ONLINE APPENDIX

Liability Insurance: Equilibrium Contracts under Monopoly and Competition

Jorge Lemus, Emil Temnyalov and John L. Turner

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Jorge Lemus, University of Illinois Urbana-Champaign. jalemus@illinois.edu.
Emil Temnyalov, University Technology Sydney. emil.temnyalov@uts.edu.au.
John L. Turner, University of Georgia. jlturner@uga.edu.

Online Appendix: ADDITIONAL MATERIAL

B1. Insurance for a Risk-Averse Agent

Risk aversion introduces novel elements that are absent in the baseline case. First, insurance reduces the risk of going to litigation. Second, the agent's wealth may determine the agent's level of risk aversion, which affects the equilibrium transfer under bargaining. In addition, generally there is no separability between the cost of insurance for the agent and the settlement payoff. So even in the absence of wealth effects (e.g., CARA utility), the price of insurance may alter the bargaining core. Third, the settlement fee paid by the agent, as well as the willingness to pay for insurance, do not generally have closed-form solutions. As a result, in general, the main analysis the model under risk aversion is not analytically tractable.

Consider a risk-averse agent with initial wealth w covered by an insurance policy $\alpha = (\alpha_S, \alpha_L\, \alpha_D)$, bought at some price Q, and with preferences over lotteries represented by an increasing and concave Bernouilli utility function $u(\cdot)$. If the third party and the agent go to litigation, the expected payoff of the agent is

$$\text{(B1)}\quad u(CE(p,\alpha,Q)) \equiv pu(w-c_A+\alpha_L-d+\alpha_D-Q)+(1-p)u(w-c_A+\alpha_L-Q),$$

where $CE(p,\alpha,Q)$ denotes the certainty equivalent of the risky litigation outcome under insurance policy α bought at price Q. Under risk neutrality, we showed that an uninformed insurer fully cover litigation costs, i.e., $\alpha_L = c_A$. However, under risk aversion this is not necessarily true. The reason is that α_L increases the payoff in both states of the world, which reduces the value of a larger α_D to decrease the variance of the lottery.[25] Additionally, the certainty equivalent of going to litigation is affected by the price of the insurance and the level of wealth of the agent, whereas in the risk-neutral case the agent's wealth and the price of insurance do not affect the decision to litigate.

Under risk aversion parties are also better off by avoiding litigation: they save on litigation cost, and the agent avoids the risky litigation outcome. This means that risk aversion provides stronger settlement incentives to the parties. A feasible settlement agreement is a transfer T from the agent to the third party such that $0 < pd-c \leq T$ and $u(CE(p,\alpha,Q)) \leq u(w-Q-\max\{T-\alpha_S,0\})$ or, equivalently,

$$T_{\min}(p) \equiv pd - c \leq T \leq w - Q - CE(p,\alpha,Q) + \min\{\alpha_S, T\}.$$

If $w - Q - CE(p,\alpha,Q) < 0$, the agent will not accept a settlement agreement and parties litigate. If $w - Q - CE(p,\alpha,Q) \geq 0$, the agent accepts a settlement agreement as long as $T \leq T_{\max}(p,\alpha,Q) \equiv w - Q - CE(p,\alpha,Q) + \alpha_S$. Without

[25] We have $\frac{\partial^2}{\partial\alpha_L\partial\alpha_D}u(CE(p,\alpha,Q)) < 0$. Under risk-neutrality this cross-derivative is zero.

insurance parties always settle because $u(CE(p,0,0)) \leq u(w-T)$ for any transfer $T \geq pd - c$.[26]

When $w - Q - CE(p,\alpha,Q) \geq 0$ and $T_{\min}(p) \leq T_{\max}(p,\alpha,Q)$ the agent and the thrid party settle. In this case, the settlement fee is given by the solution to the maximization of the Nash-product. As in the case of risk neutrality, efficiency of Nash Bargaining implies that settlement transfer must be larger than α_S, so we can write the problem as:

(B2)

$$T^{\alpha}(p,Q) \in \arg\max_{T}(u(w - Q - T + \alpha_S) - u(CE(p,\alpha,Q)))^{\theta}(T - (pd - c))^{1-\theta}$$
$$\text{subject to } \max\{\alpha_S, pd - c\} \leq T \leq w - Q - CE(p,\alpha,Q) + \alpha_S.$$

Under risk neutrality, we showed that any contract with $T_{\min}(p) = T_{\max}(p,\alpha,Q)$ is optimal, and under imperfect information the insurer sets $\alpha_L = c_A$ and $\alpha_S = 0$. Under risk aversion, however, this result may no longer hold given the non-linearity of $Q + CE(p,\alpha,Q)$, as a function of Q (in the risk neutral case, $Q + CE(p,\alpha,Q)$ is independent of Q). The price of insurance could increase by reducing the litigation cost coverage ($\alpha_L < c_A$), reducing the payment of the litigation lottery in each state of nature, which increases the value of damage coverage (α_D), which decreases risk. In addition, to increase incentives to settle, the insurer may set a positive α_S. Covering settlements has two effects. First, increasing α_S increases the settlement transfer, which lowers the agent's willingness to pay. Second, increasing α_S may provide incentives to settle.

For the general class of risk-averse preferences, the outcome of bargaining may depend upon wealth w and the price of insurance Q. Our model is then not analytically tractable for analysis beyond the complete information case. We can simulate outcomes for certain classes of utility functions, but this introduces a taxonomy of possible cases to consider (e.g., increasing risk aversion, decreasing risk aversion, etc.). Analyzing all these cases for a class of distributions of types is beyond the scope of this paper. For this reason, in the next section we focus on one class of risk-preferences that allow us to gain some analytical tractability.

Mean-Variance Preferences. — For mean-variance preferences we can obtain some analytic results. An agent with these preferences evaluates lottery X according to

$$U(X) = E(X) - \frac{\sigma \mathrm{Var}(X)}{2}.$$

[26]To see this, note that $u(CE(p,0,0)) \leq u(w-\tau)$, where $\tau = pd + c_A > pd - c$.

Under insurance policy $\alpha = (\alpha_S, \alpha_L, \alpha_D)$, the certainty equivalent under litigation is[27]

$$CE(p, \alpha) = w - (c_A - \alpha_L) - p(d - \alpha_D) - \frac{\sigma p(1-p)(d-\alpha_D)^2}{2}.$$

The only difference with the risk neutral case is the last term $RP(p, \alpha_D) \equiv \frac{\sigma p(1-p)(d-\alpha_D)^2}{2}$, which corresponds to the agent's risk-premium. The bargaining surplus is

$$S_B = \min\{T, \alpha_S\} + c + c_A - \alpha_L - p\alpha_D + RP(p, \alpha_D).$$

Mirroring the proof of the risk neutral case, it can be shown that $T \geq \alpha_S$. Then, the bargaining surplus for type p is simply

$$S_B(p) = \alpha_S + c + c_A - \alpha_L - p\alpha_D + RP(p, \alpha_D).$$

When $S_B(p) \geq 0$ the agent of type p and the third party settle, otherwise they litigate. In the risk-neutral case, the bargaining surplus $S_{RN}(p) = \alpha_S + c + c_A - \alpha_L - p\alpha_D$ is linear and strictly decreasing in p. Under risk aversion, the bargaining surplus is concave and may be non-monotone in p, so in principle it is not clear that we can define a threshold type p^*.[28] Figure B1 illustrates the bargaining surplus as a function of the agent's type for the cases of risk neutrality and risk aversion.

The next lemma guarantees the existence of a unique threshold type p^* such that $S_B(p^*) = 0$.

LEMMA 6: *There is a unique positive value that solves the equation $S_B(p) = 0$.*

PROOF:

$S_B(0) = \alpha_S + c + c_A - \alpha_L > 0$. By concavity, $S_B(p)$ has a unique positive root. QED

Lemma 6 allows us to define a litigation-threshold type p^* such that types $p \leq p^*$ settle and types $p > p^*$ litigate, where p^* is the unique positive solution to the equation

$$p^* = \frac{c + c_A - \alpha_L + \alpha_S}{\alpha_D} + \frac{RP(p^*, \alpha_D)}{\alpha_D}.$$

When $RP(p^*, \alpha_D) > 0$, the threshold p^* is larger than this threshold for a risk neutral agent. This is shown in Figure B1 (for the case $\alpha_S = 0$, where the threshold for a risk-averse agent p^*_σ is larger than this threshold for a risk neutral agent p^*_0. This is, a risk-averse agent covered by contract α has weakly larger incentives to settle than a risk-neutral agent covered by the same contract. Intuitively, risk aversion pushes the agent to settle to avoid the risky litigation outcome. Only

[27] The price Q is paid up-front, so under these preferences the term $Q + CE(p, \alpha, Q)$ is independent of Q.

[28] By concavity, the bargaining surplus is strictly decreasing for $\sigma < \frac{2\alpha_D}{(d-\alpha_D)^2}$.

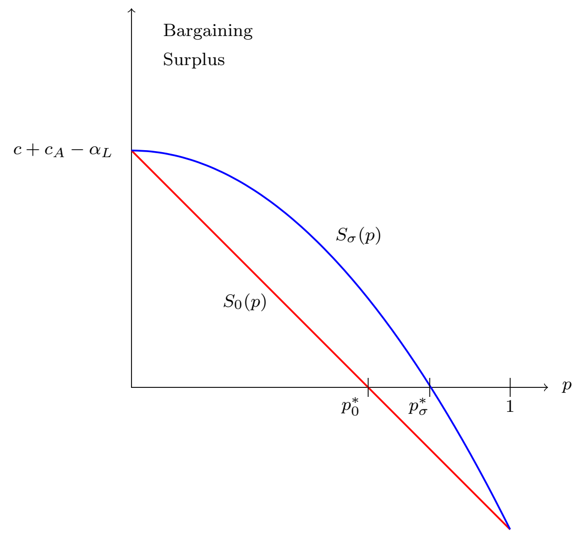

Figure B1. : Barganing surplus as a function of the agent's type for the case of risk neutrality ($\sigma = 0$) and risk aversion ($\sigma > 0$). The thresholds p_0^* (and p_σ^*) correspond to the type that is indifferent between settlement and litigation under risk neutrality (and under risk aversion).

under full insurance ($\alpha_D = d$) this force disappears, i.e., $RP(p^*, \alpha_D) = 0$. The next result study how a contract α affects the bargaining surplus.

LEMMA 7: *$S_B(\cdot)$ is strictly decreasing in α_D and α_L, and strictly increasing in α_S. Also, $\frac{\partial S_B(p^*)}{\partial p} < 0$, and sign $\frac{\partial p^*}{\partial \alpha_j} =$ sign $\frac{\partial S_B}{\partial \alpha_j}$ for $j \in \{S, L, D\}$.*

PROOF:

We have $\frac{\partial S_B(p^*)}{\partial p} = \frac{\sigma}{2}(d - \alpha_D)^2(1 - 2p^*) - \alpha_D$. From the definition of p^* we have $\frac{\sigma}{2}(d - \alpha_D)^2 = \frac{p^*\alpha_D - (\alpha_S + c + c_A - \alpha_L)}{p^*(1-p^*)}$. Replacing we get

$$\begin{aligned}\frac{\partial S_B(p^*)}{\partial p} &= \left[\frac{p^*\alpha_D - (\alpha_S + c + c_A - \alpha_L)}{p^*(1-p^*)}\right](1 - 2p^*) - \alpha_D \\ &= -\left[\frac{\alpha_S + c + c_A - \alpha_L}{p^*} + \frac{RP(p^*, \alpha_D)}{1 - p^*}\right] < 0.\end{aligned}$$

To compute the partial derivative with respect to the contract parameters we use that

$$S_B(p^*) = 0 \Rightarrow \frac{\partial S_B(p^*)}{\partial \alpha_j} + \frac{\partial S_B(p^*)}{\partial p}\frac{\partial p^*}{\partial \alpha_j} = 0.$$

Given that $\frac{\partial S_B(p^*)}{\partial p} < 0$, we have sign $\frac{\partial p^*}{\partial \alpha_j} =$ sign $\frac{\partial S_B}{\partial \alpha_j}$ for $j \in \{S, L, D\}$. QED

Lemma 7 shows that increasing α_D or α_L, or decreasing α_S reduces the bargaining surplus. Given that the agent's payment to the third party is proportional to the bargaining surplus, the insurer would like to make the bargaining surplus as small as possible. But this presents a tradeoff for the insurer: decreasing the bargaining will also induce more litigation (by decreasing p^*). The insurer faces this tradeoff when selling insurance agents that are risk neutral or risk averse. The main difference is that damages insurance (α_D) reduces the bargaining surplus and the litigation threshold at a faster rate under risk aversion.

Let $T_\alpha(p) = pd - c + (1-\theta)S_B(p)$ be the Nash bargaining transfer for contract α. An agent of type p pays a settlement transfer equal to $T(p) = \min\{T_\alpha(p) - \alpha_S, 0\}$, i.e., the agent is fully covered by the insurer if $T_\alpha(p) < \alpha_S$. In the risk-neutral case, the Nash bargaining transfer is strictly increasing in p, but under risk aversion this transfer may be non-monotone. The reason is that types around $p = 1/2$ value insurance more than types closer to $p = 0$ or $p = 1$. We need to impose a condition over σ to guarantee that the Nash bargaining transfer is increasing in p.

LEMMA 8: *When $\sigma < \frac{2}{d(1-\theta)}$, the Nash bargaining transfer is increasing, and there is a unique threshold type p^{**} is defined by the condition $pd - c + (1 - \theta)S(p^{**}) = \alpha_S$ such that agents $p \leq p^{**}$ settle and their settlement offer is fully covered by the insurance.*

PROOF:

We have $T'_\alpha(p) = d - \alpha_D(1-\theta) + (1-\theta)\frac{\sigma(d-\alpha_D)^2}{2}(1-2p)$. It's clear that for any p such that $2p \leq 1$ we have $T'_\alpha(p) > 0$. Consider then p such that $2p > 1$, so

$$T'_\alpha(p) = d - (1-\theta)\left[\alpha_D + \frac{\sigma(d-\alpha_D)^2}{2}(2p-1)\right].$$

A sufficient condition for this to hold is $d > (1-\theta)\left[\alpha_D + \frac{\sigma(d-\alpha_D)^2}{2}\right]$. The RHS of this inequality as a function of α_D is convex, so the maximum is either $\alpha_D = 0$ or $\alpha_D = d$. When $\alpha_D = d$, this holds. When $\alpha_D = 0$, this holds as long as $\sigma < \frac{2}{d(1-\theta)}$. QED

For σ sufficiently small, $T(p)$ is weakly increasing, which allow us to find a unique solution to the equation $T_\alpha(p) = \alpha_S$. We will impose this condition for the remainder of the analysis.

ASSUMPTION 2: $\sigma < \frac{2}{d(1-\theta)}$.

This expression states that willingness to pay is monotone increasing provided the risk aversion parameter σ is less than an amount proportional to the inverse of the damages. Hence, when damages are higher, we need a stronger assumption on the risk parameter σ.

How do p^* and p^{**} compare? We have that p^* solves the equation $S(p^*) = 0$ and p^{**} solves the equation $(1-\theta)S(p^{**}) = \alpha_S - (p^{**}d - c)$ with $\alpha_S > pd - c$, so $S(p^{**}) = \frac{\alpha_S - (p^{**}d-c)}{1-\theta} > 0$. Given that $S(0) > 0$, and S is concave, it is clear that p^{**} must be smaller than p^*. Therefore, we can write,

(B3)

$$W(p,\alpha) = \begin{cases} T_0(p) & \frac{c}{d} \leq p \leq p^{**} \\ T_0(p) + \alpha_S - T_\alpha(p) & \text{if } p^{**} < p \leq p^* \\ T_0(p) - p(d-\alpha_D) - (c_A - \alpha_L) - RP(p,\alpha_D) & \text{if } p > p^* \end{cases}$$

LEMMA 9: *$W(p,\alpha)$ is continuous for any level of risk aversion.*

PROOF:

First, it is easy to verify that $W(p,\alpha)$ is continuous for $p \neq p^{**}$ or $p \neq p^*$. Second, by definition p^{**} is such that $T_\alpha(p^{**}) = \alpha_S$, so $W(p,\alpha)$ is continuous at $p = p^{**}$. Third, from the definition of p^* we have that $T_\alpha(p^*) = p^*d - c$ and $p^*\alpha_D = \alpha_S + c + c_A - \alpha_L + R(p^*,\alpha_D)$. Thus, $-p^*(d-\alpha_D) - (c_A - \alpha_L) - RP(p^*,\alpha_D) = \alpha_S - T_\alpha(p^*)$. QED

Type p's willingness to pay for contract α is

(B4)

$$W(p,\alpha) = \begin{cases} pd - c + (1-\theta)\left(c + c_A + \frac{\sigma d^2}{2}p(1-p)\right) & \text{if } \frac{c}{d} \leq p \leq p^{**}, \\ \theta\alpha_S + (1-\theta)\left[\alpha_L + p\alpha_D + \frac{\sigma\alpha_D(2d-\alpha_D)}{2}p(1-p)\right] & \text{if } p^{**} < p \leq p^*, \\ \alpha_L + p\alpha_D + \frac{\sigma p(1-p)}{2}(\alpha_D(2d-\alpha_D) - \theta d^2) - \theta\,(c + c_A) & \text{if } p > p^*. \end{cases}$$

When $\frac{c}{d} < p \leq p^{**}$ the agent settles and pays nothing—the insurer fully covers the agent's settlement transfer. Thus, the agent's willingness to pay for insurance in this case is $T_0(p)$, the amount that an agent without insurance would have paid to TP under a settlement agreement. An agent of type $p^{**} < p \leq p^*$ settles litigation. The willingness to pay of this agent is the sum of the direct lump-sum transfer to cover settlement α_S plus the TP's share of the bargaining surplus not captured in the negotiation due to insurance $(1-\theta)[\alpha_L - \alpha_S + p\alpha_D + 0.5\sigma\alpha_D(2d-\alpha_D)p(1-p)]$. The first component in the term inside the bracket is the value of insurance $\alpha_L - \alpha_S + p\alpha_D$ realized with or without risk aversion. Intuitively, insurance allows the agent to extract additional surplus by improving its own threat point and reducing TP's payoff towards its own threat point. The second component is the *risk-premium reduction* effect of insurance:

(B5) $$\frac{\sigma p(1-p)\alpha_D(2d-\alpha_D)}{2} = \frac{\sigma p(1-p)d^2}{2} - \frac{\sigma p(1-p)(d-\alpha_D)^2}{2}.$$

With damages insurance α_D, the agent faces a loss of just $d - \alpha_D$ instead of d when TP wins the case. Because the agent ultimately settles with or without insurance, this gain enters the bargaining surplus directly, and the agent captures share $1-\theta$ of it.

An agent of type $p > p^*$ litigates. The term $\alpha_L + p\alpha_D$ is the direct value of insurance. Because the agent litigates, there is no bargaining surplus and it does not share this benefit with the third party.

The second term is the risk-premium reduction:

(B6)
$$\frac{p(1-p)\sigma}{2}\left[(\alpha_D(2d-\alpha_D)) - \theta d^2\right] = (1-\theta)\frac{\sigma p(1-p)d^2}{2} - \frac{\sigma p(1-p)(d-\alpha_D)^2}{2}.$$

This is lower under litigation than under settlement, by $\theta\left(\frac{\sigma p(1-p)(d-\alpha_D)^2}{2}\right)$, because the agent endures the entire variance when it owns insurance (and litigates). When $\theta = 0$, the variance-reduction effect is the same under settlement and litigation. Intuitively, the variance part of the agent's utility under insurance is the same (for $\theta = 0$) when the agent settles and when it litigates. As θ increases, the variance part of the agent's utility under insurance (which enters negatively) stays the same under litigation, but declines under settlement. For sufficiently high $\theta > \frac{d^2}{\alpha_D(2d-\alpha_D)}$, the variance-reduction effect is negative.

The third term is the litigation cost effect. This is the part of the agent's payoff under no insurance that accrues from settling and avoiding litigation costs. This is surrendered under litigation.

In contrast to the risk neutral case, $W(p,\alpha)$ may not be increasing in p the region $[p^*, p^{**}]$. The reason is that agents whose types are around $p = 1/2$ value insurance more than agents whose types are closer to $p = 0$ or $p = 1$. Figure B2 shows the shape of the willingness to pay when risk aversion is relatively low (left panel) and high (right panel). Differentiating with respect to p, we have

$$\text{(B7)} \qquad \frac{W(p,\alpha)}{dp} = \begin{cases} d + (1-\theta)\frac{\sigma d^2(1-2p)}{2} & \text{if } \frac{c}{d} \le p \le p^{**}, \\ (1-\theta)\left[\alpha_D + \frac{\sigma(1-2p)}{2}(\alpha_D(2d-\alpha_D))\right] & \text{if } p^{**} < p \le p^*, \\ \alpha_D + \frac{\sigma(1-2p)}{2}(\alpha_D(2d-\alpha_D) - \theta d^2) & \text{if } p > p^*. \end{cases}$$

LEMMA 10: *$W(p,\alpha)$ is strictly increasing in p for $\sigma \le \frac{1}{d}$.*

PROOF:

It can be shown that $\frac{W(p,\alpha)}{dp}|_{p^{**}<p\le p^*} > 0$ is the hardest condition to satisfy. Using that $\alpha_D \le d$, it is easy to see that this term is positive when $\sigma \le \frac{1}{d}$. QED

Note that Assumption 2 does not guarantee the condition in Lemma 10.

The net surplus from serving type p with policy α is

(B8)
$$(W-K)(p,\alpha) = \begin{cases} pd - c + (1-\theta)\left(c + c_A + \frac{\sigma d^2}{2}p(1-p)\right) - \alpha_S & \text{if } \frac{c}{d} \le p \le p^{**} \\ (1-\theta)\left[\alpha_L - \alpha_S + p\alpha_D + \frac{\sigma\alpha_D(2d-\alpha_D)}{2}p(1-p)\right] & \text{if } p^{**} < p \le p^* \\ -\theta\left(c + c_A\right) + \frac{\sigma p(1-p)}{2}\left(\alpha_D(2d-\alpha_D) - \theta d^2\right) & \text{if } p > p^* \end{cases}$$

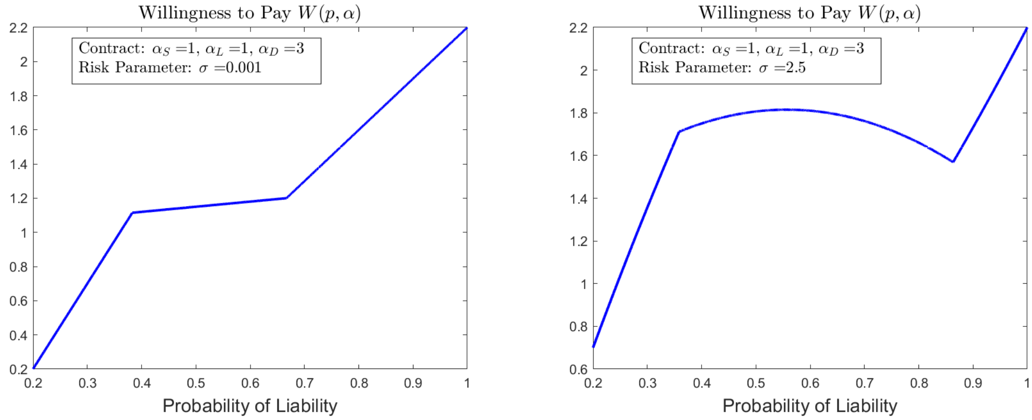

Figure B2. : $W(p, \alpha)$ for contract $\alpha_S = 1$, $\alpha_L = 1$, and $\alpha_D = 3$ and model parameters: $c = c_A = 1$, $d = 5$, $\theta = 0.8$.

In contrast to the risk neutral case, it is not always true that the net surplus is negative for types that litigate ($p > p^*$); for instance, this term will be positive when θ is small. Using the definition of p^*, we can write

(B9)

$$(W-K)(p,\alpha) = \begin{cases} pd - c + (1-\theta)(c + c_A + RP(p,0)) - \alpha_S & \text{if } \frac{c}{d} \le p \le p^{**} \\ (1-\theta)\left[c + c_A + RP(p,0) + (p-p^*)\alpha_D + RP(p^*,\alpha_D) - RP(p,\alpha_D)\right] & \text{if } p^{**} < p \le p^* \\ -\theta(c + c_A) + (1-\theta)RP(p,0) - RP(p,\alpha_D) & \text{if } p > p^* \end{cases}$$

Equation B9 describes the willingness to pay as a function of the threshold types p^* and p^{**} generated by contract α.

LEMMA 11: *Consider two contracts:* $\alpha = (\alpha_S, \alpha_L, \alpha_D)$ *that generates threshold* p^* *and* p^{**}, *and* $\hat{\alpha} = (\hat{\alpha}_S, \hat{\alpha}_L, \alpha_D)$ *that generates threshold* $\hat{p}^* = p^*$ *and* $\hat{p}^{**} < p^{**}$. *If* $\hat{\alpha}_S < \alpha_S$, *then* $(W-K)(p,\alpha) = (W-K)(p,\hat{\alpha})$ *for all* $p \in [p^{**}, 1]$ *and* $(W-K)(p,\alpha) < (W-K)(p,\hat{\alpha})$ *for all* $p < p^{**}$.

PROOF:

Both contracts generate the same litigation threshold p^* and both contracts cover the same amount in damages α_D. Thus, we have that $(W-K)(p,\alpha) = (W-K)(p,\hat{\alpha})$ for all $p \in [p^{**}, 1]$. For any $p \le \hat{p}^{**}$ we have that $(W-K)(p,\alpha) < (W-K)(p,\hat{\alpha})$ because $\hat{\alpha}_S < \alpha_S$. Finally, consider any $p \in (\hat{p}^{**}, p^{**})$. In this region, $(W-K)(p,\alpha) < (W-K)(p,\hat{\alpha})$ is equivalent to

$$pd - c - \alpha_S < (1-\theta)\left[(p-p^*)\alpha_D + RP(p^*,\alpha_D) - RP(p,\alpha_D)\right].$$

It is easy to see that this inequality corresponds to $T(p) < \alpha_S$, which holds for any $p < p^{**}$. QED

Lemma 11 implies that given to contracts that induce the same threshold p^* and cover the same amount for damages α_D, the insurer will prefer the contract with the smallest p^{**}.[29] Thus, we can show that some contracts are dominated.

PROPOSITION 7: *For a given contract* $\alpha = (\alpha_S, \alpha_L, \alpha_D)$ *define the contract* $\hat{\alpha} = (\max\{\alpha_S - \alpha_L, 0\}, \max\{\alpha_L - \alpha_S, 0\}, \alpha_D)$. *Then,* $(W - K)(p, \alpha) \leq (W - K)(p, \hat{\alpha})$ *for any* p.

PROOF:

First, note that both contracts α and $\hat{\alpha}$ generate the same threshold p^* because $\alpha_S - \alpha_L = \max\{\alpha_S - \alpha_L, 0\} - \max\{\alpha_L - \alpha_S, 0\}$. Second, note that $\max\{\alpha_S - \alpha_L, 0\} \leq \alpha_S$. Thus, by Lemma 11 we have $(W - K)(p, \alpha) \leq (W - K)(p, \hat{\alpha})$ for any p. QED

Proposition 7 implies that, if the goal is to maximize $W - K$ pointwise, the insurer will never offer contracts that cover simultaneously settlements and litigation costs, because these contracts are pointwise dominated. Thus, we can focus on contracts of the form $(\alpha_S, 0, \alpha_D)$ or $(0, \alpha_L, \alpha_D)$.

Under risk neutrality, Proposition 3 (in the main text) shows that we can go one step further: To maximize $(W-K)$ pointwise in p we can restrict to contracts of the form $\alpha = (0, c_A, c/p^*)$, for some $p^* > c/d$. The reason is that for any fixed p^* and any contract that satisfy $p^*\alpha_D = c + c_A - \alpha_L + \alpha_S$ we have: $\frac{\partial(W-K)}{\partial\alpha_D} < 0$ for $p \in (p^{**}, p^*)$ and zero otherwise; $\frac{\partial(W-K)}{\partial\alpha_S} < 0$ for $p < p^{**}$ and zero otherwise; and, $\frac{\partial(W-K)}{\partial\alpha_L} = 0$ for all p. Thus, the insurer has a preference for reducing α_S and α_D as much as possible, as long as the contract parameters satisfy $\alpha_L = c_A + c - p^*\alpha_D + \alpha_S$ with $\alpha_L \leq c_A$. Therefore, the insurer sets $\alpha_S = 0$ and $\alpha_D = \frac{c}{p^*}$.

Under risk aversion, for any fixed p^* and any contract that satisfy $p^*\alpha_D = c + c_A - \alpha_L + \alpha_S + RP(\alpha_D, p^*)$ we still have that: $\frac{\partial(W-K)}{\partial\alpha_S} < 0$ for $p < p^{**}$ and zero otherwise; and that $\frac{\partial(W-K)}{\partial\alpha_L} = 0$ for all p. However, the partial derivative with respect to α_D is no longer weakly negative:

$$\frac{\partial(W-K)}{\partial\alpha_D}(p,\alpha) =$$
$$= \begin{cases} 0 & \text{if } \frac{c}{d} \leq \frac{c}{d} \leq p \leq p^{**} \\ (1-\theta)\left[(p-p^*)\alpha_D - \sigma(d-\alpha_D)(p^*(1-p^*) - p(1-p))\right] & \text{if } p^{**} < p \leq p^* \\ \sigma(d-\alpha_D)p(1-p) & \text{if } p > p^* \end{cases}$$

We have $\frac{\partial(W-K)}{\partial\alpha_D}(p,\alpha) < 0$ for $p \in [p^*, p^{**}]$ and, when $\alpha_D < d$, $\frac{\partial(W-K)}{\partial\alpha_D}(p,\alpha) >$ 0for $p > p^*$. Thus, in contrast to the risk neutral case, it is not true that the insurer wants to reduce α_D as much as possible. This implies that even under

[29] Given that $T'(p) > 0$ we have $\frac{\partial p^{**}}{\partial\alpha_S} = \frac{\theta}{T'(p^{**})} > 0$, so lowering α_S will also lower p^{**}.

Assumption 2 there may not exist a contract that maximizes $W - K$ pointwise in p.

The insurer's cost of serving type p with policy α depends on the agent's level of risk aversion through the threshold p^*.

$$K(p, \alpha) = \begin{cases} \alpha_S & \text{if } p \leq p^*, \\ \alpha_L + p\alpha_D & \text{if } p > p^*. \end{cases} \tag{B10}$$

In contrast to the risk neutral case, the contract that maximizes $W - K$ at p for a risk-averse agent, for a fixed litigation threshold p^*, in some cases depends on p; this also explains why under risk aversion it is impossible to find a contract that maximizes $W - K$ pointwise for all p. To further hone the contrast between risk neutrality and risk aversion we present an example. In the example, we explain intuitively how to find the contract that maximizes $(W - K)$ at p for a fixed litigation threshold p^*.

Example. Consider the set of contracts that generate a fixed litigation threshold p^*. Within that set of contracts let us find the contract that maximizes $(W - K)$ at some point p. Let $z \equiv \alpha_L - \alpha_S \in (\infty, c_A]$.

1. **Risk neutrality.** This case is easily illustrated using a simple consumer choice framework, where the "consumer" is the agent+insurer (A+I) and whose utility is $W - K$ and must choose over the "budget set" $z = c_A + c - \alpha_D p^*$ subject to $z \leq c_A$. This constraint is represented by the two-piece black line in Figure B3. When $p < p^*$, A+I's willingness to pay $(W-K)(p) = (1-\theta)(z+p\alpha_D)$ is represented by the red indifference curves, which have slope p. As a result, the optimal mix of z and α_D is always at the corner where $z = c_A$, which implies that $\alpha_L = c_A$ and $\alpha_S = 0$. Intuitively, (A+I)'s marginal rate of substitution p is lower than p^*. As a result, litigation costs insurance is more valuable given the tradeoff implied by p^*, and the optimal mix of z and α_D is always at the corner where $\alpha_L = c_A$. Finally, when $p > p^*$ (i.e., types that litigate) we have that $(W - K)(p)$ is always negative and is constant, so it is affected by the choice of p^* but is otherwise unaffected by the choices of z and α_D.

2. **Risk aversion**. Fixing p^* yields a non-linear constraint illustrated in Figure B4. The $z-$axis intercept of the formula for the $S(p^*) = 0$ constraint is higher because of the risk premium. The additional constraint $z \leq c_A$ still must hold, and it binds at an $\alpha_D > \frac{c}{p^*}$ because of the risk premium. In addition, the risk premium makes the constraint a convex function of α_D. The slope of this constraint is $-p^*[1 + (1 - p^*)\sigma(d - \alpha_D)]$.

Indifference curves for an agent of type p—where $p < p^*$—similarly have a convex shape, with slope $-p[1 + (1 - p)\sigma(d - \alpha_D)]$. The indifference curve is tangent to the constraint at

$$\hat{\alpha}_D = d - \left[\frac{p - p^*}{\sigma\left(p^*(1 - p^*) - p(1 - p)\right)} \right]. \tag{B11}$$

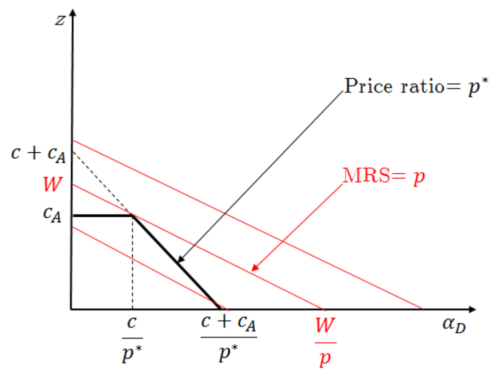

Figure B3. : Maximization of $W - K$ for $p < p^*$ in the case of risk neutrality.

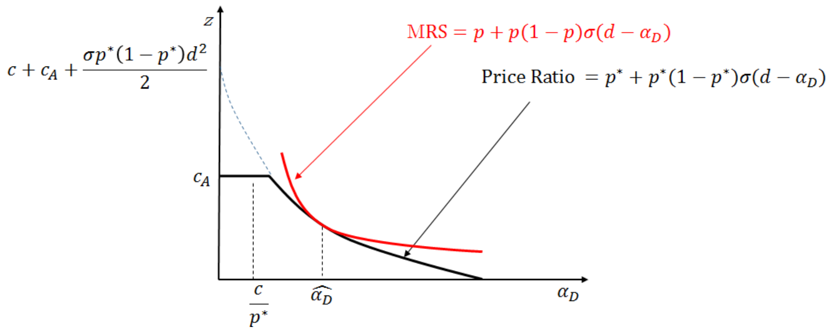

Figure B4. : Maximization of $W - K$ for $p < p^*$ in the case of risk aversion.

If this tangency occurs for $z < c_A$, as in Figure B4, then the $\hat{\alpha}_D$ given in (B11) optimizes W conditional on p^*. Otherwise the optimal coverage obtains at the upper boundary. Now, it is impossible for $\hat{\alpha}_D$ to be negative. When this condition implies a negative $\hat{\alpha}_D$, then we know that the $\alpha_L = c_A$ kink binds. We can use this condition to determine a restriction on σ that guarantees that this kink binds:

$$\sigma < \frac{1}{d}\left(\frac{1}{p^* + p - 1}\right). \tag{B12}$$

Because we restrict attention to cases where the second fraction yields a positive number (p closer to .5 than p^*), and to cases where $p \leq p^*$, the lowest that the second fraction can be is 1. Hence, we need only restrict attention to $\sigma < \frac{1}{d}$ to guarantee that $z = \alpha_C = c_A$ is pointwise optimal for types that settle.

For higher σ, of course, this is not the case. To see the intuition for that case most clearly, let $p^* > .5$. First, note that the difference in the slopes of the curves in Figure B4 depend both on p and $p(1-p)$. If $p < 1-p^*$, then p is so low that it is farther from $p = .5$ than p^*. Then both terms in the slope are of lower magnitude for the indifference curves than for the constraint. The indifference curves are then shallower than the constraint everywhere, so the optimal insurance occurs at the $\alpha_L = c_A$ kink. As p increases, it eventually reaches a point where it is closer to .5 than p^*. Then, the second term in the slope is of greater magnitude for the indifference curve than for the constraint. It is then possible for the red indifference curve to be steeper than the constraint at low α_D and shallower at high α_D.

For types that litigate, a lower α_D exposes the agent to more risk when it litigates, which reduces $W - K$ for $p > p^*$. Recall total A+I profit for types that litigate is

$$W - K = -\theta (c + c_A) + \frac{\sigma p(1-p)}{2} \left(\alpha_D (2d - \alpha_D) - \theta d^2 \right). \tag{B13}$$

Obviously, this is maximized for the *highest* level of damages, $\alpha_D = d$, and this is independent of p^* because α_S is not bounded above. The particular value of p^* does matter for z through the $S(p^*) = 0$ constraint, which implies $z = c + c_A - p^* d$.

Table B2 shows the optimal policy for type p within the set of contracts that generate a litigation threshold p^* (in the table, $p^* = 0.8$). Under risk neutrality ($\sigma = 0$), each type p optimally is allocated the same insurance, $\alpha_L = c_A$ and $\alpha_D = \frac{c}{p^*}$. This is strictly optimal for types that settle, and weakly optimal for types that litigate.

For low $\sigma = 0.1$, each type that settles optimally receives the same level of litigation costs insurance as under risk neutrality. Because of the risk premium, the implied level of damages insurance is higher. And, more importantly, each type that litigates now optimally receives maximum damages coverage $\alpha_D = 5$, and the implied $z = -2$ for all $p > .8$. Hence, even for the low-σ case, there is no contract that maximizes $W - K$ pointwise in p.

For high $\sigma = 1.25$, the lack of pointwise maximization is even more pronounced. There are essentially three different regions of p. For low $p \leq .4875$, the $\alpha_L = c_A$ kink binds. For $p \in (.4875, p^*]$, the optimal level of z is below c_A. It is positive for $p \in (.4875, .6]$, and negative for $p \in (.6, p^*]$. For $p > p^*$, types litigate. Essentially, the ability to sell coverage for settlements slacks the constraint on how big α_D can be, which is valuable when σ and p are high. As in the $\sigma = 0.1$ case, each type that litigates now receives maximum damages coverage $\alpha_D = 5$, and the implied $z = -2$ for all $p > .8$. Note that damages coverage strictly increases for $p \in [.4875, p^*]$.

It turns out that the function in (B11) is generally monotone increasing in p for $p \in [1 - p^*, p^*]$. After some algebra, we find

$$\frac{d\hat{\alpha}_D}{dp} = \frac{1}{\sigma \left(p^* + p - 1\right)^2} \tag{B14}$$

The implication is that there is no peak in $\hat{\alpha}_D$ at $p = .5$. Intuitively, increases in p above .5 increase the direct value of insurance and decrease the risk-premium reduction. But the former effect dominates.

	$\sigma = 0$		$\sigma = 0.1$		$\sigma = 1.25$	
p	z	α_D	z	α_D	z	α_D
0.3	1.00	1.25	1.00	1.38	1.00	2.22
0.4	1.00	1.25	1.00	1.38	1.00	2.22
0.5	1.00	1.25	1.00	1.38	0.85	2.33
0.6	1.00	1.25	1.00	1.38	0.00	3.00
0.7	1.00	1.25	1.00	1.38	-0.46	3.40
0.8	1.00	1.25	1.00	1.38	-0.75	3.67
0.9	1.00	1.25	-2.00	5.00	-2.00	5.00
1.0	1.00	1.25	-2.00	5.00	-2.00	5.00

Table B1—: Contract that maximizes $W - K$ for type p, conditional on $p^* = .8$, $\sigma = 1.25$, $c = c_A = 1$, $\theta = 0.8$, $d = 5$.

To summarize, pointwise maximization obtains under risk neutrality but not under risk aversion. With low levels of σ, litigation costs coverage remains more valuable than damages insurance for types that settle. But not for types that litigate. With higher levels of σ, litigation costs coverage is not necessarily more valuable than damages insurance for types that settle.

Complete Information. — If the insurer is informed about the agent's type, then it will offer a contract that targets that type. This is the same result under risk neutrality.

PROPOSITION 8: *The optimal contract for type p under complete information is any contract such that $p^*(\alpha) = p$.*

PROOF:

We say that contract α targets p if $S(p) = 0$, i.e., the litigation threshold implied by this contract is $p^* = p$.

First, by Lemma 11 we can restrict to contracts such that $p > p^{**}$. If not, we can always reduce α_S to achieve this condition. Second, the value of $(W - K)(p, \alpha)$ for a contract α that targets p is independent of the contract parameters and equal to

$$\pi_T \equiv (1 - \theta)\left[c + c_A + \frac{\sigma d^2}{2} p(1 - p)\right]. \tag{B15}$$

Third, under complete information, the optimal contract for type p does not induce type p to litigate. If type p litigates, it means the insurer offers some contract targets p^* with $p^* < p$. The value of $W - K$ for this contract at type p is then

$$\pi_L = -\theta(c + c_A) + (1 - \theta)RP(p, 0) - RP(p, \alpha_D),$$

while Equation B15 is the value of $W - K$ at p for a contract that targets type p. Note that

$$\begin{aligned} \pi_T - \pi_L &= (1-\theta)(c + c_A) + (1-\theta)RP(p, 0) + \theta(c + c_A) - (1-\theta)RP(p, 0) + RP(p, \alpha_D) \\ &= (c + c_A) + RP(p, \alpha_D) > 0. \end{aligned}$$

Therefore, it is never optimal to induce type p to litigate under complete information, so the optimal contract for type p should be such that $p^{**} < p \leq p^*$.

Fourth, $(W - K)(p, \hat{\alpha})$ is higher for a contract $\hat{\alpha}$ that targets type $\hat{p} > p$, than for a contract α that targets p iff

$$(p - \hat{p})\widehat{\alpha}_D + RP(\hat{p}, \widehat{\alpha}_D) - RP(p, \widehat{\alpha}_D) > 0,$$

where $\widehat{\alpha}_D$ is the contract parameter associated to contract $\hat{p}$. This can be written as

$$(\hat{p} - p)\widehat{\alpha}_D < \frac{\sigma}{2}(d - \widehat{\alpha}_D)^2[\hat{p}(1-\hat{p}) - p(1-p)].$$

Note that $\hat{p}(1-\hat{p}) - p(1-p) = (\hat{p} - p)(1 - (\hat{p} + p))$, and given that $p < \hat{p}$ we have

$$\widehat{\alpha}_D < \frac{\sigma}{2}(d - \widehat{\alpha}_D)^2(1 - (\hat{p} + p))$$

Thus, for any $p > 0.5$, this condition never holds because the RHS is negative. Consider $p \leq 0.5$. The RHS of the inequality is

$$\frac{\sigma}{2}(d - \widehat{\alpha}_D)^2(1 - (\hat{p} + p^*)) \leq \frac{\sigma}{2}(d - \widehat{\alpha}_D)^2(1 - \hat{p}).$$

By the definition of $\hat{p}$ we have:

$$\frac{\sigma}{2}(d - \widehat{\alpha}_D)^2\hat{p}(1-\hat{p}) = \hat{p}\hat{\alpha}_D - (\hat{\alpha}_S + c + c_A - \hat{\alpha}_L) < \hat{p}\hat{\alpha}_D.$$

This implies that

$$\frac{\sigma}{2}(d - \widehat{\alpha}_D)^2(1 - \hat{p}) < \hat{\alpha}_D.$$

Therefore, there is no contract $\hat{\alpha}$ that generates a litigation threshold $\hat{p}$, with $\hat{p} > p$, such that $(W - K)(p, \hat{\alpha}) > W - K(p, \alpha)$, where α is any contract that targets type p. QED

Just like in the case of risk neutrality, Proposition 8 shows that the optimal contract under complete information for type p is any the contracts that targets this type. We have multiple contracts that are optimal, in fact, any contract such that $S(p) = 0$.

Symmetric Information. — Proposition 7 implies that we need to search for contracts where either $\alpha_S = 0$ or $\alpha_D = 0$. The problem solved be the insurer(s) is to find a contract α solution to

$$\max_{\{\alpha=(\alpha_S,\alpha_L,\alpha_D)\}} \int_{c/d}^{1} [W(p,\alpha) - K(p,\alpha)]dF(p)$$

subject to $\alpha_L \cdot \alpha_S = 0$. Table B2 shows a numerical simulation for the optimal contract under symmetric information. The table shows that for low values of risk aversion the optimal contract shares similar characteristics with the optimal contract under risk neutrality: $\alpha_S = 0$, $\alpha_L = c_A$ and $\alpha_D \geq c/p^*$. Note, however, that in general $\alpha_D > c/p^*$. The reason for the insurer to increase α_D when the agent is risk averse—relative to the case where the agent is risk neutral—is to protect the agent against the uncertain litigation outcome, which makes the agent tougher in the negotiation with the third party. However, increasing α_D too much also induces more litigation. To counteract the increase in litigation, the insurer does not cover all the litigation costs; and when not covering litigation costs is not enough to deter the agent from going to litigation, the insurer begins to cover settlements ($\alpha_S > 0$).

Optimal menu of contracts under adverse selection – Monopoly. — We restrict our attention to direct revelation mechanisms: the insurer allocates contract $\alpha(\hat{p}) = (\alpha_S(\hat{p}), \alpha_L(\hat{p}), \alpha_D(\hat{p}))$ at cost $T(\hat{p})$ to an agent reveals a type $\hat{p}$. The insurer chooses functions $\alpha : [c/d, 1] \to [0,\infty) \times [0, c_A] \times [0, d]$ and $T : [c/d, 1] \to [0,\infty)$to solve

$$\max_{\alpha(\cdot),T(\cdot)} \int_{c/d}^{1} [T(p) - K(p,\alpha(p))]f(p)dp$$

subject to truthful revelation (incentive compatibility)

$$U(p) = \max_{\hat{p}\in[c/d,1]} W(p,\alpha(\hat{p})) - T(\hat{p})$$

and participation $U(p) \geq 0$. By the envelop theorem we have $U(p) - U(c/p) = \int_{c/d}^{p} \frac{\partial W(s,\alpha(s))}{\partial p} ds$, which is equivalent to

$$T(p) = W(p,\alpha(p)) - \int_{c/d}^{p} \frac{\partial W(s,\alpha(s))}{\partial p} ds + U(c/p). \tag{B16}$$

$F(p) = 1 - (1-p)^2$

σ	α_S	α_L	α_D	p^*
10^{-8}	0	1	1.01	0.995
0.1	0	1	1.03	0.983
0.5	0	1	1.245	0.947
0.6	0	1	1.315	0.939
0.7	0	1	1.385	0.932
0.8	0	1	1.460	0.925
0.9	0	0.97	1.550	0.920
1	0	0.63	1.880	0.921
1.5	0.23	0	2.750	0.917
2	0.26	0	2.910	0.905

$F(p) = p^{0.5}$

σ	α_S	α_L	α_D	p^*
10^{-8}	0	1	1	1
0.1	0	1	1	1
0.5	0	1	1	1
0.6	0	0.88	1.12	1
0.7	0	0.33	1.68	1
0.8	0.09	0	2.09	1
0.9	0.24	0	2.24	1
1	0.26	0	2.26	1
1.5	0.36	0	2.36	1
2	0.44	0	2.44	1

Table B2—: Optimal contract under symmetric information for different values of σ and different distribution of types. Parameters $c = c_A = 1$, $\theta = 0.8$, $d = 5$.

Replacing in the objective function we obtain

$$\max_{\alpha(\cdot)} \int_{c/d}^{1} \left[W(p,\alpha(p)) - K(p,\alpha(p)) - \int_{c/d}^{p} \frac{\partial W(s,\alpha(s))}{\partial p} ds - U(c/d) \right] f(p)dp$$

It is clear that the monopolist sets $U(c/d) = 0$. Using the standard change of variables we get the following problem:

$$\text{(B17)} \quad \max_{\alpha(\cdot)} \int_{c/d}^{1} \left[W(p,\alpha(p)) - K(p,\alpha(p)) - \frac{\partial W(p,\alpha(p))}{\partial p} \left(\frac{1-F(p)}{f(p)} \right) \right] f(p)dp.$$

In a "standard problem" of mechanism design, incentive compatibility requires an increasing allocation. Our problem differs from the standard case because the allocation is multi-dimensional, although the private information is singledimensional.

LEMMA 12: *Incentive compatibility requires*

$$\text{(B18)}\quad \sum_{i=1}^{N} \frac{\partial W(s,\alpha(t))}{\partial p \partial \alpha_i}\alpha_i'(t) =$$
$$= \frac{\partial W(s,\alpha(t))}{\partial p \partial \alpha_S}\alpha_S'(t) + \frac{\partial W(s,\alpha(t))}{\partial p \partial \alpha_L}\alpha_L'(t) + \frac{\partial W(s,\alpha(t))}{\partial p \partial \alpha_D}\alpha_D'(t) \geq 0, \quad \forall s,t.$$

PROOF:

For any p, $\hat{p}$ for incentive compatibility requires:

$$W(p,\alpha(p)) - T(p) \geq W(p,\alpha(\hat{p})) - T(\hat{p}).$$

Let $\Delta(p,\hat{p}) = W(p,\alpha(p)) - T(p) - [W(p,\alpha(\hat{p})) - T(\hat{p})]$. We have

$$\begin{aligned}
\Delta(p,\hat{p}) &= W(p,\alpha(p)) - T(p) - [W(\hat{p},\alpha(\hat{p})) - T(\hat{p}) + W(p,\alpha(\hat{p})) - W(\hat{p},\alpha(\hat{p}))]. \\
&= W(p,\alpha(p)) - T(p) - [W(\hat{p},\alpha(\hat{p})) - T(\hat{p})] - [W(p,\alpha(\hat{p})) - W(\hat{p},\alpha(\hat{p}))] \\
&= \int_{c/d}^{p} \frac{\partial W(s,\alpha(s))}{\partial p} ds - \int_{c/d}^{\hat{p}} \frac{\partial W(s,\alpha(s))}{\partial p} ds - \underbrace{[W(p,\alpha(\hat{p})) - W(\hat{p},\alpha(\hat{p}))]}_{\int_{\hat{p}}^{p} \frac{\partial W(s,\alpha(\hat{p}))}{\partial p} ds} \\
&= \int_{\hat{p}}^{p} \frac{\partial W(s,\alpha(s))}{\partial p} ds - \int_{\hat{p}}^{p} \frac{\partial W(s,\alpha(\hat{p}))}{\partial p} ds \\
&= \int_{\hat{p}}^{p} \left[\frac{\partial W(s,\alpha(s))}{\partial p} - \frac{\partial W(s,\alpha(\hat{p}))}{\partial p}\right] ds
\end{aligned}$$

Therefore, $\Delta(p,\hat{p}) \geq 0 \Leftrightarrow \int_{\hat{p}}^{p} \left[\frac{\partial W(s,\alpha(s))}{\partial p} - \frac{\partial W(s,\alpha(\hat{p}))}{\partial p}\right] ds \geq 0$. A more compact form of the same IC condition is

$$\int_{\hat{p}}^{p} \left[\frac{\partial W(s,\alpha(s))}{\partial p} - \frac{\partial W(s,\alpha(\hat{p}))}{\partial p}\right] ds \geq 0 \Leftrightarrow \int_{\hat{p}}^{p} \left[\int_{\hat{p}}^{s} \frac{d}{dt}\left[\frac{\partial W(s,\alpha(t))}{\partial p}\right]_{t=u} du\right] ds \geq 0.$$

The term inside the integral must be weakly positive, because the inequality must hold for any $\hat{p}, p$. When $\alpha(t) = (\alpha_1(t), ..., \alpha_N(t))$, we have $\frac{d}{dt}\left[\frac{\partial W(s,\alpha(t))}{\partial p}\right] = \sum_{i=1}^{N} \frac{\partial W(s,\alpha(t))}{\partial p \partial \alpha_i}\alpha_i'(t)$. Therefore, the condition implied by IC (analogous to Lemma

3 in the paper) is

$$\sum_{i=1}^{N} \frac{\partial W(s,\alpha(t))}{\partial p \partial \alpha_i} \alpha_i'(t) =$$
$$= \frac{\partial W(s,\alpha(t))}{\partial p \partial \alpha_S} \alpha_S'(t) + \frac{\partial W(s,\alpha(t))}{\partial p \partial \alpha_L} \alpha_L'(t) + \frac{\partial W(s,\alpha(t))}{\partial p \partial \alpha_D} \alpha_D'(t) \geq 0, \quad \text{for all } s, t.$$

QED

Thus, the monopolist solves problem (B17) subject to condition (B18). Once we find the allocation, we use (B16) to compute the transfer.

Risk Neutrality. To contrast the problem under risk aversion with our baseline results under risk neutrality, consider first $\sigma = 0$. In this case, condition (B18) reduces simply to $\alpha_D(p)$ weakly increasing, and problem (B17) reduces to $\max \int G(p,\alpha) dF(p)$, where

$$G(p,\alpha) = W(p,\alpha) - K(p,\alpha) - \frac{\partial W(p,\alpha)}{\partial p} \left(\frac{1 - F(p)}{f(p)} \right),$$

and $h(p) = \frac{1-F(p)}{f(p)}$. Rather than solving this problem directly, we fix $p \in [c/d, 1]$ and we look for a contract that maximizes $G(p,\alpha)$ pointwise. It can be shown that any such contract satisfies $p \in (p^{**}, p^*]$ or, equivalently,

(B19) $$\theta\alpha_S - (pd - c) < (1-\theta)[c + c_A - \alpha_L - p\alpha_D].$$

and

(B20) $$p\alpha_D \leq \alpha_S + c + c_A - \alpha_L.$$

So the point-wise maximization reduces to maximize $G(p,\alpha)$ subject to (B19) and (B20).

We solve this problem in two steps:

1) Define $\bar{p}$ as the solution to $p = h(p)$. 1. When $p \leq \bar{p}$, the insurer sets $\alpha_D(p) = 0$, α_L as large as possible, and α_S as small as possible, in the region where (B20) and (B19) are satisfied. It is easy so see that contract $\alpha(p) = (0, c_A, 0)$ satisfies all of this. Thus, this is the optimal menu in the region $p \leq \bar{p}$.

2) When $p > \bar{p}$, we solve the following linear optimization problem with linear constraints:
$$\max_{\alpha_S, \alpha_L, \alpha_D} \alpha_L - \alpha_S + (p - h(p))\alpha_D$$

subject to

$$\text{(B21)} \qquad \alpha_L + p\alpha_D \leq c + c_A + \frac{pd - c - \theta\alpha_S}{1-\theta}$$

$$\text{(B22)} \qquad \alpha_L + p\alpha_D \leq c + c_A + \alpha_S$$

It can be shown that the solution to this problem is contract $\alpha = (0, c_A, c/p)$.

The problem with this solution is that it violates the monotonicity constraint of $\alpha_D(p)$. Thus, we need to use ironing. Each types $p > \bar{p}$ will get the same allocation $(0, c_A, c/p^*)$, where p^* is defined in Theorem 2 in the main text. [30]

Risk Aversion. We can follow similar steps in the case of risk aversion. Let $G_R(p,\alpha)$ be the analogous to $G(p,\alpha)$. First, from Proposition 7, we know that we want to push p^{**} as low as possible to maximize $W - K$ pointwise. This argument is still true for $G_R(p,\alpha)$ because in the region where $p \leq p^*$ we have $G_R(p,\alpha) = W(p,\alpha) - K(p,\alpha) - \alpha_S h(p)$. Therefore, a contract α is dominated by $\hat{\alpha}$ if $\hat{p}^{**} < p^{**}$, $\hat{p}^* = p^*$ and $\hat{\alpha}_D = \alpha_D$, in the sense that $G_R(p,\alpha) \leq G_R(p,\hat{\alpha})$ for any p. This implies again that we can look for contracts where $\alpha_S = 0$ or $\alpha_L = 0$.

We proceed to maximize to find a contract $\alpha^*(p)$ such that $G_R(p,\alpha(p)) \geq G_R(p,\alpha)$ for any other contract α. This solution is the optimal menu of contracts when incentive compatibility is not violated. It is not hard to see that for small values of σ, Equation B18 is equivalent to $\alpha_D(p)$ non-decreasing. We can understand qualitatively the shape of the optimal contract by looking at the contract $\alpha^*(p)$. Given that the problem is not analytically tractable, we solve it numerically.

We simulated contracts for the parameters $c = c_A = 1$, $d = 5$, $\theta = 0.8$, $\sigma = 0.076$, and different distributions of types. The contract such that $G_R(p,\alpha(p)) \geq G_R(p,\alpha)$ features no settlement coverage and full litigation cost coverage, i.e., $\alpha_S(p) = 0$ and $\alpha_L(p) = c_A$ for all p. However, the level of damages coverage changes with the distribution of types. Figure B5 shows the value of $\alpha_D(p)$ in the simulation of three scenarios.

Under risk-neutrality the highest type that is excluded is $\bar{p}$, which is the solution to $p = \frac{1-F(p)}{f(p)}$. When $F(p) = p^a$, this threshold corresponds to $\bar{p} = \left(\frac{1}{1+a}\right)^{\frac{1}{a}}$. In Figure B5, from top to bottom we have: $\bar{p} = 0.37$, $\bar{p} = 0.5$, and $\bar{p} = 0.7$. The first observation is that risk aversion *reduces* the set of types that are excluded from damages insurance, i.e., $\bar{p}_{\text{risk averse}} \leq \bar{p}_{\text{risk neutral}}$.

Second, when types are not excluded, $\alpha_D(p)$ increases in a small region and decreases thereafter. Because incentive compatibility requires $\alpha_D(p)$ to be non-decreasing, there will be some threshold type p_{Iron}, with $p_{\text{Iron}} \geq \bar{p}_{\text{risk aversion}}$ such that any type $p \geq p_{\text{Iron}}$ receives the same damages coverage. Under risk neutrality, $p_{\text{Iron}} = \bar{p}_{\text{risk neutral}}$, so the optimal menu entailed only two contracts. Under

[30] This is precesily the solution to the ironing problem.

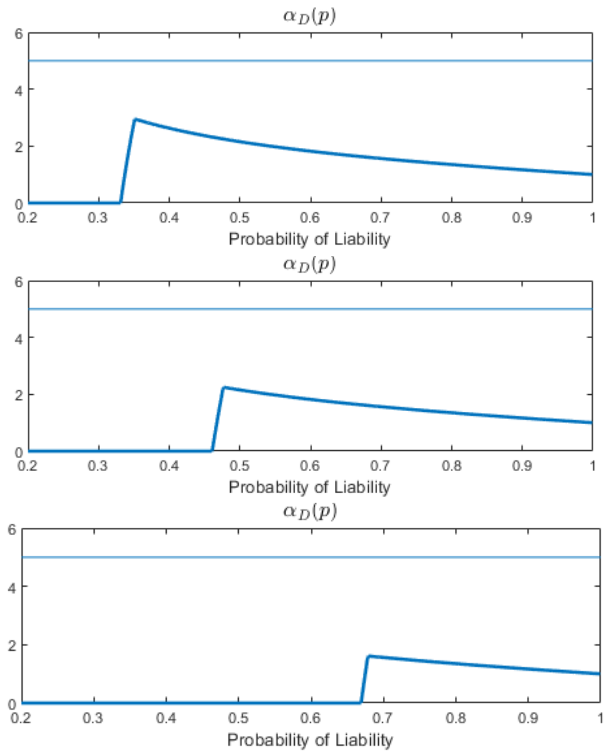

Figure B5. : Damages coverage of contracts such that $G_R(p, \alpha(p)) \geq G_R(p, \alpha)$ for any other contract α, for different distributions of types. The distribution of types in the plot at the top is $F(p) = p^{0.1}$, in the plot at middle is $F(p) = p$, and at the bottom is $F(p) = p^5$.

risk aversion, however, we have more than two contracts: Low types do not get damages coverage; types that are a little bit higher than the highest excluded type buy small amount of damages coverage, specific to their types; high types, receive the same damages coverage, and it is not specific to their types.

Thus, qualitatively, the optimal contract under risk aversion features:

1) A contract that does not cover damages for types $p \leq \bar{p}_{\text{risk averse}}$.

2) A type-dependent contract that cover both damages and litigation costs for $p \in [\bar{p}_{\text{risk averse}}, p_{\text{Iron}}]$.

3) A type-independent contract that cover both damages and litigation costs for $p > p_{\text{Iron}}$.

The main qualitative differences between this contract and the case of risk neutrality is that there are fewer types excluded from damages insurance, and that the optimal menu features more than just two contracts. Finally, for larger levels of risk aversion, as in the case of symmetric information, the optimal contract may also cover settlements. This case is more complicated because the agent's willingness to pay may not be supermodular (see Figure B2).

Perfect Competition Under Adverse Selection with Risk Aversion. — Proofs of the main results under risk neutrality rely on monotonicity of W in p and supermodularity of W in p and α_D, as well as the unprofitability of insurance sold to types that litigate. The assumption $\sigma < \frac{1}{d}$ ensures the monotonicity and supermodularity conditions, but does not guarantee unprofitability of insurance sold to types that litigate. For the latter, we need additional assumptions. For $p > p^*$, we can write

$$W - K = \left(\frac{\sigma p(1-p)}{2}\right)\left[\alpha_D(2d - \alpha_D) - \theta d^2\right] - \theta(c + c_A).$$

This is never positive for all p, and may be negative for all p. Note that $W - K$ is increasing in α_D, so imposing that $W - K$ is negative for all $p > p^*$ and $\alpha_D = d$ provides a sufficient condition for the insurer to lose money when the agent litigates. This condition corresponds to

$$\sigma < \frac{8\theta(c + c_A)}{d^2(1-\theta)}$$

Note that this condition does not imply, and is not implied by, the assumption of $\sigma < \frac{1}{d}$. The simplest way to see this is to note that the constraint is impossible to meet for $\theta = 0$ and is always met for $\theta = 1$.

Under the assumption $\sigma < \min\left\{\frac{1}{d}, \frac{8\theta(c+c_A)}{d^2(1-\theta)}\right\}$ our proofs (Lemma 1, Lemma 2, and Theorem 2 in the main text) go through essentially unchanged.

Lemma 1. This lemma states that no pooling equilibrium exists with a contract that yields $p^* < 1$. In the risk-neutral analysis, we can restrict attention to the contract $\alpha_L = c_A$ and $\alpha_D = \frac{c}{p^*}$. The result holds because there is always a profitable alternative contract. Because $p^* < 1$, the insurer incurs some costs on the types that litigate $p > p^*$. Hence, for this contract to earn zero profit for the insurer, it must be sold at a positive price. Let the break-even price, if all types buy this contract, equal $\tilde{P}$. Because of the supermodularity of W in p and α_D, there exists an alternative contract with slightly lower α_D sold at a price slightly below $\tilde{P}$ that attracts just types that settle. Because the insurer would incur no costs from these types, the contract is profitable.

In the case of risk aversion, the analog to Lemma 1 holds easily if $\sigma < \frac{1}{d}$ so that the supermodularity of W in p and α_D continues to hold. While the basic logic is the same, a couple of things are different. First, we can still focus on the $p^* < 1$ condition implied by a pooling contract, but note that we cannot restrict attention to the contract $\alpha_L = c_A$ and $\alpha_D = \frac{c}{p^*}$. The reason is that we lose pointwise maximization of W—the contract $\alpha_L = c_A$ and $\alpha_D = \frac{c}{p^*}$ maximizes W conditional on p^* for types that settle, but $\alpha_D = d$ insurance maximizes W conditional on p^* for types that litigate. So with that change, it is necessary to consider any contract that yields p^*. But note that it remains true that the insurer incurs no costs on types that settle and incurs positive costs on types that litigate. Hence, for any α_L and α_D that yield cutoff p^*, the insurer must charge a positive price to break even. Because there is cross-subsidization, this price is less than the average cost from the types that litigate. And because of the supermodularity of W in p and α_D, it remains possible to skim off the types that settle profitably.

Lemma 2. This lemma states that no separating equilibrium exists at all. Restrict attention to low-σ, where W is monotone increasing in p and $\alpha_L = c_A$ contracts dominate for types that settle. Also, consider the two-type case, $p \in \{p_L, p_H\}$.

To start, it is still true that there is no equilibrium with two contracts where the prices are both zero and all types settle, because types would not sort in that case. Under risk neutrality, the intuition is two-fold. First, because of pointwise maximization, any contract without $\alpha_L = c_A$ is dominated and cannot be part of an equilibrium. Second, because of single crossing, whichever $\alpha_L = c_A$ contract is preferred by one type will also be preferred by the other. With $\sigma < \frac{1}{d}$, pointwise maximization holds for types that settle and single-crossing holds generally. So with low σ, there is no separating equilibrium with two contracts at price 0 sold to types that settle.

So the remaining possibility includes at least one contract where types litigate. Let one such contract be α^{Lit}, and let it be sold to type p for $K(p, \alpha^{Lit}) > 0$. From Proposition 2, we know that $W - K$ is maximized for insurance α^I such that $p^*(\alpha^I) = p$. Hence, type p will always strictly prefer to pay ε for α^I insurance as long as ε is sufficiently close to 0. And this is profitable if it attracts only type p agents or if it attracts other less-costly agents. This rules out any candidate equilibrium where type p_H litigates, because α^I that is such that $p^*(\alpha^I) = p_H$ surely attracts type p_H agents, and any other agents that purchase α^I are of type p_L and also would not litigate. So now the remaining possibility includes a contract where the type p_L agent litigates, and another contract preferred by agent p_H. By our standard cream-skimming argument (which holds for $\sigma < \frac{1}{d}$), it is possible to alter the components of insurance and the price in such a way that attracts the p_L types, induces settlement by them, and does not attract the p_H agents.

With insurance possibly being profitable for types that litigate, the change is that such an insurance contract must be sold for $K(p, \alpha)$. But other than that, nothing really changes.

Now consider the continuous-density case. It remains true that two contracts that induce only settlement cannot form a separating equilibrium. It also remains true that any contract that induces some litigation is subject to cream-skimming.

What would change is the cream-skimming need not necessarily just induce settlement. Candidate contracts that induce litigation must charge break-even average prices. That means lower-p types may litigate but be less costly. They can be cream-skimmed. Any deviation that attracts lower-p types attracts still-cheaper (on average) types, so altering the less-generous coverage down to cream skim cannot be made unprofitable by inducing selection away from the more-generous coverage. We can use the old proof until we reach the case where $\mu_F(D_1(S)) = 0$. We cannot rule out such contracts with unprofitability. However, the same cream-skimming argument that the proof of Lemma 2 uses to rule out contracts that attract both types that litigate and types that settle can be used in the RA case. The reason is that $K(p, \alpha)$ is increasing in p. So any contract that attracts some measure of types that litigate, and no measure of types that settle, must sell at an average cost. If the types that buy the contract are $\underline{p}$ and $\overline{p}$, then the price would need to be $\int_{\underline{p}}^{\overline{p}} K(p, \alpha) df(p)$. But then the low-$p$ types can be sold less generous insurance for a lower price and be cream-skimmed away. For example, the $\underline{p}$ type could be sold perfect insurance for a positive price.

Theorem 2 follows from Lemmas 1 and 2, but the exact condition that guarantees that there is no profitable deviation from p^* will now change. Thus, the qualitative result is the same, but the precise conditions to sustain an equilibrium are now different.

B2. Control over the Settlement Decision

In this extension we consider the optimal assignment of control over the settlement process. In our main model we assumed that in general the agent decides whether to settle or litigate and negotiates the settlement, which is motivated by the features of actual liability insurance contracts that we observe in some industries, such as in patent litigation. In this framework the agent benefits from the ability to negotiate a better settlement with the third party, but the option to litigate gives rise to an ex post moral hazard problem. Instead, the agent and insurer may in some settings prefer an insurance contract whereby the insurer negotiates the settlement and controls the decision whether to settle or litigate, to avoid the problem of ex post moral hazard.

To study this problem, analogously to our main model, suppose that the insurer contracts with the agent, then observes p and negotiates a settlement with the third party, under the threat of litigation. The insurer offers a contract $\alpha_D \in [0, d]$ to cover the possible damages that the agent may have to pay if found liable, as in the main body of the paper. Since the insurer controls the litigation process, it pays the settlement transfer and the litigation cost c_A; alternatively, this can also be modeled analogously as the agent paying the litigation cost, and the insurance contract covering (some part of) the litigation cost.

We assume, as in the literature on litigation insurance (Meurer (1992)), that the insurer must negotiate "in good faith," a restriction which in practice is interpreted to mean that the insurer must negotiate a settlement which maximizes I and A's joint payoff. Equivalently, this can also be seen as a requirement that the insurer must leave the agent no-worse-off than if it had not bought insurance. Under both of these interpretations, since α_D is a transfer between the agent and the insurer, the parties are indifferent over all α_D. For generality, we also allow for the possibility that the insurer is better than the agent at negotiating a settlement: suppose the insurer has bargaining skill θ_I, rather than θ.

First, notice that this model of settlement is in fact analogous to our baseline model with no insurance: one party (in this case the insurer) negotiates a settlement to maximize I and A's joint payoff, which is equivalent to a model without insurance where the agent negotiates a settlement to maximize its own payoff, though possibly with different bargaining skill.

In this extension, the agent's payoff without insurance is

$$\bar{V} = -c_A - pd + \theta(c + c_A).$$

The agent's payoff with insurance (where the insurer bargains) is

$$V = -c_A - pd + \theta_I(c + c_A).$$

So the agent and insurer's net joint surplus from insurance (relative to no insurance) is

$$W = (\theta_I - \theta)(c_A + c).$$

It is clear that such insurance cannot be profitable if $\theta > \theta_I$, so we will focus on the case where $\theta_I \geq \theta$. Also, notice that this surplus is independent of p: all types value this kind of insurance contract by the same amount. With a monopolist insurer, the optimal price of this insurance is W, whereas with competition it is 0. In both settings the bargaining surplus is always positive, so there is never any litigation in equilibrium. Moreover, because this surplus is independent of p, the joint surplus from such insurance is the same across different market and information structures. Whether p is the agent's private information or not at the time of contracting with the insurer is in fact irrelevant in this case—both parties anticipate that at the time of bargaining, I knows p and bargains to maximize A and I's joint payoff (which is analogous to our baseline model where A bargains without insurance). A receives no information rents, since the net joint surplus from this insurance contract is independent of p.

For each market structure and information structure, we can now compare the insurer's overall profit in our main model against its profit from selling insurer-controlled insurance. We mainly focus on the cases where setting $p^* = 1$ is optimal, although analogous comparisons and intuitions emerge in all cases, where $p^* < 1$ may be optimal.

Monopoly and competition under symmetric informationTo begin, consider the setting with symmetric information. We show that under both monopoly and perfect competition, there exists a threshold bargaining parameter $\widetilde{\theta}_I$ such that it is optimal to assign the right to settle to the agent for any $\theta_I \leq \widetilde{\theta}_I$, and to assign it to the insurer when $\theta_I > \widetilde{\theta}_I$. Moreover, for $\theta_I = \theta$, agent-controlled settlement is optimal, confirming that the results in our main model are robust.

From Proposition 4, we can compare a monopolist insurer's profit from agent-controlled contracts, as well as a perfectly competitive insurer's profit from agent-controlled contracts–both of these relate to the net joint surplus of insurance. In the case where $p^* = 1$ is optimal, we have

$$P_M(1) = JS_C(1) = E_p(W(p,1)) = \int_{\frac{c}{d}}^{1} (1-\theta)(c_A + cp)dF(p).$$

We compare this to the I's profit and the net joint surplus from insurer-controlled contracts:

$$\tilde{P}_M = W_C \equiv \int_{\frac{c}{d}}^{1} (\theta_I - \theta)(c_A + c)dF(p).$$

With some rearranging, we have that

$$\widetilde{P}_M \geq P_M(1) = JS_C(1) \quad \Leftrightarrow \quad \int_{\frac{c}{d}}^{1} (1-\theta)c(1-p)dF(p) \geq \int_{\frac{c}{d}}^{1} (1-\theta_I)(c_A + c)dF(p)$$

Notice that for θ_I sufficiently high (e.g. $\theta_I = 1$), the right-hand side is 0 and the left-hand side is positive (independent of θ_I), so insurer-controlled insurance is optimal. On the other hand, for θ_I low enough (e.g. $\theta_I = \theta$), we have $c(1-p) < c + c_A$, so the inequality is reversed, hence agent-controlled insurance is optimal. There exists a unique threshold $\widetilde{\theta}_I$ given by

$$\int_{\frac{c}{d}}^{1} (1-\theta)c(1-p)dF(p) = \int_{\frac{c}{d}}^{1} (1-\widetilde{\theta}_I)(c_A + c)dF(p),$$

such that for $\theta_I > \widetilde{\theta}_I$, insurer-controlled contracts are optimal, whereas for $\theta_I \leq \widetilde{\theta}_I$, agent-controlled contracts are optimal. Moreover, when the agent and insurer are equally good at bargaining, i.e. $\theta = \theta_I$, agent-controlled insurance contracts are optimal. Our equilibrium results from the main model continue to hold when θ_I and θ are similar enough.

Competition under private informationNow consider a competitive market where the agent is privately informed about its type. To see whether agent-controlled or insurer-controlled insurance will be sustained as an equilibrium, we must again compare the insurer and agent's net joint surplus from each type of contract. From Lemma 2 and Theorem 2, the only possible agent-controlled equilibrium contract is a pooling contract with $p^* = 1$. I and A's joint surplus is

$$JS_C(1) \equiv E_p(W(p,1)) = \int_{\frac{c}{d}}^{1} (1-\theta)(c_A + cp)dF(p).$$

With an insurer-controlled insurance contract, I and A's joint surplus is

$$\widetilde{P}_M = W_C = \int_{\frac{c}{d}}^{1} (\theta_I - \theta)(c_A + c)dF(p).$$

Both of these are identical to the case of symmetric information, and thus our conclusions coincide: for $\theta_I > \widetilde{\theta}_I$, insurer-controlled contracts are offered in equilibrium, whereas for $\theta_I \leq \widetilde{\theta}_I$, agent-controlled contracts are offered in equilibrium. When the agent and insurer have equal (or similar enough) bargaining skill, our equilibrium results from the main model continue to hold.

Monopoly under private informationFinally, consider the monopoly setting with private information. In the case where $p^* = 1$ is optimal, from Theorem 1, the insurer offers a menu of two contracts: a contract with $\alpha_D = 0$ sold at price $(1-\theta)c_A$, for types $p \leq \bar{p}$, and one with $\alpha_D = \frac{c}{p^*}$ sold at price $(1-\theta)(c_A + c\frac{\bar{p}}{p^*})$, for types $p > \bar{p}$. The insurer's total revenue here is

$$R_M(1) \equiv \int_{\frac{c}{d}}^{\bar{p}} (1-\theta)c_A dF(p) + \int_{\bar{p}}^{1} (1-\theta)(c_A + c\frac{\bar{p}}{p^*})dF(p)$$

We compare this against the insurer's profit in this extension:

$$\widetilde{P}_M = \int_{\frac{c}{d}}^{1} (\theta_I - \theta)(c_A + c)dF(p).$$

So we have

$$\begin{aligned}\widetilde{P}_M \geq R_M(1) \Leftrightarrow & \int_{\frac{c}{d}}^{\bar{p}} (\theta_I - \theta)c dF(p) + \int_{\bar{p}}^{1} (\theta_I - \theta)c(1 - \frac{\bar{p}}{p^*})dF(p) \geq \\ & \geq \int_{\frac{c}{d}}^{\bar{p}} (1-\theta_I)c_A dF(p) + \int_{\bar{p}}^{1} (1-\theta_I)(c_A + c\frac{\bar{p}}{p^*})dF(p)\end{aligned}$$

As before, for θ_I sufficiently high (e.g. $\theta_I = 1$), the right-hand side is 0 and the left-hand side is positive, so insurer-controlled insurance is optimal. On the other hand, for θ_I low enough (e.g. $\theta_I = \theta$), the left-hand side is 0 while the right-hand side is positive, so the inequality is reversed, hence agent-controlled insurance is optimal. There exists a threshold $\bar{\theta}_I$ given by the expression

$$\int_{\frac{c}{d}}^{\bar{p}}(\bar{\theta}_I-\theta)cdF(p)+\int_{\bar{p}}^{1}(\bar{\theta}_I-\theta)c(1-\frac{\bar{p}}{p^*})dF(p)=\int_{\frac{c}{d}}^{\bar{p}}(1-\bar{\theta}_I)c_AdF(p)+\int_{\bar{p}}^{1}(1-\bar{\theta}_I)(c_A+c\frac{\bar{p}}{p^*})dF(p)$$

such that for $\theta_I > \bar{\theta}_I$, insurer-controlled contracts are optimal, whereas for $\theta_I \leq \bar{\theta}_I$, agent-controlled contracts are optimal. As in the setting with symmetric information, the results from our main model continue to hold as long as θ_I and θ are similar enough.

B3. Bargaining under Incomplete Information

When parties bargain under incomplete information, generically litigation arises in equilibrium. In the main text, when parties bargain under complete information, litigation never occurs in equilibrium. This is a well-known difference between these two models of bargaining, independent of the issue of insurance. In our baseline setting, insurance has the potential of inducing litigation in an environment that otherwise would never feature litigation.

For illustrative purposes, we consider the two-type case: A fraction λ of agents are type p_H and a fraction $1-\lambda$ are type p_L, with $0 \leq p_L < p_H \leq 1$. Assume the agent is protected by the liability insurance policy $\alpha = (\alpha_S, \alpha_L, \alpha_D)$. Following the literature, we assume the uninformed party makes a take-it-or-leave it offer to the informed party. Consider the following two offers:

$$\begin{aligned} S^L &= \alpha_S + (c_A - \alpha_L) + p^L(d - \alpha_D) \\ S^H &= \alpha_S + (c_A - \alpha_L) + p^H(d - \alpha_D) \end{aligned}$$

The low-risk type is indifferent between paying S^L and litigating, while the high-risk agent is strictly better off by accepting S^L. The settlement offer S^H leaves the high-risk type indifferent between accepting the offer or litigation but low-risk type rejects the offer and litigate. The third party's outside option is $E[p]d - c$ because it can always make a 'bad faith' settlement offer ($S^\infty = +\infty$) that forces both types to litigate. We have three cases:

1) TP makes offer S^L, both types of agents settle, and TP's payoff is

$$\pi_{TP}(S^L) = \alpha_S + (c_A - \alpha_L) + p^L(d - \alpha_D)$$

2) TP makes offer S^H, high-risk types settle but low-risk type litigate, and TP's payoff is

$$\pi_{TP}(S^H) = \lambda[\alpha_S + (c_A - \alpha_L) + p^H(d - \alpha_D)] + (1-\lambda)[p^Ld - c]$$

3) TP forces litigation by offering S^{∞} and TP's payoff in this case is

$$\pi_{TP}(S^{\infty}) = (\lambda p^H + (1-\lambda)p^L)d - c$$

It can be shown that

$$\begin{aligned}
\pi_{TP}(S^H) &= \pi_{TP}(S^{\infty}) + \underbrace{\lambda(c + c_A - \alpha_L - p^H\alpha_D + \alpha_S)}_{\equiv Y(\alpha)} \\
\pi_{TP}(S^H) &= \pi_{TP}(S^L) + \underbrace{\lambda(p^H - p^L)(d - \alpha_D) - (1-\lambda)[c + c_A - \alpha_L - p^L\alpha_D + \alpha_S]}_{\equiv Z(\alpha)} \\
\pi_{TP}(S^L) &= \pi_{TP}(S^{\infty}) + \underbrace{Y(\alpha) - Z(\alpha)}_{\equiv W(\alpha)}
\end{aligned}$$

The optimal offer, is determined by $Y(\alpha)$, $Z(\alpha)$, and $W(\alpha)$, because $S^H \succeq S^L$ is equivalent to $Y(\alpha) \geq 0$; $S^H \succeq S^{\infty}$ is equivalent to $Z(\alpha) \geq 0$; and $S^L \succeq S^{\infty}$ is equivalent to $W(\alpha) \geq 0$. In fact, the optimal offer is:

1) $S^L(\alpha)$ if and only if $\alpha \in C^L = \{\alpha \ : \ Z(\alpha) \leq 0 \text{ and } Y(\alpha) \geq Z(\alpha)\}$.

2) $S^H(\alpha)$ if and only if $\alpha \in C^H = \{\alpha \ : \ Z(\alpha) \geq 0 \text{ and } Y(\alpha) \geq 0\}$.

3) S^{∞} if and only if $\alpha \in C^{\infty} = \{\alpha \ : \ Y(\alpha) \leq 0 \text{ and } Y(\alpha) \leq Z(\alpha)\}$.

Without insurance (by setting $\alpha_S = \alpha_L = \alpha_D = 0$) we have $Y(0) = \lambda(c + c_A) > 0$ and $Z(0) = \lambda(p^H - p^L)d - (1-\lambda)(c + c_A)$, so it is optimal to offer $S^L(0)$ if $Z(0) < 0$, or to offer S^H if $Z(0) > 0$. Thus, it is possible to obtain litigation in equilibrium without insurance.

We now derive the agent's willingness to pay for insurance policy α. We first consider the case $Z(0) < 0$ or, equivalently, $\left(\frac{\lambda}{1-\lambda}\right)(p^H - p^L)d < c + c_A$. In this case, the optimal settlement offer for an agent without insurance is $S^L(0)$. Hence, every agent gets the same outside option from not buying insurance, which is to pay $S^L(0)$ as a settlement fee.

The willingness to pay of an agent of type p^L pay for insurance contract α is:

- $S^L(0) - (S^L(\alpha) - \alpha_S) = \alpha_L + p^L\alpha_D$ if $\alpha \in C^L$.
- $S^L(0) - [(c_A - \alpha_L) + p^L(d - \alpha_D)] = \alpha_L + p^L\alpha_D$ if $\alpha \notin C^L$.

This is because the low-type agent only accepts $S^L(\alpha)$ and rejects (and goes to litigation) with any other offer.

The willingness to pay of an agent of type p^H pay for insurance contract α is:

- $S^L(0) - (S^L(\alpha) - \alpha_S) = \alpha_L + p^L\alpha_D$ if $\alpha \in C^L$.
- $S^L(0) - (S^H(\alpha) - \alpha_S) = \alpha_L + p^H\alpha_D - d(p^H - p^L)$ if $\alpha \in C^H$.

- $S^L(0) - [(c_A - \alpha_L) + p^H(d - \alpha_D)] = \alpha_L + p^H \alpha_D - d(p^H - p^L)$ otherwise.

Conditional on having bought insurance policy α, the high-type agent accepts both offers $S^L(\alpha)$ and $S^H(\alpha)$, and rejects S^∞.

Complete Information between the insurer and the agent

If the agent's type is p^L, the insurer would offer a policy $\alpha \in C^L$ such that maximizes $\alpha_L + p^L\alpha_D - \alpha_S$. This is because under this policy the agent settles so the insurer has to pay α_S, but the insurer does not pay $\alpha_L + p^L\alpha_D$ (this is the gain from an improved bargaining position). In contrast, if the agent litigates, the insurer pays all the cost so the net surplus between the insurer and the agent is zero.

Thus, the optimal contract solves

$$\max_{(\alpha_S,\alpha_L,\alpha_D)} \alpha_L + p^L\alpha_D - \alpha_S$$

subject to

$$\begin{aligned} (1-\lambda)[\alpha_L - \alpha_S] + \alpha_D(p^L - \lambda p^H) &\leq (1-\lambda)(c + c_A) - \lambda d(p^H - p^L) \\ \alpha_L - \alpha_S + \alpha_D &\leq c + c_A - \lambda d(p^H - p^L) \end{aligned}$$

We can treat $\alpha_L - \alpha_S$ as a new variable $x \in (-\infty, c_A]$ and transform the problem to:

$$\max_{(x,\alpha_D)} x + p^L\alpha_D$$

subject to

$$\begin{aligned} (1-\lambda)x + \alpha_D(p^L - \lambda p^H) &\leq (1-\lambda)(c + c_A) - \lambda d(p^H - p^L) \\ x + \alpha_D &\leq c + c_A - \lambda d(p^H - p^L) \end{aligned}$$

The solution to this problem is easy to compute, although it depends on the parameters of the problem. For example if $p^H < \lambda(1-p^H)$, the solution is $\alpha_D = 0$ and $x^* = c + c_A - \left(\frac{\lambda}{1-\lambda}\right)(p^H - p^L)d$, which is positive by the assumption $Z(0) < 0$. In other words, under some assumptions on the parameters, the optimal contract under complete information for an agent of type p^L is to set $\alpha_L - \alpha_S = x^*$.

Similar to the baseline case, under complete information the insurer sells a contract that never induces litigation. In contrast to the baseline case, in this setting insurance improves welfare by reducing the amount of litigation in equilibrium. Also, as in the baseline case, there is multiplicity in the optimal contract.

Incomplete and Symmetric Information

Under incomplete and symmetric information between the agent and the insurer, the insurer would induce litigation by offering $\alpha \in C^{\infty}$, incurring in losses, so this cannot be optimal. When $\alpha \in C^H$, the insurer induces litigation for the low-type. In this case, The net surplus between the low-type agent and the insurer is zero, whereas the net surplus between the high-type agent and the insurer is $\alpha_L - \alpha_S + p^H \alpha_D - d(p^H - p^L)$. Finally, when $\alpha \in C^L$, the net surplus between both types of agents and the insurer is $\alpha_L - \alpha_S + p^L \alpha_D$.

As in the case of complete information, we can see that only the difference between α_L and α_S is relevant for the insurer's problem. Thus, the optimal contract under incomplete and symmetric information solves:

$$\max_{\alpha=(\alpha_S,\alpha_L,\alpha_D)} [x + p^L \alpha_D] 1(\alpha \in C^L) + \lambda[x + p^H \alpha_D - d(p^H - p^L)] 1(\alpha \in C^H)$$

where $x = \alpha_L - \alpha_S$. The solution to this optimization problem depends on the parameters of the problem. In contrast to the baseline case in the main text, where $\alpha_S = 0$ and $\alpha_L = c_A$, when parties bargain under incomplete information we obtain multiple optimal contracts, because α_S and α_L are "perfect substitutes."

In this setting, the insurer may not want to minimize the difference $\alpha_S - \alpha_L$, which in the main text leads to set $\alpha_S = 0$ and $\alpha_L = c_A$, because the insurer chooses a contract to extract rent from the third party, by inducing it to make a low offer that is accepted by the agent, thus raising the agent's willingness to pay for insurance.

B4. Alternative Equilibrium Concepts

One of the criticisms of the results in Rothschild and Stiglitz (1976) is that, under some conditions, equilibrium fails to exist. In our setting, when the mass of high-risk types is sufficiently low equilibrium may also fail to exist. The literature following Rothschild and Stiglitz (1976) has come up with alternative equilibrium notions under which an equilibrium exists. Riley (1979) proposes a reactive equilibrium notion, which exists in the Rothschild and Stiglitz (1976) setting. In our setting, in contrast, this equilibrium may not exist. First, if there exists a Nash equilibrium set of contracts, it is a Riley equilibrium. Consider the case where there is no Nash equilibrium. Then, there exists a contract $p_d^* < 1$ that is a profitable deviation from $p^* = 1$. But then, this new contract can be cream-skimmed. And the contract that cream-skims p_d^* can never be "safe," i.e., it can also be cream-skimmed by another contract. Therefore in our setting Riley and Nash coincide, meaning that a Riley equilibrium may not exist.

Azevedo and Gottlieb (2017) propose an equilibrium refinement of the free-entry equilibrium (there is multiplicity of free-entry equilibria in the Rothschild and Stiglitz (1976) setting). This equilibrium notion requires continuity in the costs. In our setting, however, the insurer costs are not continuous as a function of the agent's type, so we cannot directly use this equilibrium notion. Another alternative is to use mixed strategies. Farinha Luz (2017) characterizes a mixed-strategy equilibrium in the Rothschild and Stiglitz (1976) setting. The main assumption is that there is a finite number of firms, and most of the analysis is for the case of two firms.[31] We believe it may be possible to construct a mixed-strategy equilibrium in our setting, but this is beyond the scope of this paper, as we focus on pure strategies, which are more natural to interpret in our contracting environment.

Wilson (1977) proposes the notion of *anticipatory equilibrium.* A set of policies is a *Wilson* equilibrium if each policy earns nonnegative profits and there is no other set of policies which earn positive profits in the aggregate and nonnegative profits individually, after the unprofitable policies in the original set have been withdrawn. When a Nash equilibriun exists, this is also a Wilson equilibrium, because there are no profitable deviations. When a Nash equilibrium does not exists, a Wilson equilibrium may exist, and consists of pooling both types into a single contract: the contract that generates zero joint profits, and it is the most preferred contract for the low-risk type, i.e., the contract that targets the low type $p^* = p_L$, sold at price

$$Q = \lambda \left(c_A + c \frac{p_H}{p_L} \right).$$

B5. Characterization of the Symmetric Information Contract

To help characterize the solution to this problem, we consider smooth distributions for which the density may equal zero only at the boundaries of the support.[32]

ASSUMPTION 3: *Let $F(\cdot)$ be twice-continuously differentiable, with probability density $f(p) > 0$ for all $p \in (0, 1)$.*

Consider the derivative of the objective function in equation (10):
(B23)

$$\Psi'_{SI}(p^*) = \underbrace{(1-\theta)(c+c_A)f(p^*)}_{\text{marginal type}} - \underbrace{(1-\theta)\frac{c}{(p^*)^2}\int_{c/d}^{p^*} p dF(p)}_{\text{infra-marginal types}} + \underbrace{\theta(c+c_A)f(p^*)}_{\text{marginal type}}.$$

Increasing coverage has an effect on the marginal type and infra-marginal types. First, the marginal type p^* gets perfect insurance and extracts the full bargaining surplus $(c+c_A)$ from the third party. The marginal gain of increasing p^* is shown in equation (B23) in two different places: a gain of $(1-\theta)(c+c_A)$ from the leverage of types marginally below type p^*; and a gain from avoiding a loss of

[31]The paper also discusses the case of the number of firms going to infinity.
[32]These formulas also apply to the case where p has a discrete distribution with binary support, which is available upon request.

$\theta(c + c_A)$ in bargaining surplus for types marginally above p^* who would settle instead of going to court. Second, the infra-marginal types $p < p^*$ receive a level of insurance further away from their perfect level, inducing a loss in the joint surplus of the insurer and agent.

The optimal contract either precludes litigation entirely ($p^* = 1$) or balances the gain of the marginal type versus the average loss of the infra-marginal types. To further understand when it is optimal to offer a contract that induces litigation, we define the *elasticity of density*.

DEFINITION 2: *For distributions satisfying Assumption 3, the* **elasticity of density** *is*

$$\eta(p) = \frac{pf'(p)}{f(p)}.$$

It is easy to see that the following identity holds

$$\Psi''_{SI}(p)p^2 + 2\Psi'_{SI}(p)p = \frac{pf(p)}{c_A + c}\left[\eta(p) + 1 + \frac{c_A + \theta c}{c_A + c}\right].$$

Thus, if p^* is an interior solution of problem (10), the first and second order conditions, $\Psi'_{SI}(p^*) = 0$ and $\Psi''_{SI}(p^*) < 0$, respectively, imply

$$\eta(p^*) < -\left(1 + \frac{c_A + \theta c}{c_A + c}\right).$$

The elasticity of density provides us with a sufficient condition for a unique solution of problem (10).

LEMMA 13: *Under Assumption 3, the solution to problem (10) is unique and equal to $p^* = 1$ if for all $p \in \left[\frac{c}{d}, 1\right]$ we have*

$$\eta(p) \geq -\left(1 + \frac{c_A + \theta c}{c_A + c}\right).$$

For any convex distribution $F(\cdot)$, $\eta(p) \geq 0$ for all p. By Lemma 13, the unique optimal contract precludes litigation by setting $p^* = 1$. When the density function is increasing, the marginal gain dominates the infra-marginal loss, i.e., it is suboptimal to sell insurance generous enough to induce litigation by risky types. Intuitively, it is also optimal to preclude litigation when $F(p)$ is *mildly* concave.

There are many distributions where the solution to (10) induces litigation for some types. In such cases, $\eta(p)$ allows us to provide a sufficient condition for uniqueness.

LEMMA 14: *Under Assumption 3, let* $p^* < 1$ *be such that* $\Psi'_{SI}(p^*) = 0$ *and* $\Psi''_{SI}(p^*) < 0$. *Then,* p^* *is the unique interior solution if*

$$\eta(p) \leq -\left(1 + \frac{c_A + \theta c}{c + c_A}\right), \quad \textit{for all } p \in [p^*, 1]$$

When $p^* < 1$, the insurer targets a particular type p^* with perfect insurance and induces litigation by types $p > p^*$ and imperfect insurance for types $p < p^*$. In targeting, the insurer seeks a sufficiently low level of relative litigation risk associated with type p^*.[33] When the elasticity of density falls with p and the density of a high-risk type is low,[34] intuitively, the insurer prefers to induce some litigation. We have the following result.

COROLLARY 1: *If* $\eta(p)$ *is non-increasing and* $f(1) < \frac{(1-\theta)c}{c_A + c} \int_{c/d}^{1} p dF(p)$, *there exists a unique* $p^* \in \left(\frac{c}{d}, 1\right)$ *that solves (10).*

PROOF:

When $\Psi'_{SI}(1) < 0$, there exists $p^* < 1$ that solves (10). Since $\eta(p^*) < -\left(1 + \frac{c_A+\theta c}{c+c_A}\right)$ and $\eta(p)$ is non-increasing, the sufficient condition for uniqueness in Lemma 14 holds. QED

Next, we present comparative statics results.[35]

LEMMA 15: p^* *is non-decreasing in* c_A *and* θ, *and is non-increasing in* d.

Lemma 15 follows from the Topkis monotonicity theorem. An increase in the agent's litigation cost c_A increases the opportunity cost of litigation. The gain from increasing the number of types that settle is unambiguously higher, so p^* is non-decreasing in c_A. An increase in the agent's bargaining skill decreases the insurer's ability to profit from insurance: the willingness to pay for insurance falls but the cost of insurance is the same. Thus p^* is non-decreasing in θ because an increase in the agent's bargaining skill does not change the surplus gain of the marginal type, but it reduces the surplus loss of the infra-marginal types. An increase in damages d increases the number of agents exposed to credible liability claims. Thus the number of infra-marginal types increases and therefore p^* weakly decreases. The effect of the third-party's litigation cost c is ambiguous,

[33] $\eta(\cdot)$ is analogous to the Arrow-Pratt coefficient of relative risk aversion when the Bernoulli utility function is $u(x) \equiv F(x)$. A large coefficient of relative risk aversion implies that the decision-maker has very little to gain by gambling. In our environment, a large negative $\eta(p)$ means that the insurer wants a lower p, because it has *very little to lose* from gambling on relatively unlikely litigation.

[34] Note that specifying $\eta(p)$ as decreasing in p is a weaker assumption than specifying $f(p)$ to have decreasing density and to be log-concave in p.

[35] As the two-type case suggests, problem (10) may have multiple solutions, e.g. with a continuous distribution with non-monotonic $\eta(p)$. If so, the monotonicity of p^* is in the strong set order.

because it increases both the surplus gain of the marginal type and the loss in surplus of the infra-marginal types.

B6. Omitted proofs

Proof of Lemma 13

PROOF:

$p^* \neq \hat{p}' > 1$ and $p^* \neq \frac{c}{d}$ because $\Psi_{SI}(\hat{p}') < \Psi_{SI}(1)$ and $\Psi_{SI}\left(\frac{c}{d}\right) < \Psi_{SI}(1)$. With a continuous distribution $F(\cdot)$, the objective function is continuous, so a maximum exists (not necessarily unique). With a continuous density, the derivative of the $\Psi_{SI}(\cdot)$ is also continuous. If there are multiple solutions, then at least one must be an interior local maximum. The density $f(\cdot)$ is differentiable because F is twice differentiable, so the first and second order conditions imply

$$(c + c_A)f'(p^*) + \frac{f(p^*)}{p^*}\left[2c_A + (1+\theta)c\right] < 0. \tag{B24}$$

Then, if for all p^* condition (B24) is violated, we can guarantee that the solution of the problem is $p^* = 1$ because in that case there is no interior local maximum of $\Psi(\cdot)$. Hence, since a solution must exist, it must be that $p^* = 1$. QED

Proof of Lemma 14

PROOF:

Suppose $p_1 < p_2 < 1$ are two points satisfying the FOC, $\Psi'_{SI}(p_i) = 0$, and the SOC, $\Psi''_{SI}(p_i) < 0$. We have $p_i > \frac{c}{d}$ because $\Psi'_{SI}\left(\frac{c}{d}\right) > 0$. Then, by continuity of Ψ', there exists $\xi \in (p_1, p_2)$ such that $\Psi'_{SI}(\xi) = 0$ and $\Psi''_{SI}(\xi) > 0$, which implies

$$(c + c_A)f'(\xi) + \frac{f(\xi)}{\xi}\left[2c_A + (1+\theta)c\right] > 0 \Leftrightarrow \eta(\xi) > -1 - \frac{c_A + \theta c}{c_A + c}.$$

If this condition does not hold, the existence of both p_1 and p_2 is a contradiction. QED

Proof of Lemma 15

PROOF:

By Topkis' monotonicity theorem, $\dfrac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial \eta} \geq 0 \Rightarrow p^*(\cdot)$ non-decreasing in η. It is easy to show that $\dfrac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial c_A} > 0$, $\dfrac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial \theta} > 0$, and $\dfrac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial d} < 0$. We have $\dfrac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial c}(p^*) =$

$f(p^*) - \frac{(1-\theta)}{(p^*)^2}\left[\int\limits_{c/d}^{p^*} pf(p)dp - \left(\frac{c}{d}\right)^2 f\left(\frac{c}{d}\right)\right]$. As $p^* \to \frac{c}{d}$, $\frac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial c} \to \theta f\left(\frac{c}{d}\right) > 0$. Moreover, $\frac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial c}$ is increasing if $\eta(p) \geq -1$. QED