Online Appendix for Sectoral Price Facts in a Sticky-Price Model (Not for Publication)

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Contents

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Part I

Models

Here we derive the equilibrium conditions of the baseline model and the other specifications discussed in the main text.

1 Baseline specification

The baseline specification features input-output production linkages and sectoral labor market segmentation. Note that here we present a more general specification than the baseline presented in the main text, as we include a labor-supply shock. For the description of variables and parameters, see the main text.

1.1 Model

1.1.1 Representative household

The representative household maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \Gamma_t \left[\log\left(C_t\right) - \Xi_t \sum_{k=1}^{K} \omega_k \frac{H_{k,t}^{1+\varphi}}{1+\varphi} \right] \right\},$$

subject to the flow budget constraint

$$P_t C_t + E_t \left[Q_{t,t+1} B_{t+1} \right] = B_t + \sum_{k=1}^{K} W_{k,t} H_{k,t} + \sum_{k=1}^{K} \int_{\mathcal{I}_k} \Pi_{k,t}\left(i\right) di,$$

where the aggregate consumption composite C_t is defined as

$$C_t = \left[\sum_{k=1}^{K} \left(n_k D_{k,t}\right)^{1/\eta} C_{k,t}{}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}.$$

Sectoral demand shocks, $D_{k,t} > 0$ for $k = 1, 2, \cdots, K$, are relative and thus are subject to the following constraint

$$\sum_{k=1}^{K} n_k D_{k,t} = 1.$$

The price level associated with the aggregate consumption composite is given by

$$P_t = \left[\sum_{k=1}^{K} \left(n_k D_{k,t}\right) P_{k,t}^{1-\eta} \right]^{1/(1-\eta)}.$$

Given the aggregate consumption composite, C_t, and the price levels, $P_{k,t}$ and P_t, the optimal demand for the sectoral goods is one that minimizes total expenditure $P_t C_t$. This leads to the following sectoral demands:

$$C_{k,t} = n_k D_{k,t} \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} C_t.$$

Sectoral consumption $C_{k,t}$ in sector k is a composite of differentiated goods produced in the sector:

$$C_{k,t} = \left[\left(\frac{1}{n_k} \right)^{1/\theta} \int_{\mathcal{I}_k} C_{k,t}(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)}.$$

The associated sectoral price index is given by

$$P_{k,t} = \left(\frac{1}{n_k} \int_{\mathcal{I}_k} P_{k,t}(i)^{1-\theta} di \right)^{1/(1-\theta)}.$$

Given sectoral consumption $C_{k,t}$ and prices $P_{k,t}$ and $P_{k,t}(i)$, the optimal demand for type-i good in sector k, $C_{k,t}(i)$, is

$$C_{k,t}(i) = \frac{1}{n_k} \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\theta} C_{k,t}.$$

The remaining optimality conditions for the household's utility maximization problem are

$$Q_{t,t+1} = \beta \left(\frac{\Gamma_{t+1}}{\Gamma_t} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(\frac{P_t}{P_{t+1}} \right),$$
$$\frac{W_{k,t}}{P_t} = \Xi_t \omega_k H_{k,t}^{\varphi} C_t.$$

1.1.2 Firms

Consider firm i in sector k (firm ik), where $i \in \mathcal{I}_k$. Its production function is given by

$$Y_{k,t}(i) = A_t A_{k,t} H_{k,t}(i)^{1-\delta} Z_{k,t}(i)^{\delta}, \tag{1}$$

where $Z_{k,t}(i)$ is firm ik's usage of other goods as intermediate inputs, defined as follows:

$$Z_{k,t}(i) = \left[\sum_{k'=1}^{K} \left(n_{k'} D_{k',t} \right)^{1/\eta} Z_{k,k',t}(i)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}.$$

The amount of sector k' goods used as intermediate inputs by firm ik is given by

$$Z_{k,k',t}(i) = \left[\left(\frac{1}{n_{k'}} \right)^{1/\theta} \int_{\mathcal{I}_{k'}} Z_{k,k',t}(i,i')^{(\theta-1)/\theta} di' \right]^{\theta/(\theta-1)},$$

where $Z_{k,k',t}(i,i')$ is the quantity of firm $i'k'$ output purchased by firm ik. Firm ik's cost-minimization problem yields the optimal demand for intermediate inputs:

$$Z_{k,k',t}(i) = n_{k'} D_{k',t} \left(\frac{P_{k',t}}{P_t} \right)^{-\eta} Z_{k,t}(i),$$

and

$$Z_{k,k',t}(i,i') = \frac{1}{n_{k'}} \left(\frac{P_{k',t}(i')}{P_{k',t}} \right)^{-\theta} Z_{k,k',t}(i).$$

Note that firm ik's total output has to satisfy the sum of household consumption and demand by all other firms:

$$
\begin{aligned}
Y_{k,t}(i) &= C_{k,t}(i) + \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} Z_{k',k,t}(i',i)di' \\
&= C_t D_{k,t} \left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta} \left(\frac{P_{k,t}}{P_t}\right)^{-\eta} + \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} D_{k,t} \left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta} \left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Z_{k',t}(i')di' \\
&= Y_t D_{k,t} \left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta} \left(\frac{P_{k,t}}{P_t}\right)^{-\eta}, \qquad (2)
\end{aligned}
$$

where $Y_t = C_t + Z_t$ and $Z_t = \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} Z_{k',t}(i')di'$. The second term on the right hand side in the first row above represents the total demand for the intermediate input produced by firm ik.

Firm ik's nominal profit is given by

$$
\Pi_{k,t}(i) = P_{k,t}(i)Y_{k,t}(i) - W_{k,t}H_{k,t}(i) - P_t Z_{k,t}(i). \qquad (3)
$$

The cost minimization problem implies the following relationship:

$$
Z_{k,t}(i) = \frac{\delta}{1-\delta} \frac{W_{k,t}}{P_t} H_{k,t}(i),
$$

which can be substituted into the profit function (3) to yield

$$
\Pi_{k,t}(i) = P_{k,t}(i)Y_{k,t}(i) - \frac{1}{1-\delta} W_{k,t}H_{k,t}(i). \qquad (4)
$$

From the production function (1), we can write $H_{k,t}(i)$ as a function of the firm's output:

$$
H_{k,t}(i) = \frac{Y_{k,t}(i)}{A_t A_{k,t}} \left(\frac{\delta}{1-\delta}\right)^{-\delta} \left(\frac{W_{k,t}}{P_t}\right)^{-\delta}.
$$

Substitute this again into the profit function (4) to obtain

$$
\begin{aligned}
\Pi_{k,t}(i) =& P_{k,t}(i)Y_{k,t}(i) - \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta}\right)^{-\delta} \left(\frac{W_{k,t}}{P_t}\right)^{1-\delta} \frac{Y_{k,t}(i)}{A_t A_{k,t}} P_t \\
=& \left[P_{k,t}(i) - \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta}\right)^{-\delta} \left(\frac{W_{k,t}}{P_t}\right)^{1-\delta} \frac{1}{A_t A_{k,t}} P_t\right] D_{k,t} \left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta} \left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Y_t, \qquad (5)
\end{aligned}
$$

where we eliminated $Y_{k,t}(i)$ using total demand for firm ik's output (2).

Lastly, we derive the first-order condition of firm ik's profit maximization problem when prices are fully flexible:

$$
\frac{P_{k,t}(i)}{P_t} = \frac{\theta}{\theta-1} \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta}\right)^{-\delta} \left(\frac{W_{k,t}}{P_t}\right)^{1-\delta} \frac{1}{A_t A_{k,t}}, \qquad (6)
$$

where $MC_{k,t} \equiv \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta}\right)^{-\delta} \left(\frac{W_{k,t}}{P_t}\right)^{1-\delta} \frac{1}{A_t A_{k,t}}$ is the real marginal cost and $\theta/(\theta-1)$ is the price mark-up over marginal cost. It follows that, when prices are flexible, the equilibrium price is the same for all firms in a given sector. We denote that optimal flexible price by

$$
P_{k,t}^{**} \equiv \frac{\theta}{\theta-1} MC_{k,t}. \qquad (7)
$$

1.2 Price setting

Prices are sticky as in Calvo (1983). Each firm in sector k adjusts its price with probability $1-\alpha_k$ each period. A fraction α_k of firms do not change their prices. The sectoral price level $P_{k,t}$ evolves as

$$
\begin{aligned}
P_{k,t} &= \left[\frac{1}{n_k} \int_{\mathcal{I}_{k,t}^*} P_{k,t}^{*\,1-\theta} di + \frac{1}{n_k} \int_{\mathcal{I}_k - \mathcal{I}_{k,t}^*} P_{k,t-1}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \\
&= \left[(1-\alpha_k) P_{k,t}^{*\,1-\theta} + \alpha_k P_{k,t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}},
\end{aligned} \tag{8}
$$

where $P_{k,t}^*$ is the common optimal price chosen by sector k firms that adjust in period t. These firms are a random subset of all firms in the sector, and their indexes are collected in the set $\mathcal{I}_{k,t}^*$, whose measure is $n_k(1-\alpha_k)$.

Firms that adjust at time t set their prices to maximize expected discounted profits, as follows:

$$
\max_{P_{k,t}(i)} E_t \sum_{s=0}^{\infty} \alpha_k^s Q_{t,t+s} \Pi_{k,t+s}(i),
$$

where $Q_{t,t+s} = \prod_{\tau=0}^{s-1} Q_{t+\tau,t+\tau+1}$ is the nominal stochastic discount factor between time t and $t+s$, and

$$
\begin{aligned}
\Pi_{k,t+s}(i) &= P_{k,t}(i) Y_{k,t+s}(i) - W_{k,t+s} H_{k,t+s}(i) - P_{t+s} Z_{k,t+s}(i) \\
&= \left[P_{k,t}(i) - \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta} \left(\frac{W_{k,t+s}}{P_{t+s}} \right)^{1-\delta} \frac{1}{A_{t+s} A_{k,t+s}} P_{t+s} \right] D_{k,t+s} \left(\frac{P_{k,t}(i)}{P_{k,t+s}} \right)^{-\theta} \left(\frac{P_{k,t+s}}{P_{t+s}} \right)^{-\eta} Y_{t+s}
\end{aligned}
$$

is the nominal profit at time $t+s$ conditional on the price chosen at time t still being charged. So, the first-order condition for the firm's profit maximization problem is given by

$$
E_t \sum_{s=0}^{\infty} \alpha_k^s Q_{t,t+s} D_{k,t+s} \left(\frac{P_{k,t}^*}{P_{k,t+s}} \right)^{-\theta} \left(\frac{P_{k,t+s}}{P_{t+s}} \right)^{-\eta} Y_{t+s} \left[P_{k,t}^* - \left(\frac{\theta}{\theta-1} \right) MC_{k,t+s} \right] = 0,
$$

where the nominal marginal cost in period $t+s$ is given by

$$
MC_{k,t+s} = P_{t+s} \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta} \left(\frac{W_{k,t+s}}{P_{t+s}} \right)^{1-\delta} \frac{1}{A_{t+s} A_{k,t+s}}.
$$

The first-order condition above and the sectoral price level in (8) together determine equilibrium dynamics of sectoral prices. Aggregate price dynamics are then determined by aggregation of such sectoral prices.

1.3 Government policy

Monetary policy is given by either the following Taylor-type interest rate rule:

$$
I_t = \beta^{-1} I_{t-1}^{\rho_i} \left[\left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left(\frac{C_t}{C} \right)^{\phi_y} \right]^{1-\rho_i} \exp(\mu_t),
$$

or by a policy such that nominal aggregate consumption ($M_t \equiv P_t C_t$) follows an exogenous stochastic process. We abstract from fiscal policy.

1.4 Shocks

Shocks follow AR(1) processes:

$$\begin{aligned}
\log \Gamma_{t+1} &= \rho_\Gamma \log \Gamma_t + \sigma_\Gamma \varepsilon_{\Gamma,t+1}, \\
\log(\Xi_{t+1} - \Xi) &= \rho_\Xi \log(\Xi_t - \Xi) + \sigma_\Xi \varepsilon_{\Xi,t+1}, \\
\log A_{t+1} &= \rho_A \log A_t + \sigma_A \varepsilon_{A,t+1}, \\
\log A_{k,t+1} &= \rho_{A_k} \log A_{k,t} + \sigma_{A_k} \varepsilon_{A_k,t+1}, \\
\log D_{k,t+1} &= \rho_{D_k} \log D_{k,t} + \sigma_{D_k} \varepsilon_{D_k,t+1},
\end{aligned}$$

where $A_k = D_k = \Gamma = A = 1$ in the steady state, and

$$\sum_{k=1}^{K} n_k D_{k,t} = 1.$$

As for monetary policy specifications, we assume either

$$\mu_{t+1} = \rho_\mu \mu_t + \sigma_\mu \varepsilon_{\mu,t+1},$$

or

$$\log M_{t+1} = \rho_M \log M_t + \sigma_M \varepsilon_{M,t+1}.$$

In the steady state, $\mu = 0$ and $M = 1$.

1.5 Equilibrium conditions

Here we collect the equilibrium conditions of the model.

1.5.1 CES aggregates, market clearing conditions, and definitions

- Aggregate price level

$$P_t = \left[\sum_{k=1}^{K} (n_k D_{k,t}) P_{k,t}^{1-\eta}\right]^{1/(1-\eta)}.$$

- Sectoral price level

$$P_{k,t} = \left(\frac{1}{n_k} \int_{\mathcal{I}_k} P_{k,t}(i)^{1-\theta} di\right)^{1/(1-\theta)}.$$

- Aggregate consumption (value-added output)

$$C_t = \left[\sum_{k=1}^{K} (n_k D_{k,t})^{1/\eta} C_{k,t}{}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}.$$

- Sectoral consumption

$$C_{k,t} = \left[\left(\frac{1}{n_k}\right)^{1/\theta} \int_{\mathcal{I}_k} C_{k,t}(i)^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}.$$

- Intermediate input used by firm ik

$$Z_{k,t}(i) = \left[\sum_{k'=1}^{K}\left(n_{k'}D_{k',t}\right)^{1/\eta} Z_{k,k',t}(i)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}.$$

- Intermediate input produced by sector k' used by ik firm

$$Z_{k,k',t}(i) = \left[\left(\frac{1}{n_{k'}}\right)^{1/\theta}\int_{\mathcal{I}_{k'}} Z_{k,k',t}(i,i')^{(\theta-1)/\theta}di'\right]^{\theta/(\theta-1)}.$$

- Asset market clearing condition

$$B_t = 0.$$

- Sectoral labor input

$$H_{k,t} = \int_{\mathcal{I}_k} H_{k,t}(i)di.$$

- Aggregate intermediate input

$$Z_t = \sum_{k'=1}^{K}\int_{\mathcal{I}_{k'}} Z_{k',t}(i')di'.$$

- ik firm's total output

$$Y_{k,t}(i) = C_{k,t}(i) + \sum_{k'=1}^{K}\int_{\mathcal{I}_{k'}} Z_{k',k,t}(i',i)di'.$$

- Sectoral output

$$Y_{k,t} = \int_{\mathcal{I}_k} Y_{k,t}(i)\,di.$$

  Note that $Y_{k,t}$ is a simple sum of output by individual firms in sector k and thus is not the same composite (CES aggregate) as $C_{k,t}$ and $Z_{k',k,t}(i')$.

- Aggregate gross output

$$Y_t = C_t + Z_t.$$

- Aggregate wage

$$W_t = \sum_{k=1}^{K} n_k W_{k,t}.$$

- Aggregate hours index

$$H_t = \sum_{k=1}^{K} H_{k,t}.$$

1.5.2 Demand functions

- Sectoral consumption

$$C_{k,t} = n_k D_{k,t}\left(\frac{P_{k,t}}{P_t}\right)^{-\eta} C_t.$$

- Consumption of an ik good

$$C_{k,t}(i) = \frac{1}{n_k}\left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta} C_{k,t}.$$

- ik firm's demand for sector k' good

$$Z_{k,k',t}(i) = n_{k'} D_{k',t}\left(\frac{P_{k',t}}{P_t}\right)^{-\eta} Z_{k,t}(i).$$

- ik firm's demand for $i'k'$ good

$$Z_{k,k',t}(i,i') = \frac{1}{n_{k'}}\left(\frac{P_{k',t}(i')}{P_{k',t}}\right)^{-\theta} Z_{k,k',t}(i).$$

1.5.3 Household

- Consumption Euler equation / stochastic discount factor

$$Q_{t,t+1} = \beta\left(\frac{\Gamma_{t+1}}{\Gamma_t}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(\frac{P_t}{P_{t+1}}\right).$$

- Optimal labor supply condition

$$\frac{W_{k,t}}{P_t} = \Xi_t \omega_k H_{k,t}^{\varphi} C_t.$$

1.5.4 Firms

- Production function

$$Y_{k,t}(i) = A_t A_{k,t} H_{k,t}(i)^{1-\delta} Z_{k,t}(i)^{\delta}.$$

- Cost minimization

$$\frac{W_{k,t}}{P_t} = \frac{1-\delta}{\delta}\frac{Z_{k,t}(i)}{H_{k,t}(i)}.$$

- Nominal marginal cost

$$MC_{k,t+s} = P_{t+s}\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{k,t+s}}{P_{t+s}}\right)^{1-\delta}\frac{1}{A_{t+s}A_{k,t+s}}.$$

- First order condition

$$0 = E_t\sum_{s=0}^{\infty}\alpha_k^s Q_{t,t+s}\left(\frac{P_{k,t}^{*}}{P_{k,t+s}}\right)^{-\theta}\left(\frac{P_{k,t+s}}{P_{t+s}}\right)^{-\eta} Y_{t+s}\left[P_{k,t}^{*} - \left(\frac{\theta}{\theta-1}\right) MC_{k,t+s}\right].$$

- Frictionless optimal price

$$P_{k,t}^{**} = \left(\frac{\theta}{\theta-1}\right) MC_{k,t}.$$

1.6 Steady state

We solve the model by log-linearizing quilibrium conditions around a symmetric non-stochastic zero-inflation steady state, which is presented here. A non-stochastic steady-state need not be symmetric. In particular, it depends on the steady-state levels of sector-specific productivity $\{A_k\}_{k=1}^{K}$ and the sector-specific parameters that measure the relative disutilities of supplying labor, $\{\omega_k\}_{k=1}^{K}$. For simplicity, we make two assumptions that deliver a symmetric steady state: i) the steady-state levels of productivities are the same across sectors: specifically, $A_k = 1$ for all k, without loss of generality;[1] ii) $\omega_k = n_k^{-\varphi}$ for all k. The latter assumption relates the relative disutilities of labor to the size of sectors, and equalizes steady-state sectoral wages.

We solve for $\{Y, C, Z, H, W/P, \Pi/P\}$: the steady state values of aggregate gross output, aggregate value added-output (i.e. GDP), aggregate intermediate input usage, aggregate hours, real wage, and real profits. Once we obtain the steady state values of these aggregate variables, it is trivial to characterize the steady-state values for sectoral and micro variables using the symmetric nature of the steady state: $Y_k = n_k Y_k(i) = n_k Y$, $C_k = n_k C_k(i) = n_k C$, $Z_{k,k'}(i) = n_{k'} Z_{k,k'}(i, i') = n_{k'} Z$, $Z_k(i) = Z$, $H_k = n_k H_k(i) = n_k H$, $\Pi_k(i)/P = \Pi/P$, $W_k/P = W/P$, and $P(i)/P = P_k/P = 1$.

After exploiting symmetry and market-clearing conditions, the system of equilibrium conditions can be reduced to the following seven equations:

$$C = \left(\frac{W}{P}\right) H + \left(\frac{\Pi}{P}\right), \tag{9}$$

$$\left(\frac{W}{P}\right) = H^{\varphi} C, \tag{10}$$

$$Y = H^{1-\delta} Z^{\delta}, \tag{11}$$

$$Y = C + Z, \tag{12}$$

$$\left(\frac{\Pi}{P}\right) = Y - \left(\frac{W}{P}\right) H - Z, \tag{13}$$

$$Z = \frac{\delta}{1-\delta}\left(\frac{W}{P}\right) H, \tag{14}$$

$$\left(\frac{\theta}{\theta-1}\right)\chi\left(\frac{W}{P}\right)^{1-\delta} = 1, \tag{15}$$

where $\chi \equiv \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}$.

First, it is trivial to obtain the real wage from (15):

$$\left(\frac{W}{P}\right) = \left(\frac{\theta-1}{\theta}\frac{1}{\chi}\right)^{\frac{1}{1-\delta}}.$$

Next, we substitute out Z in (11) and (13) using (14), which gives:

$$Y = H\left(\frac{\delta}{1-\delta}\right)^{\delta}\left(\frac{W}{P}\right)^{\delta},$$

$$\left(\frac{\Pi}{P}\right) = Y - \left(\frac{1}{1-\delta}\right)\left(\frac{W}{P}\right) H.$$

Combining the two equations above, we substitute out H and express real profits as a function of the real wage and output:

$$\left(\frac{\Pi}{P}\right) = \left[1 - \chi\left(\frac{W}{P}\right)^{1-\delta}\right] Y.$$

[1] Similarly, we fix the steady-state level of all other exogenous processes at unity.

But, $\chi\left(\frac{W}{P}\right)^{1-\delta} = \frac{\theta-1}{\theta}$ from (15), so it follows that

$$\left(\frac{\Pi}{P}\right) = \frac{1}{\theta}Y.$$

Equation (9) indicates that aggregate value-added output should be equal to the sum of labor income and real profits:

$$C = \left(\frac{W}{P}\right)H + \left(\frac{\Pi}{P}\right) = \frac{1-\delta}{\delta}Z + \frac{1}{\theta}Y = \left[1-\delta\left(\frac{\theta-1}{\theta}\right)\right]Y.$$

Consequently, aggregate intermediate inputs are obtained as:

$$Z = Y - C = Y - \left[1-\delta\left(\frac{\theta-1}{\theta}\right)\right]Y = \delta\left(\frac{\theta-1}{\theta}\right)Y.$$

From (14), total labor hours are given by:

$$H = \frac{1-\delta}{\delta}\left(\frac{W}{P}\right)^{-1}Z = \left[\delta\left(\frac{\theta-1}{\theta}\right)\right]^{-\frac{\delta}{1-\delta}}Y.$$

So far, we have expressed the steady-state values of $\{Y, C, Z, H, \Pi/P\}$ in terms of Y, which can be obtained using (10):

$$Y = \left\{\left(\frac{1}{\chi}\right)^{\frac{1}{1-\delta}}\left(\frac{\theta-1}{\theta}\right)^{\frac{1}{1-\delta}}\left[\delta\left(\frac{\theta-1}{\theta}\right)\right]^{\frac{\delta\varphi}{1-\delta}}\left[1-\delta\left(\frac{\theta-1}{\theta}\right)\right]^{-1}\right\}^{\frac{1}{1+\varphi}}.$$

In the special case in which $\delta = 0$, the expression for aggregate gross output simplifies to $Y = \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{1+\varphi}}$, which is the standard result in models without intermediate inputs.

1.7 Log-linear approximation

We denote the log deviation of variables from the steady state by lower case letters. For example, $c_t = \log C_t - \log C$ and $\gamma_t = \log \Gamma_t - \log \Gamma$.

1.7.1 CES Aggregates, market clearing conditions, and definitions

- Aggregate price level

$$p_t = \sum_{k=1}^{K} n_k p_{k,t}.$$

- Sectoral price level

$$p_{k,t} = \frac{1}{n_k}\int_{\mathcal{I}_k} p_{k,t}(i)di.$$

- Aggregate consumption (value-added output)

$$c_t = \sum_{k=1}^{K} n_k c_{k,t}.$$

- Sectoral consumption

$$c_{k,t} = \frac{1}{n_k} \int_{\mathcal{I}_k} c_{k,t}(i) di.$$

- Intermediate input used by ik firm

$$z_{k,t}(i) = \sum_{k'=1}^{K} n_{k'} z_{k,k',t}(i).$$

- Intermediate input produced by sector k' used by ik firm

$$z_{k,k',t}(i) = \frac{1}{n_{k'}} \int_{\mathcal{I}_{k'}} z_{k,k',t}(i,i') di'.$$

- Sectoral labor input

$$h_{k,t} = \frac{1}{n_k} \int_{\mathcal{I}_k} h_{k,t}(i) di. \tag{16}$$

- Aggregate intermediate input

$$z_t = \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',t}(i') di'. \tag{17}$$

- firm ik's total output

$$y_{k,t}(i) = (1-\psi)\, c_{k,t}\,(i) + \psi \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',k,t}\,(i',i)\, di'.$$

- Sectoral output

$$y_{k,t} = \frac{1}{n_k} \int_{\mathcal{I}_k} y_{k,t}(i) di. \tag{18}$$

By the way, note that

$$\begin{aligned} y_{k,t} &= \frac{1}{n_k} \int_{\mathcal{I}_k} y_{k,t}(i) di \\ &= (1-\psi)\, c_{k,t} + \psi \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',k,t}\,(i')\, di'. \end{aligned}$$

- Aggregate gross output

$$y_t = (1-\psi) c_t + \psi z_t.$$

where $\psi \equiv \delta\left(\frac{\theta-1}{\theta}\right)$. It follows that

$$\begin{aligned} \sum_{k=1}^{K} n_k y_{k,t} &= (1-\psi) \sum_{k=1}^{K} n_k c_{k,t} + \psi \sum_{k=1}^{K} n_k \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',k,t}\,(i')\, di' \\ &= (1-\psi)\, c_t + \psi z_t \\ &= y_t. \end{aligned}$$

- Aggregate wage

$$w_t = \sum_{k=1}^{K} n_k w_{k,t}. \tag{19}$$

- Aggregate hours index

$$h_t = \sum_{k=1}^{K} n_k h_{k,t}. \tag{20}$$

1.7.2 Demand functions

- Sectoral consumption

$$c_{k,t} - c_t = -\eta \left(p_{k,t} - p_t\right) + d_{k,t}. \tag{21}$$

- Consumption of good ik

$$c_{k,t}(i) - c_{k,t} \ = -\theta \left(p_{k,t}(i) - p_{k,t}\right).$$

- Firm ik's demand for sector k' good

$$z_{k,k',t}(i) - z_{k,t}(i) = -\eta \left(p_{k',t} - p_t\right) + d_{k',t}. \tag{22}$$

- Firm ik's demand for good $i'k'$

$$z_{k,k',t}(i,i') - z_{k,k',t}(i) = -\theta \left(p_{k',t}(i') - p_{k',t}\right).$$

- Firm ik's total output. Using the demand for consumption of good ik and the demand for good ik by firm $i'k'$, we can show that

$$\begin{aligned} y_{k,t}(i) &= (1-\psi)\, c_{k,t}(i) + \psi \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',k,t}(i',i)\, di' \\ &= -\theta \left(p_{k,t}(i) - p_{k,t}\right) + y_{k,t}, \end{aligned}$$

  or

$$y_{k,t}(i) - y_{k,t} = -\theta \left(p_{k,t}(i) - p_{k,t}\right).$$

1.7.3 Household

- Consumption Euler equation

$$c_t = E_t\left(c_{t+1}\right) - \left(i_t - E_t \pi_{t+1}\right) + \left(\gamma_t - E_t \gamma_{t+1}\right).$$

- Optimal labor supply condition

$$w_{k,t} - p_t = \varphi h_{k,t} + c_t + \xi_t. \tag{23}$$

  Using the definition of the aggregate wage (19) and the aggregate hours index, we can derive the following aggregate labor supply condition

$$w_t - p_t = \varphi h_t + c_t + \xi_t.$$

1.7.4 Firms

- Firm ik's production function

$$y_{k,t}(i) = a_t + a_{k,t} + (1-\delta)\, h_{k,t}(i) + \delta z_{k,t}(i). \tag{24}$$

Using the definition of sectoral output (18) and sectoral labor input (16), we can show that

$$y_{k,t} = a_t + a_{k,t} + (1-\delta)\, h_{k,t} + \frac{\delta}{n_k} \int_{\mathcal{I}_k} z_{k,t}\,(i)\, di.$$

It follows that

$$\begin{aligned} y_t = \sum_{k=1}^{K} n_k y_{k,t} &= a_t + \sum_{k=1}^{K} n_k a_{k,t} + (1-\delta) \sum_{k=1}^{K} n_k h_{k,t} + \delta \sum_{k=1}^{K} \int_{\mathcal{I}_k} z_{k,t}\,(i)\, di \\ &= a_t + \sum_{k=1}^{K} n_k a_{k,t} + (1-\delta)\, h_t + \delta z_t, \end{aligned}$$

where we used the definition of aggregate intermediate input (17) and aggregate hours index (20).

- Cost minimization

$$w_{k,t} - p_t = z_{k,t}(i) - h_{k,t}(i). \tag{25}$$

Note that integrating both sides over $\mathcal{I}_k$ and dividing by n_k leads to

$$w_{k,t} - p_t = \frac{1}{n_k} \int_{\mathcal{I}_k} z_{k,t}\,(i)\, di - h_{k,t},$$

where we used the definition of sectoral labor input (16). From the definition of aggregate wages (19) and aggregate intermediate input (17), it follows that

$$w_t - p_t = z_t - h_t.$$

- Nominal marginal cost

$$mc_{k,t} = (1-\delta)\,(w_{k,t} - p_t) - a_{k,t} - a_t + p_t.$$

- First order condition

$$E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s p_{k,t}^* = E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s mc_{k,t+s}. \tag{26}$$

1.7.5 Derivation of the Phillips curve

From the log-linearized first order condition by the price-setting firm (26), the optimal price is given by

$$p_{k,t}^* = (1-\alpha_k\beta) E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s mc_{k,t+s},$$

or

$$p_{k,t}^* = (1-\alpha_k\beta) mc_{k,t} + \alpha_k \beta E_t \left[p_{k,t+1}^* \right]. \tag{27}$$

Loglinearizing (8) leads to

$$p_{k,t} = (1-\alpha_k)p^*_{k,t} + \alpha_k p_{k,t-1}. \tag{28}$$

Combining (27)and (28), we can derive the sectoral Phillips curve (PC)

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}(mc_{k,t} - p_{k,t}) \tag{29}$$

Now let us show how marginal cost is determined. First, note that by integrating (25) over $\mathcal{I}_k$, we obtain

$$w_{k,t} - p_t = \frac{1}{n_k}\int_{\mathcal{I}_k} z_{k,t}(i)di - h_{k,t}.$$

We can combine this eqution with (23) to obtain

$$\frac{1}{n_k}\int_{\mathcal{I}_k} z_{k,t}(i)di = (1+\varphi)h_{k,t} + c_t + \xi_t. \tag{30}$$

Also, by integrating both sides of the production function (24) over $\mathcal{I}_k$, we get

$$\begin{aligned} y_{k,t} &= a_t + a_{k,t} + (1-\delta)h_{k,t} + \delta\frac{1}{n_k}\int_{\mathcal{I}_k} z_{k,t}(i)di \\ &= a_t + a_{k,t} + (1+\delta\varphi)h_{k,t} + \delta c_t + \delta\xi_t, \end{aligned}$$

where we use (30) to obtain the second row. It follows that

$$h_{k,t} = \frac{1}{1+\delta\varphi}y_{k,t} - \frac{\delta}{1+\delta\varphi}c_t - \frac{\delta}{1+\delta\varphi}\xi_t - \frac{1}{1+\delta\varphi}a_{k,t} - \frac{1}{1+\delta\varphi}a_t,$$

and thus

$$\begin{aligned} mc_{k,t} &= (1-\delta)(w_{k,t} - p_t) - a_{k,t} - a_t + p_t \\ &= (1-\delta)(\varphi h_{k,t} + c_t + \xi_t) - a_{k,t} - a_t + p_t \\ &= \frac{(1-\delta)\varphi}{1+\delta\varphi}y_{k,t} + \frac{(1-\delta)}{1+\delta\varphi}c_t + \frac{(1-\delta)}{1+\delta\varphi}\xi_t - \frac{1+\varphi}{1+\delta\varphi}a_t - \frac{1+\varphi}{1+\delta\varphi}a_{k,t} + p_t. \end{aligned}$$

Consequently, the sectoral PC (29) can be written as

$$\begin{aligned} \pi_{k,t} =& \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\left\{\frac{(1-\delta)\varphi}{1+\delta\varphi}y_{k,t} + \frac{(1-\delta)}{1+\delta\varphi}c_t + \frac{1-\delta}{1+\delta\varphi}\xi_t \right. \\ & \left. - \frac{1+\varphi}{1+\delta\varphi}a_t - \frac{1+\varphi}{1+\delta\varphi}a_{k,t} + (p_t - p_{k,t})\right\}. \end{aligned}$$

We can write the sectoral PC in terms of sectoral consumption instead of sectoral output. Note that the consumption demand for sector k goods (21) can be rewritten as

$$p_t - p_{k,t} = \frac{1}{\eta}(c_{k,t} - c_t) - \frac{1}{\eta}d_{k,t}. \tag{31}$$

Also, using the demand function for sectoral consumption (21) and the demand for sector k good by firm $i'k'$ (22), we can show that

$$y_{k,t} = (1-\psi)\, c_{k,t} + \psi \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',k,t}\,(i')\, di'$$
$$= -\eta\,(p_{k,t} - p_t) + d_{k,t} + y_t,$$

or

$$y_{k,t} - y_t = -\eta\,(p_{k,t} - p_t) + d_{k,t}.$$

From (21), it turns out that relative consumption for sector k good is identical to relative output of sector k

$$y_{k,t} - y_t = c_{k,t} - c_t.$$

It follows that

$$\begin{aligned} y_{k,t} &= y_t + c_{k,t} - c_t \\ &= (1-\psi)\, c_t + \psi z_t + c_{k,t} - c_t \\ &= \psi z_t + c_{k,t} - \psi c_t. \end{aligned} \tag{32}$$

Thus, we can substitute out $y_{k,t}$ from the marginal cost:

$$mc_{k,t} = \frac{(1-\delta)\,\varphi}{1+\delta\varphi} c_{k,t} + \frac{(1-\delta)\,(1-\psi\varphi)}{1+\delta\varphi} c_t + \frac{(1-\delta)\,\psi\varphi}{1+\delta\varphi} z_t + \frac{(1-\delta)}{1+\delta\varphi}\xi_t - \frac{1+\varphi}{1+\delta\varphi} a_t - \frac{1+\varphi}{1+\delta\varphi} a_{k,t} + p_t. \tag{33}$$

Substituting out $p_t - p_{k,t}$ from the sectoral PC as well using (31), we obtain

$$\begin{aligned} \pi_{k,t} =& \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)\,(1-\alpha_k\beta)}{\alpha_k} \left\{ \left[\frac{(1-\delta)\,\varphi}{1+\delta\varphi} + \frac{1}{\eta} \right] c_{k,t} + \left[\frac{(1-\delta)\,(1-\psi\varphi)}{1+\delta\varphi} - \frac{1}{\eta} \right] c_t \right. \\ & \left. + \frac{(1-\delta)\,\psi\varphi}{1+\delta\varphi} z_t + \frac{1-\delta}{1+\delta\varphi}\xi_t - \frac{1+\varphi}{1+\delta\varphi} a_t - \frac{1+\varphi}{1+\delta\varphi} a_{k,t} - \frac{1}{\eta} d_{k,t} \right\}. \end{aligned}$$

Aggregate inflation is obtained by aggregation of sectoral inflation:

$$\pi_t = \sum_{k=1}^{K} n_k \pi_{k,t}.$$

1.8 Log-linear equilibrium conditions

Here we present the log-linearized equilibrium conditions necessary to characterize the equilibrium dynamics of the variables of interest. They are the following aggregate variables:

$$\{c_t, \pi_t, i_t, m_t, h_t, (w_t - p_t)\},$$

and the following sectoral variables

$$\{c_{k,t}, \pi_{k,t}\}_{k=1}^{K}.$$

The following $6+(K+2)$ equations determine the equilibrium dynamics of those variables:

$$c_t = E_t\left[c_{t+1}\right] - \left(i_t - E_t\pi_{t+1}\right) + \left(\gamma_t - E_t\gamma_{t+1}\right), \quad \text{(IS equation)}$$
$$w_t - p_t = \varphi h_t + c_t + \xi_t, \quad \text{(agg. labor supply)}$$
$$\left(1-\psi\right)c_t + \psi z_t = a_t + \sum_{k=1}^{K} n_k a_{k,t} + \left(1-\delta\right)h_t + \delta z_t, \quad \text{(agg. resource constraint)}$$
$$w_t - p_t = z_t - h_t, \quad \text{(agg. cost-minimization relation)}$$
$$\begin{aligned}\pi_{k,t} = \beta E_t\pi_{k,t+1} + \frac{\left(1-\alpha_k\right)\left(1-\alpha_k\beta\right)}{\alpha_k}&\left\{\left[\frac{\left(1-\delta\right)\varphi}{1+\delta\varphi}+\frac{1}{\eta}\right]c_{k,t} + \left[\frac{\left(1-\delta\right)\left(1-\psi\varphi\right)}{1+\delta\varphi}-\frac{1}{\eta}\right]c_t\right.\\ &\left.+\frac{\left(1-\delta\right)\psi\varphi}{1+\delta\varphi}z_t + \frac{1-\delta}{1+\delta\varphi}\xi_t - \frac{1+\varphi}{1+\delta\varphi}a_t - \frac{1+\varphi}{1+\delta\varphi}a_{k,t} - \frac{1}{\eta}d_{k,t}\right\}, \quad \text{(sectoral PC)}\end{aligned}$$
$$\pi_t = \sum_{k=1}^{K} n_k\pi_{k,t}, \quad \text{(agg. inflation)}$$
$$\Delta\left(c_{k,t+1} - c_{t+1}\right) = -\eta\left(\pi_{k,t+1} - \pi_{t+1}\right) + \Delta d_{k,t+1}. \quad \text{(sectoral consumption)}$$

For monetary policy, we consider a Taylor-type interest rate rule:

$$i_t = \rho_i i_{t-1} + \left(1-\rho_i\right)\left(\phi_\pi\pi_t + \phi_c c_t\right) + \mu_t,$$

or an exogenous stochastic process for nominal aggregate consumption:

$$m_t = p_t + c_t.$$

1.9 Derivation of key equations

Here we show how to derive important equations of the baseline specification that show how sectoral price levels evolve and how firms would set prices in the absence of pricing friction.

1.9.1 The evolution of the sectoral price levels

Recall from Section 1.7.5 that

$$p^*_{k,t} = (1-\alpha_k\beta)mc_{k,t} + \alpha_k\beta E_t\left[p^*_{k,t+1}\right], \tag{34}$$
$$p_{k,t} = (1-\alpha_k)p^*_{k,t} + \alpha_k p_{k,t-1}. \tag{35}$$

Substitute out $p^*_{k,t}$ and $p^*_{k,t+1}$ in (34) using (35), one obtains

$$\beta E_t p_{k,t+1} - \frac{1+\alpha_k^2\beta}{\alpha_k}p_{k,t} + p_{k,t-1} = -\frac{\left(1-\alpha_k\right)\left(1-\alpha_k\beta\right)}{\alpha_k}mc_{k,t}. \tag{36}$$

Note that

$$\begin{aligned}mc_{k,t} =& \frac{\left(1-\delta\right)\varphi}{1+\delta\varphi}c_{k,t} + \frac{\left(1-\delta\right)\left(1-\psi\varphi\right)}{1+\delta\varphi}c_t + \frac{\left(1-\delta\right)\psi\varphi}{1+\delta\varphi}z_t + \frac{\left(1-\delta\right)}{1+\delta\varphi}\xi_t - \frac{1+\varphi}{1+\delta\varphi}a_t - \frac{1+\varphi}{1+\delta\varphi}a_{k,t} + p_t\\ =& \frac{\left(1-\delta\right)\left(1-\psi\varphi+\varphi\right)}{1+\delta\varphi}c_t + \frac{\left(1-\delta\right)\psi\varphi}{1+\delta\varphi}z_t + \left[1+\frac{\left(1-\delta\right)\varphi\eta}{1+\delta\varphi}\right]p_t + \frac{\left(1-\delta\right)}{1+\delta\varphi}\xi_t - \frac{1+\varphi}{1+\delta\varphi}a_t\end{aligned}$$

$$-\frac{1+\varphi}{1+\delta\varphi}a_{k,t}+\frac{(1-\delta)\varphi}{1+\delta\varphi}d_{k,t}-\frac{(1-\delta)\varphi\eta}{1+\delta\varphi}p_{k,t},$$

where we substituted out $c_{k,t}$ from (33) using the demand for sectoral consumption (21). Now let

$$mc_{k,t}=\widetilde{mc}_{k,t}-\frac{(1-\delta)\varphi\eta}{1+\delta\varphi}p_{k,t}, \tag{37}$$

where

$$\begin{aligned}\widetilde{mc}_{k,t}=&\frac{(1-\delta)(1-\psi\varphi+\varphi)}{1+\delta\varphi}c_t+\frac{(1-\delta)\psi\varphi}{1+\delta\varphi}z_t+\left[1+\frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right]p_t+\frac{(1-\delta)}{1+\delta\varphi}\xi_t-\frac{1+\varphi}{1+\delta\varphi}a_t\\&-\frac{1+\varphi}{1+\delta\varphi}a_{k,t}+\frac{(1-\delta)\varphi}{1+\delta\varphi}d_{k,t}.\end{aligned}$$

Plugging (37) into (36), one obtains

$$\beta E_t p_{k,t+1}-\left\{1+\beta+\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\left[1+\frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right]\right\}p_{k,t}+p_{k,t-1}=-\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\widetilde{mc}_{k,t}. \tag{38}$$

Using the lag polynomial

$$B(L)=\beta-\left\{1+\beta+\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\left[1+\frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right]\right\}L+L^2,$$

where L is the lag operator, we can rewrite (38) as

$$E_t\left[B(L)p_{k,t+1}\right]=-\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\widetilde{mc}_{k,t}.$$

Note that the lag polynomial $B(L)$ can be factored as

$$B(L)=\beta(1-\lambda_1 L)(1-\lambda_2 L),$$

where λ_1 and λ_2 are the two roots of the characteristic polynomial

$$f(\lambda)\equiv\beta\lambda^2-\left[1+\beta+\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\left[1+\frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right]\right]\lambda+1=0.$$

Without loss of generality, suppose that $|\lambda_1|\leq|\lambda_2|$. Note that λ_1 and λ_2 are real and must satisfy $0<\lambda_1<1<1/\beta<\lambda_2$ since $f(0)>0$, $f(1)<0$, and

$$f(\lambda)<\beta\left(\lambda-\frac{1}{\beta}\right)(\lambda-1),$$

for $\lambda>0$. It follows that

$$E_t\left[(1-\lambda_1 L)(1-\lambda_2 L)p_{k,t+1}\right]=-\frac{1}{\beta}\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\widetilde{mc}_{k,t},$$

or

$$(1-\lambda_1 L)p_{k,t}=\frac{1}{\lambda_2}E_t\left[(1-\lambda_1 L)p_{k,t+1}\right]+\frac{1}{\beta\lambda_2}\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\widetilde{mc}_{k,t}.$$

Since $|\lambda_2| > 1$, one solution of this equation can be found by solving it forward as[2]

$$(1-\lambda_1 L)\, p_{k,t} = \frac{1}{\beta\lambda_2}\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\sum_{s=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^s E_t\widetilde{mc}_{k,t+s},$$

or

$$p_{k,t} = \lambda_1 p_{k,t-1} + \frac{1}{\beta\lambda_2}\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\sum_{s=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^s E_t\widetilde{mc}_{k,t+s}.$$

Since

$$\lambda_1\lambda_2 = \frac{1}{\beta},$$

it follows that

$$p_{k,t} = \lambda_1 p_{k,t-1} + \lambda_1\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\sum_{s=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^s E_t\widetilde{mc}_{k,t+s},$$

where

$$\begin{aligned}\widetilde{mc}_{k,t+s} =& \frac{(1-\delta)(1-\psi\varphi+\varphi)}{1+\delta\varphi}c_{t+s} + \frac{(1-\delta)\psi\varphi}{1+\delta\varphi}z_{t+s} + \left[1+\frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right]p_{t+s} + \frac{(1-\delta)}{1+\delta\varphi}\xi_{t+s} - \frac{1+\varphi}{1+\delta\varphi}a_{t+s}\\ &- \frac{1+\varphi}{1+\delta\varphi}a_{k,t+s} + \frac{(1-\delta)\varphi}{1+\delta\varphi}d_{k,t+s}.\end{aligned}$$

In summary, the sectoral price levels evolve as

$$\begin{aligned}p_{k,t} =& \lambda_1 p_{k,t-1} + \frac{1}{\beta\lambda_2}\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\sum_{s=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^s E_t\left\{\frac{(1-\delta)(1-\psi\varphi+\varphi)}{1+\delta\varphi}c_{t+s} + \frac{(1-\delta)\psi\varphi}{1+\delta\varphi}z_{t+s}\right.\\ &\left. + \left[1+\frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right]p_{t+s} + \frac{(1-\delta)}{1+\delta\varphi}\xi_{t+s} - \frac{1+\varphi}{1+\delta\varphi}a_{t+s} - \frac{1+\varphi}{1+\delta\varphi}a_{k,t+s} + \frac{(1-\delta)\varphi}{1+\delta\varphi}d_{k,t+s}\right\}. \end{aligned} \tag{39}$$

We then consider simple cases to inspect the mechanisms more clearly. First, when $\delta = 0$ and $\varphi = 0$, (39) is reduced to

$$p_{k,t} = \lambda_1 p_{k,t-1} + \frac{1}{\beta\lambda_2}\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\sum_{s=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^s E_t\left[c_{t+s} + p_{t+s} + \xi_{t+s} - a_{t+s} - a_{k,t+s}\right].$$

If we further assume that $m_t = p_t + c_t$, it follows that

$$p_{k,t} = \lambda_1 p_{k,t-1} + \frac{1}{\beta\lambda_2}\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\sum_{s=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^s E_t\left[m_{t+s} + \xi_{t+s} - a_{t+s} - a_{k,t+s}\right].$$

Second, when $\delta > 0$ but $\varphi = 0$, (39) is reduced to

$$p_{k,t} = \lambda_1 p_{k,t-1} + \frac{1}{\beta\lambda_2}\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\sum_{s=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^s E_t\left[(1-\delta)\,c_{t+s} + p_{t+s} + (1-\delta)\,\xi_{t+s} - a_{t+s} - a_{k,t+s}\right].$$

If we further assume that $m_t = p_t + c_t$, it follows that

$$p_{k,t} = \lambda_1 p_{k,t-1} + \frac{1}{\beta\lambda_2}\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\sum_{s=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^s E_t\left[(1-\delta)\,m_{t+s} + (1-\delta)\,\xi_{t+s} - a_{t+s} - a_{k,t+s} + \delta p_{t+s}\right].$$

[2] Since the price level is difference-stationary in this model, $\widetilde{mc}_{k,t+s}$ is also difference-stationary. However, its expectation conditional on the information set of date t is bounded and thus the infinite sum is well defined. For a discussion of the uniqueness of the solution, see Woodford (2003).

Third, when $\delta = 0$ but $\varphi > 0$, $\psi = 0$. So (39) is reduced to

$$p_{k,t} = \lambda_1 p_{k,t-1} + \frac{1}{\beta\lambda_2}\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\sum_{s=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^s E_t\left[(1+\varphi)c_{t+s} + (1+\varphi\eta)p_{t+s} + \xi_{t+s}\right.$$
$$\left. - (1+\varphi)(a_{t+s} + a_{k,t+s}) + \varphi d_{k,t+s}\right].$$

If we further assume that $m_t = p_t + c_t$, it follows that

$$p_{k,t} = \lambda_1 p_{k,t-1} + \frac{1}{\beta\lambda_2}\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\sum_{s=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^s E_t\left[(1+\varphi)m_{t+s} + \varphi(\eta-1)p_{t+s} + \xi_{t+s}\right.$$
$$\left. - (1+\varphi)(a_{t+s} + a_{k,t+s}) + \varphi d_{k,t+s}\right].$$

1.9.2 Frictionless optimal price

When firms can change prices continuously, the log-linearized optimal price is given by

$$\begin{aligned} p^{**}_{k,t} =& mc_{k,t} \\ =& \frac{(1-\delta)(1-\psi\varphi+\varphi)}{1+\delta\varphi}c_t + \frac{(1-\delta)\psi\varphi}{1+\delta\varphi}z_t + \left[1 + \frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right]p_t + \frac{(1-\delta)}{1+\delta\varphi}\xi_t - \frac{1+\varphi}{1+\delta\varphi}a_t \\ & - \frac{1+\varphi}{1+\delta\varphi}a_{k,t} + \frac{(1-\delta)\varphi}{1+\delta\varphi}d_{k,t} - \frac{(1-\delta)\varphi\eta}{1+\delta\varphi}p_{k,t}, \end{aligned}$$

where we substitute the demand for sectoral consumption (21) for $c_{k,t}$ in (33). If $m_t = p_t + c_t$, this can be written as

$$\begin{aligned} p^{**}_{k,t} =& \frac{(1-\delta)(1-\psi\varphi+\varphi)}{1+\delta\varphi}m_t + \frac{(1-\delta)\psi\varphi}{1+\delta\varphi}z_t + \left[1 - \frac{(1-\delta)(1-\psi\varphi+\varphi-\varphi\eta)}{1+\delta\varphi}\right]p_t + \frac{(1-\delta)}{1+\delta\varphi}\xi_t - \frac{1+\varphi}{1+\delta\varphi}a_t \\ & - \frac{1+\varphi}{1+\delta\varphi}a_{k,t} + \frac{(1-\delta)\varphi}{1+\delta\varphi}d_{k,t} - \frac{(1-\delta)\varphi\eta}{1+\delta\varphi}p_{k,t}. \end{aligned}$$

We then consider simple cases to understand the nature of pricing interactions more clearly. First, when $\varphi = 0$ but $\delta > 0$, the frictionless optimal price is simplified to

$$p^{**}_{k,t} = (1-\delta)m_t + \delta p_t + (1-\delta)\xi_t - a_t - a_{k,t}.$$

Second, when $\delta = 0$ but $\varphi > 0$, the frictionless optimal price is simplified to

$$p^{**}_{k,t} = (1+\varphi)m_t - (\varphi - \varphi\eta)p_t + \xi_t - (1+\varphi)(a_t + a_{k,t}) + \varphi d_{k,t} - \varphi\eta p_{k,t}.$$

Lastly, when $\delta = 0$ and $\varphi = 0$, the frictionless optimal price becomes

$$p^{**}_{k,t} = m_t + \xi_t - a_t - a_{k,t}.$$

For the frictionless optimal price with the firm-specific labor market, see Section 3.

2 Economy-wide labor market

This section describes a simple version of the baseline specification with a single, economy-wide labor market. We present only differences from the baseline model.

2.1 Model

2.1.1 Representative household

The representative household maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \Gamma_t \left[\log\left(C_t\right) - \Xi_t \frac{H_t^{1+\varphi}}{1+\varphi} \right] \right\},$$

subject to the flow budget constraint

$$P_t C_t + E_t \left[Q_{t,t+1} B_{t+1} \right] = B_t + W_t H_t + \sum_{k=1}^{K} \int_{\mathcal{I}_k} \Pi_{k,t}\left(i\right) di,$$

where H_t is the labor supply and W_t is nominal wages for a unit of labor hours.

The optimality conditions for the household's utility maximization problem are given by

$$Q_{t,t+1} = \beta \left(\frac{\Gamma_{t+1}}{\Gamma_t} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(\frac{P_t}{P_{t+1}} \right),$$

$$\frac{W_t}{P_t} = \Xi_t H_t^{\varphi} C_t.$$

2.1.2 Firms

Consider firm i in sector k (firm ik), where $i \in \mathcal{I}_k$. Its production function is given by

$$Y_{k,t}\left(i\right) = A_t A_{k,t} H_t(i)^{1-\delta} Z_{k,t}\left(i\right)^{\delta}. \tag{40}$$

Firm ik's nominal profit is given by

$$\Pi_{k,t}(i) = P_{k,t}(i) Y_{k,t}(i) - W_t H_t(i) - P_t Z_{k,t}(i). \tag{41}$$

The cost minimization problem implies the following relationship

$$Z_{k,t}(i) = \frac{\delta}{1-\delta} \frac{W_t}{P_t} H_t(i),$$

which can be substituted into the profit function (41) as

$$\Pi_{k,t}(i) = P_{k,t}(i) Y_{k,t}(i) - \frac{1}{1-\delta} W_t H_t(i). \tag{42}$$

From the production function (40), we can write $H_t(i)$ as a function of the firm's output

$$H_t(i) = \frac{Y_{k,t}(i)}{A_t A_{k,t}} \left(\frac{\delta}{1-\delta} \right)^{-\delta} \left(\frac{W_t}{P_t} \right)^{-\delta}.$$

Substitute this again into the profit function (42) to obtain

$$
\begin{aligned}
\Pi_{k,t}(i) =& P_{k,t}(i)Y_{k,t}(i) - \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_t}{P_t}\right)^{1-\delta}\frac{Y_{k,t}(i)}{A_t A_{k,t}}P_t \\
=& \left[P_{k,t}(i) - \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_t}{P_t}\right)^{1-\delta}\frac{1}{A_t A_{k,t}}P_t\right] D_{k,t}\left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta}\left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Y_t. \qquad (43)
\end{aligned}
$$

Lastly, we derive the first order condition of firm ik's profit maximization problem when prices are flexible

$$
\frac{P_{k,t}(i)}{P_t} = \left(\frac{\theta}{\theta-1}\right)\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_t}{P_t}\right)^{1-\delta}\frac{1}{A_t A_{k,t}}, \qquad (44)
$$

where $\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{k,t}}{P_t}\right)^{1-\delta}\frac{1}{A_t A_{k,t}}$ is the real marginal cost and $\theta/(\theta-1)$ is the mark-up. It follows that when prices are flexible, the equilibrium price is identical across firms within a sector.

2.2 Price setting

Firm ik that can adjust its price at time t chooses its price to maximize the expected discounted profits as follows

$$
\max_{P_{k,t}(i)} E_t \sum_{s=0}^{\infty} \alpha_k^s Q_{t,t+s} \Pi_{k,t+s}(i),
$$

where $Q_{t,t+s} = \prod_{\tau=0}^{s-1} Q_{t+\tau,t+\tau+1}$ is the nominal stochastic discount factor between time t and $t+s$, and

$$
\begin{aligned}
\Pi_{k,t+s}(i) &= P_{k,t}(i)\, Y_{k,t+s}(i) - W_{t+s} H_{t+s}(i) - P_{t+s} Z_{k,t+s}(i), \\
&= \left[P_{k,t}(i) - \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{t+s}}{P_{t+s}}\right)^{1-\delta}\frac{1}{A_{t+s}A_{k,t+s}}P_{t+s}\right] D_{k,t+s}\left(\frac{P_{k,t}(i)}{P_{k,t+s}}\right)^{-\theta}\left(\frac{P_{k,t+s}}{P_{t+s}}\right)^{-\eta} Y_{t+s},
\end{aligned}
$$

is the nominal profit at time $t+s$ under the condition that the firm's price has not been updated since time t. The nominal profit $\Pi_{k,t+s}(i)$ can be derived as $\Pi_{k,t}(i)$ in (43). So the the first order condition of the firm's profit maximization problem is given by

$$
E_t \sum_{s=0}^{\infty} \alpha_k^s Q_{t,t+s} D_{k,t+s}\left(\frac{P_{k,t}^*}{P_{k,t+s}}\right)^{-\theta}\left(\frac{P_{k,t+s}}{P_{t+s}}\right)^{-\eta} Y_{t+s}\left[P_{k,t}^* - \left(\frac{\theta}{\theta-1}\right) MC_{k,t+s}\right] = 0,
$$

where the nominal marginal cost in period $t+s$ is given by

$$
MC_{k,t+s} = P_{t+s}\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{t+s}}{P_{t+s}}\right)^{1-\delta}\frac{1}{A_{t+s}A_{k,t+s}}.
$$

2.3 Equilibrium conditions

Here we collect the equilibrium conditions specific to this specification. The other equilibrium conditions are the same as for the baseline specification.

2.3.1 CES aggregates, market clearing conditions, and definitions

- Aggregate labor input (hours index)

$$H_t = \int_0^1 H_t(i)di.$$

2.3.2 Household

- Optimal labor supply condition

$$\frac{W_t}{P_t} = \Xi_t \omega_k H_t^{\varphi} C_t.$$

2.3.3 Firms

- Production function

$$Y_{k,t}(i) = A_t A_{k,t} H_t(i)^{1-\delta} Z_{k,t}(i)^{\delta}.$$

- Cost minimization

$$\frac{W_t}{P_t} = \frac{1-\delta}{\delta} \frac{Z_{k,t}(i)}{H_t(i)}.$$

- Nominal marginal cost

$$MC_{k,t+s} = P_{t+s} \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta} \left(\frac{W_{t+s}}{P_{t+s}} \right)^{1-\delta} \frac{1}{A_{t+s} A_{k,t+s}}.$$

- Frictionless optimal price

$$P_{k,t}^{**} = \left(\frac{\theta}{\theta-1} \right) MC_{k,t}.$$

2.4 Steady state

The steady state is the same as for the baseline model.

2.5 Log-linear approximation

2.5.1 CES aggregates, market clearing conditions, and definitions

- Aggregate labor input (hours index)

$$h_t = \int_0^1 h_{k,t}(i)di.$$

2.5.2 Household

- Optimal labor supply condition

$$w_t - p_t = \varphi h_t + c_t + \xi_t. \tag{45}$$

2.5.3 Firms

- Firm ik's production function

$$y_{k,t}(i) = a_t + a_{k,t} + (1-\delta) h_t(i) + \delta z_{k,t}(i).$$

Using the definition of sectoral output (18) and sectoral labor input (16), we can show that

$$y_{k,t} = a_t + a_{k,t} + (1-\delta)\frac{1}{n_k}\int_{\mathcal{I}_k} h_t(i)\,di + \frac{\delta}{n_k}\int_{\mathcal{I}_k} z_{k,t}(i)\,di.$$

It follows that

$$\begin{aligned} y_t = \sum_{k=1}^{K} n_k y_{k,t} &= a_t + \sum_{k=1}^{K} n_k a_{k,t} + (1-\delta)\,h_t + \delta \sum_{k=1}^{K}\int_{\mathcal{I}_k} z_{k,t}(i)\,di \\ &= a_t + \sum_{k=1}^{K} n_k a_{k,t} + (1-\delta)\,h_t + \delta z_t, \end{aligned} \tag{46}$$

where we used the definition of aggregate intermediate input (17).

- Cost minimization

$$w_t - p_t = z_{k,t}(i) - h_t(i).$$

Note that integrating both sides over $\mathcal{I}_k$ and dividing by n_k leads to

$$w_t - p_t = \frac{1}{n_k}\int_{\mathcal{I}_k} z_{k,t}(i)\,di - \frac{1}{n_k}\int_{\mathcal{I}_k} h_t(i)\,di.$$

From the definition of aggregate intermediate input (17), it follows that

$$w_t - p_t = z_t - h_t. \tag{47}$$

- Nominal marginal cost

$$mc_{k,t} = (1-\delta)(w_t - p_t) - a_{k,t} - a_t + p_t.$$

2.5.4 Derivation of the Phillips curve

As in the baseline model, we can derive the sectoral Phillips curve (PC)

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}(mc_{k,t} - p_{k,t}) \tag{48}$$

Now let us show how the marginal cost is determined. First combine (45) with (47) to obtain

$$z_t = (1+\varphi)\,h_t + c_t + \xi_t,$$

which can be plugged in (46) to obtain

$$y_t = a_t + \sum_{k=1}^{K} n_k a_{k,t} + (1+\delta\varphi)\,h_t + \delta c_t + \delta\xi_t,$$

or

$$h_t = \frac{1}{1+\delta\varphi}y_t - \frac{\delta}{1+\delta\varphi}c_t - \frac{\delta}{1+\delta\varphi}\xi_t - \frac{1}{1+\delta\varphi}a_t - \frac{1}{1+\delta\varphi}\bar{a}_t,$$

where

$$\bar{a}_t = \sum_{k=1}^{K} n_k a_{k,t}.$$

It follows that

$$\begin{aligned} mc_{k,t} &= (1-\delta)(w_t - p_t) - a_{k,t} - a_t + p_t \\ &= (1-\delta)(\varphi h_t + c_t + \xi_t) - a_{k,t} - a_t + p_t \\ &= \frac{(1-\delta)\varphi}{1+\delta\varphi} y_t + \frac{(1-\delta)}{1+\delta\varphi} c_t + \frac{(1-\delta)}{1+\delta\varphi} \xi_t - \frac{1+\varphi}{1+\delta\varphi} a_t - a_{k,t} - \frac{(1-\delta)\varphi}{1+\delta\varphi} \bar{a}_t + p_t. \end{aligned}$$

Consequently, the sectoral PC (48) can be written as

$$\begin{aligned} \pi_{k,t} = &\beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k} \left\{ \frac{(1-\delta)\varphi}{1+\delta\varphi} y_t + \frac{(1-\delta)}{1+\delta\varphi} c_t + \frac{1-\delta}{1+\delta\varphi} \xi_t \right. \\ &\left. - \frac{1+\varphi}{1+\delta\varphi} a_t - a_{k,t} - \frac{(1-\delta)\varphi}{1+\delta\varphi} \bar{a}_t + (p_t - p_{k,t}) \right\}. \end{aligned}$$

Using the resource constraint

$$y_t = (1-\psi) c_t + \psi z_t, \tag{49}$$

we can substitute out y_t from the marginal cost as

$$mc_{k,t} = \frac{(1-\delta)[(1-\psi)\varphi + 1]}{1+\delta\varphi} c_t + \frac{(1-\delta)\psi\varphi}{1+\delta\varphi} z_t + \frac{(1-\delta)}{1+\delta\varphi} \xi_t - \frac{1+\varphi}{1+\delta\varphi} a_t - a_{k,t} - \frac{(1-\delta)\varphi}{1+\delta\varphi} \bar{a}_t + p_t.$$

Substituting out $p_t - p_{k,t}$ from the sectoral PC as well, we have

$$\begin{aligned} \pi_{k,t} = &\beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k} \left\{ \frac{1}{\eta} c_{k,t} + \left[\frac{(1-\delta)[(1-\psi)\varphi + 1]}{1+\delta\varphi} - \frac{1}{\eta} \right] c_t + \frac{(1-\delta)\psi\varphi}{1+\delta\varphi} z_t \right. \\ &\left. + \frac{1-\delta}{1+\delta\varphi} \xi_t - \frac{1+\varphi}{1+\delta\varphi} a_t - a_{k,t} - \frac{(1-\delta)\varphi}{1+\delta\varphi} \bar{a}_t - \frac{1}{\eta} d_{k,t} \right\}. \end{aligned}$$

2.5.5 Frictionless optimal price

When firms can change prices continuously, the log-linearized optimal price is given by

$$p_{k,t}^{**} = \frac{(1-\delta)\varphi}{1+\delta\varphi} y_t + \frac{(1-\delta)}{1+\delta\varphi} c_t + \frac{(1-\delta)}{1+\delta\varphi} \xi_t - \frac{1+\varphi}{1+\delta\varphi} a_t - a_{k,t} - \frac{(1-\delta)\varphi}{1+\delta\varphi} \bar{a}_t + p_t.$$

Using (49) and the definition that $m_t = p_t + c_t$, we can rewrite the above equation as

$$\begin{aligned} (1+\delta\varphi) p_{k,t}^{**} = &(1-\delta)[\varphi(1-\psi)+1] m_t + (1-\delta)\varphi\psi z_t + (1-\delta)\xi_t - (1+\varphi)(a_t + a_{k,t}) \\ &\{1 - (1-\delta)[\varphi(1-\psi)+1]\} p_t. \end{aligned}$$

When $\delta = 0$, $\psi = 0$ and it follows that

$$p_{k,t}^{**} = (\varphi + 1) m_t + (1-\delta)\xi_t - (1+\varphi)(a_t + a_{k,t}) - \varphi p_t.$$

2.6 Log-linear equilibrium conditions

Here we present the log-linearized equilibrium conditions necessary to characterize the equilibrium dynamics of the variables of interest. They are the following aggregate variables

$$\{c_t, \pi_t, i_t, m_t, h_t, (w_t - p_t)\}$$

and the following sectoral variables

$$\{c_{k,t}, \pi_{k,t}\}_{k=1}^{K}.$$

The following $6 + (K + 2)$ equations determine the equilibrium dynamics of those variables

$$c_t = E_t [c_{t+1}] - (i_t - E_t \pi_{t+1}) + (\gamma_t - E_t \gamma_{t+1}), \qquad \text{(IS equation)}$$

$$w_t - p_t = \varphi h_t + c_t + \xi_t, \qquad \text{(agg. labor supply)}$$

$$(1 - \psi) c_t + \psi z_t = a_t + \sum_{k=1}^{K} n_k a_{k,t} + (1 - \delta) h_t + \delta z_t, \qquad \text{(agg. resource constraint)}$$

$$w_t - p_t = z_t - h_t, \qquad \text{(agg. cost-minimization relation)}$$

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \frac{(1 - \alpha_k)(1 - \alpha_k \beta)}{\alpha_k} \left\{ \frac{1}{\eta} c_{k,t} + \left[\frac{(1 - \delta)[(1 - \psi)\varphi + 1]}{1 + \delta\varphi} - \frac{1}{\eta} \right] c_t + \frac{(1 - \delta)\psi\varphi}{1 + \delta\varphi} z_t \right.$$
$$\left. + \frac{1 - \delta}{1 + \delta\varphi} \xi_t - \frac{1 + \varphi}{1 + \delta\varphi} a_t - a_{k,t} - \frac{(1 - \delta)\varphi}{1 + \delta\varphi} \bar{a}_t - \frac{1}{\eta} d_{k,t} \right\}. \qquad \text{(sectoral PC)}$$

$$\pi_t = \sum_{k=1}^{K} n_k \pi_{k,t}, \qquad \text{(agg. inflation)}$$

$$\Delta (c_{k,t+1} - c_{t+1}) = -\eta (\pi_{k,t+1} - \pi_{t+1}) + \Delta d_{k,t+1}. \qquad \text{(sectoral consumption)}$$

For monetary policy, we consider a Taylor-type interest rate rule

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\phi_\pi \pi_t + \phi_c c_t) + \mu_t,$$

or an exogenous stochastic process for nominal aggregate consumption

$$m_t = p_t + c_t.$$

3 Firm-specific labor market

This section describes a specification where the labor market is firm-specific. We describe only differences from the baseline specification.

3.1 Model

3.1.1 Representative household

The representative household maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \Gamma_t \left[\log\left(C_t\right) - \Xi_t \sum_{k=1}^{K} \omega_k \int_{\mathcal{I}_k} \frac{H_{k,t}\left(i\right)^{1+\varphi}}{1+\varphi} di \right] \right\},$$

subject to the flow budget constraint

$$P_t C_t + E_t\left[Q_{t,t+1} B_{t+1}\right] = B_t + \sum_{k=1}^{K} \int_{\mathcal{I}_k} W_{k,t}\left(i\right) H_{k,t}\left(i\right) di + \sum_{k=1}^{K} \int_{\mathcal{I}_k} \Pi_{k,t}\left(i\right) di,$$

where $H_{k,t}\left(i\right)$ is the labor supply to firm i in sector k and $W_{k,t}\left(i\right)$ is the wage rate.

The optimality conditions for the household's utility maximization problem are given by

$$Q_{t,t+1} = \beta \left(\frac{\Gamma_{t+1}}{\Gamma_t}\right) \left(\frac{C_{t+1}}{C_t}\right)^{-1} \left(\frac{P_t}{P_{t+1}}\right),$$

$$\frac{W_{k,t}\left(i\right)}{P_t} = \Xi_t \omega_k H_{k,t}\left(i\right)^{\varphi} C_t.$$

3.1.2 Firms

Consider firm i in sector k (firm ik), where $i \in \mathcal{I}_k$. Its production function is given by

$$Y_{k,t}\left(i\right) = A_t A_{k,t} H_{k,t}(i)^{1-\delta} Z_{k,t}\left(i\right)^{\delta}. \tag{50}$$

Firm ik's nominal profit is given by

$$\Pi_{k,t}(i) = P_{k,t}(i) Y_{k,t}(i) - W_{k,t}\left(i\right) H_{k,t}(i) - P_t Z_{k,t}(i). \tag{51}$$

The cost minimization problem implies the following relationship

$$Z_{k,t}(i) = \frac{\delta}{1-\delta} \frac{W_{k,t}\left(i\right)}{P_t} H_{k,t}(i),$$

which can be substituted into the profit function (51) as

$$\Pi_{k,t}(i) = P_{k,t}(i) Y_{k,t}(i) - \frac{1}{1-\delta} W_{k,t}\left(i\right) H_{k,t}\left(i\right). \tag{52}$$

From the production function (40), we can write $H_{k,t}(i)$ as a function of the firm's output

$$H_{k,t}(i) = \frac{Y_{k,t}(i)}{A_t A_{k,t}} \left(\frac{\delta}{1-\delta}\right)^{-\delta} \left(\frac{W_{k,t}\left(i\right)}{P_t}\right)^{-\delta}.$$

Substitute this again into the profit function (52) to obtain

$$\Pi_{k,t}(i) = P_{k,t}(i) Y_{k,t}(i) - \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta}\right)^{-\delta} \left(\frac{W_{k,t}\left(i\right)}{P_t}\right)^{1-\delta} \frac{Y_{k,t}(i)}{A_t A_{k,t}} P_t$$

$$= \left[P_{k,t}(i) - \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{k,t}(i)}{P_t}\right)^{1-\delta}\frac{1}{A_t A_{k,t}} P_t \right] D_{k,t}\left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta}\left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Y_t.$$

Lastly, we derive the first order condition of firm ik's profit maximization problem when prices are flexible

$$\frac{P_{k,t}(i)}{P_t} = \left(\frac{\theta}{\theta-1}\right)\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{k,t}(i)}{P_t}\right)^{1-\delta}\frac{1}{A_t A_{k,t}},$$

where $\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{k,t}}{P_t}\right)^{1-\delta}\frac{1}{A_t A_{k,t}}$ is the real marginal cost and $\theta/(\theta-1)$ is the mark-up.

3.2 Price setting

Firm ik that can adjust its price at time t chooses its price to maximize the expected discounted profits as follows

$$\max_{P_{k,t}(i)} E_t \sum_{s=0}^{\infty} \alpha_k^s Q_{t,t+s} \Pi_{k,t+s}(i),$$

where $Q_{t,t+s} = \prod_{\tau=0}^{s-1} Q_{t+\tau,t+\tau+1}$ is the nominal stochastic discount factor between time t and $t+s$, and

$$\begin{aligned}\Pi_{k,t+s}(i) &= P_{k,t}(i) Y_{k,t+s}(i) - W_{k,t+s}(i) H_{k,t+s}(i) - P_{t+s} Z_{k,t+s}(i), \\ &= \left[P_{k,t}(i) - \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{k,t+s}(i)}{P_{t+s}}\right)^{1-\delta}\frac{1}{A_{t+s} A_{k,t+s}} P_{t+s} \right] D_{k,t+s}\left(\frac{P_{k,t}(i)}{P_{k,t+s}}\right)^{-\theta}\left(\frac{P_{k,t+s}}{P_{t+s}}\right)^{-\eta} Y_{t+s},\end{aligned}$$

is the nominal profit at time $t+s$ under the condition that the firm's price has not been updated since time t. So the the first order condition of the firm's profit maximization problem is given by

$$E_t \sum_{s=0}^{\infty} \alpha_k^s Q_{t,t+s} D_{k,t+s}\left(\frac{P_{k,t}^*}{P_{k,t+s}}\right)^{-\theta}\left(\frac{P_{k,t+s}}{P_{t+s}}\right)^{-\eta} Y_{t+s}\left[P_{k,t}^* - \left(\frac{\theta}{\theta-1}\right) MC_{k,t+s}(i) \right] = 0,$$

where the nominal marginal cost in period $t+s$ is given by

$$MC_{k,t+s}(i) = P_{t+s}\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{k,t+s}(i)}{P_{t+s}}\right)^{1-\delta}\frac{1}{A_{t+s} A_{k,t+s}}.$$

3.3 Equilibrium conditions

Here we collect the equilibrium conditions of the model.

3.3.1 CES aggregates, market clearing conditions, and definitions

- Aggregate wage

$$W_t = \sum_{k=1}^{K} \int_{\mathcal{I}_k} W_{k,t}(i)\, di.$$

3.3.2 Household

- Optimal labor supply condition

$$\frac{W_{k,t}(i)}{P_t} = \Xi_t \omega_k H_{k,t}(i)^{\varphi} C_t.$$

3.3.3 Firms

- Cost minimization

$$\frac{W_{k,t}(i)}{P_t} = \frac{1-\delta}{\delta}\frac{Z_{k,t}(i)}{H_{k,t}(i)}.$$

- Nominal marginal cost

$$MC_{k,t+s}(i) = P_{t+s}\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{k,t+s}(i)}{P_{t+s}}\right)^{1-\delta}\frac{1}{A_{t+s}A_{k,t+s}}.$$

3.4 Steady state

The steady state is the same as for the baseline model.

3.5 Log-linear approximation

3.5.1 CES aggregates, market clearing conditions, and definitions

- Aggregate wage

$$w_t = \sum_{k=1}^{K}\int_{\mathcal{I}_k} w_{k,t}(i)\,di.$$

3.5.2 Household

- Optimal labor supply condition

$$w_{k,t}(i) - p_t = \varphi h_{k,t}(i) + c_t + \xi_t. \tag{53}$$

  Note that this is integrated over sector k as

$$\frac{1}{n_k}\int_{\mathcal{I}_k} w_{k,t}(i)\,di - p_t = \varphi h_{k,t} + c_t + \xi_t,$$

  and thus the aggregate labor supply condition is derived as

$$w_t - p_t = \varphi h_t + c_t + \xi_t.$$

3.5.3 Firms

- Cost minimization

$$w_{k,t}(i) - p_t = z_{k,t}(i) - h_{k,t}(i). \tag{54}$$

  Note that integrating both sides over $\mathcal{I}_k$ and dividing by n_k leads to

$$\frac{1}{n_k}\int_{\mathcal{I}_k} w_{k,t}(i)\,di - p_t = \frac{1}{n_k}\int_{\mathcal{I}_k} z_{k,t}(i)\,di - h_{k,t},$$

  which can be further summed over all the sectors so that

$$w_t - p_t = z_t - h_t.$$

- Nominal marginal cost

$$mc_{k,t}(i) = (1-\delta)(w_{k,t}(i) - p_t) - a_{k,t} - a_t + p_t.$$

3.5.4 Derivation of the Phillips curve

Let us first show how the marginal cost is determined. Suppose that firm ik sets its price at $p^*_{k,t}$ and cannot readjust the price again. Combine (54) with (53) to obtain

$$z_{k,t}(i) = (1+\varphi)h_{k,t}(i) + c_t + \xi_t.$$

Also from the production function, it follows that

$$\begin{aligned} h_{k,t}(i) &= \frac{1}{1+\delta\varphi} y_{k,t}(i) - \frac{\delta}{1+\delta\varphi} c_t - \frac{\delta}{1+\delta\varphi} \xi_t - \frac{1}{1+\delta\varphi} a_{k,t} - \frac{1}{1+\delta\varphi} a_t \\ &= -\frac{\theta}{1+\delta\varphi}\left(p^*_{k,t} - p_{k,t}\right) + \frac{1}{1+\delta\varphi} y_{k,t} - \frac{\delta}{1+\delta\varphi} c_t - \frac{\delta}{1+\delta\varphi} \xi_t - \frac{1}{1+\delta\varphi} a_{k,t} - \frac{1}{1+\delta\varphi} a_t, \end{aligned}$$

where we used the demand function for firm ik's output to eliminate $y_{k,t}(i)$. Therefore, the nominal marginal cost of firm ik in period $t+s$ is given by

$$\begin{aligned} mc_{k,t+s}(i) &= (1-\delta)(w_{k,t+s}(i) - p_{t+s}) - a_{k,t+s} - a_{t+s} + p_{t+s} \\ &= (1-\delta)(\varphi h_{k,t+s}(i) + c_{t+s} + \xi_{t+s}) - a_{k,t+s} - a_{t+s} + p_{t+s} \\ &= -\frac{(1-\delta)\theta\varphi}{1+\delta\varphi}\left(p^*_{k,t} - p_{k,t+s}\right) + \frac{(1-\delta)\varphi}{1+\delta\varphi} y_{k,t+s} + \frac{(1-\delta)}{1+\delta\varphi} c_{t+s} + \frac{(1-\delta)}{1+\delta\varphi} \xi_{t+s} \\ &\quad - \frac{1+\varphi}{1+\delta\varphi} a_{t+s} - \frac{1+\varphi}{1+\delta\varphi} a_{k,t+s} + p_{t+s}. \end{aligned}$$

Let

$$mc_{k,t+s}(i) = -\frac{(1-\delta)\theta\varphi}{1+\delta\varphi} p^*_{k,t} + \frac{(1-\delta)\theta\varphi + 1 + \delta\varphi}{1+\delta\varphi} p_{k,t+s} + \widetilde{mc}_{k,t+s},$$

where

$$\widetilde{mc}_{k,t+s} = \frac{(1-\delta)\varphi}{1+\delta\varphi} y_{k,t+s} + \frac{(1-\delta)}{1+\delta\varphi} c_{t+s} + \frac{(1-\delta)}{1+\delta\varphi} \xi_{t+s} - \frac{1+\varphi}{1+\delta\varphi} a_{t+s} - \frac{1+\varphi}{1+\delta\varphi} a_{k,t+s} - (p_{k,t+s} - p_{t+s}).$$

From the log-linearized first order condition by the price-setting firm, the optimal price is given by

$$p^*_{k,t} = (1-\alpha_k\beta) E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s mc_{k,t+s}(i),$$

or

$$p^*_{k,t} = (1-\alpha_k\beta) p_{k,t} + \frac{(1-\alpha_k\beta)(1+\delta\varphi)}{1+\delta\varphi + (1-\delta)\theta\varphi} \widetilde{mc}_{k,t} + \alpha_k\beta E_t\left[p^*_{k,t+1}\right].$$

Loglinearizing the equation for the price index leads to

$$p_{k,t} = (1-\alpha_k)p^*_{k,t} + \alpha_k p_{k,t-1}.$$

Therefore, we can derive the sectoral Phillips curve

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k} \frac{(1+\delta\varphi)}{1+\delta\varphi+(1-\delta)\theta\varphi} \widetilde{mc}_{k,t},$$

or

$$\begin{aligned}\pi_{k,t} =& \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k} \frac{(1+\delta\varphi)}{1+\delta\varphi+(1-\delta)\theta\varphi} \left\{ \frac{(1-\delta)\varphi}{1+\delta\varphi} y_{k,t} + \frac{(1-\delta)}{1+\delta\varphi} c_t + \frac{(1-\delta)}{1+\delta\varphi} \xi_t \right. \\ & \left. - \frac{1+\varphi}{1+\delta\varphi} a_t - \frac{1+\varphi}{1+\delta\varphi} a_{k,t} + (p_t - p_{k,t}) \right\}.\end{aligned}$$

We can write the sectoral PC in terms of sectoral consumption instead of sectoral output as in the baseline model. Substituting out $y_{k,t}$ and $p_t - p_{k,t}$ from the sectoral PC, we have

$$\begin{aligned}\pi_{k,t} =& \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k} \frac{(1+\delta\varphi)}{1+\delta\varphi+(1-\delta)\theta\varphi} \left\{ \left[\frac{(1-\delta)\varphi}{1+\delta\varphi} + \frac{1}{\eta} \right] c_{k,t} + \left[\frac{(1-\delta)(1-\psi\varphi)}{1+\delta\varphi} - \frac{1}{\eta} \right] c_t \right. \\ & \left. + \frac{(1-\delta)\psi\varphi}{1+\delta\varphi} z_t + \frac{1-\delta}{1+\delta\varphi} \xi_t - \frac{1+\varphi}{1+\delta\varphi} a_t - \frac{1+\varphi}{1+\delta\varphi} a_{k,t} - \frac{1}{\eta} d_{k,t} \right\}.\end{aligned}$$

3.5.5 Frictionless optimal price

When firms can change prices continuously, the log-linearized optimal price is given by

$$\begin{aligned}p^{**}_{k,t} =& mc_{k,t} \\ =& -\frac{(1-\delta)\theta\varphi}{1+\delta\varphi} \left(p^{**}_{k,t} - p_{k,t}\right) + \frac{(1-\delta)\varphi}{1+\delta\varphi} y_{k,t} + \frac{(1-\delta)}{1+\delta\varphi} c_t + \frac{(1-\delta)}{1+\delta\varphi} \xi_t - \frac{1+\varphi}{1+\delta\varphi} a_t - \frac{1+\varphi}{1+\delta\varphi} a_{k,t} + p_t,\end{aligned}$$

or

$$[1+\delta\varphi+(1-\delta)\theta\varphi]\, p^{**}_{k,t} = (1-\delta)\theta\varphi p_{k,t} + (1-\delta)\varphi y_{k,t} + (1-\delta)c_t + (1-\delta)\xi_t - (1+\varphi)(a_t + a_{k,t}) + (1+\delta\varphi)p_t.$$

Using (32), (21), and the definition that $m_t = p_t + c_t$, we can rewrite the above equation as

$$\begin{aligned}[1+\delta\varphi+(1-\delta)\theta\varphi]\, p^{**}_{k,t} =& (1-\delta)(\theta-\eta)\varphi p_{k,t} + \{(1-\delta)[\varphi\eta - \varphi(1-\psi) - 1] + 1 + \delta\varphi\} p_t \\ & + (1-\delta)\varphi\psi z_t + (1-\delta)[\varphi(1-\psi)+1] m_t + (1-\delta)\varphi d_{k,t} + (1-\delta)\xi_t - (1+\varphi)(a_t + a_{k,t}).\end{aligned}$$

When $\delta = 0$, $\psi = 0$ and it follows that

$$(1+\theta\varphi)\, p^{**}_{k,t} = (\theta-\eta)\varphi p_{k,t} + (\varphi\eta - \varphi) p_t + (\varphi+1) m_t + \varphi d_{k,t} + \xi_t - (1+\varphi)(a_t + a_{k,t}).$$

3.6 Log-linear equilibrium conditions

Here we present the loglinearized equilibrum conditions necessary to characterize the equilibrium dynamics of the variables of interest. They are the following aggregate variables

$$\{c_t, \pi_t, i_t, m_t, h_t, (w_t - p_t)\}$$

and the following sectoral variables

$$\{c_{k,t}, \pi_{k,t}\}_{k=1}^{K}.$$

The following $6+(K+2)$ equations determine the equilibrium dynamics of those variables

$$c_t = E_t\left[c_{t+1}\right] - \left(i_t - E_t\pi_{t+1}\right) + \left(\gamma_t - E_t\gamma_{t+1}\right), \qquad \text{(IS equation)}$$

$$w_t - p_t = \varphi h_t + c_t + \xi_t, \qquad \text{(agg. labor supply)}$$

$$(1-\psi)\, c_t + \psi z_t = a_t + \sum_{k=1}^{K} n_k a_{k,t} + (1-\delta)\, h_t + \delta z_t, \qquad \text{(agg. resource constraint)}$$

$$w_t - p_t = z_t - h_t, \qquad \text{(agg. cost-minimization relation)}$$

$$\begin{aligned}\pi_{k,t} = \beta E_t\pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\frac{(1+\delta\varphi)}{1+\delta\varphi+(1-\delta)\theta\varphi}\left\{\left[\frac{(1-\delta)\varphi}{1+\delta\varphi}+\frac{1}{\eta}\right]c_{k,t} + \left[\frac{(1-\delta)(1-\psi\varphi)}{1+\delta\varphi}-\frac{1}{\eta}\right]c_t\right. \\ \left. +\frac{(1-\delta)\psi\varphi}{1+\delta\varphi}z_t + \frac{1-\delta}{1+\delta\varphi}\xi_t - \frac{1+\varphi}{1+\delta\varphi}a_t - \frac{1+\varphi}{1+\delta\varphi}a_{k,t} - \frac{1}{\eta}d_{k,t}\right\}, \qquad \text{(sectoral PC)}\end{aligned}$$

$$\pi_t = \sum_{k=1}^{K} n_k \pi_{k,t}, \qquad \text{(agg. inflation)}$$

$$\Delta\left(c_{k,t+1} - c_{t+1}\right) = -\eta\left(\pi_{k,t+1} - \pi_{t+1}\right) + \Delta d_{k,t+1}. \qquad \text{(sectoral consumption)}$$

For monetary policy, we consider a Taylor-type interest rate rule

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi \pi_t + \phi_c c_t\right) + \mu_t,$$

or an exogenous stochastic process for nominal aggregate consumption

$$m_t = p_t + c_t.$$

4 Extension with habit formation in sectoral consumption

This section describes an extension of the baseline model that features habit formation in sectoral consumption. We describe only what changes from the baseline model.

4.1 Model

4.1.1 Representative household

The representative household derives utility from an aggregate consumption index X_t defined by

$$X_t = \left[\sum_{k=1}^{K} \left(n_k D_{k,t}\right)^{1/\eta}\left(C_{k,t} - \tau_k \tilde{C}_{k,t-1}\right)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)},$$

where $C_{k,t}$ is consumption of sector k goods, $\tilde{C}_{k,t-1}$ is the consumption index of sector k goods in period $t-1$, which the household takes as exogenously given, and $0 \le \tau_k \le 1$ is a parameter that determines the extent of habit formation in consumption of sector k goods. In equilibrium, we have that $\tilde{C}_{k,t-1} = C_{k,t-1}$. The optimal level of $C_{k,t}$ given $\tilde{C}_{k,t-1}$ is a solution to the expenditure minimization problem and given by

$$C_{k,t} = n_k D_{k,t}\left(\frac{P_{k,t}}{P_t}\right)^{-\eta} X_t + \tau_k \tilde{C}_{k,t-1},$$

where the aggregate price level is given by

$$P_t = \left[\sum_{k=1}^{K} (n_k D_{k,t}) \, P_{k,t}^{1-\eta} \right]^{1/(1-\eta)}.$$

At the optimum, we have that

$$P_t X_t = \sum_{k=1}^{K} P_{k,t} \left(C_{k,t} - \tau_k \tilde{C}_{k,t-1} \right).$$

The representative household maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \Gamma_t \left[\log (X_t) - \Xi_t \sum_{k=1}^{K} \omega_k \frac{H_{k,t}^{1+\varphi}}{1+\varphi} \right] \right\},$$

subject to the flow budget constraint

$$\sum_{k=1}^{K} P_{k,t} C_{k,t} + E_t \left[Q_{t,t+1} B_{t+1} \right] = B_t + \sum_{k=1}^{K} W_{k,t} H_{k,t} + \sum_{k=1}^{K} \int_{\mathcal{I}_k} \Pi_{k,t} (i) \, di.$$

Note that the sectoral demand shocks are relative and thus subject to the following constraint

$$\sum_{k=1}^{K} n_k D_{k,t} = 1,$$

where $D_{k,t} > 0$ for $k = 1, 2, \cdots, K$.

Sectoral consumption is defined as a composite of differentiated goods in each sector

$$C_{k,t} = \left[\left(\frac{1}{n_k} \right)^{1/\theta} \int_{\mathcal{I}_k} C_{k,t}(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)}.$$

Then the sectoral price index is derived as

$$P_{k,t} = \left(\frac{1}{n_k} \int_{\mathcal{I}_k} P_{k,t}(i)^{1-\theta} di \right)^{1/(1-\theta)}.$$

Given $C_{k,t}$, the optimal demand for type-i good in sector k, $C_{k,t}(i)$, would be

$$C_{k,t}(i) = \frac{1}{n_k} \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\theta} C_{k,t}.$$

The optimality conditions for the household's utility maximization problem are given by

$$Q_{t,t+1} = \beta \left(\frac{\Gamma_{t+1}}{\Gamma_t} \right) \left(\frac{X_{t+1}}{X_t} \right)^{-1} \left(\frac{P_t}{P_{t+1}} \right),$$

$$\frac{W_{k,t}}{P_t} = \Xi_t \omega_k H_{k,t}^{\varphi} X_t.$$

Lastly, we define aggregate consumption as the real consumption expenditure

$$C_t = \frac{1}{P_t}\sum_{k=1}^{K} P_{k,t}C_{k,t},$$

which implies that in equilibrium

$$P_tC_t = P_tX_t + \sum_{k=1}^{K} \tau_k P_{k,t}C_{k,t-1}.$$

Also note that

$$P_tC_t = \sum_{k=1}^{K} P_{k,t}C_{k,t} = \sum_{k=1}^{K}\int_{\mathcal{I}_k} P_{k,t}\left(i\right)C_{k,t}\left(i\right)di. \tag{55}$$

4.1.2 Firms

Consider firm i in sector k (firm ik), where $i \in \mathcal{I}_k$. Its production function is given by

$$Y_{k,t}\left(i\right) = A_tA_{k,t}H_{k,t}(i)^{1-\delta}Z_{k,t}\left(i\right)^{\delta}, \tag{56}$$

where $Z_{k,t}\left(i\right)$ is the firm ik's usage of other goods as intermediate inputs defined as follows

$$Z_{k,t}(i) = \left[\sum_{k'=1}^{K}\left(n_{k'}D_{k',t}\right)^{1/\eta} Z_{k,k',t}(i)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)},$$

where the sectoral intermediate input, or the amount of firm ik's usage of sector k' goods as intermediate inputs, is given by

$$Z_{k,k',t}(i) = \left[\left(\frac{1}{n_{k'}}\right)^{1/\theta}\int_{\mathcal{I}_{k'}} Z_{k,k',t}(i,i')^{(\theta-1)/\theta}di'\right]^{\theta/(\theta-1)},$$

with the quantity of input produced by $i'k'$ firm and purchased by ik firm $Z_{k,k',t}\left(i,i'\right)$. The cost-minimization problem of firm ik yields the optimal demand for intermediate inputs as follows

$$Z_{k,k',t}(i) = n_{k'}D_{k',t}\left(\frac{P_{k',t}}{P_t}\right)^{-\eta} Z_{k,t}(i),$$

and

$$Z_{k,k',t}(i,i') = \frac{1}{n_{k'}}\left(\frac{P_{k',t}(i')}{P_{k',t}}\right)^{-\theta} Z_{k,k',t}(i).$$

Note that the firm ik's total output is the sum of household consumption and the intermediate input

$$Y_{k,t}(i) = C_{k,t}(i) + \underbrace{\sum_{k'=1}^{K}\int_{\mathcal{I}_{k'}} Z_{k',k,t}(i',i)di'}_{\substack{\text{total demand for the intermediate}\\ \text{input produced by firm } ik}}$$

$$= X_tD_{k,t}\left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta}\left(\frac{P_{k,t}}{P_t}\right)^{-\eta} + \frac{\tau_k}{n_k}\left(\frac{P_{k,t}\left(i\right)}{P_{k,t}}\right)^{-\theta}C_{k,t-1} + \sum_{k'=1}^{K}\int_{\mathcal{I}_{k'}} D_{k,t}\left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta}\left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Z_{k',t}(i')di'$$

$$= \Omega_{k,t} D_{k,t} \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\theta} \left(\frac{P_{k,t}}{P_t} \right)^{-\eta}, \tag{57}$$

where $Z_t = \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} Z_{k',t}(i')di'$ and $\Omega_{k,t}$ is the sector k habit-adjusted aggregate demand which is given by

$$\Omega_{k,t} = X_t + \left[n_k D_{k,t} \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} \right]^{-1} \tau_k C_{k,t-1} + Z_t.$$

We imposed the equilibrium condition $\tilde{C}_{k,t-1} = C_{k,t-1}$.

Firm ik's nominal profit is given by

$$\Pi_{k,t}(i) = P_{k,t}(i) Y_{k,t}(i) - W_{k,t} H_{k,t}(i) - P_t Z_{k,t}(i). \tag{58}$$

The cost minimization problem implies the following relationship

$$Z_{k,t}(i) = \frac{\delta}{1-\delta} \frac{W_{k,t}}{P_t} H_{k,t}(i),$$

which can be substituted into the profit function (58) as

$$\Pi_{k,t}(i) = P_{k,t}(i) Y_{k,t}(i) - \frac{1}{1-\delta} W_{k,t} H_{k,t}(i). \tag{59}$$

From the production function (56), we can write $H_{k,t}(i)$ as a function of the firm's output

$$H_{k,t}(i) = \frac{Y_{k,t}(i)}{A_t A_{k,t}} \left(\frac{\delta}{1-\delta} \right)^{-\delta} \left(\frac{W_{k,t}}{P_t} \right)^{-\delta}.$$

Substitute this again into the profit function (59) to obtain

$$\begin{aligned} \Pi_{k,t}(i) =& P_{k,t}(i) Y_{k,t}(i) - \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta} \left(\frac{W_{k,t}}{P_t} \right)^{1-\delta} \frac{Y_{k,t}(i)}{A_t A_{k,t}} P_t \\ =& \left[P_{k,t}(i) - \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta} \left(\frac{W_{k,t}}{P_t} \right)^{1-\delta} \frac{1}{A_t A_{k,t}} P_t \right] D_{k,t} \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\theta} \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} \Omega_{k,t}, \end{aligned} \tag{60}$$

where we eliminated $Y_{k,t}(i)$ using the total demand for firm ik's output (57).

Lastly, we define aggregate gross output as the nominal gross value of output by all the firms deflated by the aggregate price level

$$Y_t = \frac{1}{P_t} \sum_{k=1}^{K} \int_{\mathcal{I}_k} P_{k,t}(i) Y_{k,t}(i) di.$$

Note that the nominal gross expenditure on intermediate goods by all the firms is given by

$$\begin{aligned} \sum_{k=1}^{K} \int_{\mathcal{I}_k} P_{k,t}(i) \left[\sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} Z_{k',k,t}(i',i) di' \right] di &= \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} \left[\sum_{k=1}^{K} \int_{\mathcal{I}_k} P_{k,t}(i) Z_{k',k,t}(i',i) di \right] di' \\ &= \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} \left[\sum_{k=1}^{K} P_{k,t} Z_{k',k,t}(i') \right] di' \end{aligned}$$

$$= \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} P_t Z_{k',t}(i')\, di'$$
$$= P_t Z_t. \tag{61}$$

Therefore, from (55), (57), and (61), it follows that

$$Y_t = C_t + Z_t.$$

4.2 Price setting

Prices are set as in Calvo (1983). The firms in sector k can adjust their prices with probability $1-\alpha_k$ each period. The remaining fraction α_k of prices remains unchanged. Then the sectoral price level $P_{k,t}$ evolves as

$$\begin{aligned} P_{k,t} &= \left[\frac{1}{n_k}\int_{\mathcal{I}^*_{k,t}} {P^*_{k,t}}^{1-\theta} di + \frac{1}{n_k}\int_{\mathcal{I}_k - \mathcal{I}^*_{k,t}} P_{k,t-1}(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \\ &= \left[(1-\alpha_k)\, {P^*_{k,t}}^{1-\theta} + \alpha_k P^{1-\theta}_{k,t-1}\right]^{\frac{1}{1-\theta}}, \end{aligned} \tag{62}$$

where $P^*_{k,t}$ is the common optimal price chosen by firms in $\mathcal{I}^*_{k,t}$. The set $\mathcal{I}^*_{k,t}$, whose measure is $n_k(1-\alpha_k)$, is a randomly chosen subset of $\mathcal{I}_k$, which collects the indexes of firms that adjust their prices at time t.

Firm ik that can adjust its price at time t chooses its price to maximize the expected discounted profits as follows

$$\max_{P_{k,t}(i)} E_t \sum_{s=0}^{\infty} \alpha_k^s Q_{t,t+s} \Pi_{k,t+s}(i),$$

where $Q_{t,t+s} = \prod_{\tau=0}^{s-1} Q_{t+\tau,t+\tau+1}$ is the nominal stochastic discount factor between time t and $t+s$, and

$$\begin{aligned} \Pi_{k,t+s}(i) =& P_{k,t}(i)\, Y_{k,t+s}(i) - W_{k,t+s} H_{k,t+s}(i) - P_{t+s} Z_{k,t+s}(i), \\ =& \left[P_{k,t}(i) - \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{k,t+s}}{P_{t+s}}\right)^{1-\delta}\frac{1}{A_{t+s}A_{k,t+s}} P_{t+s}\right] D_{k,t+s}\left(\frac{P_{k,t}(i)}{P_{k,t+s}}\right)^{-\theta}\left(\frac{P_{k,t+s}}{P_{t+s}}\right)^{-\eta}\Omega_{k,t+s}, \end{aligned}$$

is the nominal profit at time $t+s$ under the condition that the firm's price has not been updated since time t. The nominal profit $\Pi_{k,t+s}(i)$ can be derived as $\Pi_{k,t}(i)$ in (60). So the the first order condition of the firm's profit maximization problem is given by

$$E_t \sum_{s=0}^{\infty} \alpha_k^s Q_{t,t+s} D_{k,t+s}\left(\frac{P^*_{k,t}}{P_{k,t+s}}\right)^{-\theta}\left(\frac{P_{k,t+s}}{P_{t+s}}\right)^{-\eta}\Omega_{k,t+s}\left[P^*_{k,t} - \left(\frac{\theta}{\theta-1}\right) MC_{k,t+s}\right] = 0,$$

where the nominal marginal cost in period $t+s$ is given by

$$MC_{k,t+s} = P_{t+s}\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{k,t+s}}{P_{t+s}}\right)^{1-\delta}\frac{1}{A_{t+s}A_{k,t+s}}.$$

The first order condition above and (62) together determine equilibrium dynamics of the sectoral prices. The aggregate price dynamics is then determined by aggregation of such sectoral prices.

Let $P_{k,t}^{**}(i)$ denote the optimal price when firm ik can set the prices freely. Then

$$P_{k,t}^{**}(i) = \left(\frac{\theta}{\theta-1}\right) MC_{k,t},$$

It follows that when prices are flexible, the equilibrium price is identical across firms within a sector. So we can drop the firm index i and write

$$P_{k,t}^{**} = \left(\frac{\theta}{\theta-1}\right) MC_{k,t}, \tag{63}$$

4.3 Government policy

We abstract from any influences of fiscal policy on equilibrium. For monetary policy, we consider a Taylor-type interest rate rule

$$I_t = \beta^{-1} I_{t-1}^{\rho_i} \left[\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{C_t}{C}\right)^{\phi_y} \right]^{1-\rho_i} \exp(\mu_t),$$

or a policy such that nominal aggregate consumption follows an exogenous stochastic process

$$M_t = P_t C_t,$$

where M_t is an exogenous process. Note that C_t is value-added output in this economy.

4.4 Shocks

The shocks are assumed to follow an AR(1) process

$$\begin{aligned}
\log \Gamma_{t+1} &= \rho_\Gamma \log \Gamma_t + \sigma_\Gamma \varepsilon_{\Gamma,t+1}, \\
\log(\Xi_{t+1} - \Xi) &= \rho_\Xi \log(\Xi_t - \Xi) + \sigma_\Xi \varepsilon_{\Xi,t+1}, \\
\log A_{t+1} &= \rho_A \log A_t + \sigma_A \varepsilon_{A,t+1}, \\
\log A_{k,t+1} &= \rho_{A_k} \log A_{k,t} + \sigma_{A_k} \varepsilon_{A_k,t+1}, \\
\log D_{k,t+1} &= \rho_{D_k} \log D_{k,t} + \sigma_{D_k} \varepsilon_{D_k,t+1},
\end{aligned}$$

where $A_k = D_k = \Gamma = A = 1$ in the steady state, and

$$\sum_{k=1}^{K} n_k D_{k,t} = 1.$$

For monetary policy, we assume

$$\mu_{t+1} = \rho_\mu \mu_t + \sigma_\mu \varepsilon_{\mu,t+1},$$

or

$$\log M_{t+1} = \rho_M \log M_t + \sigma_M \varepsilon_{M,t+1}.$$

In the steady state, $\mu = 0$ and $M = 1$.

4.5 Equilibrium conditions

Here we collect the equilibrium conditions of the model.

4.5.1 CES aggregates, market clearing conditions, and definitions

- Aggregate price level

$$P_t = \left[\sum_{k=1}^{K} (n_k D_{k,t}) P_{k,t}^{1-\eta} \right]^{1/(1-\eta)}.$$

- Sectoral price level

$$P_{k,t} = \left(\frac{1}{n_k} \int_{\mathcal{I}_k} P_{k,t}(i)^{1-\theta} di \right)^{1/(1-\theta)}.$$

- Aggregate consumption index

$$X_t = \left[\sum_{k=1}^{K} (n_k D_{k,t})^{1/\eta} (C_{k,t} - \tau_k C_{k,t-1})^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)},$$

- Real consumption expenditure (value-added output)

$$C_t = \frac{1}{P_t} \sum_{k=1}^{K} P_{k,t} C_{k,t}.$$

- Sectoral consumption

$$C_{k,t} = \left[\left(\frac{1}{n_k} \right)^{1/\theta} \int_{\mathcal{I}_k} C_{k,t}(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)}.$$

- Intermediate input used by firm ik

$$Z_{k,t}(i) = \left[\sum_{k'=1}^{K} \left(n_{k'} D_{k',t} \right)^{1/\eta} Z_{k,k',t}(i)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}.$$

- Intermediate input produced by sector k' used by ik firm

$$Z_{k,k',t}(i) = \left[\left(\frac{1}{n_{k'}} \right)^{1/\theta} \int_{\mathcal{I}_{k'}} Z_{k,k',t}(i,i')^{(\theta-1)/\theta} di' \right]^{\theta/(\theta-1)}.$$

- Asset market clearing condition

$$B_t = 0.$$

- Sectoral labor input

$$H_{k,t} = \int_{\mathcal{I}_k} H_{k,t}(i) di.$$

- Aggregate intermediate input

$$Z_t = \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} Z_{k',t}(i') di'.$$

- ik firm's total output

$$Y_{k,t}(i) = C_{k,t}(i) + \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} Z_{k',k,t}(i',i) di'.$$

- Sectoral output

$$Y_{k,t} = \int_{\mathcal{I}_k} Y_{k,t}\,(i)\,di.$$

Note that $Y_{k,t}$ is a simple sum of output by individual firms in sector k and thus is not the same consumption composite (CES aggregate) as $C_{k,t}$ and $Z_{k',k,t}\,(i')$.

- Aggregate gross output

$$Y_t = C_t + Z_t.$$

- Aggregate wage

$$W_t = \sum_{k=1}^{K} n_k W_{k,t}.$$

- Aggregate hours index

$$H_t = \sum_{k=1}^{K} H_{k,t}.$$

4.5.2 Demand functions

- Sectoral consumption

$$C_{k,t} = n_k D_{k,t} \left(\frac{P_{k,t}}{P_t}\right)^{-\eta} X_t + \tau_k C_{k,t-1}.$$

- Consumption of an ik good

$$C_{k,t}(i) = \frac{1}{n_k} \left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta} C_{k,t}.$$

- ik firm's demand for sector k' good

$$Z_{k,k',t}(i) = n_{k'} D_{k',t} \left(\frac{P_{k',t}}{P_t}\right)^{-\eta} Z_{k,t}(i).$$

- ik firm's demand for $i'k'$ good

$$Z_{k,k',t}(i,i') = \frac{1}{n_{k'}} \left(\frac{P_{k',t}(i')}{P_{k',t}}\right)^{-\theta} Z_{k,k',t}(i).$$

4.5.3 Household

- Consumption Euler equation / stochastic discount factor

$$Q_{t,t+1} = \beta \left(\frac{\Gamma_{t+1}}{\Gamma_t}\right) \left(\frac{X_{t+1}}{X_t}\right)^{-1} \left(\frac{P_t}{P_{t+1}}\right).$$

- Optimal labor supply condition

$$\frac{W_{k,t}}{P_t} = \Xi_t \omega_k H_{k,t}^{\varphi} X_t.$$

4.5.4 Firms

- Production function

$$Y_{k,t}(i) = A_t A_{k,t} H_{k,t}(i)^{1-\delta} Z_{k,t}\,(i)^{\delta}\,.$$

- Cost minimization
$$\frac{W_{k,t}}{P_t} = \frac{1-\delta}{\delta}\frac{Z_{k,t}(i)}{H_{k,t}(i)}.$$

- Nominal marginal cost
$$MC_{k,t+s} = P_{t+s}\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W_{k,t+s}}{P_{t+s}}\right)^{1-\delta}\frac{1}{A_{t+s}A_{k,t+s}}.$$

- First order condition
$$0 = E_t\sum_{s=0}^{\infty}\alpha_k^s Q_{t,t+s}D_{k,t+s}\left(\frac{P_{k,t}^*}{P_{k,t+s}}\right)^{-\theta}\left(\frac{P_{k,t+s}}{P_{t+s}}\right)^{-\eta}\Omega_{k,t+s}\left[P_{k,t}^* - \left(\frac{\theta}{\theta-1}\right)MC_{k,t+s}\right],$$

  where
$$\Omega_{k,t+s} = X_{t+s} + \frac{\tau_k}{n_k D_{k,t+s}}\left(\frac{P_{k,t+s}}{P_{t+s}}\right)^{\eta}C_{k,t+s-1} + Z_{t+s}.$$

- Frictionless optimal price
$$P_{k,t}^{**} = \left(\frac{\theta}{\theta-1}\right)MC_{k,t}.$$

4.6 Steady state

We solve the model by log-linearizing the equilibrium conditions around a symmetric non-stochastic zero-inflation steady state, which is derived here. A non-stochastic steady-state need not be symmetric. In particular, it depends on the steady-state levels of sector-specific productivity $\{A_k\}_{k=1}^K$ and the sector-specific parameters that measure the relative disutilities of supplying hours, $\{\omega_k\}_{k=1}^K$. For simplicity, we make two assumptions that deliver a symmetric steady state: i) the steady-state levels of sector-specific productivities are the same across sectors: specifically, $A_k = 1$ for all k, without loss of generality;[3] ii) $\omega_k = n_k^{-\varphi}$ for all k. The latter assumption relates the relative disutilities of labor to the size of the sectors and equalizes steady-state sectoral wages.

Let us solve for $\{Y, X, C, Z, H, W/P, \Pi/P\}$: the steady state values of aggregate gross output, aggregate value added-output (i.e. GDP), aggregate intermediate input usage, aggregate hours, real wage, and real profits. Once we obtain the steady state values of these aggregate variables, it is trivial to characterize the steady-state values for sectoral and micro variables using the symmetric nature of the steady state (i.e. $Y_k = n_k Y_k(i) = n_k Y$, $C_k = n_k C_k(i) = n_k C$, $Z_{k,k'}(i) = n_{k'}Z_{k,k'}(i,i') = n_{k'}Z$, $Z_k(i) = Z$, $H_k = n_k H_k(i) = n_k H$, $\Pi_k(i)/P = \Pi/P$, $W_k/P = W/P$, and $P(i)/P = P_k/P = 1$).

After exploiting the symmetry and the market-clearing conditions, the system of equilibrium conditions can be reduced to the following seven equations:

$$\left[\sum_{k=1}^K n_k\left(1-\tau_k\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} C = X, \tag{64}$$

$$C = \left(\frac{W}{P}\right)H + \left(\frac{\Pi}{P}\right), \tag{65}$$

$$\left(\frac{W}{P}\right) = H^{\varphi}X, \tag{66}$$

$$Y = H^{1-\delta}Z^{\delta}, \tag{67}$$

[3] Similarly, we fix the steady-state level of all other exogenous processes at unity.

$$Y = C + Z, \tag{68}$$

$$\left(\frac{\Pi}{P}\right) = Y - \left(\frac{W}{P}\right) H - Z, \tag{69}$$

$$Z = \frac{\delta}{1-\delta}\left(\frac{W}{P}\right) H, \tag{70}$$

$$1 = \left(\frac{\theta}{\theta - 1}\right)\chi\left(\frac{W}{P}\right)^{1-\delta}, \tag{71}$$

where $\chi \equiv \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}$.

First, it is trivial to obtain the real wage from (71):

$$\left(\frac{W}{P}\right) = \left(\frac{\theta-1}{\theta}\frac{1}{\chi}\right)^{\frac{1}{1-\delta}}.$$

Next, we substitute out Z in (67) and (69) using (70), which gives:

$$Y = H\left(\frac{\delta}{1-\delta}\right)^{\delta}\left(\frac{W}{P}\right)^{\delta},$$

$$\left(\frac{\Pi}{P}\right) = Y - \left(\frac{1}{1-\delta}\right)\left(\frac{W}{P}\right) H.$$

Combining the two equations above, we substitute out H and express real profits as a function of the real wage and output:

$$\left(\frac{\Pi}{P}\right) = \left[1 - \chi\left(\frac{W}{P}\right)^{1-\delta}\right] Y.$$

But, $\chi\left(\frac{W}{P}\right)^{1-\delta} = \frac{\theta-1}{\theta}$ from (71), and consequently we obtain:

$$\left(\frac{\Pi}{P}\right) = \frac{1}{\theta}Y.$$

Equation (65) indicates that aggregate value-added output should be equal to the sum of labor income and real profits:

$$C = \left(\frac{W}{P}\right) H + \left(\frac{\Pi}{P}\right) = \frac{1-\delta}{\delta}Z + \frac{1}{\theta}Y = \left[1 - \delta\left(\frac{\theta-1}{\theta}\right)\right] Y.$$

Consequently, aggregate intermediate input usage is obtained as:

$$Z = Y - C = Y - \left[1 - \delta\left(\frac{\theta-1}{\theta}\right)\right] Y = \delta\left(\frac{\theta-1}{\theta}\right) Y.$$

From (70), total labor hours are given by:

$$H = \frac{1-\delta}{\delta}\left(\frac{W}{P}\right)^{-1} Z = \left[\delta\left(\frac{\theta-1}{\theta}\right)\right]^{-\frac{\delta}{1-\delta}} Y.$$

So far, we have expressed the steady-state values of $\{Y, X, C, Z, H, \Pi/P\}$ in terms of Y, which can be obtained using

(66) as:

$$Y = \left\{ \left(\frac{1}{\chi}\right)^{\frac{1}{1-\delta}} \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{1-\delta}} \left[\delta\left(\frac{\theta-1}{\theta}\right)\right]^{\frac{\delta\varphi}{1-\delta}} \left[1-\delta\left(\frac{\theta-1}{\theta}\right)\right]^{-1} (1-\tau)^{-1} \right\}^{\frac{1}{1+\varphi}}.$$

4.7 Log-linear approximation

4.7.1 CES aggregates, market clearing conditions, and definitions

- Aggregate price level

$$p_t = \sum_{k=1}^{K} n_k p_{k,t}.$$

- Sectoral price level

$$p_{k,t} = \frac{1}{n_k} \int_{\mathcal{I}_k} p_{k,t}(i) di.$$

- Aggregate consumption index

$$x_t = \left(\frac{C}{X}\right)^{\frac{\eta-1}{\eta}} \sum_{k=1}^{K} n_k (1-\tau_k)^{\frac{\eta-1}{\eta}} \left[\frac{1}{\eta-1} d_{k,t} + \frac{1}{1-\tau_k}(c_{k,t} - \tau_k c_{k,t-1})\right],$$

  where $(X/C)^{(\eta-1)/\eta} = \sum_{k=1}^{K} n_k (1-\tau_k)^{(\eta-1)/\eta}$.

- Real consumption expenditure (value-added output)

$$c_t = \sum_{k=1}^{K} n_k c_{k,t}.$$

- Sectoral consumption

$$c_{k,t} = \frac{1}{n_k} \int_{\mathcal{I}_k} c_{k,t}(i) di.$$

- Intermediate input used by ik firm

$$z_{k,t}(i) = \sum_{k'=1}^{K} n_{k'} z_{k,k',t}(i).$$

- Intermediate input produced by sector k' used by ik firm

$$z_{k,k',t}(i) = \frac{1}{n_{k'}} \int_{\mathcal{I}_{k'}} z_{k,k',t}(i,i') di'.$$

- Sectoral labor input

$$h_{k,t} = \frac{1}{n_k} \int_{\mathcal{I}_k} h_{k,t}(i) di. \tag{72}$$

- Aggregate intermediate input

$$z_t = \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',t}(i') di'. \tag{73}$$

- firm ik's total output

$$y_{k,t}(i) = (1-\psi)\, c_{k,t}\,(i) + \psi \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',k,t}\,(i',i)\, di'.$$

- Sectoral output

$$y_{k,t} = \frac{1}{n_k} \int_{\mathcal{I}_k} y_{k,t}(i) di. \tag{74}$$

By the way, note that

$$\begin{aligned} y_{k,t} &= \frac{1}{n_k} \int_{\mathcal{I}_k} y_{k,t}(i) di \\ &= (1-\psi)\, c_{k,t} + \psi \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',k,t}\,(i')\, di'. \end{aligned}$$

- Aggregate gross output

$$y_t = (1-\psi) c_t + \psi z_t.$$

where $\psi \equiv \delta\left(\frac{\theta-1}{\theta}\right)$. It follows that

$$\begin{aligned} \sum_{k=1}^{K} n_k y_{k,t} &= (1-\psi) \sum_{k=1}^{K} n_k c_{k,t} + \psi \sum_{k=1}^{K} n_k \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',k,t}\,(i')\, di' \\ &= (1-\psi)\, c_t + \psi z_t \\ &= y_t. \end{aligned}$$

- Aggregate wage

$$w_t = \sum_{k=1}^{K} n_k w_{k,t}. \tag{75}$$

- Aggregate hours index

$$h_t = \sum_{k=1}^{K} n_k h_{k,t}. \tag{76}$$

4.7.2 Demand functions

- Sectoral consumption

$$c_{k,t} = \frac{X}{C} \left[d_{k,t} - \eta\,(p_{k,t} - p_t) + x_t\right] + \tau_k c_{k,t-1}, \tag{77}$$

where $X/C = \left[\sum_{k=1}^{K} n_k\,(1-\tau_k)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}$.

- Consumption of good ik

$$c_{k,t}(i) - c_{k,t} \; = -\theta\,(p_{k,t}(i) - p_{k,t})\,.$$

- Firm ik's demand for sector k' good

$$z_{k,k',t}(i) - z_{k,t}(i) = -\eta\,(p_{k',t} - p_t) + d_{k',t}. \tag{78}$$

- Firm ik's demand for good $i'k'$

$$z_{k,k',t}(i,i') - z_{k,k',t}(i) = -\theta\left(p_{k',t}(i') - p_{k',t}\right).$$

- Firm ik's total output. Using the demand for consumption of good ik and the demand for good ik by firm $i'k'$, we can show that

$$\begin{aligned} y_{k,t}(i) &= (1-\psi)\, c_{k,t}(i) + \psi \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',k,t}(i',i)\, di' \\ &= -\theta\left(p_{k,t}(i) - p_{k,t}\right) + y_{k,t}, \end{aligned}$$

or

$$y_{k,t}(i) - y_{k,t} = -\theta\left(p_{k,t}(i) - p_{k,t}\right).$$

4.7.3 Household

- Consumption Euler equation

$$x_t = E_t\left(x_{t+1}\right) - \left(i_t - E_t\pi_{t+1}\right) + \left(\gamma_t - E_t\gamma_{t+1}\right).$$

- Optimal labor supply condition

$$w_{k,t} - p_t = \varphi h_{k,t} + x_t + \xi_t. \tag{79}$$

Using the definition of the aggregate wage (75) and the aggregate hours index, we can derive the following aggregate labor supply condition

$$w_t - p_t = \varphi h_t + x_t + \xi_t.$$

4.7.4 Firms

- Firm ik's production function

$$y_{k,t}(i) = a_t + a_{k,t} + (1-\delta)\, h_{k,t}(i) + \delta z_{k,t}(i). \tag{80}$$

Using the definition of sectoral output (74) and sectoral labor input (72), we can show that

$$y_{k,t} = a_t + a_{k,t} + (1-\delta)\, h_{k,t} + \frac{\delta}{n_k} \int_{\mathcal{I}_k} z_{k,t}(i)\, di.$$

It follows that

$$\begin{aligned} y_t = \sum_{k=1}^{K} n_k y_{k,t} &= a_t + \sum_{k=1}^{K} n_k a_{k,t} + (1-\delta) \sum_{k=1}^{K} n_k h_{k,t} + \delta \sum_{k=1}^{K} \int_{\mathcal{I}_k} z_{k,t}(i)\, di \\ &= a_t + \sum_{k=1}^{K} n_k a_{k,t} + (1-\delta)\, h_t + \delta z_t, \end{aligned}$$

where we used the definition of aggregate intermediate input (73) and aggregate hours index (76).

- Cost minimization

$$w_{k,t} - p_t = z_{k,t}(i) - h_{k,t}(i). \tag{81}$$

Note that integrating both sides over $\mathcal{I}_k$ and dividing by n_k leads to

$$w_{k,t} - p_t = \frac{1}{n_k} \int_{\mathcal{I}_k} z_{k,t}(i)\, di - h_{k,t},$$

where we used the definition of sectoral labor input (72). From the definition of aggregate wages (75) and aggregate intermediate input (73), it follows that

$$w_t - p_t = z_t - h_t.$$

- Nominal marginal cost

$$mc_{k,t} = (1-\delta)(w_{k,t} - p_t) - a_{k,t} - a_t + p_t.$$

- First order condition

$$E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s p_{k,t}^* = E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s mc_{k,t+s}. \tag{82}$$

4.7.5 Derivation of the Phillips curve

From the log-linearized first order condition by the price-setting firm (82), the optimal price is given by

$$p_{k,t}^* = (1-\alpha_k\beta) E_t \sum_{s=0}^{\infty} \alpha_k^s \beta^s mc_{k,t+s},$$

or

$$p_{k,t}^* = (1-\alpha_k\beta) mc_{k,t} + \alpha_k \beta E_t \left[p_{k,t+1}^*\right]. \tag{83}$$

Loglinearizing (62) leads to

$$p_{k,t} = (1-\alpha_k) p_{k,t}^* + \alpha_k p_{k,t-1}. \tag{84}$$

Combining (83)and (84), we can derive the sectoral Phillips curve (PC)

$$\pi_{k,t} = \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}(mc_{k,t} - p_{k,t}) \tag{85}$$

Now let us show how the marginal cost is determined. First note that by integrating (81) over $\mathcal{I}_k$, we obtain

$$w_{k,t} - p_t = \frac{1}{n_k} \int_{\mathcal{I}_k} z_{k,t}(i) di - h_{k,t}.$$

We can combine this with (79) to obtain

$$\frac{1}{n_k} \int_{\mathcal{I}_k} z_{k,t}(i) di = (1+\varphi) h_{k,t} + x_t + \xi_t. \tag{86}$$

Also by integrating both sides of the production function (80) over $\mathcal{I}_k$, we get

$$y_{k,t} = a_t + a_{k,t} + (1-\delta) h_{k,t} + \delta \frac{1}{n_k} \int_{\mathcal{I}_k} z_{k,t}(i) di$$

$$= a_t + a_{k,t} + (1+\delta\varphi)\, h_{k,t} + \delta x_t + \delta \xi_t,$$

where we use (86) to obtain the second line. It follows that

$$h_{k,t} = \frac{1}{1+\delta\varphi} y_{k,t} - \frac{\delta}{1+\delta\varphi} x_t - \frac{\delta}{1+\delta\varphi}\xi_t - \frac{1}{1+\delta\varphi} a_{k,t} - \frac{1}{1+\delta\varphi} a_t,$$

and thus

$$\begin{aligned} mc_{k,t} &= (1-\delta)\,(w_{k,t} - p_t) - a_{k,t} - a_t + p_t \\ &= (1-\delta)\,(\varphi h_{k,t} + x_t + \xi_t) - a_{k,t} - a_t + p_t \\ &= \frac{(1-\delta)\,\varphi}{1+\delta\varphi} y_{k,t} + \frac{(1-\delta)}{1+\delta\varphi} x_t + \frac{(1-\delta)}{1+\delta\varphi}\xi_t - \frac{1+\varphi}{1+\delta\varphi} a_t - \frac{1+\varphi}{1+\delta\varphi} a_{k,t} + p_t. \end{aligned}$$

Consequently, the sectoral PC (85) can be written as

$$\begin{aligned} \pi_{k,t} =& \beta E_t \pi_{k,t+1} + \frac{(1-\alpha_k)\,(1-\alpha_k\beta)}{\alpha_k} \left\{ \frac{(1-\delta)\,\varphi}{1+\delta\varphi} y_{k,t} + \frac{(1-\delta)}{1+\delta\varphi} x_t + \frac{1-\delta}{1+\delta\varphi}\xi_t \right. \\ & \left. - \frac{1+\varphi}{1+\delta\varphi} a_t - \frac{1+\varphi}{1+\delta\varphi} a_{k,t} + (p_t - p_{k,t}) \right\}. \end{aligned}$$

We can write the sectoral PC in terms of sectoral consumption instead of sectoral output. Note that the consumption demand for sector k goods (77) can be rewritten as

$$p_t - p_{k,t} = \frac{1}{\eta}\left[\frac{C}{X}\,(c_{k,t} - \tau_k c_{k,t-1}) - d_{k,t} - x_t\right]. \tag{87}$$

Also, using the demand function for sectoral consumption (77) and the demand for sector k good by firm $i'k'$ (78), we can show that

$$\begin{aligned} y_{k,t} &= (1-\psi)\, c_{k,t} + \psi \sum_{k'=1}^{K} \int_{\mathcal{I}_{k'}} z_{k',k,t}\,(i')\, di' \\ &= \left[(1-\psi)\,\frac{X}{C} + \psi\right] \left[-\eta\,(p_{k,t} - p_t) + d_{k,t}\right] + (1-\psi)\left[\frac{X}{C} x_t + \tau_k c_{k,t-1}\right] + \psi z_t. \end{aligned}$$

From (77), it follows that

$$y_{k,t} = \psi z_t - \psi x_t + \left[(1-\psi) + \psi\frac{C}{X}\right] c_{k,t} - \psi\tau_k \frac{C}{X} c_{k,t-1}.$$

Thus we can substitute out $y_{k,t}$ from the marginal cost as

$$\begin{aligned} mc_{k,t} =& \frac{(1-\delta)\,\varphi}{1+\delta\varphi}\left\{\left[(1-\psi) + \psi\frac{C}{X}\right] c_{k,t} - \psi\tau_k\frac{C}{X} c_{k,t-1}\right\} + \frac{(1-\delta)\,(1-\psi\varphi)}{1+\delta\varphi} x_t + \frac{(1-\delta)\,\psi\varphi}{1+\delta\varphi} z_t \\ & + \frac{(1-\delta)}{1+\delta\varphi}\xi_t - \frac{1+\varphi}{1+\delta\varphi} a_t - \frac{1+\varphi}{1+\delta\varphi} a_{k,t} + p_t. \end{aligned} \tag{88}$$

Substituting out $p_t - p_{k,t}$ from the sectoral PC as well using (87), we have

$$\pi_{k,t} = \beta E_t \pi_{k,t+1}$$

$$+\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\left\{\left[\frac{(1-\delta)\varphi}{1+\delta\varphi}\left[(1-\psi)+\psi\frac{C}{X}\right]+\frac{1}{\eta}\frac{C}{X}\right]c_{k,t}-\tau_k\frac{C}{X}\left(\frac{(1-\delta)\varphi\psi}{1+\delta\varphi}+\frac{1}{\eta}\right)c_{k,t-1}\right.$$
$$\left.+\left[\frac{(1-\delta)(1-\psi\varphi)}{1+\delta\varphi}-\frac{1}{\eta}\right]x_t+\frac{(1-\delta)\psi\varphi}{1+\delta\varphi}z_t+\frac{1-\delta}{1+\delta\varphi}\xi_t-\frac{1+\varphi}{1+\delta\varphi}a_t-\frac{1+\varphi}{1+\delta\varphi}a_{k,t}-\frac{1}{\eta}d_{k,t}\right\}.$$

Aggregate inflation can be obtained by aggregation of sectoral inflation as follows

$$\pi_t=\sum_{k=1}^{K}n_k\pi_{k,t}.$$

4.8 Log-linear equilibrium conditions

Here we present the loglinearized equilibrum conditions necessary to characterize the equilibrium dynamics of the variables of interest. They are the following aggregate variables

$$\{c_t,\pi_t,i_t,m_t,h_t,(w_t-p_t)\}$$

and the following sectoral variables

$$\{c_{k,t},\pi_{k,t}\}_{k=1}^{K}.$$

The following $6+(K+2)$ equations determine the equilibrium dynamics of those variables

$$x_t=\left(\frac{C}{X}\right)^{\frac{\eta-1}{\eta}}\sum_{k=1}^{K}n_k(1-\tau_k)^{\frac{\eta-1}{\eta}}\left[\frac{1}{\eta-1}d_{k,t}+\frac{1}{1-\tau_k}(c_{k,t}-\tau_k c_{k,t-1})\right], \quad \text{(habit formation)}$$

$$x_t=E_t[x_{t+1}]-(i_t-E_t\pi_{t+1})+(\gamma_t-E_t\gamma_{t+1}), \quad \text{(IS equation)}$$

$$w_t-p_t=\varphi h_t+x_t+\xi_t, \quad \text{(agg. labor supply)}$$

$$(1-\psi)c_t+\psi z_t=a_t+\sum_{k=1}^{K}n_k a_{k,t}+(1-\delta)h_t+\delta z_t, \quad \text{(agg. resource constraint)}$$

$$w_t-p_t=z_t-h_t, \quad \text{(agg. cost-minimization relation)}$$

$$\pi_{k,t}=\beta E_t\pi_{k,t+1}+\frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}\left\{\left[\frac{(1-\delta)\varphi}{1+\delta\varphi}\left[(1-\psi)+\psi\frac{C}{X}\right]+\frac{1}{\eta}\frac{C}{X}\right]c_{k,t}-\tau_k\frac{C}{X}\left(\frac{(1-\delta)\varphi\psi}{1+\delta\varphi}+\frac{1}{\eta}\right)c_{k,t-1}\right.$$
$$\left.+\left[\frac{(1-\delta)(1-\psi\varphi)}{1+\delta\varphi}-\frac{1}{\eta}\right]x_t+\frac{(1-\delta)\psi\varphi}{1+\delta\varphi}z_t+\frac{1-\delta}{1+\delta\varphi}\xi_t-\frac{1+\varphi}{1+\delta\varphi}a_t-\frac{1+\varphi}{1+\delta\varphi}a_{k,t}-\frac{1}{\eta}d_{k,t}\right\}, \quad \text{(sectoral PC)}$$

$$\pi_t=\sum_{k=1}^{K}n_k\pi_{k,t}, \quad \text{(agg. inflation)}$$

$$\Delta\left[c_{k,t+1}-\frac{X}{C}x_{t+1}-\tau_k c_{k,t}\right]=\frac{X}{C}\left[-\eta(\pi_{k,t+1}-\pi_{t+1})+\Delta d_{k,t+1}\right]. \quad \text{(sectoral consumption)}$$

For monetary policy, we consider a Taylor-type interest rate rule

$$i_t=\rho_i i_{t-1}+(1-\rho_i)(\phi_\pi\pi_t+\phi_c c_t)+\mu_t,$$

or an exogenous stochastic process for nominal aggregate consumption

$$m_t=p_t+c_t.$$

5 Alternative monetary policy rule

We consider an alternative specification of the monetary policy rule, in which the interest rate responds to smoother measures of inflation and consumption (growth). When log-linearized, it specifies that the nominal interest rate is adjusted as follows:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\sum_{j=0}^{3} \frac{\phi_\pi}{4}\pi_{t-j} + \frac{\phi_c}{4}(c_t - c_{t-4})\right] + \mu_t. \tag{89}$$

Part II

Sectoral price facts, model fit, and other results

6 Sectoral price facts under alternative specifications

This section reports the estimated sectoral price facts under alternative specifications.[4]

6.1 Variants of the baseline specification

We first report the price facts of the variants of the baseline specification. All these specifications are estimated on the 27-sector quarterly data as the baseline specification and do not include Sector 11 (Gasoline and other energy goods) in estimation as explained in the main text. Specifically, we consider 1) a specification where $\delta = 0$ and thus intermediate inputs are not used; 2) a specification where nominal aggregate consumption is exogenous ($M_t = P_t C_t$); 3) a specification without labor market segmentation so there is a common labor market across the economy; 4) a specification where the labor market is firm-specific; 5) a specification that features habit formation in sectoral consumption; 6) a specification that features habit formation in sectoral consumption and where η and δ are estimated; and 7) a specification where monetary policy follows an alternative monetary policy rule. Note that all the other features of these specifications except the described feature are identical to those of the baseline specification. The alternative monetary policy rule is described in Equation (89).

As for the baseline specification, we estimate these extra specifications using Bayesian methods with the same prior distribution for the parameters, simulate 12,000 observations from each of the specifications at the posterior mode, discard first 2,000 observations, and estimate FAVAR on the remaining 10,000 observations to compute the price facts. FAVAR includes 2 latent common components and 4 lags of them in their VAR process. The price facts are reported in Tables 1 and 2.

[4] We do not report the Bayesian estimation results of all these specifications to save on space.

Table 1: Price facts for variants of the baseline specification: Speed of sectoral price responses

Specifications		Mean	Median	Standard deviations	Correlation with $1-\alpha_k$	Correlation
(1)	To common component	0.364	0.366	0.062	0.958	0.920
	To sector-specific component	0.841	0.800	0.235	0.903	
(2)	To common component	0.736	0.733	0.093	0.979	0.950
	To sector-specific component	0.847	0.897	0.237	0.956	
(3)	To common component	0.270	0.279	0.042	0.919	0.834
	To sector-specific component	0.745	0.758	0.267	0.802	
(4)	To common component	0.251	0.246	0.019	0.708	0.679
	To sector-specific component	0.829	0.845	0.235	0.932	
(5)	To common component	0.483	0.502	0.046	0.878	0.750
	To sector-specific component	0.732	0.731	0.225	0.811	
(6)	To common component	0.444	0.453	0.069	0.802	0.765
	To sector-specific component	0.843	0.889	0.237	0.955	
(7)	To common component	0.267	0.266	0.006	0.758	0.661
	To sector-specific component	0.776	0.813	0.248	0.870	

Note: Specifications are: (1) a specification where $\delta = 0$ and thus intermediate inputs are not used; (2) a specification where nominal aggregate consumption is exogenous ($M_t = P_t C_t$); (3) a specification without labor market segmentation so there is a common labor market across the economy; (4) a specification where the labor market is firm-specific; (5) a specification that features habit formation in sectoral consumption; (6) a specification that features habit formation in sectoral consumption and where η and δ are estimated; and (7) a specification where monetary policy follows an alternative monetary policy rule. $1-\alpha_k$ is the price change probability estimated in each specification.

Table 2: Price facts for variants of the baseline specification: Correlations between components of prices and quantities

Specifications		Mean	Median	Max	Min
(1)	Common component	-0.101	-0.073	0.171	-0.383
	Sector-specific component	-0.316	-0.363	0.022	-0.591
(2)	Common component	-0.663	-0.786	-0.024	-0.979
	Sector-specific component	-0.403	-0.459	0.09	-0.623
(3)	Common component	-0.146	-0.145	0.212	-0.561
	Sector-specific component	-0.589	-0.646	-0.135	-0.739
(4)	Common component	-0.078	-0.045	-0.015	-0.301
	Sector-specific component	-0.435	-0.465	0.064	-0.665
(5)	Common component	-0.384	-0.543	0.472	-0.811
	Sector-specific component	-0.325	-0.359	0.125	-0.688
(6)	Common component	-0.421	-0.414	-0.124	-0.800
	Sector-specific component	-0.424	-0.484	0.071	-0.649
(7)	Common component	-0.346	-0.382	0.076	-0.569
	Sector-specific component	-0.259	-0.264	0.207	-0.608

Note: See the note in Table 1.

6.2 Other specifications I

Next we report estimated price facts based on specifications that differ from the baseline specification in terms of the sectors included in estimation or the level of disaggregation. Specifically, we consider 8) a specification that includes Sector 11 and thus all sectors; 9) a specification that is estimated on the 15-sector quarterly data. In specification 9), we drop the sector of gasoline and other energy goods from estimation but keep the sector in the model as in the baseline specification. Note that all the other features of these extra specifications are identical to those of the baseline specification.

We compute the price facts in the same way as before and report them in Table 3 and 4. FAVAR includes 2 latent common components and 4 lags of them in their VAR process.

Table 3: Price facts for other specifications I: Speed of sectoral price responses

Specifications		Mean	Median	Standard deviations	Correlation with $1-\alpha_k$	Correlation
(8)	To common component	0.318	0.306	0.079	0.535	0.357
	To sector-specific component	0.853	0.914	0.231	0.934	
(9)	To common component	0.271	0.269	0.011	0.520	0.722
	To sector-specific component	0.818	0.785	0.178	0.804	

Note: Specifications are: (8) a specification that includes Sector 11 and thus all sectors; (9) a specification that is estimated on the 15-sector quarterly data. $1-\alpha_k$ is the price change probability estimated in each specification.

Table 4: Price facts for other specifications I: Correlations between components of prices and quantities

Specifications		Mean	Median	Max	Min
(8)	Common component	-0.092	-0.035	0.045	-0.910
	Sector-specific component	-0.411	-0.455	0.094	-0.656
(9)	Common component	-0.086	-0.068	0.035	-0.279
	Sector-specific component	-0.392	-0.419	0.077	-0.628

Note: See the note in Table 3.

6.3 Other specifications II

Lastly we report price facts based on a specification that includes hours worked data in estimation. This specification, numbered as (10), is identical to the baseline specification except that it includes the labor supply shock as described in Section 1 of this appendix and is estimated using data on hours worked as well.

We compute the price facts in the same way as before and report them in Tables 5 and 6. For comparison, we also present the price facts computed in FAVAR estimated on actual data that includes hours worked data. FAVAR includes 2 latent common components and 4 lags of them in their VAR process. Note that sector 11 is not included in estimation of the structural model and FAVAR as in the baseline specification.

Table 5: Price facts for other specifications II: Speed of sectoral price responses

Specifications		Mean	Median	Standard deviations	Correlation with $1-\alpha_k$	Correlation
(10)	To common component	0.300	0.291	0.031	0.938	0.745
	To sector-specific component	0.799	0.826	0.275	0.723	
Actual data (with hours)	To common component	0.274	0.279	0.043	0.484	0.302
	To sector-specific component	0.851	0.864	0.238	0.426	

Note: Specification (10) is identical to the baseline specification except that it includes the labor supply shock and is estimated on hours worked as well. $1-\alpha_k$ is the price change probability estimated in each specification.

Table 6: Price facts for other specifications II: Correlations between components of prices and quantities

Specifications		Mean	Median	Max	Min
(10)	Common component	0.035	0.051	0.978	-0.920
	Sector-specific component	0.048	0.036	0.162	-0.040
Actual data (with hours)	Common component	-0.203	-0.174	0.585	-0.858
	Sector-specific component	-0.259	-0.268	0.113	-0.630

Note: See the note in Table 5.

7 Model fit

This section presents various statistics to assess the fit of the baseline model and compares it with the other specifications. We report the statistics for volatility, persistence and comovement of the observable variables in the data and those implied by the estimated specifications. The statistics for the estimated specifications are computed using simulated data with model parameters fixed at the posterior mode. It is actually the same simulated data used to estimate FAVAR. For each specification, we simulate 12,000 observations, discard the first 2,000 observations and use the last 10,000 observations to compute the statistics.

7.1 Volatility

We report the standard deviation of the observable variables in the data and in the estimated specifications to assess the fit of the models in terms of volatility. The baseline specification fits volatility of the data reasonably well. Specifications (5) and (6) that extend the baseline specification and allow for habit formation in sectoral consumption fit volatility of sectoral consumption quite well.

Table 7: Volatility - standard deviations of observable variables

Variable	Data	Baseline	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Nominal interest rates	0.006	0.009	0.004	0.004	0.010	0.009	0.007	0.007	0.004
Sectoral consumption growth rates									
Sector 1	0.063	0.062	0.061	0.061	0.062	0.062	0.061	0.061	0.061
Sector 2	0.060	0.062	0.062	0.062	0.083	0.062	0.061	0.061	0.062
Sector 3	0.013	0.017	0.011	0.014	0.017	0.018	0.013	0.012	0.015
Sector 4	0.021	0.026	0.020	0.022	0.028	0.027	0.020	0.019	0.025
Sector 5	0.023	0.024	0.022	0.021	0.025	0.024	0.022	0.022	0.022
Sector 6	0.020	0.023	0.018	0.020	0.025	0.024	0.018	0.018	0.021
Sector 7	0.007	0.017	0.009	0.014	0.018	0.017	0.011	0.008	0.015
Sector 8	0.012	0.021	0.012	0.019	0.024	0.022	0.015	0.013	0.019
Sector 9	0.014	0.019	0.013	0.017	0.020	0.020	0.014	0.013	0.018
Sector 10	0.014	0.019	0.013	0.016	0.020	0.020	0.015	0.013	0.018
Sector 11	0.013				NA				
Sector 12	0.016	0.021	0.017	0.017	0.021	0.021	0.016	0.016	0.018
Sector 13	0.010	0.016	0.010	0.012	0.015	0.017	0.011	0.010	0.014
Sector 14	0.007	0.014	0.008	0.010	0.012	0.015	0.009	0.008	0.012
Sector 15	0.014	0.042	0.019	0.041	0.057	0.043	0.025	0.019	0.041
Sector 16	0.004	0.015	0.007	0.009	0.012	0.015	0.011	0.008	0.011
Sector 17	0.028	0.036	0.027	0.035	0.046	0.037	0.028	0.029	0.035
Sector 18	0.005	0.016	0.009	0.011	0.015	0.017	0.018	0.010	0.013
Sector 19	0.015	0.017	0.013	0.013	0.016	0.018	0.014	0.013	0.015
Sector 20	0.020	0.041	0.022	0.040	0.055	0.042	0.026	0.021	0.041
Sector 21	0.009	0.015	0.009	0.011	0.013	0.016	0.011	0.010	0.012
Sector 22	0.010	0.017	0.012	0.012	0.014	0.017	0.011	0.010	0.013
Sector 23	0.021	0.026	0.019	0.025	0.032	0.026	0.022	0.020	0.025
Sector 24	0.012	0.031	0.015	0.029	0.039	0.032	0.024	0.017	0.030

Table 7: Volatility - standard deviations of observable variables (continued)

Variable		Data	Baseline	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Sector 25	0.013	0.021	0.013	0.019	0.024	0.022	0.016	0.013	0.020
	Sector 26	0.006	0.015	0.008	0.010	0.013	0.016	0.008	0.008	0.012
	Sector 27	0.007	0.014	0.008	0.009	0.012	0.015	0.009	0.009	0.011
Sectoral inflation										
	Sector 1	0.005	0.009	0.006	0.007	0.010	0.009	0.006	0.008	0.006
	Sector 2	0.020	0.020	0.022	0.019	0.020	0.020	0.022	0.019	0.019
	Sector 3	0.005	0.009	0.006	0.007	0.010	0.009	0.009	0.009	0.005
	Sector 4	0.011	0.011	0.009	0.010	0.013	0.011	0.011	0.011	0.009
	Sector 5	0.005	0.009	0.008	0.007	0.011	0.009	0.007	0.009	0.006
	Sector 6	0.009	0.010	0.009	0.009	0.012	0.010	0.012	0.010	0.008
	Sector 7	0.006	0.009	0.007	0.008	0.011	0.009	0.011	0.009	0.006
	Sector 8	0.009	0.010	0.008	0.009	0.013	0.010	0.012	0.011	0.008
	Sector 9	0.008	0.010	0.008	0.009	0.012	0.010	0.013	0.011	0.008
	Sector 10	0.007	0.010	0.008	0.009	0.012	0.010	0.013	0.010	0.007
	Sector 11	0.063				NA				
	Sector 12	0.006	0.009	0.006	0.006	0.010	0.009	0.008	0.008	0.005
	Sector 13	0.006	0.009	0.006	0.006	0.010	0.009	0.009	0.008	0.005
	Sector 14	0.005	0.008	0.005	0.006	0.010	0.008	0.009	0.008	0.005
	Sector 15	0.021	0.018	0.017	0.017	0.023	0.018	0.021	0.022	0.016
	Sector 16	0.003	0.008	0.004	0.004	0.009	0.008	0.005	0.006	0.003
	Sector 17	0.014	0.014	0.015	0.013	0.017	0.014	0.017	0.015	0.012
	Sector 18	0.006	0.008	0.004	0.004	0.010	0.008	0.006	0.007	0.004
	Sector 19	0.003	0.008	0.005	0.005	0.010	0.008	0.007	0.007	0.004
	Sector 20	0.021	0.018	0.018	0.018	0.022	0.018	0.023	0.022	0.017
	Sector 21	0.003	0.008	0.005	0.005	0.009	0.008	0.007	0.008	0.004
	Sector 22	0.002	0.008	0.004	0.003	0.008	0.008	0.005	0.006	0.003
	Sector 23	0.010	0.012	0.013	0.011	0.014	0.012	0.017	0.013	0.010
	Sector 24	0.013	0.013	0.013	0.013	0.016	0.013	0.018	0.015	0.012
	Sector 25	0.008	0.010	0.009	0.009	0.012	0.010	0.013	0.011	0.008
	Sector 26	0.004	0.008	0.004	0.004	0.010	0.008	0.006	0.006	0.004
	Sector 27	0.002	0.008	0.004	0.004	0.009	0.008	0.006	0.006	0.004
Aggregate consumption growth rates		0.007	0.013	0.006	0.006	0.009	0.014	0.007	0.007	0.010
Aggregate inflation		0.003	0.007	0.003	0.005	0.009	0.007	0.007	0.006	0.003

Note: Specifications are: (1) a specification where $\delta = 0$ and thus intermediate inputs are not used; (2) a specification where nominal aggregate consumption is exogenous ($M_t = P_t C_t$); (3) a specification without labor market segmentation where there is a common labor market across the economy; (4) a specification where the labor market is firm-specific; (5) a specification that features habit formation in sectoral consumption; (6) a specification that features habit formation in sectoral consumption and where η and δ are estimated; and (7) a specification where monetary policy follows an alternative monetary policy rule. Note that estimation did not use aggregate consumption growth and aggregate inflation data. We aggregate sectoral data and compute the statistics for aggregated data.

The root mean squared error between model-predicted volatility and volatility in the data is computed for sectoral consumption growth and sectoral inflation across sectors and reported in Table 8. The baseline specification tends to overpredict volatility of sectoral consumption growth. Habit formation in sectoral consumption (specifications 5 and 6) however reduces sectoral consumption volatility predicted by the baseline specification so that it is close to that in the data.

Table 8: Root mean squared error of the model-implied volatility against the volatility in the data

Specifications	Baseline	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Sectoral consumption growth	0.010	0.002	0.008	0.015	0.011	0.005	0.002	0.009
Sectoral inflation	0.003	0.002	0.002	0.005	0.003	0.004	0.003	0.002

Note: See the note in Table 7.

7.2 Persistence

To assess the fit of the models in terms of persistence, we estimate a univariate AR(4) model for each of the observable variables and report the sum of the four AR coefficients. The baseline specification fits persistence of sectoral consumption growth and sectoral inflation reasonably well. Specifications (5) and (6) that allow for habit formation in sectoral consumption further improve the fit in terms of persistence.

Table 9: Persistence - sum of AR coefficients for observable variables

Variable		Data	Baseline	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Nominal interest rates		0.955	0.975	0.931	0.945	0.979	0.974	0.884	0.924	0.946
Sectoral consumption growth rates										
	Sector 1	-0.511	-0.429	-0.482	-0.415	-0.288	-0.385	-0.411	-0.382	-0.435
	Sector 2	-0.584	-0.152	-0.265	-0.163	0.008	-0.170	-0.151	-0.192	-0.151
	Sector 3	0.506	0.016	0.095	0.002	0.010	-0.015	0.284	0.361	0.054
	Sector 4	0.609	0.178	0.217	0.296	0.337	0.200	0.348	0.340	0.242
	Sector 5	0.012	-0.085	-0.060	-0.131	-0.068	-0.097	-0.107	-0.050	-0.050
	Sector 6	0.201	0.101	0.158	0.120	0.205	0.145	0.214	0.223	0.137
	Sector 7	-0.132	-0.050	-0.074	-0.027	0.098	-0.037	0.163	0.393	0.034
	Sector 8	0.443	-0.068	-0.019	-0.083	0.092	-0.061	0.336	0.458	-0.027
	Sector 9	-0.291	-0.052	-0.268	-0.085	-0.006	-0.047	0.049	0.158	-0.032
	Sector 10	0.040	-0.070	-0.235	-0.147	-0.048	-0.060	-0.060	0.085	-0.070
	Sector 11					NA				
	Sector 12	0.358	0.029	0.169	0.099	0.172	0.024	0.100	0.090	0.106
	Sector 13	0.236	0.012	0.010	0.160	0.168	0.074	0.234	0.264	0.147
	Sector 14	0.384	-0.102	-0.013	-0.008	-0.047	-0.074	0.227	0.485	0.077
	Sector 15	0.193	-0.103	-0.105	-0.106	-0.007	-0.099	0.131	0.238	-0.089
	Sector 16	0.652	0.003	0.036	0.238	0.087	0.018	0.880	0.844	0.266
	Sector 17	-1.295	-0.011	-0.163	0.032	0.086	0.022	-0.078	-0.292	0.011
	Sector 18	0.350	0.018	-0.036	0.278	0.398	0.066	0.883	0.757	0.225
	Sector 19	0.801	0.038	0.202	0.162	0.040	0.034	0.730	0.770	0.129
	Sector 20	0.011	-0.089	-0.089	-0.072	-0.085	-0.109	0.054	0.145	-0.100
	Sector 21	0.470	0.003	0.189	0.280	-0.045	0.009	0.533	0.586	0.203
	Sector 22	-0.027	0.052	0.256	0.251	0.152	0.026	0.194	0.248	0.218
	Sector 23	-0.152	-0.216	-0.199	-0.195	-0.217	-0.234	-0.016	0.078	-0.211

Table 9: Persistence - sum of AR coefficients for observable variables (continued)

Variable		Data	Baseline	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Sector 24	0.249	-0.270	-0.190	-0.317	-0.388	-0.255	0.687	0.130	-0.285
	Sector 25	0.468	-0.023	-0.014	-0.075	0.014	-0.015	0.277	0.378	0.007
	Sector 26	0.698	0.057	0.100	0.322	0.300	0.091	0.487	0.618	0.265
	Sector 27	0.612	0.013	0.139	0.251	0.063	0.018	0.587	0.737	0.236
Sectoral inflation										
	Sector 1	0.781	0.857	0.599	0.544	0.897	0.857	0.705	0.787	0.601
	Sector 2	0.439	0.329	0.420	0.025	0.592	0.339	0.497	0.429	-0.001
	Sector 3	0.620	0.892	0.524	0.271	0.889	0.886	0.522	0.759	0.442
	Sector 4	0.861	0.788	0.529	0.502	0.813	0.793	0.643	0.722	0.506
	Sector 5	0.624	0.842	0.543	0.344	0.865	0.843	0.619	0.745	0.434
	Sector 6	0.779	0.797	0.477	0.338	0.813	0.792	0.537	0.692	0.423
	Sector 7	0.514	0.864	0.391	0.126	0.844	0.858	0.404	0.697	0.295
	Sector 8	0.399	0.764	0.303	0.118	0.756	0.760	0.345	0.579	0.200
	Sector 9	0.434	0.790	0.244	0.060	0.789	0.790	0.318	0.607	0.152
	Sector 10	0.517	0.791	0.244	0.044	0.799	0.783	0.313	0.610	0.199
	Sector 11					NA				
	Sector 12	0.845	0.910	0.703	0.542	0.919	0.905	0.748	0.841	0.647
	Sector 13	0.806	0.899	0.570	0.367	0.893	0.895	0.611	0.788	0.511
	Sector 14	0.659	0.908	0.569	0.273	0.905	0.904	0.525	0.782	0.460
	Sector 15	0.431	0.404	-0.024	0.007	0.391	0.404	0.079	0.298	0.010
	Sector 16	0.824	0.959	0.794	0.700	0.971	0.958	0.857	0.924	0.763
	Sector 17	0.240	0.607	0.254	0.180	0.652	0.628	0.380	0.472	0.210
	Sector 18	0.935	0.949	0.744	0.658	0.954	0.949	0.823	0.912	0.735
	Sector 19	0.647	0.935	0.757	0.564	0.940	0.933	0.699	0.857	0.672
	Sector 20	-0.022	0.395	-0.053	0.005	0.380	0.390	0.034	0.239	0.024
	Sector 21	0.607	0.932	0.694	0.538	0.961	0.929	0.732	0.857	0.635
	Sector 22	0.871	0.967	0.882	0.806	0.983	0.966	0.905	0.948	0.848
	Sector 23	0.080	0.647	0.088	-0.107	0.667	0.655	0.106	0.392	-0.062
	Sector 24	0.142	0.597	0.010	-0.103	0.524	0.567	-0.070	0.303	-0.058
	Sector 25	0.414	0.786	0.294	0.121	0.764	0.782	0.284	0.573	0.234
	Sector 26	0.788	0.938	0.722	0.614	0.944	0.937	0.795	0.891	0.691
	Sector 27	0.505	0.958	0.792	0.622	0.976	0.954	0.780	0.913	0.745
Aggregate consumption growth rates		0.604	-0.044	-0.074	0.173	-0.141	-0.040	0.412	0.624	0.155
Aggregate inflation		0.911	0.968	0.808	0.446	0.956	0.966	0.652	0.895	0.695

Note: Specifications are: (1) a specification where $\delta = 0$ and thus intermediate inputs are not used; (2) a specification where nominal aggregate consumption is exogenous ($M_t = P_t C_t$); (3) a specification without labor market segmentation where there is a common labor market across the economy; (4) a specification where the labor market is firm-specific; (5) a specification that features habit formation in sectoral consumption; (6) a specification that features habit formation in sectoral consumption and where η and δ are estimated; and (7) a specification where monetary policy follows an alternative monetary policy rule. Note that estimation did not use aggregate consumption growth and aggregate inflation data. We aggregate sectoral data and compute the statistics for aggregated data.

The root mean squared error between model-predicted persistence and persistence in the data is computed for sectoral consumption growth and sectoral inflation across sectors and reported in Table 10. The baseline specification tends to overpredict persistence but as for volatility, habit formation in sectoral consumption (specifications 5 and 6) improves the fit in terms of persistence.

Table 10: Root mean squared error of the model-implied persistence against the persistence in the data

Specifications	Baseline	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Sectoral consumption growth	0.470	0.412	0.421	0.458	0.467	0.329	0.294	0.412
Sectoral inflation	0.284	0.181	0.297	0.286	0.282	0.149	0.172	0.229

Note: See the note in Table 9.

7.3 Comovement

This section reports the correlation of pairs of variables to assess the fit of the models in terms of comovement. To save on space, we report correlations with aggregate variables only. Note that aggregate consumption growth and inflation are not included in estimation but they are computed by aggregating their sectoral components. The results are reported in Table 11.

In terms of the sign and magnitude of comovement, the baseline specification fits the data well. It predicts negative comovement between nominal interest rates and aggregate consumption growth as opposed to positive comovement estimated in the data. However, habit formation in sectoral consumption (specification 6) helps fit positive comovement between nominal interest rates and aggregate consumption growth in the data without distorting comovement of the other variables.

Table 11: Comovement - correlations of selected pairs of the observable variables

Variable pairs	Data	Baseline	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Nominal interest rates and									
Agg. consumption growth	0.026	-0.050	0.037	0.011	-0.032	-0.061	-0.081	0.032	0.048
Agg. inflation	0.617	0.945	0.718	-0.034	0.936	0.942	0.414	0.714	0.420
Aggregate consumption growth and									
Agg. inflation	-0.226	-0.066	0.011	-0.514	-0.083	-0.088	-0.367	-0.317	-0.429
Sectoral consumption growth in Sector 1	0.680	0.423	0.535	0.457	0.389	0.428	0.528	0.443	0.416
Sector 2	0.117	0.208	0.150	0.159	0.151	0.223	0.160	0.138	0.190
Sector 3	0.663	0.682	0.305	0.365	0.396	0.694	0.505	0.442	0.560
Sector 4	0.728	0.703	0.535	0.550	0.588	0.706	0.599	0.586	0.653
Sector 5	0.500	0.491	0.228	0.189	0.273	0.503	0.225	0.228	0.371
Sector 6	0.568	0.523	0.220	0.218	0.278	0.530	0.335	0.313	0.406
Sector 7	0.299	0.711	0.338	0.437	0.490	0.725	0.596	0.568	0.630
Sector 8	0.296	0.532	0.199	0.267	0.258	0.548	0.453	0.409	0.409
Sector 9	0.528	0.614	0.290	0.370	0.373	0.638	0.514	0.449	0.529
Sector 10	0.395	0.567	0.213	0.279	0.280	0.581	0.415	0.327	0.450
Sector 11					NA				
Sector 12	0.207	0.569	0.244	0.159	0.311	0.570	0.272	0.243	0.408
Sector 13	0.405	0.695	0.308	0.325	0.396	0.710	0.460	0.445	0.557
Sector 14	0.557	0.761	0.363	0.383	0.462	0.773	0.593	0.570	0.629
Sector 15	0.090	0.272	0.109	0.147	0.131	0.271	0.303	0.260	0.193

Table 11: Comovement - correlations of selected pairs of the observable variables (continued)

Variable pairs	Data	Baseline	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Sector 16	0.249	0.834	0.533	0.288	0.600	0.852	0.154	0.653	0.689
Sector 17	0.142	0.399	0.217	0.261	0.265	0.401	0.331	0.270	0.337
Sector 18	0.012	0.770	0.455	0.288	0.532	0.785	0.287	0.572	0.614
Sector 19	0.487	0.676	0.310	0.217	0.412	0.702	0.435	0.506	0.529
Sector 20	0.188	0.304	0.126	0.180	0.141	0.318	0.328	0.264	0.244
Sector 21	0.343	0.754	0.372	0.290	0.505	0.766	0.462	0.532	0.619
Sector 22	0.255	0.742	0.377	0.125	0.475	0.756	0.346	0.392	0.540
Sector 23	-0.012	0.398	0.134	0.191	0.165	0.414	0.319	0.237	0.306
Sector 24	0.269	0.511	0.288	0.387	0.385	0.526	0.360	0.443	0.439
Sector 25	0.252	0.530	0.237	0.293	0.291	0.568	0.478	0.402	0.455
Sector 26	0.212	0.765	0.399	0.207	0.505	0.779	0.454	0.591	0.595
Sector 27	0.399	0.814	0.443	0.257	0.537	0.825	0.527	0.612	0.652
Aggregate inflation and									
Sectoral inflation in Sector 1	0.540	0.788	0.412	0.492	0.883	0.788	0.579	0.753	0.437
Sector 2	0.208	0.400	0.181	0.336	0.464	0.391	0.362	0.370	0.224
Sector 3	0.469	0.863	0.524	0.770	0.889	0.857	0.820	0.801	0.596
Sector 4	0.840	0.734	0.479	0.775	0.780	0.728	0.672	0.670	0.566
Sector 5	0.522	0.789	0.382	0.608	0.847	0.785	0.616	0.738	0.457
Sector 6	0.653	0.714	0.366	0.563	0.748	0.706	0.656	0.623	0.391
Sector 7	0.325	0.849	0.619	0.789	0.855	0.845	0.840	0.780	0.626
Sector 8	0.297	0.721	0.431	0.607	0.724	0.717	0.686	0.604	0.422
Sector 9	0.524	0.767	0.515	0.707	0.797	0.760	0.760	0.693	0.539
Sector 10	0.440	0.747	0.416	0.651	0.784	0.738	0.738	0.668	0.445
Sector 11					NA				
Sector 12	0.596	0.850	0.413	0.605	0.861	0.839	0.666	0.773	0.457
Sector 13	0.624	0.861	0.562	0.735	0.871	0.855	0.789	0.784	0.553
Sector 14	0.532	0.889	0.643	0.790	0.902	0.888	0.856	0.829	0.607
Sector 15	0.117	0.421	0.229	0.378	0.410	0.423	0.505	0.345	0.232
Sector 16	0.455	0.947	0.769	0.701	0.938	0.946	0.743	0.882	0.664
Sector 17	0.115	0.563	0.312	0.530	0.605	0.568	0.584	0.506	0.357
Sector 18	0.859	0.924	0.679	0.698	0.897	0.920	0.678	0.784	0.612
Sector 19	0.516	0.898	0.500	0.682	0.923	0.893	0.833	0.851	0.578
Sector 20	0.219	0.429	0.249	0.395	0.438	0.427	0.510	0.357	0.258
Sector 21	0.456	0.905	0.609	0.723	0.919	0.899	0.754	0.829	0.623
Sector 22	0.591	0.938	0.576	0.512	0.921	0.934	0.627	0.741	0.560
Sector 23	0.231	0.627	0.293	0.538	0.671	0.629	0.644	0.553	0.337
Sector 24	0.325	0.647	0.469	0.617	0.631	0.640	0.696	0.544	0.506
Sector 25	0.255	0.737	0.433	0.633	0.745	0.738	0.744	0.638	0.455
Sector 26	0.469	0.900	0.662	0.633	0.897	0.893	0.715	0.755	0.542
Sector 27	0.591	0.937	0.698	0.741	0.925	0.932	0.831	0.797	0.632

Note: Specifications are: (1) a specification where $\delta = 0$ and thus intermediate inputs are not used; (2) a specification where nominal aggregate consumption is exogenous ($M_t = P_t C_t$); (3) a specification without labor market segmentation where there is a common labor market across the economy; (4) a specification where the labor market is firm-specific; (5) a specification that features habit formation in sectoral consumption; (6) a specification that features habit formation in sectoral consumption and where η and δ are estimated; and (7) a specification where monetary policy follows an alternative monetary policy rule. Note that estimation did not use aggregate consumption growth and aggregate inflation data. We aggregate sectoral data and compute the statistics for aggregated data.

8 Number of latent common components in FAVAR

The baseline specification of FAVAR includes two latent common components (factors) in addition to an observable common component (the Federal Funds rate). Here we present some evidence to support this specification of FAVAR.[5] BGM specify five latent common factors in FAVAR but they use more disaggregated data (190 sectors) than us (27 sectors in the baseline specification). So a smaller number of latent common components in our FAVAR than theirs would be desirable in order to prevent the common components from being contaminated by some sector-specific components. On the other hand, Maćkowiak et al. (2009, MMW henceforth) use sectoral inflation data only and include a single common factor (aggregate shock) in their baseline specification of DFM.

The principal component analysis (PCA) shows that the first two factors contribute relatively more to the variation of sectoral consumption growth and sectoral inflation than the other factors, whether Sector 11 is included or not. We also do PCA on the 15-sector quarterly data and draw similar conclusions. This is also the case for the simulated data from the estimated baseline specification. Actually in this simulated data, the first two factors explain more than 50% of the variance of sectoral consumption growth and sectoral inflation. The result is shown in Table 12.

Table 12: Contribution of the first ten factors to the variance of sectoral consumption growth and inflation

Factor	Actual data: 27 sectors				Actual data: 15 sectors		Simulated data	
	All sectors included		Sector 11 excluded					
	Proportion	Cumulative	Proportion	Cumulative	Proportion	Cumulative	Proportion	Cumulative
1	15.40	15.40	15.97	15.97	18.46	18.46	32.10	32.10
2	10.69	26.09	10.73	26.70	13.12	31.58	19.80	51.90
3	5.50	31.59	5.59	32.29	6.87	38.45	2.49	54.39
4	5.00	36.59	4.88	37.18	5.91	44.36	2.41	56.80
5	4.73	41.32	4.60	41.78	5.54	49.90	1.97	58.77
6	4.12	45.44	4.05	45.83	5.13	55.03	1.94	60.71
7	3.60	49.04	3.68	49.51	4.35	59.38	1.90	62.61
8	3.49	52.53	3.54	53.05	3.85	63.23	1.83	64.43
9	3.16	55.69	3.12	56.17	3.57	66.80	1.81	66.24
10	2.94	58.63	2.96	59.12	3.45	70.25	1.75	68.00

Note: In percentage. The frequency of the data is quarterly. Results are shown only for the first ten factors in the descending order of the proportion of the variance explained by each factor. PCA is done after standardization of the data, which includes sectoral consumption growth, sectoral inflation and the Federal Funds rate. Simulated data is generated from the estimated baseline DSGE model at the posterior mode. We draw 12,000 observations and discard the first 2,000.

We also report R^2 produced by each of the factors to assess whether the latent common components are really aggregate (common across sectors) or clustered only in a few sectors. To compute R^2, we estimate the following orthogonal factor model using PCA

$$y_{it}^j = \beta_i f_t + \varepsilon_{it}^j,$$

for $i = 1, \cdots, K$, where y_{it}^j is sectoral consumption growth ($j = dc$), sectoral inflation ($j = \pi$), or the Federal Funds rate ($j = ffr$) for sector i, $f_t = (f_{1t}, \cdots, f_{mt})$ includes $m \times 1$ common factors that are orthogonal to each other, $\beta_i^j = \left(\beta_{i1}^j, \cdots, \beta_{im}^j\right)$ is an $m \times 1$ vector of factor loadings and ε_{it}^j is the error term with $\text{var}\left(\varepsilon_{it}^j\right) = \left(\sigma_i^j\right)^2$ and $\text{cov}\left(\varepsilon_{i,t}^j, f_{kt}\right) = 0$ for $k = 1, \cdots, m$. Then the variation of y_{it}^j explained by factor f_{kt} can be computed as

$$\frac{\left(\beta_{ij}^j\right)^2 \text{var}\left(f_{jt}\right)}{\sum_{k=1}^m \left(\beta_{ij}^j\right)^2 \text{var}\left(f_{kt}\right) + \left(\sigma_i^j\right)^2}.$$

[5] We thank a referee for suggesting us to do these exercises to decide on the number of latent common components in FAVAR.

The results are reported in Table 13. Indeed the first two factors explain a large portion of the variation in the data in most of the sectors as we hope for latent common factors in FAVAR. They are not clustered in a small number of sectors. When we exclude Sector 11 as in the baseline specification, R^2 by the first two factors is less than 0.2 only in 4 sectors among 26 sectors and mostly greater than 0.5 for sectoral consumption growth and is less than 0.2 in 6 sectors among 26 sectors and mostly greater than 0.5 for sectoral inflation.

Table 13: R^2 produced by the factors: actual data (27 sectors, quarterly)

		All sectors included						Sector 11 excluded					
Data	Sector	Factor	1	2	3	4	(1 + 2)	Factor	1	2	3	4	(1 + 2)
Federal Funds rates			0.63	0.19	0.00	0.00	0.81		0.62	0.19	0.01	0.15	0.80
Sectoral	1		0.52	0.21	0.03	0.21	0.73		0.47	0.16	0.03	0.08	0.64
consumption growth	2		0.12	0.57	0.02	0.01	0.69		0.09	0.46	0.01	0.29	0.56
	3		0.56	0.36	0.03	0.05	0.92		0.50	0.31	0.03	0.02	0.81
	4		0.81	0.05	0.03	0.00	0.86		0.81	0.05	0.05	0.09	0.86
	5		0.79	0.00	0.11	0.02	0.79		0.82	0.00	0.08	0.10	0.82
	6		0.88	0.00	0.00	0.12	0.88		0.90	0.00	0.00	0.00	0.90
	7		0.12	0.02	0.29	0.51	0.14		0.13	0.01	0.25	0.18	0.14
	8		0.27	0.16	0.29	0.14	0.43		0.27	0.18	0.31	0.13	0.45
	9		0.76	0.04	0.12	0.03	0.80		0.78	0.04	0.11	0.05	0.82
	10		0.62	0.00	0.16	0.03	0.62		0.62	0.00	0.12	0.15	0.62
	11		0.00	0.18	0.04	0.73	0.18						
	12		0.89	0.04	0.02	0.04	0.92		0.88	0.03	0.03	0.01	0.91
	13		0.97	0.02	0.00	0.01	0.99		0.96	0.02	0.00	0.00	0.98
	14		0.67	0.10	0.01	0.02	0.77		0.68	0.10	0.02	0.20	0.78
	15		0.09	0.00	0.07	0.01	0.09		0.08	0.00	0.05	0.87	0.08
	16		0.60	0.14	0.01	0.12	0.73		0.67	0.12	0.02	0.08	0.79
	17		0.01	0.13	0.18	0.63	0.14		0.01	0.11	0.23	0.00	0.12
	18		0.90	0.06	0.02	0.00	0.95		0.90	0.05	0.03	0.01	0.95
	19		0.91	0.01	0.00	0.05	0.93		0.82	0.01	0.00	0.01	0.83
	20		0.04	0.03	0.03	0.81	0.07		0.04	0.08	0.02	0.01	0.12
	21		0.74	0.04	0.03	0.04	0.78		0.74	0.03	0.02	0.20	0.77
	22		0.71	0.00	0.11	0.02	0.72		0.69	0.00	0.08	0.20	0.69
	23		0.25	0.48	0.10	0.01	0.73		0.20	0.40	0.06	0.10	0.59
	24		0.19	0.14	0.02	0.01	0.33		0.19	0.12	0.02	0.60	0.31
	25		0.22	0.01	0.27	0.39	0.23		0.22	0.00	0.32	0.17	0.22
	26		0.37	0.01	0.12	0.06	0.38		0.38	0.00	0.22	0.40	0.39
	27		0.77	0.03	0.10	0.08	0.80		0.73	0.04	0.07	0.06	0.77
Sectoral inflation	1		0.17	0.53	0.10	0.05	0.70		0.16	0.49	0.05	0.08	0.65
	2		0.12	0.01	0.05	0.70	0.13		0.04	0.00	0.01	0.12	0.04
	3		0.25	0.53	0.21	0.01	0.78		0.25	0.55	0.19	0.01	0.80
	4		0.62	0.31	0.03	0.02	0.92		0.62	0.30	0.02	0.01	0.92
	5		0.17	0.43	0.38	0.01	0.60		0.16	0.43	0.34	0.02	0.59
	6		0.27	0.58	0.13	0.02	0.85		0.28	0.57	0.15	0.00	0.85

Table 13: R^2 produced by the factors: actual data (27 sectors, quarterly) (continued)

			All sectors included						Sector 11 excluded				
Data	Sector	Factor	1	2	3	4	(1 + 2)	Factor	1	2	3	4	(1 + 2)
	7		0.00	0.82	0.11	0.06	0.82		0.00	0.89	0.07	0.02	0.89
	8		0.18	0.19	0.30	0.31	0.38		0.20	0.27	0.38	0.02	0.47
	9		0.10	0.59	0.26	0.00	0.70		0.10	0.59	0.24	0.06	0.69
	10		0.11	0.48	0.26	0.09	0.58		0.11	0.55	0.21	0.11	0.66
	11		0.00	0.38	0.13	0.47	0.38						
	12		0.28	0.23	0.11	0.03	0.52		0.28	0.18	0.10	0.38	0.46
	13		0.06	0.46	0.23	0.24	0.52		0.06	0.53	0.23	0.00	0.58
	14		0.17	0.61	0.14	0.00	0.78		0.17	0.61	0.15	0.05	0.77
	15		0.05	0.28	0.18	0.01	0.33		0.04	0.25	0.17	0.51	0.29
	16		0.05	0.66	0.23	0.01	0.72		0.05	0.64	0.21	0.07	0.70
	17		0.01	0.06	0.65	0.28	0.07		0.01	0.04	0.71	0.00	0.05
	18		0.01	0.01	0.90	0.01	0.02		0.01	0.01	0.61	0.05	0.02
	19		0.00	0.83	0.14	0.00	0.83		0.00	0.78	0.14	0.05	0.78
	20		0.08	0.75	0.15	0.02	0.83		0.06	0.59	0.12	0.01	0.65
	21		0.03	0.77	0.09	0.05	0.80		0.03	0.76	0.09	0.07	0.79
	22		0.00	0.47	0.19	0.21	0.47		0.00	0.38	0.14	0.04	0.38
	23		0.00	0.07	0.02	0.66	0.07		0.00	0.13	0.01	0.12	0.13
	24		0.00	0.13	0.14	0.23	0.13		0.00	0.09	0.08	0.25	0.09
	25		0.10	0.06	0.38	0.05	0.16		0.10	0.07	0.46	0.36	0.17
	26		0.03	0.27	0.12	0.17	0.29		0.03	0.29	0.19	0.46	0.32
	27		0.01	0.78	0.18	0.01	0.80		0.02	0.77	0.21	0.01	0.78

9 Robustness exercises for FAVAR

9.1 Number of latent common components and number of lags

We perform robustness checks for FAVAR with respect to the number of latent common components and the number of lags in the VAR for the common components and report the results in Tables 14-16. Here we only present the price facts in terms of the speed of sectoral price responses.

The price facts summarized in Section 2.3 of the main text are quite robust, either estimated on actual data or estimated on the simulated data from the estimated baseline specification.

Table 14: Price facts in FAVAR with different numbers of the latent common components and different numbers of lags in the VAR for the common components: Estimated on actual data that *includes all sectors*

Number of common factors	Number of lags		Speed of sectoral price responses				
			Mean	Median	Standard deviation	Correlation with $1-\alpha_k$	Correlation
1	4	To the common component	0.243	0.248	0.057	-0.108	-0.126
		To the sector-specific component	0.769	0.757	0.271	0.339	
2	4	To the common component	0.281	0.286	0.046	0.490	0.352
		To the sector-specific component	0.861	0.915	0.237	0.441	
3	4	To the common component	0.334	0.316	0.080	0.574	0.126
		To the sector-specific component	0.863	0.912	0.251	0.323	
2	3	To the common component	0.290	0.291	0.046	0.502	0.286
		To the sector-specific component	0.855	0.851	0.237	0.336	
2	5	To the common component	0.289	0.296	0.046	0.461	0.391
		To the sector-specific component	0.845	0.818	0.253	0.444	

Note: $1-\alpha_k$ is the frequency of price changes in sector k. Sectoral frequencies of price changes are constructed by aggregating up from the ELI-level price-setting statistics reported by Nakamura and Steinsson (2008), using time-averaged consumption expenditures shares as weights.

Table 15: Price facts in FAVAR with different numbers of the common components and different numbers of lags in the VAR for the common components: Estimated on actual data that *includes all sectors but Sector 11*

Number of common factors	Number of lags		Speed of sectoral price responses: Mean	Median	Standard deviation	Correlation with $1-\alpha_k$	Correlation
1	4	To the common component	0.243	0.248	0.057	-0.108	-0.126
		To the sector-specific component	0.769	0.757	0.271	0.339	
2 (baseline)	4	To the common component	0.281	0.283	0.046	0.460	0.363
		To the sector-specific component	0.852	0.884	0.241	0.435	
3	4	To the common component	0.330	0.318	0.079	0.539	0.086
		To the sector-specific component	0.858	0.905	0.251	0.316	
2	3	To the common component	0.287	0.288	0.044	0.485	0.342
		To the sector-specific component	0.853	0.875	0.242	0.377	
2	5	To the common component	0.285	0.288	0.046	0.468	0.389
		To the sector-specific component	0.830	0.806	0.252	0.385	

Note: $1-\alpha_k$ is the price change probability estimated in Nakamura and Steinsson (2008).

Table 16: Price facts in the FAVAR with different numbers of the common components and different numbers of lags in the VAR for the common components: Estimated on simulated data from the estimated baseline DSGE specification

Number of common factors	Number of lags		Speed of sectoral price responses: Mean	Median	Standard deviation	Correlation with $1-\alpha_k$	Correlation
1	4	To the common component	0.184	0.177	0.020	-0.128	0.001
		To the sector-specific component	0.705	0.668	0.242	0.917	
2 (baseline)	4	To the common component	0.251	0.244	0.019	0.726	0.702
		To the sector-specific component	0.853	0.874	0.229	0.947	
3	4	To the common component	0.264	0.244	0.061	0.515	0.422
		To the sector-specific component	0.854	0.874	0.229	0.947	
2	3	To the common component	0.243	0.239	0.015	0.703	0.686
		To the sector-specific component	0.831	0.849	0.212	0.964	
2	5	To the common component	0.257	0.250	0.022	0.727	0.691
		To the sector-specific component	0.875	0.891	0.250	0.934	

Note: $1-\alpha_k$ is the frequency of price changes in sector k estimated in the baseline DSGE specification.

9.2 Other specifications

We now report the price facts estimated in FAVAR with different levels of disaggregation, different frequencies, and different datasets. These other specifications of FAVAR include 1) a specification that is identical to BGM's FAVAR;

2) a specification estimated on 50-sector quarterly data; 3) a specification estimated on 15-sector quarterly data; 4) a specification estimated on 27-sector quarterly data with hours worked data included as well; 5) a specification estimated on 50-sector monthly data; 6) a specification estimated on 27-sector monthly data; 7) a specification estimated on 15-sector monthly data; 8) a specification that includes only sectoral inflation data and allows for a single latent common component as in MMW; and 9) a specification that includes only sectoral inflation data as in MMW and allows for two latent common components. Note that all these FAVAR specifications are estimated on actual data with all sectors included. Specification (1) uses exactly the same specification as in BGM: five latent common components. For specifications (2)-(7), we assume that there are two latent common components in the FAVAR and the common components follows VAR(4) as in the baseline FAVAR specification. 15-sector data and 50-sector data uses the PCE data at the 2nd and 3rd level of disaggregation of PCE, respectively. For specification (8), we assume that there is a single latent common component in the FAVAR and the common component follow AR(4) while for specification (9), we assume that there are two latent common components in the FAVAR and they follow VAR(4). In specification (5), the first two factors are found to explain relatively more of the variation of data in PCA. Here we only present the price facts in terms of the speed of sectoral price responses.

Again the main price facts are robust across these specifications.

Table 17: Price facts in the other FAVAR specifications: Estimated on actual data that includes all sectors

Specifications		Speed of sectoral price responses				
		Mean	Median	Standard deviation	Correlation with $1-\alpha_k$	Correlation
(1)	To the common component	0.274	0.261	0.066	0.422	0.306
	To the sector-specific component	1.010	1.001	0.281	0.188	
(2)	To the common component	0.294	0.293	0.051	0.243	-0.020
	To the sector-specific component	0.879	0.910	0.248	0.238	
(3)	To the common component	0.304	0.309	0.039	0.104	0.620
	To the sector-specific component	0.827	0.837	0.212	0.107	
(4)	To the common component	0.277	0.281	0.046	0.541	0.298
	To the sector-specific component	0.861	0.881	0.234	0.442	
(5)	To the common component	0.334	0.329	0.120	0.416	0.375
	To the sector-specific component	0.862	0.875	0.278	0.286	
(6)	To the common component	0.313	0.337	0.071	0.359	0.185
	To the sector-specific component	0.869	0.900	0.304	0.190	
(7)	To the common component	0.338	0.327	0.068	0.119	0.619
	To the sector-specific component	0.847	0.813	0.296	0.024	
(8)	To the common component	0.286	0.286	NA	-0.277	0.087

Table 17: Price facts in the other FAVAR specifications: Estimated on actual data that includes all sectors (continued)

Specifications		Speed of sectoral price responses				
		Mean	Median	Standard deviation	Correlation with $1-\alpha_k$	Correlation
	To the sector-specific component	0.755	0.726	0.229	0.432	
(9)	To the common component	0.323	0.298	0.051	0.647	0.405
	To the sector-specific component	0.814	0.810	0.237	0.585	

Note: Specifications are 1) a specification that is identical to BGM's FAVAR; 2) a specification estimated on 50-sector quarterly data; 3) a specification estimated on 15-sector quarterly data; 4) a specification estimated on 27-sector quarterly data with hours worked data included as well; 5) a specification estimated on 50-sector monthly data; 6) a specification estimated on 27-sector monthly data; 7) a specification estimated on 15-sector monthly data; 8) a specification that includes only sectoral inflation data and allows for a single latent common component as in MMW; and 9) a specification that includes only sectoral inflation data as in MMW and allows for two latent common components. $1-\alpha_k$ is the price change probability estimated in Nakamura and Steinsson (2008) except in (1) where the correlation is based on a mapping between 108 (out of 192) PCE price series included in BGM's FAVAR and Bils and Klenow (2004).

10 Comparison of FAVAR and DFM regarding the dispersion of the speeds of sectoral price responses

This section presents simulation studies to explain the discrepancy between Maćkowiak et al. (2009, MMW henceforth)'s DFM and our results regarding the cross-sectional dispersion of the speeds of sectoral price responses to different shocks.

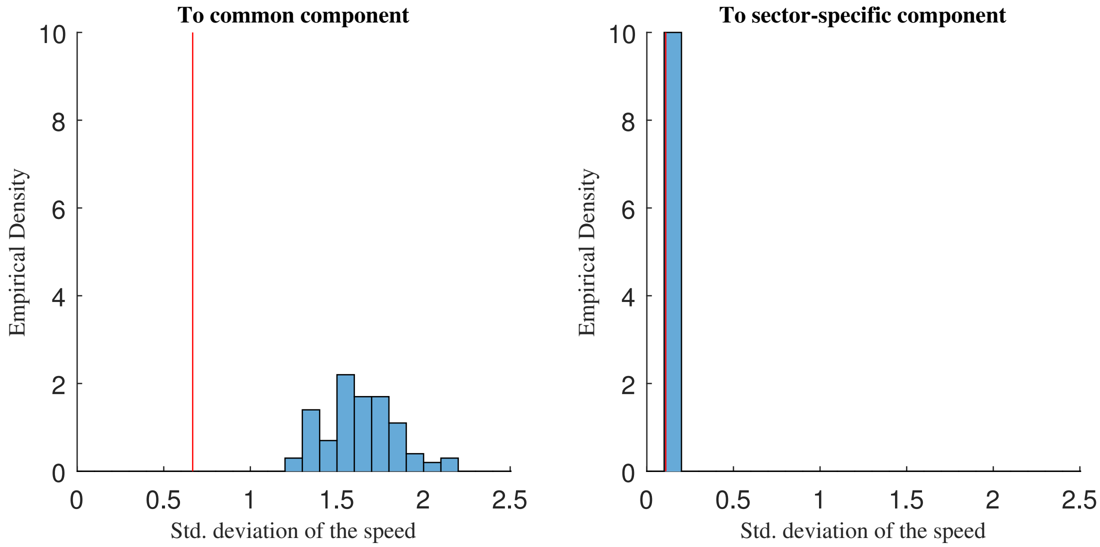
Simulations results indicate that MMW's DFM tends to overestimate the cross-sectional dispersion of the speed of sectoral price responses to shocks to the common component relatively more than to sector-specific components. To show this, we simulate data from DFM at the posterior median of its parameters and estimating the same DFM again on these simulated data. In MMW's DFM, at the posterior median, the standard deviation of the speed of sectoral price responses is 0.6670 to the common component and 0.1092 to the sector-specific component, respectively.

We simulate 100 samples with 245 observations (the same size as the actual dataset used in the baseline specification of MMW) each, estimate DFM with the same specification as in MMW on each of these samples, and then compute the cross-sectional standard deviation of the speeds of sectoral price responses to shocks to the common component and to the sector-specific component, respectively.

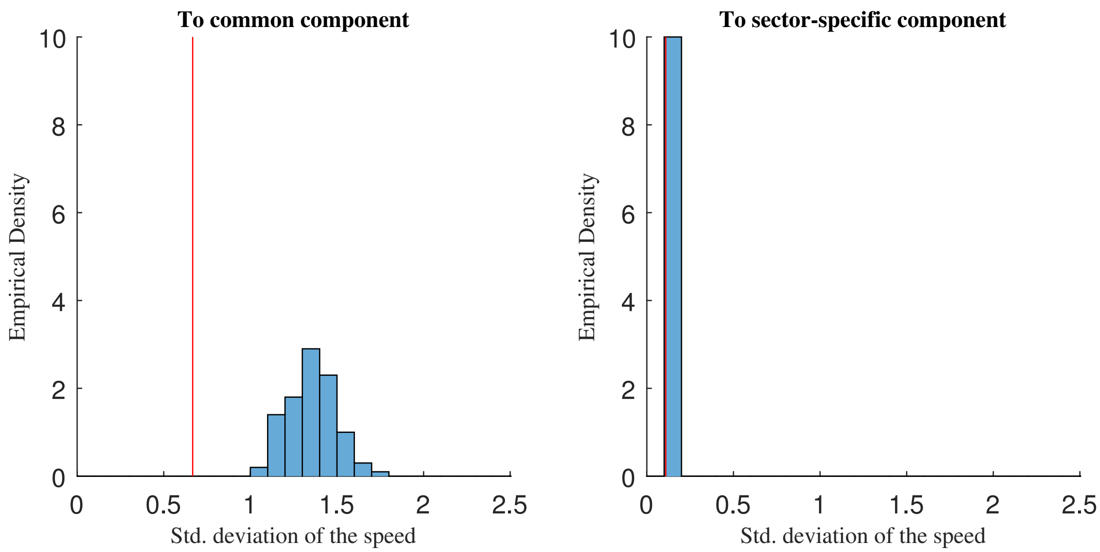
We compute the cross-sectional standard deviation in two different ways and compare them: one that accounts for both dispersion across sectors and parameter uncertainty as in MMW, by pooling all the speeds of sectoral price responses across sectors obtained for all posterior draws and another that is computed across sectors *for each of the parameter draws* from the posterior distribution. The second approach generates the correct posterior distribution of the cross-sectional dispersion of the speeds of sectoral price responses. With negligible parameter uncertainty - i.e. in a very large sample - the two approaches yield the same cross-sectional standard deviation. In finite samples, however, the second approach delivers a smaller standard deviation of the speed of sectoral price responses across sectors than the first approach.

Figure 1 presents the distribution of the cross-sectional standard deviation of the speed of sectoral price responses. Panel (a) presents the histogram of the cross-sectional standard deviation across the 100 simulated samples. For each sample, the cross-sectional standard deviation is computed by taking into account dispersion across sectors and parameter uncertainty as in MMW. Panel (b) shows the histogram of the posterior median of the cross-sectional standard deviation across the 100 simulated samples. For each sample, the posterior median of the standard deviation of the speed is computed from the posterior distribution of the standard deviation of the speed.

Two results are noteworthy. First, the cross-sectional dispersion of the speed of sectoral price responses is more exacerbated when both differences across sectors and parameter uncertainty are accounted for, as in MMW. Second, even for the appropriate posterior distributions of the objects of interest (panel b), DFM overestimates the cross-sectional dispersion to the common component substantially more than to the sector-specific component. In panel (b), across the 100 simulated samples, the median and 90% probability interval of the standard deviation is 1.3575 and [0.7832, 2.6845] to the common component and 0.1420 and [0.1357, 0.1496] to the sector-specific component, respectively. Note that in the data generating process, the standard deviation of the speed of sectoral price responses is 0.6670 to the common component and 0.1092 to the sector-specific component, respectively.



(a) Distribution when both uncertainty across sectors and parameter uncertainty are accounted for as in MMW



(b) Distribution of the posterior median of the cross-sectional standard deviation

Figure 1: Distribution of the standard deviation of the speed of sectoral price responses across 100 simulated samples
Note: The standard deviation in the data generating process is marked in red vertical lines.

From here on, we use the second approach to compute the cross-sectional dispersion of the speed of sectoral price responses.

We now do another simulation exercise varying sample sizes. As before, we simulate 100 samples at the posterior median of MMW's DFM, estimate DFM on each of the simulated samples, and compute the cross-sectional standard deviation of the speed of sectoral price responses to the common component and to the sector-specific component, respectively. We repeat this for different sample sizes. Table 18 reports the posterior median with 90% probability intervals of the standard deviation of the speed of sectoral price responses. In small samples, the cross-sectional dispersion is estimated to be larger than their data generating process (DGP) values and this overestimation is much more severe when looking at shocks to the common component than when considering sector-specific shocks. In larger samples, the difference becomes smaller. We conjecture that as the sample size increases, parameter uncertainty becomes smaller, which works relatively more in favor of the common component so that the standard deviation of the speed of sectoral price responses to the common component shrinks more. This is because the speed of sectoral price responses to the common component involves more coefficients than to the sector-specific component.[6]

Table 18: Standard deviation of the speed of sectoral price responses in DFM estimated on DFM-simulated data: posterior median and 90% probability interval

Sample size	To the common component		To the sector-specific component	
	Median	[5%, 95% quantiles]	Median	[5%, 95% quantiles]
100	1.7597	[0.9738, 3.3581]	0.1357	[0.1287, 0.1451]
245	1.3575	[0.7832, 2.6845]	0.1420	[0.1357, 0.1496]
500	1.1500	[0.6768, 2.3089]	0.1473	[0.1419, 0.1533]
1K	1.0487	[0.6272, 2.1147]	0.1502	[0.1461, 0.1548]
5K	0.8190	[0.5157, 1.5983]	0.1527	[0.1507, 0.1549]
DGP	0.6670		0.1092	

Note: The numbers are the mean of the posterior median, 5% and 95% quantiles, which is computed across 100 simulated samples. DGP is MMW's DFM at the posterior median of the parameters. The posterior distribution is obtained by computing the standard deviation of the speed of sectoral price responses across sectors conditional on each posterior draw of the parameters of DFM.

The posterior median of the parameters of DFM in MMW implies that the cross-sectional dispersion of the speed of sectoral price responses is larger to the common component (0.6670) than to the sector-specific component (0.1092). To show that DFM overestimates the cross-sectional dispersion to the common component more than to the sector-specific component even if the speed of sectoral price responses is the same between to the common component and to the sector-specific component, we set the parameter values of DFM so that the speed of sectoral price responses to both components is identical and so is the cross-sectional dispersion of the speed.[7] As before we simulate 100 samples with 245 observations at these parameter values, estimate DFM on each of the simulated samples and compute the posterior median of the cross-sectional standard deviation of the speed of sectoral price responses for each sample.

The histogram across the 100 samples is displayed in Figure 2. It shows that DFM still overestimates the cross-sectional distribution of the speed of sectoral price responses to the common component more than to the sector-specific component. Across the 100 simulated samples, the median and 90% probability interval of the standard deviation is 1.5244 and [0.7169, 3.2727] to the common component and 0.1426 and [0.1362, 0.1504] to the sector-specific component. Interestingly, the cross-sectional standard deviation is more dispersed than at the posterior median of the parameters reported above.

[6]The baseline specification has 24 coefficients on the common component and its lags and 6 AR coefficients on the sector-specific component.

[7]We do this by setting the MA coefficients on the lags of the common component in each sector at the impulse response of sectoral inflation to the sector-specific component of each sector in Equation (1) of MMW.

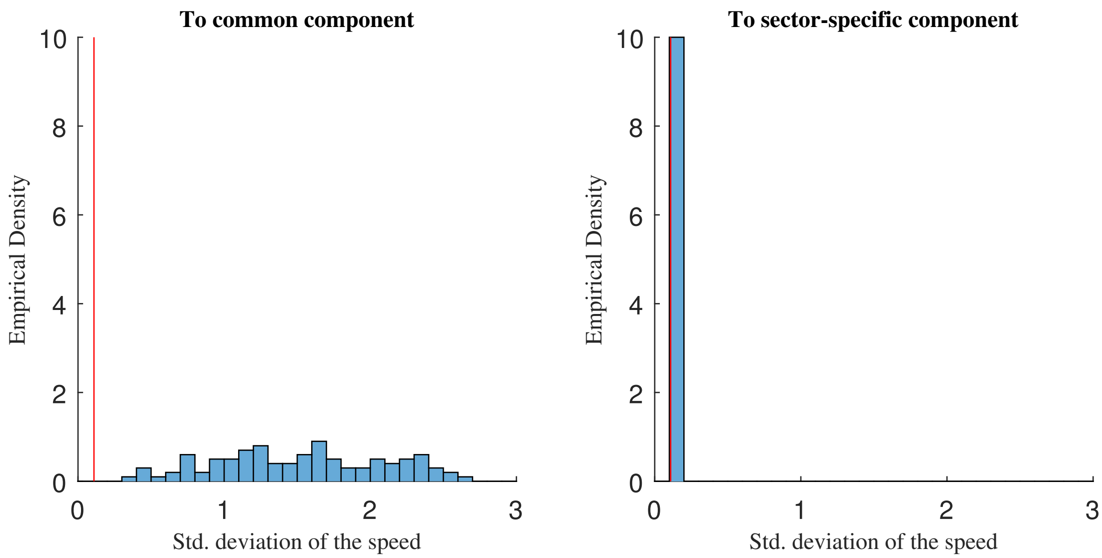


Figure 2: Distribution of the posterior median of the cross-sectional standard deviation of the speed of sectoral price responses across 100 simulated samples when DGP dispersion is identical
Note: The standard deviation in the data generating process is marked in red vertical lines.

We also did a simulation exercise with different sample sizes as above. The result is reported in Table 19. We can observe a similar pattern across different sample sizes. The cross-sectional dispersion is much larger than their DGP values in small samples and this difference becomes smaller as the sample size increases.

Table 19: Standard deviation of the speed of sectoral price responses in DFM estimated on DFM-simulated data when DGP dispersion is identical: posterior median and 90% probability interval

Sample size	To the common component		To the sector-specific component	
	Median	[5%, 95% quantiles]	Median	[5%, 95% quantiles]
100	2.2231	[1.0691, 4.2371]	0.1363	[0.1291, 0.1463]
245	1.5244	[0.7169, 3.2727]	0.1426	[0.1362, 0.1504]
500	0.7229	[0.3999, 1.8731]	0.1468	[0.1414, 0.1528]
1K	0.3683	[0.2773, 0.6519]	0.1496	[0.1454, 0.1541]
5K	0.2010	[0.1817, 0.2240]	0.1524	[0.1504, 0.1546]
DGP	0.1092		0.1092	

Note: The numbers are the mean of the posterior median, 5% and 95% quantiles, which is computed across 100 simulated samples. The posterior distribution is obtained by computing the standard deviation of the speed of price responses across sectors conditional on each posterior draw of the parameters of DFM.

Now we repeat a similar simulation exercise using our estimated baseline DSGE model as DGP. We simulate samples with different sizes, 100 samples for each sample size, from the baseline DSGE model at the posterior mode and estimate DFM on each of these samples. Since the frequency of the baseline DSGE model is quarterly, we modify DFM accordingly as described in MMW. Note that we include only sectoral inflation to estimate DFM as in MMW. The posterior median and 90% probability interval of the standard deviation of the speed of sectoral price responses across sectors are presented in Table 20. As for the cases where DFM is DGP, DFM overestimates the cross-sectional dispersion of the speed of sectoral price responses more severely to the common component than to the sector-specific component. And the overestimation becomes smaller as the sample size increases. Importantly, when the sample size is large, the cross-sectional dispersion of speeds of price responsed is smaller when shocks to the common component are considered, relative to when sector-specific shocks are considered.

Table 20: Standard deviation of the speed of sectoral price responses in DFM estimated on DSGE-simulated data: posterior median and 90% probability interval

Sample size	To the common component		To the sector-specific component	
	Median	[5%, 95% quantiles]	Median	[5%, 95% quantiles]
100	0.5738	[0.2549, 5.4147]	0.2510	[0.2303, 0.2731]
245	0.3031	[0.1827, 1.1597]	0.2571	[0.2408, 0.2741]
500	0.2327	[0.1671, 0.3784]	0.2573	[0.2449, 0.2699]
1K	0.2053	[0.1619, 0.2758]	0.2585	[0.2491, 0.2678]
5K	0.1921	[0.1720, 0.2163]	0.2587	[0.2544, 0.2631]
10K	0.1879	[0.1742, 0.2030]	0.2586	[0.2554, 0.2618]

Note: The numbers are the mean of the posterior median, 5% and 95% quantiles, which is computed across 100 simulated samples. DGP is the baseline sectoral DSGE model at the posterior mode. The posterior distribution is obtained by computing the standard deviation of the speed of price responses across sectors conditional on each posterior draw of the parameters of DFM. We simulate 100 samples per each size and estimate DFM on each sample.

Overestimation is less severe in FAVARs. We estimate a FAVAR on the same 100 simulated samples from the baseline DSGE model at the posterior mode as above. The results regarding the cross-sectional dispersion are presented in Table 21. Note that the cross-sectional dispersion of the speeds of price responses is smaller when shocks to the common component are considered, relative to when sector-specific shocks are considered - and this holds even for a small sample.

Table 21: Standard deviation of the speed of sectoral price responses in FAVAR estimated on DSGE-simulated data: posterior median and 90% probability interval

Sample size	To the common component		To the sector-specific component	
	Median	[5%, 95% quantiles]	Median	[5%, 95% quantiles]
100	0.1123	[0.0561, 0.1918]	0.2616	[0.2208, 0.3020]
245	0.0876	[0.0539, 0.1298]	0.2438	[0.2146, 0.2762]
500	0.0831	[0.0549, 0.1234]	0.2412	[0.2189, 0.2628]
1K	0.0756	[0.0543, 0.1107]	0.2367	[0.2208, 0.2540]
5K	0.0671	[0.0596, 0.0778]	0.2348	[0.2270, 0.2440]
10K	0.0641	[0.0590, 0.0697]	0.2340	[0.2278, 0.2410]

Note: The numbers are the median, 5% and 95% quantiles of the standard deviation of the speed of sectoral price responses that is computed across 100 simulated samples. DGP is the baseline sectoral DSGE model at the posterior mode. We simulate 100 samples per each sample size and estimate FAVAR on each sample.

Part III

Bayesian estimation and convergence

11 Details of the estimation algorithm

We estimate the structural model as follows. For each specification, we first run numerical optimization routines in order to find the posterior mode, which is used as a starting point of the Markov chain. We carry out an extensive search for the posterior mode by starting the optimization routine at many different points. Specifically, we generate 20,000 draws from the prior distribution of the parameters, keep 20 draws with the highest posterior densities and 5 draws randomly chosen among the 20,000 draws, and start the optimization routine at these 25 draws. Since some of the parameters have a bounded support, the numerical optimization is executed in two different ways: one way where the support of the prior distribution is directly imposed and another where it is imposed through usual reparameterization. For each optimization run, we use two different numerical optimization routines sequentially. The first optimization routine is `csminwel` by Chris Sims and the second is `fminsearch` from Matlab's optimization toolbox which starts at the point found by `csminwel`.

To characterize the posterior distribution of the parameters, we simulate their draws from the posterior distribution using the random-walk Metropolis algorithm. The starting point of the algorithm is the posterior mode. The proposal distribution of the algorithm is a student t-distribution. The mean of the proposal distribution is the previous draw. The covariance matrix of the proposal distribution is found as follows. We first compute the inverse of -1 times the Hessian of the log posterior density at the posterior mode, which provides a first crude estimate of the covariance matrix of the proposal distribution. Then we run a so-called *adaptive phase* of the MCMC, with four sub-phases of 100, 200, 600, and 100 thousand iterations, respectively. At the end of each sub-phase we discard the first half of the draws and compute a sample covariance matrix of the parameter draws to be used in the proposal distribution in the next sub-phase. In each sub-phase we rescale the covariance matrix inherited from the previous sub-phase in order to get a *fine-tuned covariance matrix* that yields the average acceptance rate as close as possible to 0.23. In the adaptive phases, we did not find a point with a higher posterior density than the posterior mode numerically found previously. Next, we run the so-called *fixed phase* of the MCMC. We take the estimate of the posterior mode and the fine-tuned sample covariance matrix from the adaptive phase for the proposal distribution and run a chain of 1.5 million iterations. To initialize each chain we draw from a candidate normal distribution centered on the posterior mode estimate, with three times the covariance matrix given by the fine-tuned covariance matrix found in the adaptive phase. In each chain, we generate 1.5 million draws, discard the first one third, and keep the last out of every 5 draws among the remaining 1 million draws for thinning of the chain. This gives us 0.2 million draws for every chain. We generate 5 parallel chains, for a total of 1 million draws.

12 Estimation results

Moments of prior and posterior distributions of the parameters of the baseline specification are provided in Table 9 of the main text. Among other specifications we estimate, we report prior and posterior distributions of the specification that features habit formation in sectoral consumption in Table 22.

Table 22: Prior and posterior distributions for the specification with sectoral habit formation

Parameters	**Prior distribution**			**Posterior distribution**				
	Type	**Mean**	**Std. deviation**	**Mode**	**Mean**	**[5%**	**Median**	**95%]**
ϕ_π	Normal	1.5	0.25	2.127	2.238	1.896	2.237	2.580
ϕ_c	Normal	0.5	0.05	0.483	0.466	0.385	0.466	0.547
τ_1	Beta	0.5	0.2	0.042	0.063	0.017	0.058	0.128
τ_2	Beta	0.5	0.2	0.054	0.085	0.022	0.077	0.176
τ_3	Beta	0.5	0.2	0.307	0.357	0.229	0.354	0.493
τ_4	Beta	0.5	0.2	0.278	0.386	0.255	0.389	0.508
τ_5	Beta	0.5	0.2	0.179	0.249	0.096	0.240	0.435
τ_6	Beta	0.5	0.2	0.173	0.192	0.085	0.189	0.308
τ_7	Beta	0.5	0.2	0.067	0.101	0.031	0.094	0.193
τ_8	Beta	0.5	0.2	0.368	0.379	0.257	0.379	0.500
τ_9	Beta	0.5	0.2	0.117	0.128	0.050	0.125	0.216
τ_{10}	Beta	0.5	0.2	0.062	0.097	0.029	0.091	0.186
τ_{11}	Beta	0.5	0.2	0.872	0.871	0.850	0.871	0.890
τ_{12}	Beta	0.5	0.2	0.045	0.072	0.018	0.064	0.153
τ_{13}	Beta	0.5	0.2	0.120	0.144	0.051	0.139	0.255
τ_{14}	Beta	0.5	0.2	0.139	0.169	0.082	0.166	0.268
τ_{15}	Beta	0.5	0.2	0.223	0.241	0.089	0.236	0.413
τ_{16}	Beta	0.5	0.2	0.965	0.963	0.948	0.964	0.976
τ_{17}	Beta	0.5	0.2	0.044	0.065	0.017	0.059	0.134
τ_{18}	Beta	0.5	0.2	0.911	0.870	0.744	0.886	0.937
τ_{19}	Beta	0.5	0.2	0.758	0.777	0.718	0.778	0.829
τ_{20}	Beta	0.5	0.2	0.184	0.205	0.082	0.201	0.340
τ_{21}	Beta	0.5	0.2	0.414	0.530	0.329	0.539	0.702
τ_{22}	Beta	0.5	0.2	0.065	0.085	0.024	0.079	0.165
τ_{23}	Beta	0.5	0.2	0.157	0.182	0.064	0.176	0.321
τ_{24}	Beta	0.5	0.2	0.933	0.930	0.907	0.931	0.950
τ_{25}	Beta	0.5	0.2	0.326	0.360	0.201	0.356	0.535
τ_{26}	Beta	0.5	0.2	0.267	0.297	0.191	0.297	0.403
τ_{27}	Beta	0.5	0.2	0.511	0.545	0.434	0.546	0.651
α_1	Beta	0.519	0.1	0.885	0.879	0.841	0.881	0.906
α_2	Beta	0.12	0.1	0.563	0.578	0.460	0.583	0.683
α_3	Beta	0.483	0.1	0.555	0.608	0.509	0.608	0.709
α_4	Beta	0.463	0.1	0.648	0.687	0.603	0.688	0.765
α_5	Beta	0.691	0.1	0.823	0.810	0.736	0.813	0.871
α_6	Beta	0.551	0.1	0.583	0.617	0.520	0.617	0.712
α_7	Beta	0.318	0.1	0.409	0.436	0.337	0.437	0.532
α_8	Beta	0.492	0.1	0.489	0.562	0.433	0.557	0.711
α_9	Beta	0.314	0.1	0.416	0.480	0.351	0.480	0.605
α_{10}	Beta	0.409	0.1	0.405	0.493	0.350	0.489	0.653
α_{11}	Beta	0.12	0.1	0.003	0.023	0.002	0.018	0.063

Table 22: Prior and posterior distributions for the specification with sectoral habit formation (continued)

Parameters	Prior distribution			Posterior distribution				
	Type	Mean	Std. deviation	Mode	Mean	[5%	Median	95%]
α_{12}	Beta	0.613	0.1	0.760	0.774	0.700	0.775	0.841
α_{13}	Beta	0.755	0.1	0.627	0.687	0.590	0.689	0.774
α_{14}	Beta	0.582	0.1	0.538	0.592	0.499	0.591	0.687
α_{15}	Beta	0.307	0.1	0.199	0.217	0.120	0.215	0.323
α_{16}	Beta	0.722	0.1	0.929	0.929	0.911	0.931	0.943
α_{17}	Beta	0.212	0.1	0.412	0.432	0.327	0.434	0.532
α_{18}	Beta	0.857	0.1	0.886	0.853	0.726	0.870	0.924
α_{19}	Beta	0.503	0.1	0.690	0.712	0.644	0.713	0.778
α_{20}	Beta	0.181	0.1	0.155	0.176	0.067	0.171	0.299
α_{21}	Beta	0.727	0.1	0.726	0.756	0.676	0.757	0.831
α_{22}	Beta	0.85	0.1	0.900	0.903	0.843	0.907	0.951
α_{23}	Beta	0.247	0.1	0.251	0.330	0.179	0.328	0.495
α_{24}	Beta	0.78	0.1	0.107	0.118	0.074	0.117	0.167
α_{25}	Beta	0.313	0.1	0.355	0.386	0.276	0.386	0.496
α_{26}	Beta	0.822	0.1	0.795	0.840	0.757	0.842	0.914
α_{27}	Beta	0.862	0.1	0.752	0.770	0.698	0.765	0.863
ρ_i	Beta	0.7	0.15	0.815	0.823	0.769	0.826	0.866
ρ_μ	Beta	0.7	0.15	0.728	0.633	0.372	0.644	0.857
ρ_γ	Beta	0.7	0.15	0.769	0.777	0.717	0.778	0.833
ρ_a	Beta	0.7	0.15	0.478	0.513	0.336	0.514	0.689
ρ_{A_1}	Beta	0.7	0.15	0.491	0.496	0.338	0.492	0.671
ρ_{A_2}	Beta	0.7	0.15	0.974	0.893	0.711	0.948	0.986
ρ_{A_3}	Beta	0.7	0.15	0.872	0.799	0.635	0.811	0.920
ρ_{A_4}	Beta	0.7	0.15	0.975	0.961	0.933	0.963	0.983
ρ_{A_5}	Beta	0.7	0.15	0.344	0.362	0.196	0.354	0.559
ρ_{A_6}	Beta	0.7	0.15	0.945	0.931	0.872	0.935	0.975
ρ_{A_7}	Beta	0.7	0.15	0.978	0.950	0.881	0.960	0.988
ρ_{A_8}	Beta	0.7	0.15	0.807	0.716	0.509	0.731	0.869
ρ_{A_9}	Beta	0.7	0.15	0.826	0.733	0.541	0.743	0.888
$\rho_{A_{10}}$	Beta	0.7	0.15	0.814	0.707	0.487	0.719	0.885
$\rho_{A_{11}}$	Beta	0.7	0.15	0.871	0.863	0.792	0.868	0.921
$\rho_{A_{12}}$	Beta	0.7	0.15	0.961	0.943	0.879	0.950	0.983
$\rho_{A_{13}}$	Beta	0.7	0.15	0.904	0.849	0.717	0.860	0.945
$\rho_{A_{14}}$	Beta	0.7	0.15	0.898	0.826	0.686	0.834	0.939
$\rho_{A_{15}}$	Beta	0.7	0.15	0.927	0.917	0.855	0.920	0.968
$\rho_{A_{16}}$	Beta	0.7	0.15	0.819	0.819	0.759	0.821	0.872
$\rho_{A_{17}}$	Beta	0.7	0.15	0.985	0.979	0.959	0.981	0.994
$\rho_{A_{18}}$	Beta	0.7	0.15	0.896	0.915	0.848	0.918	0.973
$\rho_{A_{19}}$	Beta	0.7	0.15	0.986	0.981	0.965	0.982	0.994
$\rho_{A_{20}}$	Beta	0.7	0.15	0.912	0.907	0.840	0.910	0.963

Table 22: Prior and posterior distributions for the specification with sectoral habit formation (continued)

Parameters	Prior distribution			Posterior distribution				
	Type	**Mean**	**Std. deviation**	**Mode**	**Mean**	**[5%**	**Median**	**95%]**
$\rho_{A_{21}}$	Beta	0.7	0.15	0.980	0.969	0.940	0.971	0.991
$\rho_{A_{22}}$	Beta	0.7	0.15	0.967	0.955	0.907	0.960	0.986
$\rho_{A_{23}}$	Beta	0.7	0.15	0.797	0.720	0.526	0.733	0.871
$\rho_{A_{24}}$	Beta	0.7	0.15	0.817	0.831	0.734	0.836	0.914
$\rho_{A_{25}}$	Beta	0.7	0.15	0.873	0.905	0.817	0.908	0.977
$\rho_{A_{26}}$	Beta	0.7	0.15	0.953	0.914	0.823	0.923	0.974
$\rho_{A_{27}}$	Beta	0.7	0.15	0.979	0.969	0.938	0.972	0.991
ρ_{D_1}	Beta	0.7	0.15	0.777	0.763	0.682	0.764	0.838
ρ_{D_2}	Beta	0.7	0.15	0.870	0.888	0.800	0.877	0.988
ρ_{D_3}	Beta	0.7	0.15	0.951	0.945	0.901	0.948	0.981
ρ_{D_4}	Beta	0.7	0.15	0.845	0.944	0.800	0.981	0.994
ρ_{D_5}	Beta	0.7	0.15	0.862	0.822	0.701	0.831	0.912
ρ_{D_6}	Beta	0.7	0.15	0.925	0.927	0.871	0.929	0.974
ρ_{D_7}	Beta	0.7	0.15	0.979	0.969	0.936	0.972	0.992
ρ_{D_8}	Beta	0.7	0.15	0.981	0.967	0.931	0.971	0.991
ρ_{D_9}	Beta	0.7	0.15	0.888	0.891	0.832	0.893	0.941
$\rho_{D_{10}}$	Beta	0.7	0.15	0.832	0.834	0.738	0.837	0.919
$\rho_{D_{12}}$	Beta	0.7	0.15	0.964	0.960	0.921	0.963	0.991
$\rho_{D_{13}}$	Beta	0.7	0.15	0.946	0.936	0.886	0.938	0.979
$\rho_{D_{14}}$	Beta	0.7	0.15	0.936	0.924	0.870	0.927	0.970
$\rho_{D_{15}}$	Beta	0.7	0.15	0.938	0.927	0.874	0.929	0.974
$\rho_{D_{16}}$	Beta	0.7	0.15	0.984	0.980	0.959	0.982	0.994
$\rho_{D_{17}}$	Beta	0.7	0.15	0.873	0.871	0.797	0.873	0.941
$\rho_{D_{18}}$	Beta	0.7	0.15	0.988	0.986	0.974	0.987	0.996
$\rho_{D_{19}}$	Beta	0.7	0.15	0.915	0.898	0.828	0.901	0.959
$\rho_{D_{20}}$	Beta	0.7	0.15	0.873	0.872	0.793	0.875	0.943
$\rho_{D_{21}}$	Beta	0.7	0.15	0.976	0.866	0.600	0.918	0.988
$\rho_{D_{22}}$	Beta	0.7	0.15	0.943	0.933	0.882	0.935	0.979
$\rho_{D_{23}}$	Beta	0.7	0.15	0.912	0.910	0.844	0.913	0.966
$\rho_{D_{24}}$	Beta	0.7	0.15	0.727	0.709	0.598	0.709	0.820
$\rho_{D_{25}}$	Beta	0.7	0.15	0.969	0.952	0.907	0.955	0.984
$\rho_{D_{26}}$	Beta	0.7	0.15	0.969	0.907	0.782	0.922	0.982
$\rho_{D_{27}}$	Beta	0.7	0.15	0.910	0.922	0.853	0.926	0.974
σ_{μ}	Inverted Gamma	0.00125	0.005	0.001	0.001	0.000	0.001	0.001
σ_{γ}	Inverted Gamma	0.02	0.5	0.015	0.017	0.013	0.017	0.022
σ_{a}	Inverted Gamma	0.02	0.5	0.009	0.009	0.006	0.009	0.012
σ_{A_1}	Inverted Gamma	0.02	0.5	0.132	0.132	0.065	0.127	0.215
σ_{A_2}	Inverted Gamma	0.02	0.5	0.037	0.045	0.033	0.043	0.064
σ_{A_3}	Inverted Gamma	0.02	0.5	0.011	0.015	0.010	0.014	0.024
σ_{A_4}	Inverted Gamma	0.02	0.5	0.022	0.027	0.020	0.026	0.036

Table 22: Prior and posterior distributions for the specification with sectoral habit formation (continued)

Parameters	Prior distribution			Posterior distribution				
	Type	Mean	Std. deviation	Mode	Mean	[5%	Median	95%]
σ_{A_5}	Inverted Gamma	0.02	0.5	0.086	0.087	0.038	0.077	0.166
σ_{A_6}	Inverted Gamma	0.02	0.5	0.018	0.021	0.015	0.021	0.031
σ_{A_7}	Inverted Gamma	0.02	0.5	0.009	0.011	0.008	0.010	0.013
σ_{A_8}	Inverted Gamma	0.02	0.5	0.018	0.026	0.015	0.023	0.052
σ_{A_9}	Inverted Gamma	0.02	0.5	0.014	0.018	0.012	0.017	0.028
$\sigma_{A_{10}}$	Inverted Gamma	0.02	0.5	0.013	0.019	0.012	0.017	0.035
$\sigma_{A_{11}}$	Inverted Gamma	0.02	0.5	0.040	0.041	0.033	0.041	0.051
$\sigma_{A_{12}}$	Inverted Gamma	0.02	0.5	0.017	0.021	0.013	0.019	0.032
$\sigma_{A_{13}}$	Inverted Gamma	0.02	0.5	0.012	0.019	0.011	0.017	0.031
$\sigma_{A_{14}}$	Inverted Gamma	0.02	0.5	0.009	0.012	0.008	0.011	0.018
$\sigma_{A_{15}}$	Inverted Gamma	0.02	0.5	0.022	0.024	0.020	0.023	0.028
$\sigma_{A_{16}}$	Inverted Gamma	0.02	0.5	0.066	0.071	0.043	0.070	0.105
$\sigma_{A_{17}}$	Inverted Gamma	0.02	0.5	0.021	0.022	0.018	0.022	0.027
$\sigma_{A_{18}}$	Inverted Gamma	0.02	0.5	0.034	0.032	0.010	0.027	0.073
$\sigma_{A_{19}}$	Inverted Gamma	0.02	0.5	0.010	0.011	0.008	0.011	0.014
$\sigma_{A_{20}}$	Inverted Gamma	0.02	0.5	0.023	0.024	0.020	0.024	0.029
$\sigma_{A_{21}}$	Inverted Gamma	0.02	0.5	0.012	0.015	0.010	0.015	0.023
$\sigma_{A_{22}}$	Inverted Gamma	0.02	0.5	0.019	0.027	0.012	0.022	0.061
$\sigma_{A_{23}}$	Inverted Gamma	0.02	0.5	0.015	0.019	0.013	0.018	0.028
$\sigma_{A_{24}}$	Inverted Gamma	0.02	0.5	0.011	0.012	0.010	0.012	0.014
$\sigma_{A_{25}}$	Inverted Gamma	0.02	0.5	0.012	0.013	0.010	0.013	0.017
$\sigma_{A_{26}}$	Inverted Gamma	0.02	0.5	0.015	0.029	0.012	0.023	0.069
$\sigma_{A_{27}}$	Inverted Gamma	0.02	0.5	0.009	0.011	0.007	0.010	0.020
σ_{D_1}	Inverted Gamma	0.02	0.5	0.122	0.131	0.115	0.130	0.148
σ_{D_2}	Inverted Gamma	0.02	0.5	0.132	0.143	0.125	0.142	0.163
σ_{D_3}	Inverted Gamma	0.02	0.5	0.022	0.023	0.020	0.023	0.026
σ_{D_4}	Inverted Gamma	0.02	0.5	0.032	0.035	0.031	0.035	0.040
σ_{D_5}	Inverted Gamma	0.02	0.5	0.044	0.047	0.041	0.047	0.053
σ_{D_6}	Inverted Gamma	0.02	0.5	0.033	0.035	0.031	0.035	0.040
σ_{D_7}	Inverted Gamma	0.02	0.5	0.013	0.014	0.012	0.013	0.015
σ_{D_8}	Inverted Gamma	0.02	0.5	0.019	0.021	0.018	0.021	0.024
σ_{D_9}	Inverted Gamma	0.02	0.5	0.020	0.022	0.019	0.022	0.025
$\sigma_{D_{10}}$	Inverted Gamma	0.02	0.5	0.022	0.024	0.021	0.024	0.027
$\sigma_{D_{12}}$	Inverted Gamma	0.02	0.5	0.032	0.034	0.030	0.034	0.039
$\sigma_{D_{13}}$	Inverted Gamma	0.02	0.5	0.017	0.018	0.016	0.018	0.020
$\sigma_{D_{14}}$	Inverted Gamma	0.02	0.5	0.011	0.011	0.010	0.011	0.013
$\sigma_{D_{15}}$	Inverted Gamma	0.02	0.5	0.039	0.041	0.036	0.040	0.046
$\sigma_{D_{16}}$	Inverted Gamma	0.02	0.5	0.010	0.010	0.009	0.010	0.011
$\sigma_{D_{17}}$	Inverted Gamma	0.02	0.5	0.059	0.062	0.055	0.062	0.070
$\sigma_{D_{18}}$	Inverted Gamma	0.02	0.5	0.013	0.013	0.011	0.013	0.014

Table 22: Prior and posterior distributions for the specification with sectoral habit formation (continued)

Parameters	Prior distribution			Posterior distribution				
	Type	Mean	Std. deviation	Mode	Mean	[5%	Median	95%]
$\sigma_{D_{19}}$	Inverted Gamma	0.02	0.5	0.014	0.015	0.013	0.015	0.017
$\sigma_{D_{20}}$	Inverted Gamma	0.02	0.5	0.041	0.043	0.038	0.042	0.048
$\sigma_{D_{21}}$	Inverted Gamma	0.02	0.5	0.014	0.015	0.013	0.015	0.017
$\sigma_{D_{22}}$	Inverted Gamma	0.02	0.5	0.019	0.021	0.018	0.021	0.023
$\sigma_{D_{23}}$	Inverted Gamma	0.02	0.5	0.041	0.044	0.038	0.043	0.049
$\sigma_{D_{24}}$	Inverted Gamma	0.02	0.5	0.040	0.041	0.037	0.041	0.047
$\sigma_{D_{25}}$	Inverted Gamma	0.02	0.5	0.024	0.026	0.023	0.026	0.029
$\sigma_{D_{26}}$	Inverted Gamma	0.02	0.5	0.009	0.009	0.008	0.009	0.011
$\sigma_{D_{27}}$	Inverted Gamma	0.02	0.5	0.010	0.010	0.009	0.010	0.012

Note: there are no parameters for $d_{11,t}$ because data for Sector 11 are used in the estimation.

13 Convergence statistics of Bayesian estimations

We check for convergence for the draws of all 5 chains per each estimation by calculating the potential scale reduction factor (PSRF). The PSRF is the square root of the ratio of an estimate of the marginal posterior variance to the mean of the marginal posterior variance within each chain. This factor expresses the potential reduction in the scaling of the estimated marginal posterior variance relative to the true distribution expected when increasing the number of iterations in the Markov-chain algorithm. Hence, as the PSRF for a parameter approaches unity, it is a sign of convergence of the Markov-chain for the parameter. See Gelman and Rubin (1992) for more information.

The estimates of the baseline specification are presented in Table 23. For most of the parameters, the PSRF estimates and the upper bound of their 95% confidence intervals are effectively 1. The PSRF estimates for all the parameters but three are less than 1.1, which is the rule-of-thumb value commonly used in the literature as an upper limit for good convergence. Parameters ρ_μ, σ_μ, and σ_{A_5} is estimated to have a relatively large PSRF. Comparative statics analysis shows, however, that this parameter has little effect on overall model fit and on its ability to match sectoral price facts.

Table 23: Potential scale reduction factor estimates of the baseline specification

Parameter	PSRF	Upper CI	Parameter	PSRF	Upper CI	Parameter	PSRF	Upper CI
ϕ_π	1.05	1.13	$\rho_{A_{16}}$	1.00	1.01	σ_{A_8}	1.00	1.01
ϕ_c	1.02	1.06	$\rho_{A_{17}}$	1.00	1.01	σ_{A_9}	1.00	1.01
α_1	1.00	1.01	$\rho_{A_{18}}$	1.01	1.01	$\sigma_{A_{10}}$	1.00	1.00
α_2	1.00	1.00	$\rho_{A_{19}}$	1.00	1.00	$\sigma_{A_{11}}$	1.06	1.14
α_3	1.00	1.01	$\rho_{A_{20}}$	1.00	1.00	$\sigma_{A_{12}}$	1.01	1.02
α_4	1.01	1.03	$\rho_{A_{21}}$	1.00	1.01	$\sigma_{A_{13}}$	1.00	1.01
α_5	1.03	1.06	$\rho_{A_{22}}$	1.00	1.01	$\sigma_{A_{14}}$	1.00	1.01
α_6	1.00	1.00	$\rho_{A_{23}}$	1.00	1.01	$\sigma_{A_{15}}$	1.00	1.00
α_7	1.00	1.00	$\rho_{A_{24}}$	1.00	1.01	$\sigma_{A_{16}}$	1.02	1.05
α_8	1.00	1.00	$\rho_{A_{25}}$	1.00	1.00	$\sigma_{A_{17}}$	1.00	1.00

Table 23: Potential scale reduction factor estimates of the baseline specification (continued)

Parameter	PSRF	Upper CI	Parameter	PSRF	Upper CI	Parameter	PSRF	Upper CI
α_9	1.00	1.00	$\rho_{A_{26}}$	1.01	1.02	$\sigma_{A_{18}}$	1.01	1.01
α_{10}	1.00	1.00	$\rho_{A_{27}}$	1.00	1.01	$\sigma_{A_{19}}$	1.00	1.01
α_{11}	1.01	1.03	ρ_{D_1}	1.00	1.00	$\sigma_{A_{20}}$	1.00	1.00
α_{12}	1.00	1.01	ρ_{D_2}	1.00	1.00	$\sigma_{A_{21}}$	1.00	1.00
α_{13}	1.00	1.01	ρ_{D_3}	1.00	1.00	$\sigma_{A_{22}}$	1.01	1.03
α_{14}	1.00	1.00	ρ_{D_4}	1.00	1.00	$\sigma_{A_{23}}$	1.00	1.00
α_{15}	1.00	1.00	ρ_{D_5}	1.00	1.00	$\sigma_{A_{24}}$	1.00	1.00
α_{16}	1.01	1.04	ρ_{D_6}	1.00	1.00	$\sigma_{A_{25}}$	1.00	1.00
α_{17}	1.00	1.01	ρ_{D_7}	1.00	1.00	$\sigma_{A_{26}}$	1.01	1.02
α_{18}	1.00	1.00	ρ_{D_8}	1.00	1.00	$\sigma_{A_{27}}$	1.01	1.02
α_{19}	1.00	1.00	ρ_{D_9}	1.00	1.00	σ_{D_1}	1.00	1.00
α_{20}	1.00	1.00	$\rho_{D_{10}}$	1.00	1.00	σ_{D_2}	1.00	1.00
α_{21}	1.00	1.00	$\rho_{D_{12}}$	1.00	1.01	σ_{D_3}	1.00	1.00
α_{22}	1.01	1.03	$\rho_{D_{13}}$	1.00	1.00	σ_{D_4}	1.00	1.00
α_{23}	1.00	1.00	$\rho_{D_{14}}$	1.00	1.00	σ_{D_5}	1.00	1.01
α_{24}	1.00	1.00	$\rho_{D_{15}}$	1.00	1.00	σ_{D_6}	1.00	1.00
α_{25}	1.00	1.00	$\rho_{D_{16}}$	1.00	1.00	σ_{D_7}	1.00	1.00
α_{26}	1.00	1.01	$\rho_{D_{17}}$	1.00	1.00	σ_{D_8}	1.00	1.00
α_{27}	1.00	1.01	$\rho_{D_{18}}$	1.00	1.00	σ_{D_9}	1.00	1.00
ρ_i	1.01	1.04	$\rho_{D_{19}}$	1.00	1.00	$\sigma_{D_{10}}$	1.00	1.00
ρ_μ	1.12	1.31	$\rho_{D_{20}}$	1.00	1.00	$\sigma_{D_{12}}$	1.00	1.01
ρ_γ	1.01	1.01	$\rho_{D_{21}}$	1.00	1.00	$\sigma_{D_{13}}$	1.00	1.00
ρ_a	1.00	1.00	$\rho_{D_{22}}$	1.00	1.01	$\sigma_{D_{14}}$	1.00	1.00
ρ_{A_1}	1.00	1.00	$\rho_{D_{23}}$	1.00	1.00	$\sigma_{D_{15}}$	1.00	1.00
ρ_{A_2}	1.00	1.00	$\rho_{D_{24}}$	1.00	1.00	$\sigma_{D_{16}}$	1.00	1.00
ρ_{A_3}	1.00	1.00	$\rho_{D_{25}}$	1.00	1.00	$\sigma_{D_{17}}$	1.00	1.00
ρ_{A_4}	1.01	1.01	$\rho_{D_{26}}$	1.00	1.00	$\sigma_{D_{18}}$	1.00	1.00
ρ_{A_5}	1.06	1.14	$\rho_{D_{27}}$	1.00	1.00	$\sigma_{D_{19}}$	1.00	1.00
ρ_{A_6}	1.00	1.00	σ_μ	1.10	1.24	$\sigma_{D_{20}}$	1.00	1.01
ρ_{A_7}	1.00	1.00	σ_γ	1.03	1.05	$\sigma_{D_{21}}$	1.00	1.00
ρ_{A_8}	1.00	1.00	σ_a	1.01	1.03	$\sigma_{D_{22}}$	1.00	1.00
ρ_{A_9}	1.00	1.00	σ_{A_1}	1.00	1.01	$\sigma_{D_{23}}$	1.00	1.00
$\rho_{A_{10}}$	1.00	1.01	σ_{A_2}	1.01	1.01	$\sigma_{D_{24}}$	1.00	1.00
$\rho_{A_{11}}$	1.07	1.17	σ_{A_3}	1.00	1.01	$\sigma_{D_{25}}$	1.00	1.00
$\rho_{A_{12}}$	1.04	1.06	σ_{A_4}	1.00	1.01	$\sigma_{D_{26}}$	1.00	1.00
$\rho_{A_{13}}$	1.00	1.00	σ_{A_5}	1.14	1.29	$\sigma_{D_{27}}$	1.00	1.01
$\rho_{A_{14}}$	1.00	1.00	σ_{A_6}	1.00	1.01			
$\rho_{A_{15}}$	1.00	1.00	σ_{A_7}	1.00	1.00			

Notes: PSRF columns report a point estimate of the potential scale reduction factor and Upper CI columns report the upper bound of its 95% confidence interval.

Then we compute the effective sample size of the posterior draws, which is the sample size adjusted for autocorrelations in the MCMC draws. The estimates of the effective sample sizes in the baseline specification are reported in Table 24. Except for ρ_μ, $\rho_{A_{11}}$, σ_μ, and $\sigma_{A_{11}}$, all the parameters have more than 1,000 effective sample sizes and many of them actually have more than 3,000 effective sample sizes, which is sufficiently large.

Table 24: Estimated effective sample sizes of the posterior draws in the baseline specification

Parameter	Effective size	Parameter	Effective size	Parameter	Effective size
ϕ_π	2854.8	$\rho_{A_{16}}$	5459.9	σ_{A_8}	5374.5
ϕ_c	2860.9	$\rho_{A_{17}}$	5215.9	σ_{A_9}	5038.7
α_1	4867.5	$\rho_{A_{18}}$	3075.5	$\sigma_{A_{10}}$	5854.6
α_2	2983.7	$\rho_{A_{19}}$	3794.5	$\sigma_{A_{11}}$	284.5
α_3	5476.7	$\rho_{A_{20}}$	7020.2	$\sigma_{A_{12}}$	3602.9
α_4	4722.1	$\rho_{A_{21}}$	3079.9	$\sigma_{A_{13}}$	3574.2
α_5	3027.4	$\rho_{A_{22}}$	4719.3	$\sigma_{A_{14}}$	5506.2
α_6	4901.9	$\rho_{A_{23}}$	4720.1	$\sigma_{A_{15}}$	5017.6
α_7	4805.7	$\rho_{A_{24}}$	5376.8	$\sigma_{A_{16}}$	3746.2
α_8	5454.4	$\rho_{A_{25}}$	4498.3	$\sigma_{A_{17}}$	5045.1
α_9	5601.0	$\rho_{A_{26}}$	4346.2	$\sigma_{A_{18}}$	2684.1
α_{10}	5361.5	$\rho_{A_{27}}$	4526.3	$\sigma_{A_{19}}$	3189.2
α_{11}	2422.4	ρ_{D_1}	5822.5	$\sigma_{A_{20}}$	5277.7
α_{12}	4501.6	ρ_{D_2}	3867.1	$\sigma_{A_{21}}$	2834.2
α_{13}	4219.3	ρ_{D_3}	5697.1	$\sigma_{A_{22}}$	1435.6
α_{14}	5422.9	ρ_{D_4}	5477.2	$\sigma_{A_{23}}$	5161.6
α_{15}	5201.1	ρ_{D_5}	5130.8	$\sigma_{A_{24}}$	5147.9
α_{16}	4564.0	ρ_{D_6}	5499.3	$\sigma_{A_{25}}$	4369.2
α_{17}	5239.2	ρ_{D_7}	5007.8	$\sigma_{A_{26}}$	1887.0
α_{18}	3617.5	ρ_{D_8}	5495.2	$\sigma_{A_{27}}$	2763.3
α_{19}	4153.3	ρ_{D_9}	5358.9	σ_{D_1}	5641.5
α_{20}	5559.2	$\rho_{D_{10}}$	5860.6	σ_{D_2}	4262.3
α_{21}	4207.9	$\rho_{D_{12}}$	5321.4	σ_{D_3}	5713.8
α_{22}	2635.6	$\rho_{D_{13}}$	5631.9	σ_{D_4}	6872.4
α_{23}	4998.7	$\rho_{D_{14}}$	5069.8	σ_{D_5}	5467.1
α_{24}	5890.3	$\rho_{D_{15}}$	4967.4	σ_{D_6}	5393.3
α_{25}	4662.6	$\rho_{D_{16}}$	6034.7	σ_{D_7}	5779.0
α_{26}	2879.4	$\rho_{D_{17}}$	5734.2	σ_{D_8}	5916.8
α_{27}	3472.8	$\rho_{D_{18}}$	5117.5	σ_{D_9}	5262.1
ρ_i	3702.0	$\rho_{D_{19}}$	4417.8	$\sigma_{D_{10}}$	5445.7
ρ_μ	194.3	$\rho_{D_{20}}$	5143.0	$\sigma_{D_{12}}$	5468.5
ρ_γ	2130.8	$\rho_{D_{21}}$	3927.0	$\sigma_{D_{13}}$	5654.2
ρ_a	6155.8	$\rho_{D_{22}}$	5026.0	$\sigma_{D_{14}}$	5433.3
ρ_{A_1}	4516.6	$\rho_{D_{23}}$	5679.8	$\sigma_{D_{15}}$	6477.1
ρ_{A_2}	4352.5	$\rho_{D_{24}}$	5581.5	$\sigma_{D_{16}}$	5239.2
ρ_{A_3}	5205.5	$\rho_{D_{25}}$	5535.1	$\sigma_{D_{17}}$	5742.9

Table 24: Estimated effective sample sizes of the posterior draws in the baseline specification (continued)

Parameter	Effective size	Parameter	Effective size	Parameter	Effective size
ρ_{A_4}	5198.9	$\rho_{D_{26}}$	4963.9	$\sigma_{D_{18}}$	5686.8
ρ_{A_5}	2707.6	$\rho_{D_{27}}$	5635.7	$\sigma_{D_{19}}$	5246.9
ρ_{A_6}	4847.8	σ_{μ}	96.4	$\sigma_{D_{20}}$	5573.0
ρ_{A_7}	5287.9	σ_{γ}	1305.4	$\sigma_{D_{21}}$	5450.1
ρ_{A_8}	5554.9	σ_{a}	3291.6	$\sigma_{D_{22}}$	5401.8
ρ_{A_9}	4762.4	σ_{A_1}	6092.1	$\sigma_{D_{23}}$	5333.5
$\rho_{A_{10}}$	5287.1	σ_{A_2}	3022.7	$\sigma_{D_{24}}$	6106.3
$\rho_{A_{11}}$	203.7	σ_{A_3}	4532.0	$\sigma_{D_{25}}$	5064.8
$\rho_{A_{12}}$	2238.6	σ_{A_4}	4420.9	$\sigma_{D_{26}}$	6087.0
$\rho_{A_{13}}$	3753.3	σ_{A_5}	1731.1	$\sigma_{D_{27}}$	5527.0
$\rho_{A_{14}}$	5577.1	σ_{A_6}	3672.3		
$\rho_{A_{15}}$	5841.8	σ_{A_7}	4054.0		

In addition to monitoring convergence using the PSRF and the effective sample size of the posterior draws, we also visually inspect plots of the cumulative means of the Markov-chain draws for each parameter, which are not reported for conciseness. Upon convergence, the draws of all 5 chains are combined to form a posterior sample of 1 million draws per each estimation.

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