set $\alpha = 1$, $\gamma_{\{i,j\}} = 1$. The optimal intervention strategy is decapitation or amputation.*
(i) Vertical networks C and TA favor amputation, whereas source-central networks S and TS favor decapitation:

- $SH_{TA}(0, \bar{\mu}) < min\{SH_S(0, \bar{\mu}), SH_{TS}(0, \bar{\mu})\}$;
- $SH_C(0, \bar{\mu}) < SH_{TS}(0, \bar{\mu})$ *and, if h_S is not too high relative to h_A, or if h_S is high but $\delta < (1 + \bar{\mu})/(1 + 2\bar{\mu})$,* $SH_C(0, \bar{\mu}) < SH_S(0, \bar{\mu})$.

(ii) Expected harms from amputation are lower

- (vertical networks): *in C than TA if $\delta < (1 - \bar{\mu})h_A/(h_S + h_A)$;*
- (source-central networks): *in TS than S if $\delta \leq \bar{\mu}/(1 + \bar{\mu})$, or if $\delta \in (\bar{\mu}/(1 + \bar{\mu}), 1/(1 + \bar{\mu}))$ and h_S is sufficiently small relative to h_A.*

The recommendation to prioritize the source or the tail (Proposition 2) extends to trafficking networks. Because expected harms from decapitation are identical and given by $SH(\bar{\mu}, 0) = (1 - \bar{\mu})(h_S + 3h_A)$, the comparisons in Proposition 8 hinge on expected harms from amputation. There are two insights.

First, *amputation is more effective in vertical networks.* In such networks, as TA and C, inter-agent connectivity is high relative to source-central networks such as S and TS. Detection of a tail-central agent with direct links to the source and many other agents, who can thus be traced back from others, will seriously disrupt the network. This makes C, and in particular TA, vulnerable to amputation, whereas networks S and TS in which the source has three and two direct links cause larger harms because amputation does not scrutinize the source.

Second, *high detection contiguity amplifies the effectiveness of amputation in all networks. The magnitude of the effect, however, depends on link structure.* If δ is small, amputation effectiveness is higher (expected harms smaller) in vertical networks than those that expand by branch formation. In the limit case of $\delta = 0$, expected harms under amputation rank, from small to large, as: C, TA, TS, S. Conversely, if detections are highly contiguous, amputation is more effective in the tail tree TA than the chain C, in the star network S than the source tree TS. Thus, if δ is high, the network type in which amputation is most effective and preferred over decapitation is TA. Next comes the chain C or the star network S depending on the harm h_S caused by the source relative to the harm from the agents, h_A. Amputation would become quite an effective strategy in the star network as well because any single agent detection can deliver the source and eliminate the harms. Introducing link size effects will of course qualify these conclusions depending on geographic distances.

G. Dynamics

The interaction between law enforcement and crime chains has dynamic aspects that are not directly captured in this model, some favoring amputation, others favoring decapitation.

In the aftermath of a successful kingpin operation, downstream harms h_A may fall temporarily for lack of supplies but the vacuum at the source can produce extreme violence and raise the harms h_S above pre-decapitation levels. Multiple and less visible source units may emerge to form their own thinner trafficking chains.[34] Decapitating this new upstream source structure will be more costly.

Second, crime chains can regenerate. This aspect makes amputation less attractive because tail units would regenerate faster than a detected source. The impact of an increase in tail regeneration rate can be captured in this model by smaller values of α, effectiveness of the enforcement dollar invested in detecting tail units relative to investing it to detect the source.[35]

Third, in the course of its activities the enforcement authority's information set will change, which may lead it to conduct focused interventions along the tail, or at the source, by re-allocating resources.[36] The risk is larger for trafficking structures with nodes where branches congregate, such as tree-form organizations with central units (A_2 in TA, Figure 6).[37] Responding to the expected improvements in the authority's information, the organization may gradually eliminate its tail-central nodes. These conjectures, however, must be verified in formal dynamic models.

[34]The historic domination of cocaine cartels in Colombia were broken by large scale interventions and disruptions in the production scenes. Cartels were then replaced by a fragmented structure where numerous upstream actors established their own contacts and supply chains (Europol, 2019).

[35]In the case of drug trafficking, a dynamic enforcement perspective may also take into account short-run and long-run price elasticities (see Olmstead et al. (2015) for estimates of elasticities). Tail segments that quickly recover from detections would maintain supply whereas a successful source intervention that disrupts the supply for a longer time, due to a more elastic long-run demand, can generate an extra demand reduction benefit in the longer run.

[36]This is typical in implementation of amputation strategies. To give one example, preparations for Operation Pollino targeting the European tail controlled by the Calabrian Ndrangheta crime chain, trafficking drugs, mainly cocaine, began in 2016. Final execution came in December 2018 (Europol, 2019).

[37]Another important dynamic aspect bears on internal enforcement mechanisms of trafficking organizations to sustain cooperation between its members and deter defections. Our knowledge about the dynamic implications of these mechanisms on the structure of trafficking chains is rough. Minimizing direct information links reduces detection risks but may compromise from effective cooperation. Baccara and Bar-Isaac (2008) study this tradeoff in an abstract framework that does not capture trafficking activities. The link structure of a crime chain suggests a delegated internal enforcement structure where $\mathcal{S}$ would control A_1, who in turn would control A_2, and so on.

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Appendix A: Properties of source- and tail-detection probabilities

In light of Definition 2, $\mu^+_{i,k} = \delta[\alpha\mu + (1-\alpha\mu)\delta[\cdots\delta[\alpha\mu + (1-\alpha\mu)\delta[\alpha\mu]]...]$. Thus,

$$\mu^+_{i,k} = \alpha\mu\sum_{j=1}^{k-i}\delta^j(1-\alpha\mu)^{j-1} \;=\; \delta\alpha\mu\frac{1-\delta^{k-i}(1-\alpha\mu)^{k-i}}{1-\delta(1-\alpha\mu)}. \tag{A1}$$

As for the expression $\mu^+_{i,k} = \delta[\alpha\mu + (1-\alpha\mu)\delta[\cdots\delta[\alpha\mu + (1-\alpha\mu)\delta[\alpha\mu]]...]$, it can be written as

(A2)

$$\mu^-_{i,k} = \mu_S\delta^i(1-\alpha\mu)^{i-1} + \alpha\mu\sum_{j=1}^{i-1}\delta^j(1-\alpha\mu)^{j-1} \;=\; \mu_S\delta^i(1-\alpha\mu)^{i-1} + \delta\alpha\mu\frac{1-\delta^{i-1}(1-\alpha\mu)^{i-1}}{1-\delta(1-\alpha\mu)}.$$

Note that $\mu^+_{k,k} = \mu^-_{0,k} = 0$ and $\mu^-_{1,k} = \delta\mu_S$.

LEMMA 3: *Consider C_k. Let $\delta > 0$ and fix (μ_S, μ) with strictly positive components. Then,*

$$\begin{aligned} \mu^+_{i,k} &= \delta[\alpha\mu + (1-\alpha\mu)\mu^+_{i+1,k}] \quad \textit{for } 0 \le i \le k-1, \\ \mu^-_{i,k} &= \delta[\alpha\mu + (1-\alpha\mu)\mu^-_{i-1,k}] \quad \textit{for } i \ge 2. \end{aligned} \tag{A3}$$

The following properties hold:

(i) For any $0 \le i < j \le k$, $\mu^+_{i,k} > \mu^+_{j,k}$ and $\mu^-_{i,k} < \mu^-_{j,k}$.

(ii) $\frac{\partial\mu^+_{i,k}}{\partial\mu} > 0$, $\frac{\partial\mu^+_{i,k}}{\partial\mu_S} = 0$; $\frac{\partial\mu^-_{i,k}}{\partial\mu} > 0$, $\frac{\partial\mu^-_{i,k}}{\partial\mu_S} > 0$.

(iii) (a) $\mu^+_{i,k} = \mu^+_{i+1,k+1}$ and $\mu^-_{i,k} = \mu^-_{i,k+1}$, (b) $\mu^+_{i,k+1} - \mu^+_{i,k} = \alpha\mu\delta^{k-i+1}(1-\alpha\mu)^{k-i}$.

Proof. The identities in (A3) follow from (A1) and (A2).

(i) Since $\mu^+_{k,k} = 0$, for $i = k-1$ the relationship in (A3) becomes $\mu^+_{k-1,k} = \delta\alpha\mu = \mu^+_{k,k}$. Moving to $i = k-2$, $\mu^+_{k-2,k} = \delta[\alpha\mu + (1-\alpha\mu)\mu^+_{k-1,k}]$, hence, $\mu^+_{k-2,k} > \mu^+_{k-1,k}$. Successive applications of (A3) yields $\mu^+_{i,k} > \mu^+_{i+1,k}$, for $i < k-2$. The claim $\mu^-_{i,k} < \mu^-_{i+1,k}$ follows from similar arguments.

(ii) Differentiate the first equality in (A3) with respect to μ. This yields

$$\frac{\partial \mu^+_{i,k}}{\partial \mu} = \delta[\alpha - \alpha\mu^+_{i+1,k} + (1-\alpha\mu)\frac{\partial \mu^+_{i+1,k}}{\partial \mu}]. \tag{A4}$$

For $i = k-1$, $\partial\mu^+_{k-1,k}/\partial\mu = \delta\alpha > 0$. Using this fact, for $i = k-2$ equation (A4) becomes $\partial\mu^+_{k-2,k}/\partial\mu = \delta[\alpha - \alpha\mu^+_{k-1,k} + (1-\alpha\mu)\delta\alpha]$, which is strictly positive. Thus, for $i = k-3$, $\partial\mu^+_{k-3,k}/\partial\mu = \delta[\alpha - \alpha\mu^+_{k-2,k} + (1-\alpha\mu)(\partial\mu^+_{k-2,k}/\partial\mu)]$ must also be strictly positive. Successive application of this argument yields $\partial\mu^+_{i,k}/\partial\mu > 0$ for all $i < k$.

The proof of the claim $\partial\mu^-_{i,k}/\partial\mu > 0$ for $i > 1$ is similar, using (A3): Since $\partial\mu^-_{1,k}/\partial\mu = 0$, it follows that $\partial\mu^-_{2,k}/\partial\mu > 0$, which implies $\partial\mu^-_{3,k}/\partial\mu > 0$. Proceeding forward up to $i = k$ yields the result. The signs of the partial derivatives with respect to μ_S follow from (A1) and (A2).

(iii)-(a) Since the number of links from $A_{i,k}$ to $A_{k,k}$ and from $A_{i+1,k+1}$ to $A_{k+1,k+1}$ are identical, tail-detection probabilities are identical: $\mu^+_{i,k} = \mu^+_{i+1,k+1}$. By the same token, $\mu^-_{i,k} = \mu^-_{i,k+1}$, because $A_{i,k}$ and $A_{i,k+1}$ are equidistant to $\mathcal{S}$.

(iii)-(b) Using (A1) and the fact $\mu^+_{i+1,k+1} = \mu^+_{i,k}$, we observe

$$\begin{aligned}
[\mu^+_{i,k+1} - \mu^+_{i,k}] &= \delta(\alpha\mu + (1-\alpha\mu)\mu^+_{i+1,k+1}) - \mu^+_{i,k} \\
&= \delta\alpha\mu - (1-\delta(1-\alpha\mu))\sum_{j=1}^{k-i}\delta^j(1-\alpha\mu)^{j-1}\alpha\mu \\
&= \delta\alpha\mu\big[1 - (1-\delta(1-\alpha\mu))\sum_{j=1}^{k-i}\delta^{j-1}(1-\alpha\mu)^{j-1}\big] \\
&= \delta\alpha\mu\big[1 - (1-\delta(1-\alpha\mu))\frac{1-\delta^{k-i}(1-\alpha\mu)^{k-i}}{1-\delta(1-\alpha\mu)}\big] \\
&= \alpha\mu\delta^{k-i+1}(1-\alpha\mu)^{k-i}.
\end{aligned}$$

□

Derivation of Equation (12)**.** The expected sanction of the new recruit is $\mu_{k+1,k+1}.s_A = [\alpha\mu + (1-\alpha\mu)\mu^-_{k+1,k+1}].s_A$ where, using (A2),

$$\mu^-_{k+1,k+1} = \mu_S\delta^{k+1}(1-\alpha\mu)^k + \delta\alpha\mu\frac{1-\delta^k(1-\alpha\mu)^k}{1-\delta(1-\alpha\mu)}.$$

Hence, $\mu_{k+1,k+1} = \alpha\mu + \mu_S \delta^{k+1}(1-\alpha\mu)^{k+1} + \delta\alpha\mu(1-\alpha\mu)\dfrac{1-\delta^k(1-\alpha\mu)^k}{1-\delta(1-\alpha\mu)}$.

Rearranging terms and multiplying by s_A yields (12).

Appendix B: Proofs

Proof of Lemma 1. Using the inequalities $\mu^+_{i-1,k} > \mu^+_{i,k}$ and $\mu^+_{i,k+1} > \mu^+_{i,k}$ in (4), the result follows. □

Proof of Proposition 1. (i). The sign of $\partial p_{i,k}/\partial\mu_S$ is straightforward from (4).

(ii). $\partial p_{k,k}/\partial\mu < 0$ can be verified from (4). For $1 \le i \le k-1$, set $\alpha = 1$. Thus,

$$\begin{aligned}\frac{\partial p_{i,k}}{\partial\mu} &= (1-\mu_S)(1-\delta)\Bigg\{\frac{(1-\mu)^i - \delta^{k-i}(1-\mu)^k}{1-\delta(1-\mu)} \\ &\quad + \mu\left[\frac{-i.(1-\mu)^{i-1} + k.\delta^{k-i}(1-\mu)^{k-1}}{1-\delta(1-\mu)} - \delta.\frac{(1-\mu)^i - \delta^{k-i}(1-\mu)^k}{[1-\delta(1-\mu)]^2}\right]\Bigg\}.\end{aligned}$$

Simplifying and arranging the terms, the condition for $\partial p_{i,k}/\partial\mu < 0$ can be written as $\left[(1-\mu)^{i-1}(1-\mu-i.\mu) - \delta^{k-i}(1-\mu)^{k-1}(1-\mu-k.\mu)\right][1-\delta(1-\mu)] - \delta\mu[(1-\mu)^i - \delta^{k-i}(1-\mu)^k] < 0$, which can be simplified further:

$$(1-\mu)^{i-1}\left[(1-\mu-i.\mu)(1-\delta(1-\mu)) - \delta\mu(1-\mu)\right] < \delta^{k-i}(1-\mu)^{k-1}\left[(1-\mu-k.\mu)(1-\delta(1-\mu)) - \delta\mu(1-\mu)\right].$$

Thus, $\partial p_{i,k}/\partial\mu < 0$ if $(1-\mu)(1-\delta) - i.\mu(1-\delta(1-\mu)) < \delta^{k-i}(1-\mu)^{k-i}\left[(1-\mu)(1-\delta) - k.\mu(1-\delta(1-\mu))\right]$.

Grouping the i and k-index terms at the right hand side yields

$$\text{(B1)} \qquad \frac{\partial p_{i,k}}{\partial\mu} < 0 \quad \text{if} \quad \frac{(1-\mu)(1-\delta)}{\mu(1-\delta(1-\mu))} < \frac{i - k\delta^{k-i}(1-\mu)^{k-i}}{1-\delta^{k-i}(1-\mu)^{k-i}} \equiv \psi_i(\mu,\delta,k).$$

The left hand side of (B1) is positive, independent of i and k, and decreasing in μ. The schedule $\psi_i(\mu,\delta,k)$ defined on the right hand side is increasing in i and μ, and is positive if $i > k\delta^{k-i}(1-\mu)^{k-i}$.

Given these facts, consider the claim $\psi_i(\mu,\delta,k) < \psi_{i+1}(\mu,\delta,k)$, or, using (B1),

$$\psi_i(\mu,\delta,k) = \frac{i - k\delta^{k-i}(1-\mu)^{k-i}}{1-\delta^{k-i}(1-\mu)^{k-i}} < \frac{i+1-k\delta^{k-i-1}(1-\mu)^{k-i-1}}{1-\delta^{k-i-1}(1-\mu)^{k-i-1}} = \psi_{i+1}(\mu,\delta,k).$$

Cross multiplying and simplifying the terms yields $(k-i)\delta^{k-i-1}(1-\mu)^{k-i-1} < (k-i-1)\delta^{k-i}(1-\mu)^{k-i} + 1$, or, $\delta^{k-i-1}(1-\mu)^{k-i-1} < (k-i-1)\delta(1-\mu) + 1$. This condition holds because the left hand side is smaller, whereas the right hand side is larger, than one. Therefore $\psi_i(\mu,\delta,k) < \psi_{i+1}(\mu,\delta,k)$, implying $\partial p_{i,k}/\partial\mu < 0 \Rightarrow \partial p_{i+1,k}/\partial\mu < 0$.

The rest of the proof establishes existence of the schedules $\tilde{\mu}_{i,k}(\delta)$ defining the boundary of the sets $M_{i,k}$, as $\tilde{\mu}_{i,k}(\delta) : [0,1] \to [0,1]$, such that $\partial p_{i,k}/\partial\mu = 0$.

Set $i = k-1$ and consider (B1). Note that $\psi_{k-1}(\mu,\delta,k) > 0$ if $\mu > [1-k(1-\delta)]/k\delta$. Next, observe that at $\mu = 1/k$ condition (B1) becomes an equality, where $\psi_{k-1}(\frac{1}{k},\delta,k) > 0$ for all $\delta < 1$ (since $1/k \geq [1-k(1-\delta)]/k\delta$, with strict inequality holding for $\delta < 1$). Because the left hand side is decreasing whereas the right hand side is increasing in μ, (B1) will be violated, and thus

$$\frac{(1-\mu)(1-\delta)}{\mu(1-\delta(1-\mu))} > k - \frac{1}{1-\delta(1-\mu)} \text{ if and only if } \mu < 1/k.$$

So, for $\mu \leq 1/k \equiv \tilde{\mu}_{k-1}$, $\partial p_{k-1,k}/\partial\mu \geq 0$.

Since $\psi_1(\mu,\delta,k) < \cdots < \psi_{k-1}(\mu,\delta,k)$, if $\mu \leq 1/k$, then $\partial p_{i,k}/\partial\mu > 0$ for all $i < k-1$.

Now set $i = k-2$ and consider $\mu > 1/k$. Since $\psi_{k-2}(\tilde{\mu}_{k-1},\delta,k) < \psi_{k-1}(\tilde{\mu}_{k-1},\delta,k)$, the inequality in (B1) is violated at $\mu = 1/k+\epsilon$, for ϵ arbitrarily small. Given a fixed $\delta < 1$ and the fact that $\psi_{k-2}(\mu,\delta,k)$ is increasing whereas the left hand side of (B1) is decreasing in μ, by the intermediate value theorem there exists $\tilde{\mu}_{k-2}(\delta) \in (\frac{1}{k},1)$ such that (B1) holds if and only if $\mu > \tilde{\mu}_{k-2}(\delta)$. Note that $\psi_{k-2}(\tilde{\mu}_{k-2}(\delta),\delta,k)$ must be positive.

Existence of $\tilde{\mu}_{k-3}(\delta) \in (\tilde{\mu}_{k-2}(\delta),1)$ is established along the same lines as $\tilde{\mu}_{k-2}(\delta)$, by using the monotonicity properties of the expressions in (B1). Repeating the procedure for all $i < k-2$ generates the declining sequence $\tilde{\mu}_{k-1}(\delta) < \tilde{\mu}_{k-2}(\delta) < \cdots < \tilde{\mu}_1(\delta) < 1$, which implies the set inclusions $M_{1,k} \subset M_{2,k} \subset \cdots M_{k-1,k} \subset M_{k,k} = [0,1)^2$.

(iii). $p_{\emptyset,k} + \sum_{i=1}^{k} p_{i,k} = 1$ implies $\sum_{i=1}^{k} \partial p_{i,k}/\partial\mu = -\partial p_{\emptyset,k}/\partial\mu$ and $\sum_{i=1}^{k} \partial p_{i,k}/\partial\mu_S = -\partial p_{\emptyset,k}/\partial\mu_S$. Since $\partial p_{i,k}/\partial\mu_S < 0$ for all $0 < i \leq k$, $\sum_{i=1}^{k} \partial p_{i,k}/\partial\mu_S < 0$, implying $\partial p_{\emptyset,k}/\partial\mu_S > 0$ and also, $\sum_{i=1}^{k}[\partial p_{i,k}/\partial\mu_S](h_S + i.h_A) < 0$.

Consider now $p_{\emptyset,k} = \mu_S + (1-\mu_S)[\alpha\mu + (1-\alpha\mu)\mu^+_{1,k}]$, thus, $\partial p_{\emptyset,k}/\partial\mu = (1-\mu_S)[\alpha(1-\mu^+_{1,k}) + (1-\alpha\mu)(\partial\mu^+_{1,k}/\partial\mu)]$. By Lemma 3-(ii), $\partial\mu^+_{1,k}/\partial\mu > 0$. Thus, $\partial p_{\emptyset,k}/\partial\mu > 0$ which implies $\sum_{i=1}^{k} \partial p_{i,k}/\partial\mu < 0$.

On the other hand, we know that $\sum_{i=1}^{k} \partial p_{i,k}/\partial\mu < 0$ and $\partial p_{i,k}/\partial\mu \leq 0 \Rightarrow \partial p_{i+1,k}/\partial\mu < 0$. Therefore, $(\partial p_{i,k}/\partial\mu)(h_S + i.h_A) \leq 0 \Rightarrow (\partial p_{i+1,k}/\partial\mu)(h_S + (i+1).h_A) < 0$. Thus, if $\sum_{i=1}^{k} \partial p_{i,k}/\partial\mu < 0$, then $\sum_{i=1}^{k}(\partial p_{i,k}/\partial\mu)(h_S + i.h_A) < 0$. □

Proof of Proposition 2. Consider the claim that $(\mu_S^F,\mu^F) = (\bar{\mu},0)$ or $(\mu_S^F,\mu^F) = (0,\bar{\mu})$.

The State's objective function SH in (B2) below is obtained by using (4) and substituting for $\mu_S = \bar{\mu} - \mu$ from the budget constraint in (1).

Given $\bar{\mu} \in (0,1)$, the State choses $\mu \in [0, \bar{\mu}]$ to minimize

$$\text{(B2)} \quad SH = \sum_{i=1}^{k} p_{i,k}(h_S + i.h_A)$$
$$= (1-\bar{\mu}+\mu)(1-\delta)\alpha\mu \sum_{i=1}^{k-1} \Big[(1-\alpha\mu)^i + \delta(1-\alpha\mu)^{i+1} + \cdots$$
$$+\delta^{k-i-1}(1-\alpha\mu)^{k-1}\Big](h_S + i.h_A) + (1-\bar{\mu}+\mu)(1-\alpha\mu)^k(h_S + k.h_A).$$

For $i \le k-1$, let us write $p_{i,k}(h_S + i.h_A) = (1-\bar{\mu}+\mu)f_i(\mu)$, thus the expected harm as

$$\text{(B3)} \quad SH(\mu) = (1-\bar{\mu}+\mu)\left[\sum_{i=1}^{k-1} f_i(\mu) + (1-\alpha\mu)^k(h_S + k.h_A)\right]$$

where

$$f_i(\mu) = (1-\delta)\alpha\mu\left[(1-\alpha\mu)^i + \delta(1-\alpha\mu)^{i+1} + \cdots + \delta^{k-i-1}(1-\alpha\mu)^{k-1}\right](h_S + i.h_A).$$

Observe that $f_i(\mu)$ is the sum of $k-i$ functions, $f_{i,j}(\mu) = (1-\delta)\delta^j\alpha\mu(1-\alpha\mu)^{i+j}(h_S + i.h_A)$ where $j = 0, \cdots, k-i-1$. Each $f_{i,j}(\mu)$ is strictly concave and strictly positive for $\mu \in (0, \bar{\mu}]$, with $f_{i,j}(0) = 0$ and $f_{i,j}(\bar{\mu}) > 0$, given $\alpha\bar{\mu} < 1$.[38] These properties extend to the sum $\sum_{i=1}^{k-1} f_i(\mu)$. Since $\sum_{i=1}^{k-1} f_i(0) = 0$ and $\sum_{i=1}^{k-1} f_i(\mu)$ is strictly concave for $\mu \in [0.1]$ and strictly positive for $\mu \in (0, \bar{\mu}]$, the minimand of the function $\sum_{i=1}^{k-1} f_i(\mu)$ cannot lie in the interior domain $(0, \bar{\mu})$. This holds for the function $(1-\bar{\mu}+\mu)\sum_{i=1}^{k-1} f_i(\mu)$ as well.

The last component of (B3) is $(1-\bar{\mu}+\mu)(1-\alpha\mu)^k(h_S + k.h_A)$, or, $[(1-\bar{\mu})(1-\alpha\mu)^k + \mu(1-\alpha\mu)^k](h_S + k.h_A)$. The first part decreases continuously in the interval $[1-\bar{\mu}, (1-\bar{\mu})(1-\alpha\bar{\mu})^k]$ whereas the second part has value 0 at $\mu = 0$ and increases thereafter as a strictly concave curve, to reach a unique maximum if $\bar{\mu}$ is large enough, and gets the value $(1-\alpha\bar{\mu})^k$ at $\mu = \bar{\mu}$. Given these properties, the minimum of $(1-\bar{\mu}+\mu)(1-\alpha\mu)^k(h_S + k.h_A)$ is either at $\mu = 0$, or at $\mu = \bar{\mu}$.

It follows that the minimand of (B3) cannot lie in the open interval $(0, \bar{\mu})$. The solution to (B2) is thus $(\mu_S^F, \mu^F) = (\bar{\mu}, 0)$ or $(\mu_S^F, \mu^F) = (0, \bar{\mu})$.

(i) For $k = 1$, $SH(\alpha\mu) = (1-\bar{\mu}+\mu)(1-\alpha\mu)(h_S + h_A)$. Note that $SH(0) < SH(\alpha\bar{\mu})$ if $\alpha < 1$ and $SH(0) = SH(\alpha\bar{\mu})$ if $\alpha = 1$, implying $\alpha_1 = 1$.

[38]Writing its component that depends on μ and differentiating twice yields $f'_{i,j}(\mu) = \alpha(1-\alpha\mu)^{i+j} - \alpha^2\mu(i+j)(1-\alpha\mu)^{i+j-1}$ and $f''_i(\mu) = -2\alpha^2(i+j)(1-\alpha\mu)^{i+j-1} + \alpha^3\mu(i+j)(i+j-1)(1-\alpha\mu)^{i+j-2}$, which is negative because $\alpha\mu < 2/(i+j+1)$ for $i \ge 1$ and $j \ge 0$.

Let $k \geq 2$. Using (B2) the condition $SH(\alpha\bar{\mu}) < SH(0)$ can be written as

$$\text{(B4)} \qquad SH(\alpha\bar{\mu}) = (1-\delta)\alpha\bar{\mu}\sum_{i=1}^{k-1}\Big[(1-\alpha\bar{\mu})^i + \delta(1-\alpha\bar{\mu})^{i+1} + \cdots + \delta^{k-i-1}(1-\alpha\bar{\mu})^{k-1}\Big](h_S + i.h_A) + (1-\alpha\bar{\mu})^k(h_S + k.h_A) < (1-\bar{\mu})(h_S + k.h_A) = SH(0).$$

Recall, by Proposition 1-(iii), $\partial SH/\partial\mu < 0$ and $\partial SH/\partial\alpha < 0$.

Observe that (B4) is violated at $\alpha = 0$, so, let $\alpha \in (0, 1]$. Summing the terms, the expression of $SH(\alpha\bar{\mu})$ can be arranged and simplified in several steps, as follows:

$$\begin{aligned}
\text{(B5)} \quad SH(\alpha\bar{\mu}) &= (1-\delta)\alpha\bar{\mu}\sum_{i=1}^{k-1}(1-\alpha\bar{\mu})^i\left[\frac{1-\delta^{k-i}(1-\alpha\bar{\mu})^{k-i}}{1-\delta(1-\alpha\bar{\mu})}(h_S + i.h_A)\right] \\
&\quad + (1-\alpha\bar{\mu})^k(h_S + k.h_A) \\
&= \frac{(1-\delta)\alpha\bar{\mu}}{1-\delta(1-\alpha\bar{\mu})}\Bigg[\sum_{i=1}^{k-1}(1-\alpha\bar{\mu})^i(h_S + i.h_A) \\
&\quad - (1-\alpha\bar{\mu})^k\sum_{i=1}^{k-1}\delta^{k-i}(h_S + i.h_A)\Bigg] + (1-\alpha\bar{\mu})^k(h_S + k.h_A).
\end{aligned}$$

Substituting above for $(h_S + i.h_A)$ the larger term $(h_S + k.h_A)$, as if the same maximal harm occurs in every S-connection outcome, yields the expression $\widehat{SH}(\alpha\bar{\mu})$, which is larger than $SH(\alpha\bar{\mu})$ but also monotonically decreasing in α. Thus,

$$\begin{aligned}
SH(\alpha\bar{\mu}) &< \tfrac{(1-\delta).\alpha\bar{\mu}}{1-\delta(1-\alpha\bar{\mu})}\left[\textstyle\sum_{i=1}^{k-1}(1-\alpha\bar{\mu})^i(h_S + k.h_A) - (1-\alpha\bar{\mu})^k\sum_{i=1}^{k-1}\delta^{k-i}(h_S + k.h_A)\right] \\
&\quad + (1-\alpha\bar{\mu})^k(h_S + k.h_A) \equiv \widehat{SH}(\alpha\bar{\mu}) \\
&= \left[\tfrac{(1-\delta)(1-\alpha\bar{\mu}-(1-\alpha\bar{\mu})^k)-\alpha\bar{\mu}\delta(1-\delta^{k-1})(1-\alpha\bar{\mu})^k}{1-\delta(1-\alpha\bar{\mu})} + (1-\alpha\bar{\mu})^k\right](h_S + k.h_A) \\
&= \left[\tfrac{(1-\delta)(1-\alpha\bar{\mu})-(1-\alpha\bar{\mu})^k[1-\delta+\alpha\bar{\mu}\delta(1-\delta^{k-1})}{1-\delta(1-\alpha\bar{\mu})} + (1-\alpha\bar{\mu})^k\right](h_S + k.h_A).
\end{aligned}$$

Further arranging this expression, the condition $\widehat{SH}(\alpha\bar{\mu}) < SH(0)$ can be written as

$$\widehat{SH}(\alpha\bar{\mu}) = \left[1-(1-\alpha\bar{\mu})^k - \frac{\alpha\bar{\mu}(1-\delta^k(1-\alpha\bar{\mu})^k)}{1-\delta(1-\alpha\bar{\mu})} + (1-\alpha\bar{\mu})^k\right](h_S + k.h_A)$$

$$< (1-\bar{\mu})(h_S + k.h_A) = SH(0). \quad \text{(B6)}$$

This inequality in turn simplifies to

$$\alpha\frac{1-\delta^k(1-\alpha\bar{\mu})^k}{1-\delta(1-\alpha\bar{\mu})} = \alpha\left[1+\delta(1-\alpha\bar{\mu})+\cdots+\delta^{k-1}(1-\alpha\bar{\mu})^{k-1}\right] > 1. \quad \text{(B7)}$$

Note that (B6), or equivalently (B7), implies (B4), given α. The left hand side of the inequality in (B7) is continuously increasing in α. Moreover (B7) holds at $\alpha = 1$ but is violated at $\alpha = 1/k$, since $1+\delta(1-\frac{1}{k}\bar{\mu})+\cdots+\delta^{k-1}(1-\frac{1}{k}\bar{\mu})^{k-1} < k$. By continuity of $\widehat{SH}(\alpha\bar{\mu})$ in α, there exists a unique $0 < \widehat{\alpha}_k < 1$ such that $SH(0) = \widehat{SH}(\widehat{\alpha}_k\bar{\mu})$. Since $\widehat{SH}(\widehat{\alpha}_k\bar{\mu}) > SH(\widehat{\alpha}_k\bar{\mu})$ and $SH(\alpha\bar{\mu})$ is decreasing in α, there exists $\alpha_k < \widehat{\alpha}_k$ such that $SH(\alpha_k\bar{\mu}) = SH(0)$, and $SH(\alpha\bar{\mu}) > SH(0)$ if and only if $\alpha > \alpha_k$.

(ii) Evaluating the left-hand side of (B4) for $k+1$, the following difference in expected harm in C_{k+1} and C_k can be formed, under amputation ($\mu = \bar{\mu}$):

$$\begin{aligned}\Delta SH_k(\bar{\mu}) &= SH_{k+1}(\bar{\mu}) - SH_k(\bar{\mu})\\ &= (1-\delta)\alpha\bar{\mu}(1-\alpha\bar{\mu})^k\Big[h_S + k.h_A\\ &\quad +\delta(h_S+(k-1)h_A)+\cdots+\delta^{k-1}(h_S+h_A)\Big]\\ &\quad +(1-\alpha\bar{\mu})^{k+1}(h_S+(k+1)h_A) - (1-\alpha\bar{\mu})^k(h_S+k.h_A).\end{aligned}$$

The expression $h_S+k.h_A+\delta(h_S+(k-1)h_A)+\cdots+\delta^{k-1}(h_S+h_A)$ can be grouped as $\frac{1-\delta^k}{1-\delta}h_S + \sum_{i=1}^k \frac{1-\delta^i}{1-\delta}h_A$. Therefore

$$\begin{aligned}\Delta SH_k(\bar{\mu}) &= (1-\delta)\alpha\bar{\mu}(1-\alpha\bar{\mu})^k\left[\tfrac{1-\delta^k}{1-\delta}h_S + \textstyle\sum_{i=1}^k \tfrac{1-\delta^i}{1-\delta}h_A\right]\\ &\quad +(1-\alpha\bar{\mu})^k[-\alpha\bar{\mu}h_S + ((1-\alpha\bar{\mu})(k+1)-k)h_A],\\ &= (1-\alpha\bar{\mu})^k\left[(-\alpha\bar{\mu}\delta^k)h_S + \left[(1-\alpha\bar{\mu}) - \alpha\bar{\mu}(k-\textstyle\sum_{i=1}^k(1-\delta^i))\right]h_A\right].\end{aligned}$$

Under decapitation, $SH_k(0) = (1-\bar{\mu})(h_S+k.h_A)$. Thus, $\Delta SH_k(0) = (1-\bar{\mu})h_A$. Comparing $\Delta SH_k(0)$ with $\Delta SH_k(\bar{\mu})$ and arranging the terms yields the result: $\Delta SH_k(\bar{\mu}) < \Delta SH_k(0)$ if and only if (6) holds.

(iii) In (B5), the part of $SH(\bar{\mu})$ that depends on δ,

$$\frac{(1-\delta)\alpha\bar{\mu}}{1-\delta(1-\alpha\bar{\mu})}\left[\sum_{i=1}^{k-1}(1-\alpha\bar{\mu})^i(h_S+i.h_A)-(1-\alpha\bar{\mu})^k\sum_{i=1}^{k-1}\delta^{k-i}(h_S+i.h_A)\right].$$

is decreasing in δ. It follows that $SH(\bar{\mu})$ is decreasing in δ.

(iv) Observing that $\partial SH(\alpha\bar{\mu})/\partial h_S = \partial\widehat{SH}(\alpha\bar{\mu})/\partial h_S$ and using (B6) yields,

$$\frac{\partial\widehat{SH}(\alpha\bar{\mu})}{\partial h_S} < \frac{\partial SH(0)}{\partial h_S} \text{ if and only if } \alpha\left[1+\delta(1-\alpha\bar{\mu})+\cdots+\delta^{k-1}(1-\alpha\bar{\mu})^{k-1}\right] > 1.$$

The LHS of the second inequality above is continuous and increasing in α. Its value at $\alpha = 0$ is zero whereas its value at $\alpha = 1$ is larger than one, implying existence of $0 < \alpha'_k < 1$ at which the inequality condition becomes equality. Thus, if $\alpha > \alpha'_k$, $\partial SH(\alpha\bar{\mu})/\partial h_S < \partial SH(0)/\partial h_S$. □

Proof of Proposition 3. The first part of the proof derives the expressions in (8), (9) and (11). We have

$$\text{(B8)}\quad \Delta Es_A(k) = \sum_{i=1}^{k+1} Es_{i,k+1} - \sum_{i=1}^{k} Es_{i,k} = \big[\sum_{i=1}^{k}(\mu_{i,k+1}-\mu_{i,k})+\mu_{k+1,k+1}\big]s_A,$$

$$\text{(B9)}\quad \Delta R(k) = \sum_{i=1}^{k+1} p_{i,k+1}[i.v] - \sum_{i=1}^{k} p_{i,k}[i.v].$$

Setting $i = 0$ in the expression of $\mu_{i,k+1}-\mu_{i,k}$ and using Lemma 3(iii) yields the expression for $\Delta Es_S(k)$ in (8). To see that (B8) can be expressed as (9), use the fact $\mu^-_{i,k} = \mu^-_{i,k+1}$ in the expression of $\mu_{i,k}$ in (3), which yields $\mu_{i,k+1}-\mu_{i,k} = (1-\alpha\mu)(1-\mu^-_{i,k})[\mu^+_{i,k+1}-\mu^+_{i,k}]$. Since $\mu^+_{i,k+1}-\mu^+_{i,k} = \alpha\mu\delta^{k-i+1}(1-\alpha\mu)^{k-i}$ by Lemma 3(iii),

$$\text{(B10)}\quad \Delta Es_i(k) = \mu_{i,k+1}-\mu_{i,k} = (1-\alpha\mu)(1-\mu^-_{i,k})\alpha\mu\delta^{k-i+1}(1-\alpha\mu)^{k-i} > 0.$$

Summing over the first k terms and adding $\mu_{k+1,k+1}$ yields (9). The claims $\Delta Es_S(k) > 0$, $\Delta Es_A(k) > 0$ are obvious.

Using Lemma 3(iii) for $i \le k-1$ we have

$$\text{(B11)}\quad [p_{i,k+1}-p_{i,k}][i.v] = (1-\mu_S)(1-\alpha\mu)^i(1-\delta)\alpha\mu\delta^{k-i}(1-\alpha\mu)^{k-i}[i.v] > 0,$$

$$\text{(B12)}\quad p_{k+1,k+1}v + [p_{k,k+1}-p_{k,k}]v = (1-\mu_S)(1-\alpha\mu)^k[1-\alpha\mu-\alpha\mu\delta k]v.$$

Summing over the expressions in (B11) from $i = 1$ to $k-1$ and adding (B12),

$$\text{(B13)}\quad \Delta R(k) = (1-\mu_S)(1-\alpha\mu)^k \left[1 - \alpha\mu - \alpha\mu\delta k + (1-\delta)\alpha\mu \sum_{i=1}^{k-1}(k-i)\delta^i\right] v.$$

The summation term in (B13) can be written as

$$\begin{aligned}\sum_{i=1}^{k-1}(k-i)\delta^i &= \sum_{i=1}^{k-1}\delta^i + \sum_{i=1}^{k-2}\delta^i + \cdots + \sum_{i=1}^{2}\delta^i + \delta \\ &= \frac{1}{1-\delta}\left[(\delta-\delta^k) + (\delta-\delta^{k-1}) + \cdots + (\delta-\delta^2)\right] \\ &= \frac{1}{1-\delta}\left[(k-1)\delta - \delta^k - \delta^{k-1} - \cdots - \delta^2\right].\end{aligned}$$

Using this expression for $\sum_{i=1}^{k-1}(k-i)\delta^i$ the term in the squared brackets in (B13) becomes

$1 - \alpha\mu - \alpha\mu\delta k + \alpha\mu\left[(k-1)\delta - \delta^k - \delta^{k-1} - \cdots - \delta^2\right]$

$= 1 - \alpha\mu - \delta k - \alpha\mu\delta(\delta + \delta^2 + \cdots + \delta^{k-2} + \delta^{k-1}) = 1 - \alpha\mu - \alpha\mu\delta\frac{1-\delta^k}{1-\delta}$.

Substituting in (B13) yields (11), which is positive if and only if

$$\alpha\mu < \frac{1}{1+\delta\frac{1-\delta^k}{1-\delta}}.$$

To determine the chain size k_G, consider the schedule $m(\delta,k)$. Observe that $m(\delta,k)$ forms a decreasing sequence in k, starting from $m(\delta,0) = 1$, $m(\delta,1) = 1/(1+\delta)$, ... , down to its limit $m(\delta,\infty) = 1-\delta$ as $k \to \infty$, given $\delta > 0$. So, a sufficient condition for $\Delta R(k) > 0$ to hold for all k is $\alpha\mu \leq 1-\delta$. Given this declining sequence, if $\alpha\mu > 1-\delta$ the intermediate value theorem implies existence of a positive k_G such that $\Delta R(k_G) \geq 0$ but $\Delta R(k_G+1) < 0$. □

Proof of Proposition 4. Existence of an integer $k^* \in \{0,\cdots,\bar{k}\}$ maximizing $\Pi(k)$ is guaranteed. If $k^* \notin \{0,\bar{k}\}$, it must satisfy $\Delta\Pi(k^*) \geq 0 > \Delta\Pi(k^*+1)$. If $0 < k^* < \bar{k}$, $\Delta\Pi(k_G+1) = \Delta R(k_G+1) - \Delta Es_S(k_G+1) - \Delta Es_A(k_G+1) < 0$, because $\Delta R(k_G+1) < 0$ whereas $\Delta Es_S(k_G+1) > 0$ and $\Delta Es_A(k_G+1) > 0$. It follows that $k^* \leq k_G$. The optimal k^* for $(\mu_S = \bar{\mu}, \mu = 0)$, in (14), is derived in the text. □

Proof of Proposition 5. Note that the crime chain's equilibrium strategy is given by its best response in Proposition 4. Given this, and the fact that the State's optimal intervention strategy is either $(\mu_S,\mu) = (\bar{\mu},0)$ or $(\mu_S,\mu) = (0,\bar{\mu})$ for any $k \in \{0,\cdots\bar{k}\}$, there can be but three equilibrium paths of play. Two of these paths involve $(\mu_S,\mu) = (\bar{\mu},0)$ (decapitation), one followed by $k^* = 0$ and the other by $k^* = \bar{k}$. As for the third path, it is induced by the amputation strategy of the State, $(\mu_S, \mu) = (0, \bar{\mu})$.

If $\bar{\mu} \geq \bar{\mu}^c$, the State has no incentive to deviate from $(\mu_S, \mu) = (\bar{\mu}, 0)$ because the chain's response is $k^* = 0$ and thus $SH = 0$. If $\bar{\mu} < \bar{\mu}^c$ the chain's response to $(\bar{\mu}, 0)$ in (14) dictates $k^* = \bar{k}$ and the State's objective becomes $SH(\{\bar{\mu}, 0\}) = (1-\bar{\mu})(h_S+\bar{k}h_A)$. Deviating to the amputation strategy, given the chain's optimal size response, yields the expected harm $SH(\{0, \bar{\mu}\}, \alpha)$. The next step, stated in Lemma 2, shows that there is a unique α^c that satisfies (16), that is, such that $SH(\{\bar{\mu}, 0\}) = SH(\{0, \bar{\mu}\}, \alpha^c)$.

Proof of Lemma 2. Clearly, the decapitation harm $SH(\{\bar{\mu}, 0\}) = (1 - \bar{\mu})(h_S + k.h_A) > 0$ is independent of α. Consider the amputation harm $SH(\{0, \bar{\mu}\}, \alpha)$. Holding k constant, By Proposition 1-(iii), $SH(\{0, \bar{\mu}\}, \alpha)$ is continuously decreasing in α. On the other hand, $SH(\{0, \bar{\mu}\}, \alpha)$ is increasing in k whereas the chain's size response to amputation, $k^*(0, \bar{\mu})$, is nonincreasing in α. These two facts reinforce each other, implying that $SH(\{0, \bar{\mu}\}, \alpha)$ is decreasing in α. Evaluating $SH(\{0, \bar{\mu}\}, \alpha)$ at extreme values of α reveals that it is maximal and equal to $SH(\{0, \bar{\mu}\}, 0) = h_S + \bar{k}h_A$ at $\alpha = 0$, implying $SH(\{\bar{\mu}, 0\}) < SH(\{0, \bar{\mu}\}, 0)$. On the other hand one can clearly pick a sufficiently large $\alpha\bar{\mu}$ that will induce the chain to set $k^* < \bar{k}$ under amputation, such that $SH(\{\bar{\mu}, 0\}) > SH(\{0, \bar{\mu}\}, \alpha)$. Thus, α^c as defined in (16) is unique. This completes the proof of Lemma 2.

The equilibrium intervention strategy at each α, then, depends on the comparison between $SH(\{\bar{\mu}, 0\})$ and $SH(\{0, \bar{\mu}\}, \alpha)$, the State opting for decapitation if $SH(\{\bar{\mu}, 0\}) \geq SH(\{0, \bar{\mu}\}, \alpha)$ and amputation otherwise, as characterized in the text for the case $\delta = 0$. The proof for $\delta > 0$ is based on the same arguments. □

Proof of Proposition 6. Given an allocation $\{\mu^d_{S,k}, (\mu^d_{1,k}, \cdots, \mu^d_{k,k})\}$, let us focus first on the intra-agent allocation $\mu^d_{1,k}, \cdots, \mu^d_{k,k}$, taking $\mu^d_{S,k}$ fixed. Set $\alpha = 1$ without loss of generality (to be relaxed later).

To show that the optimal allocation must set $\mu^d_{2,k} = \cdots = \mu^d_{k,k} = 0$, assume the contrary, $\mu^d_{k,k} = m > 0$. Thus $p_{k,k} = (1 - \mu^d_{S,k}) \prod_{j=1}^{k-2}(1 - \mu^d_{j,k})(1 - \mu^d_{k-1,k})(1 - m) > 0$. Now transfer all the budget for target $A_{k,k}$ to target $A_{k-1,k}$, inducing the change in direct detection probabilities $\Delta\mu^d_{k,k} = -m$, $\Delta\mu^d_{k-1,k} = m$. This adjustment will reduce $p_{k,k}$ by

$$\text{(B14)} \qquad \Delta p_{k,k} = -(1 - \mu^d_{S,k}) \prod_{j=1}^{k-2}(1 - \mu^d_{j,k})\mu^d_{k-1,k}m.$$

It will also reduce $p_{k-1,k}$, to zero, because the tail-detection probability of $A_{k-1,k}$ falls zero. The fall in $p_{k-1,k}$ is thus equal to its pre-adjustment level,

$$\text{(B15)} \qquad \Delta p_{k-1,k} = -(1 - \mu^d_{S,k}) \prod_{j=1}^{k-2}(1 - \mu^d_{j,k})(1 - \mu^d_{k-1,k})(1 - \delta)m.$$

Moving to the S-connection outcome with $k-2$ agents, $p_{k-2,k} = (1-\mu^d_{S,k})\prod_{j=1}^{k-2}(1-\mu^d_{j,k})(1-\delta)[\mu^d_{k-1,k} + (1-\mu^d_{k-1,k})\delta m]$ becomes, after the adjustment, $p_{k-2,k} = (1-\mu^d_{S,k})\prod_{j=1}^{k-2}(1-\mu^d_{j,k})(1-\delta)[\mu^d_{k-1,k} + m]$, thus,

$$\Delta p_{k-2,k} = (1-\mu^d_{S,k})\prod_{j=1}^{k-2}(1-\mu^d_{j,k})(1-\delta)m[1-(1-\mu^d_{k-1,k})\delta].$$

The probabilities $p_{i,k}$, for $i \leq k-2$, are affected similarly. Proceeding thus, the pre-adjustment level of $p_{k-3,k} = (1-\mu^d_{S,k})\prod_{j=1}^{k-3}(1-\mu^d_{j,k})(1-\delta)[\mu^d_{k-2,k} + (1-\mu^d_{k-2,k})\delta(\mu^d_{k-1,k}+(1-\mu^d_{k-1,k})\delta m)]$ becomes $(1-\mu^d_{S,k})\prod_{j=1}^{k-3}(1-\mu^d_{j,k})(1-\delta)[\mu^d_{k-2,k}+(1-\mu^d_{k-2,k})\delta(\mu^d_{k-1,k}+m)]$. The change is:

$$\Delta p_{k-3,k} = (1-\mu^d_{S,k})\prod_{j=1}^{k-2}(1-\mu^d_{j,k})(1-\delta)\delta m[1-\delta(1-\mu^d_{k-1,k})].$$

By the same token, $\Delta p_{k-4,k} = (1-\mu^d_{S,k})\prod_{j=1}^{k-2}(1-\mu^d_{j,k})(1-\delta)\delta^2 m[1-\delta(1-\mu^d_{k-1,k})]$ and thus

$$\Delta p_{i,k} = (1-\mu^d_{S,k})\prod_{j=1}^{k-2}(1-\mu^d_{j,k})(1-\delta)\delta^{k-i-2}m[1-\delta(1-\mu^d_{k-1,k})].$$

Combining these terms, the impact of the resource adjustment $\Delta\mu^d_{k,k} = -m$, $\Delta\mu^d_{k-1,k} = m$ on the sum probabilities of S-connection outcomes is $\sum_{i=1}^{k}\Delta p_{i,k} = \sum_{i=1}^{k-2}\Delta p_{i,k}+\Delta p_{k-1,k}+\Delta p_{k,k}$ or, using (B14) and (B15) and regrouping the terms,

$$\sum_{i=1}^{k}\Delta p_{i,k} = (1-\mu^d_{S,k})\prod_{j=1}^{k-2}(1-\mu^d_{j,k})\Big[(1-\delta)m\big[(1-\delta(1-\mu^d_{k-1,k}))\sum_{i=1}^{k-1}\delta^{i-1}-1\big]-m\mu^d_{k-1,k}\Big]$$

Since $\sum_{i=1}^{i-1}\delta^{i-1} = (1-\delta^{k-1})/(1-\delta)$, the terms in the squared brackets simplify to $m([1-\delta(1-\mu^d_{k-1,k})(1-\delta^{k-1})]-1)-m\mu^d_{k-1,k}$, which is negative. Therefore, the adjustment $\Delta\mu^d_{k,k} = -m$, $\Delta\mu^d_{k-1,k} = m$ leads to $\sum_{i=1}^{k}\Delta p_{i,k} < 0$. By Proposition 1-(iii), $\sum_{i=1}^{k}\Delta p_{i,k}(h_S + i.h_A) < 0$ and thus $\mu^d_{k,k} > 0$ cannot be optimal.

The proof that $\mu^d_{k-1,k} = 0$ involves the same argument. Assume, contrary to the claim in the proposition, $\mu^d_{k-1,k} = m > 0$, and consider the adjustment $\Delta\mu^d_{k-1,k} = -m$, $\Delta\mu^d_{k-2,k} = m$. Exactly same arguments apply to show that

$\sum_{i=1}^{k-1} \Delta p_{i,k} < 0$. Also,

$$\Delta p_{k,k} = -(1-\mu^d_{S,k}) \prod_{j=1}^{k-3} (1-\mu^d_{j,k})\mu^d_{k-2,k} m < 0,$$

implying $\sum_{i=1}^{k} \Delta p_{i,k} < 0$. Hence, $\mu^d_{k-1,k} = 0$. Iterated application of this procedure establishes that $\mu^d_{i,k} = 0$ for $i \geq 2$.

Given $\mu^d_{i,k} = 0$ for $i \geq 2$, the State's problem is reduced to

$$\min_{\{\mu^d_{S,k},\mu^d_{1,k}\}} SH = (1-\mu^d_{S,k})(1-\alpha\mu^d_{1,k})(h_S+k.h_A), \quad \text{subject to} \quad \mu^d_{S,k}+\mu^d_{1,k} = \bar{\mu} < 1.$$

The solution is straightforward: $\mu^{dF}_{1,k} = 0$ if $\alpha \leq 1$ and $\mu^{dF}_{1,k} = \bar{\mu}$ if $\alpha > 1$, as stated in (19). □

Proof of Proposition 7. The problem of the State under Assumption 1 is now a special case of Proposition 2 where $p_{1,k} = \cdots = p_{k-1,k} = 0$. Substituting the budget constraint into the objective function yields $SH_{amput} = (1-\alpha\bar{\mu})^k(h_S+h_A)$ under amputation, $SH_{decap} = (1-\bar{\mu})(h_S+h_A)$ under decapitation. The former is smaller if $\alpha > [1-(1-\bar{\mu})^{1/k}]/\bar{\mu} \equiv \alpha^T_k$ and the optimal enforcement strategy in (20) follows.

The claim $\alpha^T_{k+1} < \alpha^T_k$ is shown by simplifying and rearranging $[1-(1-\bar{\mu})^{\frac{1}{k+1}}]/\bar{\mu} < [1-(1-\bar{\mu})^{\frac{1}{k}}]/\bar{\mu}$, as $(1-\bar{\mu})^{\frac{1}{k+1}} > (1-\bar{\mu})^{\frac{1}{k}}$, or as $(1-\bar{\mu}) < (1-\bar{\mu})(1-\bar{\mu})^{\frac{1}{k}}$, implying $1 < (1-\bar{\mu})^{\frac{1}{k}}$. □

Proof of Proposition 8. Given an intervention strategy (μ_S, μ), the expected harms from network $n \in \{C, TA, TS, S\}$, $SH_n = \sum_{i=1}^{3} p^n_{i,3}[h_S + ih_A]$, are respectively:

$$\begin{aligned}
SH_{TA} &= (1-\mu_S)(1-\mu)\Big[(1-\mu)^2(h_S+3h_A)+2\mu(1-\mu)(1-\delta)(h_S+2h_A) \\
&\quad +(1-\delta)^2\mu^2((h_S+h_A)\Big]; \\
SH_{TS} &= (1-\mu_S)(1-\mu)\Big[(1-\mu)^2(h_S+3h_A)+2\mu(1-\mu)(1-\delta)(h_S+2h_A) \\
&\quad +(1-\delta)\mu[1+\delta(1-\mu)+(1-\delta)\mu](h_S+h_A)\Big]; \\
SH_{S} &= (1-\mu_S)(1-\mu)\Big[(1-\mu)^2(h_S+3h_A)+3\mu(1-\mu)(1-\delta)(h_S+2h_A) \\
&\quad +3(1-\delta)^2\mu^2((h_S+h_A)\Big]; \\
SH_{C} &= (1-\mu_S)(1-\mu)\Big[(1-\mu)^2(h_S+3h_A)+(1-\mu)(1-\delta)\mu(h_S+2h_A) \\
&\quad +(1-\delta)[\mu+(1-\mu)\delta\mu]((h_S+h_A)\Big].
\end{aligned}$$

Using the budget constraint $\mu_S = \bar{\mu} - \mu$, first observe that all four expected harm expressions are increasing in μ at $\mu = 0$. Second, in each expected harm expression, each of the three terms is either monotonically increasing in μ or has a unique interior maximum in $[0, \bar{\mu}]$, where $\bar{\mu} < 1$. It follows that each expected harm expression is minimized at $\mu = 0$ (decapitation) or at $\mu = \bar{\mu}$ (amputation).

(i) Setting $\mu = 0$ in all four networks yields the expected harms from decapitation stated in the text. Consider the amputation strategy, $\mu_S = 0$, $\mu = \bar{\mu}$. Excluding the expected harm in the S-connection outcome with three agents (same in all four networks), the remaining two expected harm components are

$$\begin{aligned}
&\bar{\mu}(1-\bar{\mu})(1-\delta)\Big[(1-\bar{\mu})(h_S+2h_A)+[1+(1-\bar{\mu})\delta](h_S+h_A)\Big] \qquad \text{in } C; \\
&\bar{\mu}(1-\bar{\mu})(1-\delta)\Big[3(1-\bar{\mu})(h_S+2h_A)+3(1-\delta)\bar{\mu}(h_S+h_A)\Big] \qquad \text{in } S; \\
&\bar{\mu}(1-\bar{\mu})(1-\delta)\Big[2(1-\bar{\mu})(h_S+2h_A)+(1-\delta)\bar{\mu}(h_S+h_A)\Big] \qquad \text{in } TA; \\
&\bar{\mu}(1-\bar{\mu})(1-\delta)\Big[2(1-\bar{\mu})(h_S+2h_A)+[1+\delta(1-\bar{\mu})+(1-\delta)\bar{\mu}](h_S+h_A)\Big] \qquad \text{in } TS.
\end{aligned}$$

Observe that $SH_{TA}(0,\bar{\mu}) < SH_{TS}(0,\bar{\mu})$, $SH_{TA}(0,\bar{\mu}) < SH_S(0,\bar{\mu})$, and $SH_C(0,\bar{\mu}) <$

$SH_{TS}(0, \bar{\mu})$. On the other hand, $SH_C(0, \bar{\mu}) < SH_S(0, \bar{\mu})$ if

$$2(1 - \bar{\mu})(h_S + 2h_A) + [3(1 - \delta)\bar{\mu} - 1 - (1 - \bar{\mu})\delta](h_S + h_A) > 0.$$

The coefficient of h_A in this expression is strictly positive. The h_S component is positive if $2(1-\bar{\mu})+3(1-\delta)\bar{\mu} > 1+(1-\bar{\mu})\delta$, which requires $\delta < (1+\bar{\mu})/(1+2\bar{\mu})$. Therefore, $SH_C(0, \bar{\mu}) < SH_S(0, \bar{\mu})$ if h_S is not too large, or if $\delta < (1+\bar{\mu})/(1+2\bar{\mu})$.

(ii) Forming the expression $SH_{TA}(0, \bar{\mu}) - SH_C(0, \bar{\mu}) > 0$ and simplifying the terms yields the result. Finally, $SH_S(0, \bar{\mu}) - SH_{TS}(0, \bar{\mu}) > 0$ if $(1 - \bar{\mu})(h_S + 2h_A) + [2(1 - \delta)\bar{\mu}) - 1 - \delta(1 - \bar{\mu})](h_S + h_A) > 0$. The coefficient of h_S is positive if $\delta < \bar{\mu}/(1 + \bar{\mu})$ and the coefficient of h_A is positive if $\delta < 1/(1 + \bar{\mu})$. Therefore, $SH_S(0, \bar{\mu}) - SH_{TS}(0, \bar{\mu}) > 0$ if $\delta \leq \bar{\mu}/(1 + \bar{\mu})$, or if $\delta \in (\bar{\mu}/(1 + \bar{\mu}), 1/(1 + \bar{\mu}))$ and h_S is small relative to h_A. □

Appendix C: Pattern of expected sanctions and revenues

The following lemma referred to in Section IV describes the evolution of expected sanctions and revenues as chain size k increases.

LEMMA 4: *The changes in the components of profits by successive recruitments relate as follows:*

$$\text{(C1)}\qquad \Delta E s_S(k+1) = \delta(1 - \alpha\mu)\Delta E s_S(k),$$

$$\text{(C2)}\qquad \Delta E s_A(k+1) = \delta(1 - \alpha\mu)\Big[\Delta E s_A(k) + \Big(\alpha\mu + (1 - \alpha\mu)\alpha\mu\delta(1 - \mu^-_{k+1,k+1})\Big) s_A\Big],$$

$$\text{(C3)}\qquad \Delta R(k+1) = (1 - \alpha\mu)\left[\frac{1 - \delta - \alpha\mu(1 - \delta^{k+2})}{1 - \delta - \alpha\mu(1 - \delta^{k+1})}\right]\Delta R(k).$$

Proof. Using the fact $\mu^-_{i,k+1} = \mu^-_{i,k}$ for $i \leq k$ and (8), the change in i'th agent's expected sanction by extension $C_{k+1} \to C_{k+2}$ is $\Delta E s_i(k+1) = [\alpha\mu(1 - \mu^-_{i,k})\delta^{k-i+2}(1 - \alpha\mu)^{k-i+2}]s_A = \delta(1 - \alpha\mu)\Delta E s_i(k)$, for $i \leq k$, where the last equality follows from (B10). Therefore, the total change in the first k agents' expected sanctions from extension $C_{k+1} \to C_{k+2}$ is

$$\text{(C4)}\qquad \delta(1 - \alpha\mu)\alpha\mu\sum_{i=1}^{k}(1 - \mu^-_{i,k})\delta^{k-i+1}(1 - \alpha\mu)^{k-i+1}.s_A$$

On the other hand the change in the $k+1$'th agent's expected sanction is

$$\begin{aligned}[\mu_{k+1,k+2} - \mu_{k+1,k+1}]s_A &= (1-\alpha\mu)(1-\mu^-_{k+1,k+1})(\mu^+_{k+1,k+2} - \mu^+_{k+1,k+1})s_A \\ &= (1-\alpha\mu)(1-\mu^-_{k+1,k+1})\delta\alpha\mu . s_A. \qquad \text{(C5)}\end{aligned}$$

The last equality follows, because $\mu^+_{k+1,k+1} = 0$ and $\mu^+_{k+1,k+2} = \delta\alpha\mu$. Using equations (C4), (C5), the fact that $\mu_{k+2,k+2} = \alpha\mu + (1-\alpha\mu)\mu^-_{k+2,k+2} = \alpha\mu + (1-\alpha\mu)\delta\mu_{k+1,k+1}$ and rearranging terms, the total change in expected sanctions from extension $C_{k+1} \to C_{k+2}$ can be written as:
$\Delta E s_A(k+1) = \delta(1-\alpha\mu)\Delta E s_A(k) + \delta(1-\alpha\mu)\left[\alpha\mu + (1-\alpha\mu)\alpha\mu\delta(1-\mu^-_{k+1,k+1})\right]s_A.$

Finally, $\Delta E s_S(k+1) = \delta(1-\alpha\mu)\Delta E s_S(k)$ is derived from (8) and the relationship between $\Delta R(k)$ and $\Delta R(k+1)$ in (C3), from (11) by arranging the terms.

□