Appendix: Why does the U.S. have the best research universities?

W. Bentley MacLeod and Miguel Urquiola*

One goal of our paper is to highlight the role of faculty incentives in explaining universities' research output. Specifically, we are interested in the impact that specialization has on performance. To explore this, this appendix sets out a simple agency model building on the multi-tasking framework developed by Holmström and Milgrom (1991). To explore the effects of competition, we augment this with a tournament model along the lines of Lazear and Rosen (1981) and Green and Stokey (1983).

Suppose, as discussed in the main text, that universities can enhance the demand for their services by improving their reputation for research production. To keep matters simple, suppose that a university hires a single professor, i, who can exert research effort, denoted r_i. This produces the following payoff for the school:

$$\Pi = V_r \times r_i - w_i, \tag{1}$$

where V_r is the value the university assigns to research and w_i is the compensation paid to the professor. Part of the latter may be in amenities such as lab space, reduced teaching assignments, etc.

Faculty performance and preferences

We suppose that when evaluating a candidate, the university observes a measure of individual research performance. Specifically, in considering an individual i who works in field f, the school observes:

$$s_i = r_i + \alpha_i + \gamma_f,$$

where r_i is research effort, α_i is ability (assumed to be non-random for simplicity), and $\gamma_f \sim N(0, \eta_f)$ is field-specific noise. This last component reflects the fact that non-experts—say a university hiring or tenure committee—have trouble assessing professors' research performance. This is particularly true in making comparisons across fields. For example, in the humanities faculty tend to write books, while articles are the norm in the sciences. Similarly, a chemistry professor may be listed as a co-author on all papers produced by her laboratory, while an economic theorist writes alone—it is hard to tell who has higher performance.

* MacLeod: Columbia Unversity and NBER, wbmacleod@wbmacleod.net; Urquiola: Columbia University and NBER, msu2101@columbia.edu.

One way to tackle this problem is by comparing a candidate to other professors in her field. To see this, suppose the university observes a second signal that consists of the average performance in a candidate's field:

$$s_f = \bar{r}_f + \alpha_f + \gamma_f.$$

The first term $\bar{r}_f$ is the average effort in the field; this will be determined as an equilibrium outcome below. The random variable α_f represents the average ability of individuals in the field, and we let this be given by $\alpha_f \sim N(\bar{\alpha}_f, \Omega_f)$; i.e, Ω_f captures the variance in ability in field f. As before, $\gamma_f \sim N(0, \eta_f)$.[1]

Using both signals, the university can create a signal of *relative* performance—comparing individuals against their field (Lazear and Rosen 1981). In particular, it can compute:

$$\begin{aligned}\delta_{if} &= s_i - s_f \\ &= r_i - \bar{r}_f + \alpha_i - \alpha_f.\end{aligned}$$

A key point is that comparing within field causes the term γ_f to drop out. The variance of this signal is thus Ω_f.

Following Lazear and Rosen (1981) and Holmström and Milgrom (1991), compensation can then take the form:

$$w_i = \bar{w}_i + b_i s_i + b_f \delta_{if},$$

which features a base wage and bonuses paid as a function of the individual and the relative performance measures.[2] Let $\vec{p} = \{\bar{w}, b_i, b_f\}$ summarize the contract offered by the university.

Suppose that in addition to doing research, professors can engage in non-research, "outside" activities. Broadly, this could include consulting work, but also staying active in alumni networks or, in the 1800s, religious groups. Formally we assume that the effort exerted by a professor is given by:

$$\vec{e}_i = [r_i, o_i]^T,$$

where r_i is effort on research and o_i is effort on outside activities. This highlights the fact that effort is fundamentally a resource allocation problem—how to allocate one's time. To keep things tractable, we posit a quadratic effort cost function

[1] It would be possible to add an additional subscript t to γ, exploring over-time variation. We leave that for future work.

[2] We have assumed that measures are normally distributed, and hence this equation can be viewed as a measure of log wages.

(Holmstrom and Milgrom 1991):

$$C(\vec{e}_i) = \vec{e}_i^T \begin{bmatrix} 1 & d \\ d & 1 \end{bmatrix} \vec{e}_i/2.$$

The off-diagonal terms, $d \in (0,1)$, imply that the two types of effort are substitutes—an increase in outside activity, o_i, increases the cost of supplying research effort and vice versa. Note that the matrix $A = \begin{bmatrix} 1 & d \\ d & 1 \end{bmatrix}/2$ is invertible and positive definite, and hence $C(\vec{e}_i) \geq 0$ for any effort and is zero if and only if:

$$\vec{e}_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The payoff for professor i takes the form:

$$U_i = E\{w_i\} + V_o \times o_i + V_{ri} \times r_i - C(\vec{e}_i) - \frac{\rho}{2} var(w_i).$$

In words, the parameters that determine faculty performance are the marginal value of outside activities, V_o, the intrinsic value the professor attaches to research, V_{ri}, and relative risk aversion, ρ. In order to have a smooth trade-off between research and outside actitives, it is assumes that the value of outside activities are sufficiently similar that professors always choose a combination of research and outside activities:

$$1 \geq min\left\{\frac{V_o}{V_r}, \frac{V_r}{V_o}\right\} > d > 0.$$

Finally, note that this setup does not model the issue of matching faculty to appointments. There is work showing that some individuals have an intrinsic taste for research that leads them to seek university positions (Stern 2004; Agarwal and Ohyama 2013).. Our focus is on the design of rewards.

Optimal Compensation

We now derive the optimal contract for the university—the problem facing it is to choose a pay package, $\vec{p}_i = \{\bar{w}_i, b_i, b_f\}$, subject to participation and incentive constraints.[3]

Relying on the assumption that the expected value of the noise term γ_f is zero,

[3] To keep matters simple, we fix the candidate's alternative utility to the average ability of individuals in the field, and hence $U^0 = \bar{\alpha}_f$.

the professor's payoff given compensation, $\vec{p}_i = \{\bar{w}_i, b_i, b_f\}$, is:

$$
\begin{aligned}
(2)\quad U_i(\vec{e}_i, \vec{p}_i) &= E\{w_i\} + V_{ri}r_i + V_0 o_i - C(\vec{e}_i) - \frac{\rho}{2} var(w_i) \\
&= \bar{w}_i + b_i(r_i + \alpha_i) + b_f(r_i - \bar{r}_f + \alpha_i - \bar{\alpha}_f) + V_{ri}r_i + V_0 o_i - C(\vec{e}_i) \\
&- \frac{\rho}{2}\left(b_i^2 \eta_f + b_f^2 \Omega_f\right).
\end{aligned}
$$

Let $B_i \equiv (b_i + b_f)$ be the total reward for effort provided by the university. Notice that it is a function of both the university's reward system and the professor's intrinsic motivation, V_{ri}. We suppose that parameters are set so we have an interior solution with positive levels of both types of effort, r_i and o_i. Then we can set:

$$
\vec{e}_i^{\,*}(B_i) = \begin{bmatrix} r_i(B_i) \\ o_i(B_i) \end{bmatrix} \tag{3}
$$

$$
= argmax_{\vec{e}} U_i(\vec{e}, \vec{p}), \tag{4}
$$

$$
= \frac{1}{1-d^2}\begin{bmatrix} B_i + V_{ri} - dV_o \\ V_o - d(B_i + V_{ri}) \end{bmatrix} \tag{5}
$$

Thus, as the total reward for research, B_i, increases, the effort devoted to research goes up, while that devoted to outside activities declines. The payoff for the university is given by:

$$
\begin{aligned}
\Pi(r_i, \vec{p}_i) &= V_r \times r_i - (\bar{w}_i + b_i(r_i + \alpha_i) + b_f(r_i - \bar{r}_f + (\alpha_i - \bar{\alpha}_f))), \\
&= V_r \times r_i - (\bar{w}_i + B_i(r_i + \alpha_i) - b_f(\bar{\alpha}_f + \bar{r}_f)).
\end{aligned}
$$

Note that variable employment costs thus have two terms. One term is the direct incentive cost that rewards total productivity, $r_i + \alpha_i$. The second term reflects the relative performance evaluation. If the professor is above average, that creates an additional cost. As we shall see, the optimal compensation also depends upon the risk the professor must face, which affects the base wage, $\bar{w}$.

The determination of bonus pay is assumed to maximize university welfare subject to the professor's participation constraint:

$$
\begin{aligned}
&max_{\vec{p}_i} \Pi(\vec{p}_i) \\
&\qquad \vec{e}_i \in argmax_{\vec{e}_i} U_i(\vec{e}_i, \vec{p}_i), \\
&U_i(\vec{e}_i, \vec{p}_i) \geq U^0 = \bar{\alpha}.
\end{aligned} \tag{6}
$$

The solution to this problem is relatively straightforward. We can substitute the closed form solution for effort from (3). The Lagrangian for the optimal contract is:

$$
L(\vec{p}_i, \lambda) = \Pi(r_i(\vec{p}_i), \vec{p}_i) + \lambda\{U_i(\vec{e}_i^{\,*}(\vec{p}_i), \vec{p}_i) - \bar{\alpha}_f\}.
$$

In order to have an optimal solution for the fixed payment $\bar{w}$ it must be the case that $\lambda = 1$. Using (2) and (3) this becomes:

$$\begin{aligned} L\left(\vec{p}_f, 1\right) &= \left(V_r + V_{ri}\right) r_i\left(B_i\right) + V_o \times o_i\left(B_i\right) - C\left(\vec{e}_i^*\left(B_i\right)\right) - \bar{\alpha}_f \\ &\quad - \frac{\rho}{2}\left(b_i^2 \eta_f + b_f^2 \Omega_f\right). \end{aligned}$$

Note that the first line depends only upon total performance based compensation, B_i. The second line measures the trade-off between the two forms of compensation, based upon the relative precision of the performance measures. Given B_i we follow Grossman and Hart (1983) and define the risk premium that must be paid to faculty as a function of performance pay:

$$RPC\left(B_i\right) = min_{B_i \leq b_i + b_f} \frac{\rho}{2}\left(b_i^2 \eta_f + b_f^2 \Omega_f\right).$$

This is increasing in B_i with each component given by:

$$b_i\left(B_i\right) = \frac{\Omega_f}{\left(\Omega_f + \eta_f\right)} B_i, \tag{7}$$

$$b_f\left(B_i\right) = \frac{\eta_f}{\left(\Omega_f + \eta_f\right)} B_i. \tag{8}$$

Thus the ratio of relative performance pay (b_{if}) to performance pay is:

$$\frac{b_f}{b_i} = \frac{\eta_f}{\Omega_f}. \tag{9}$$

In other words, the use of relative performance pay increases with η_f, i.e., with the difficulty non-experts have in assessing performance in a field f. From this we can compute the overall costs of performance pay:

$$\begin{aligned} RPC\left(B_i\right) &= \frac{\rho}{2} \frac{\Omega_f \eta_f}{\left(\Omega_f + \eta_f\right)} B_i. \\ &\equiv \kappa B_i \end{aligned} \tag{10}$$

where κ measures the marginal cost of performance pay upon faculty payoffs. Thus, the cost of performance pay falls with more precise signals. If η_f increases due to the increase in the complexity of a field, then costs can be reduced if the variance of individual performance, Ω_f, can be reduced. This can be achieved with hierarchical sorting by skill. From this we can compute both the optimal level of performance pay and the total expected compensation to faculty as the solution to $\frac{dL(B_i)}{dB_i} = 0$, where:

$$\begin{aligned} L\left(B_i\right) &= \left(V_r + V_{ri}\right) \times r_i^*\left(B\right) + V_o \times o_i^*\left(B_i\right) - C\left(\vec{e}_i^*\left(B_i\right)\right) \\ &\quad - \bar{\alpha}_f - RP\left(B_i\right). \end{aligned} \tag{11}$$

Taking the first order condition we get:

$$\begin{aligned}\frac{dL(B_i)}{dB_i} = & \left(V_r + V_{ri} - \frac{\partial C}{\partial r_i}\right)\frac{dr_i}{dB_i} \\ & + \left(V_o - \frac{\partial C}{\partial o_i}\right)\frac{do_i}{dB_i} \\ & - \kappa.\end{aligned}$$

Using the closed form solution for effort we get:

$$B_i^* = V_r - \kappa\left(1 - d^2\right), \tag{12}$$

$$= V_r - \frac{\rho}{2}\frac{\Omega_f \eta_f}{(\Omega_f + \eta_f)}\left(1 - d^2\right). \tag{13}$$

By construction, faculty utility is fixed at $\bar{\alpha}_f$, the average quality of individuals in the peer pool. Rewards have to compensate individuals choosing effort above their normal level given by the intrinsic return to research, V_{ri}. Total expected compensation is given by:

$$w_i = \bar{\alpha}_f + U^0(B_i^*) + \kappa B_i^*.$$

Thus, the faculty member receives additional pay to compensate her for additional effort,

$$U^0(B_i^*) = V_{ri} r_i^*(B_i^*) + V_o o_i^*(B_i^*) - C(\vec{e}_i^{\,*}(B_i^*)), \tag{14}$$

and the costs associated with relative performance evaluation, κB_i^*.

Implications

This yields two propositions. Consider first the situation in which there is no bonus pay ($B_i = 0$). In that case result (5) implies:

PROPOSITION 1: *Suppose faculty are paid a fixed wage that ensures participation (with no bonus pay for research performance). Then research effort is given by:*

$$r_i = \frac{V_{ri} - dV_o}{1 - d^2}.$$

Hence research is increasing in the intrinsic preference for research, and decreasing in the return to outside activities.

It is optimal for colleges to reward faculty as a function of their research performance. The amount of reward is given by (7), (8) and (12). Hence, we have:

PROPOSITION 2: *Optimal performance pay is decreasing with the variance of the ability of the candidates. The use of relative performance evaluation increases rewards for research, and the level of research activities.*

Notice that the level of performance pay is determined by the value of κ:

$$\kappa = \frac{\rho}{2} \frac{\Omega_f \eta_f}{(\Omega_f + \eta_f)}.$$

A smaller κ leads to lower costs, and this can only be achieved with universities becoming increasingly selective in their pool of applicants which determines the value of Ω_f. Thus we have:

PROPOSITION 3: *If departments can draw from a more homogeneous pool of candidates (smaller Ω_f), this decreases κ, and hence increases the optimal bonus (from 12), and in turn increases research output (from 5).*

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