Online Appendix:
An index of competitiveness and cooperativeness for normal-form games

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I Relationship between coco value and CCI

We now illustrate the differences between the CCI and the Coco value (Kalai and Kalai, 2013) in terms of computation and predictions. Consider the following three games, of which the first is the hot dog vendor game in Kalai and Kalai (2013):

Game I

	A	B
A	$20, 40$	$40, 200$
B	$100, 80$	$50, 100$

Game II

	A	B
A	$10, 60$	$95, 145$
B	$40, 90$	$15, 65.$

Game III

	A	B
A	$350, -150$	$-180, 420$
B	$-220, 380$	$350, -150.$

In all three games, the cooperative component as defined in Kalai and Kalai (2013) has a max-max value of (120,120) in the strategy profile (A,B). The zero-sum component has a min-max value of (-25,25) in all three games. Thus, the coco-values are (95,145) for all

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three games. Intuitively, both players coordinate on the solution with the maximal joint surplus (A,B) and obtain their respective coco-values after side payments.

To determine the CCI, we first need to mean-normalize the games:

M-N Game I

	A	B
A	$-32.5, -65$	$-12.5, 95$
B	$47.5, -25$	$-2.5, -5$

M-N Game II

	A	B
A	$-30, -30$	$55, 55$
B	$0, 0$	$-25, -25.$

M-N Game III

	A	B
A	$275, -275$	$-255, 295$
B	$-295, 255$	$275, -275.$

For Game I we obtain:

$$CCI = \frac{32.5^2 + 107.5^2 + 72.5^2 + 2.5^2}{2(32.5^2 + 12.5^2 + 47.5^2 + 2.5^2 + 65^2 + 95^2 + 25^2 + 5^2)} \approx 0.514$$

A similar computation gives us $CCI = 0$ for Game II and $CCI \approx 0.997$ for Game III.

The coco-value takes the opportunity to agree with side payments as given. Thus, the coco-value predictions are uniform across the three games. On the other hand, the CCI considers a non-cooperative game and tries to analyze how difficult it is to reach a cooperative outcome without side payments. The hot dog vendor game yields an intermediate CCI. In Game II it is easy to coordinate on (A,B) in line with a low CCI. In contrast, Game III is close to a zero-sum game which makes cooperation difficult.

II Derivation for general continuous games and proofs of Lemma 1 and Proposition 1

Consider a finite set of N players and for every player a set of strategies S_i. Unlike in the main text, we now also allow the sets S_i to be infinite. Let $S = \prod_{i \leq N} S_i$ be the set of strategy profiles. Let $\mathcal{S}$ be a σ-algebra on S such that $(S, \mathcal{S})$ is a measurable space. In addition, let Q be a measure on $(S, \mathcal{S})$.

Let $\mathcal{G}$ be the set of games $g = \{\pi_i : i \leq N\}$ such that for all $i \leq N$, $\pi_i \in L_2(Q)$.[1] For

[1]Here $L_2(Q)$ is the set of Q-measurable functions $\pi_i : S \to \mathbb{R}$ such that $\int_S (\pi_i)^2 dQ < \infty$.

$g = \{\pi_i : i \leq N\}, h = \{\tilde{\pi} : i \leq N\} \in \mathcal{G}$, we define the following inner product $\langle ., . \rangle$ on $\mathcal{G}$:

$$\langle g, h \rangle = \sum_{i=1}^{N} \int_S \pi_i(s)\tilde{\pi}_i(s)\, Q(ds) \equiv \sum_{i=1}^{N} \int_S \pi_i \tilde{\pi}_i dQ.$$

By varying the measure Q we can give more or less weight to certain strategies. For the applications in the main part of the paper, we used the Lebesgue measure for Q.

Observe that the norm $\|g\| = \sqrt{\langle g, g \rangle}$ treats two games in the same way if their payoff functions are equal Q-a.s.. In other words, in order to be technically correct, we should define the set of games $\mathcal{G}$ only on the partition induced by the measure Q. For ease of notation, however, we omit this from the notation, but keep in mind that all statements in the Appendix only hold Q-a.s..

Compared to the main text, the sets $\mathcal{K}, \mathcal{M}, \mathcal{Z}$ and $\mathcal{C}$ differ marginally,

$$\mathcal{K} = \{g = \{\pi_i : i \leq N\} \in \mathcal{G} : \forall i \leq N, \pi_i \text{ is constant }\},$$
$$\mathcal{M} = \left\{g = \{\pi_i : i \leq N\} \in \mathcal{G} : \forall i \leq N, \int_S \pi_i dQ = 0\right\},$$
$$\mathcal{Z} = \left\{g = \{\pi_i : i \leq N\} \in \mathcal{G} : \forall s \in S, \sum_{i \leq N} \pi_i = 0\right\},$$
$$\mathcal{C} = \{g = \{\pi_i : i \leq N\} \in \mathcal{G} : \forall i, j \leq N, \pi_i = \pi_j\}.$$

The proof the following lemma includes the proof Lemma **??** in the main text.

Lemma 1.

1. *$\mathcal{M}^\perp = \mathcal{K}$ and $\mathcal{Z}^\perp = \mathcal{C}$.*

2. *Every game g can be uniquely decomposed as*

$$g = \hat{g} + \tilde{g},$$

where $\hat{g} \in \mathcal{M}$ and $\tilde{g} \in \mathcal{K}$. In particular $\hat{g} = Mg$ and $\tilde{g} = Kg$. Moreover, in this case,

$$\|g\|^2 = \|\hat{g}\|^2 + \|\tilde{g}\|^2.$$

3. *Every game g can be uniquely decomposed as,*

$$g = \hat{g} + \tilde{g},$$

where $\hat{g} \in \mathcal{Z}$ and $\tilde{g} \in \mathcal{C}$. In particular $\hat{g} = Zg$ and $\tilde{g} = Cg$. Moreover, in this case,

$$\|g\|^2 = \|\hat{g}\|^2 + \|\tilde{g}\|^2.$$

4. *For $g = \{\pi_i : i \leq N\}$ we have that*
   - *$Kg = \{\tilde{\pi}_i : i \leq N\}$ is such that, $\tilde{\pi}_i = \frac{\int_S \pi_i dQ}{\int_S dQ}$.*
   - *$Cg = \{\tilde{\pi}_i : i \leq N\}$ is such that, $\tilde{\pi}_i = \frac{\sum_{j \leq N} \pi_j}{N}$.*

5. *The operators $(M, Z), (M, C), (K, Z)$ and (K, C) are commutative. In other words, for all $g \in \mathcal{G}$, $MZg = ZMg$, $KZg = ZKg$, $MCg = CMg$ and $KCg = CKg$.*

Proof. **Part 1:** We will start by showing the equivalence $\mathcal{K}^\perp = \mathcal{M}$. Let $\tilde{g} = \{\tilde{\pi} : i \leq N\} \in \mathcal{K}^\perp$. Then, for all $g = \{\pi_i : i \leq N\} \in \mathcal{K}$,

$$\langle \tilde{g}, g \rangle = 0 = \sum_{i \leq N} \int_S \pi_i \tilde{\pi}_i dQ.$$

Let g be such that for $i \leq N$, $\pi_i = 1 \in \mathbb{R}$ and for $j \neq i$, $\pi_j = 0$. Then,

$$0 = \int_S \pi_i dQ,$$

which shows that $\tilde{g} \in \mathcal{M}$.

Also, if $\tilde{g} = \{\tilde{\pi}_i : i \leq N\} \in \mathcal{M}$ and $g = \{\pi_i : i \leq N\} \in \mathcal{K}$ with $\pi_i = c_i \in \mathbb{R}$, then,

$$\langle \tilde{g}, g \rangle = \sum_{i \leq N} \int_S \pi \tilde{\pi} dQ = \sum_{i \leq N} c_i \int_S \tilde{\pi}_i dQ = 0.$$

Next, we will show that $\mathcal{C} = \mathcal{Z}^\perp$. Let $\tilde{g} = \{\tilde{\pi} : i \leq N\} \in \mathcal{Z}^\perp$ and $g = \{\pi : i \leq N\} \in \mathcal{Z}$. Then,

$$\langle g, \tilde{g} \rangle = \sum_{i \leq N} \int_S \pi \tilde{\pi} dQ.$$

Let B be a measurable set and let g be such that for $k \neq i, j$, $\pi_k(s) = 0$ for all $s \in S$, let for $s \in B$, $\pi_i(s) = -\pi_j(s)$ and let for $s' \notin B$, $\pi_i(s') = \pi_j(s') = 0$. Then,

$$0 = \langle g, \tilde{g} \rangle = \sum_{i \leq N} \int_S \pi_i \tilde{\pi}_i dQ,$$
$$\leftrightarrow \int_B \tilde{\pi}_i(s) dQ = \int_B \tilde{\pi}_j(a) dQ.$$

Given that B was arbitrary, it follows that $\pi_i = \pi_j$.

Parts 2 and 3: Observe that $g = Mg + (g - Mg)$. By the theory on linear operators, $Kg = g - Mg \in \mathcal{M}^\perp = \mathcal{K}$. To show uniqueness, let $\hat{g}, \hat{g}' \in \mathcal{K}$ and $\tilde{g}, \tilde{g}' \in \mathcal{M}$ and assume that $g = \hat{g} + \tilde{g} = \hat{g}' + \tilde{g}'$. Then,

$$g - g = \hat{g} - \tilde{g} + \hat{g}' - \tilde{g}'.$$

As $\mathcal{K}$ and $\mathcal{M}$ are vector spaces, $\hat{g} - \hat{g}' \in \mathcal{M}$ and $\tilde{g} - \tilde{g}' \in \mathcal{K}$, so,

$$\|g - g\|^2 = 0 = \|\hat{g} - \hat{g}'\|^2 + \|\tilde{g} - \tilde{g}'\|^2,$$

which shows that $\hat{g} = \hat{g}'$ and $\tilde{g} = \tilde{g}'$. The proof of Part 3, i.e., the decomposition of g into Zg and Cg, is similar and thus omitted.

Part 4: Let $g = \{\pi_i : i \leq N\}$, $Kg = \{\tilde{\pi}_i : i \leq N\}$ and $Mg = \{\hat{\pi}_i : i \leq N\}$. Then, by Part 2,

$$\pi_i = \tilde{\pi}_i + \hat{\pi}_i.$$

Integrating over S gives

$$\int_S \pi_i dQ = \int_S \tilde{\pi}_i dQ + \int_S \hat{\pi}_i dQ = \int_S \tilde{\pi}_i dQ = c_i \int_S dQ,$$

where $\tilde{\pi}_i = c_i \in \mathbb{R}$. So,

$$c_i = \tilde{\pi}_i = \frac{\int_S \pi_i dQ}{\int_S dQ}.$$

Next, let $Zg = \{\hat{\pi}_i : i \leq N\}$ and $Cg = \{\tilde{\pi}_i : i \leq N\}$. Then again,

$$\pi_i = \hat{\pi}_i + \tilde{\pi}_i.$$

Summing over all $i \leq N$ gives

$$\sum_{j \leq N} \pi_j = \sum_{j \leq N} \hat{\pi}_j + \sum_{j \leq N} \tilde{\pi}_j = \sum_{j \leq N} \tilde{\pi}_j.$$

Given that $Cg \in \mathcal{C}$, i.e. $\tilde{\pi}_i = \tilde{\pi}_j$, we have,

$$\tilde{\pi}_i = \frac{\sum_{j \leq N} \pi_j}{N}.$$

Part 5: Consider $g = \{\pi_i : i \leq N\}$ then $Kg = \{\tilde{\pi}_i : i \leq N\}$ is such that

$$\tilde{\pi}_i = \frac{\int_S \pi_i dQ}{\int_S dQ},$$

and therefore $CKg = \{\hat{\pi}_i : i \leq N\}$ is such that

$$\hat{\pi}_i = \frac{\sum_{j \leq N} \tilde{\pi}_j}{N} = \frac{\sum_{j \leq N} \int_S \pi_j dQ}{N \int_S dQ}.$$

Similarly, $KCg = \{\pi_i^* : i \leq N\}$ is such that

$$\pi_i^* = \frac{\int_S \sum_{j \leq N} \pi_j dQ}{N \int_S dQ}.$$

By exchanging summation and integration, we obtain $CKg = KCg$.

Next, $Cg = C(Kg + Mg) = CKg + CMg = KCg + CMg$. This implies that

$$CMg = Cg - KCg = MCg,$$

which shows that C and M are also commutative. Next, $Mg = ZMg + CMg = ZMg + MCg$, so

$$ZMg = Mg - MCg = M(g - Cg) = MZg,$$

which shows that Z and M also commute. Finally, $Zg = MZg + KZg = ZMg + KZg$ so,

$$KZg = Zg - ZMg = Z(g - Mg) = ZKg,$$

which shows that Z and K commute.

$\square$

III Additional Details for the Examples

Prisoner's Dilemma

Correlation coefficients among variables of interest (data from Mengel, 2017)

	CCI	TEMPT	RISK	EFF
CCI	1			
TEMPT	-0.0109	1		
	(0.1031)			
RISK	0.6413	-0.0029	1	
	(0.0791)	(0.1031)		
EFF	-0.3215	-0.0638	-0.2046	1
	(0.0977)	(0.1092)	(0.1010)	

Standard errors in parentheses

Contests

Derivations for the Tullock contest: When $\alpha = 3$, we obtain:

$$\pi_i = \int_0^v \int_0^v \frac{1}{v}\frac{1}{v}\left(\left(\frac{x_i^3}{x_i^3 + x_j^3}\right) v - x_i\right) \mathrm{d}x_j \mathrm{d}x_i = 0.$$

Thus, the game is already mean-normalized, i.e., $\bar{\pi}_i = 0$. The calculation of our index yields:

$$\mathrm{CCI}(g) = \frac{||\pi_1(x_1, x_2) - \pi_2(x_1, x_2)||^2}{2(||\pi_1(x_1, x_2)||^2 + ||\pi_2(x_1, x_2)||^2)} \approx \frac{0.15354}{0.32027} \approx 0.48.$$

Derivations for the contest of Durham, Hirshleifer, and Smith: We first need to mean-normalize the payoffs of the game by Durham, Hirshleifer, and Smith (1998). The average payoff is $\frac{v}{8}$. Thus, the mean-normalized payoff functions are:

$$\pi_1 - \overline{\pi}_1 = \frac{x_1^r}{x_1^r + x_2^r}(v - x_1)(v - x_2) - \frac{v}{8} \text{ and } \pi_2 - \bar{\pi}_2 = \frac{x_2^r}{x_1^r + x_2^r}(v - x_1)(v - x_2) - \frac{v}{8}$$

We will henceforth set $v = 20$ as in the experiment. For $r = 1$, we obtain:

$$\begin{aligned} \|\pi_1 - \bar{\pi}_1 - (\pi_2 - \bar{\pi}_2)\|^2 &= \int_0^{20} \int_0^{20} \left(\frac{x_1 - x_2}{x_1 + x_2}(20 - x_1)(20 - x_2)\right)^2 \mathrm{d}x_2 \mathrm{d}x_1 \\ &\approx 2.07342 * 10^6, \end{aligned}$$

$$\|\pi_1 - \bar{\pi}_1\|^2 = \|\pi_2 - \bar{\pi}_2\|^2 = \int_0^{20}\int_0^{20} \left(\frac{x_1}{x_1+x_2}(20-x_1)(20-x_2) - \frac{20}{8}\right)^2 \mathrm{d}x_2\mathrm{d}x_1$$
$$\approx 2.198863 * 10^6.$$

Thus, we get $\mathrm{CCI}_{1,20}(g) \approx 0.236$ for the game with $r = 1$ and $v = 20$.

For $r = 4$, we obtain:

$$\|\pi_1 - \bar{\pi}_1 - (\pi_2 - \bar{\pi}_2)\|^2 = \int_0^{20}\int_0^{20} \left(\frac{x_1^4 - x_2^4}{x_1^4 + x_2^4}(20-x_1)(20-x_2)\right)^2 \mathrm{d}x_2\mathrm{d}x_1,$$
$$\approx 5.13881 * 10^6,$$

$$\|\pi_1 - \bar{\pi}_1\|^2 = \|\pi_2 - \bar{\pi}_2\|^2 = \int_0^{20}\int_0^{20} \left(\frac{x_1^4}{x_1^4+y_1^4}(20-x_1)(20-x_2) - \frac{20}{8}\right)^2 \mathrm{d}x_2\mathrm{d}x_1,$$
$$\approx 2.96298 * 10^6.$$

Thus, we get $\mathrm{CCI}_{4,20}(g) \approx 0.433$ for the game with $r = 4$ and $v = 20$.

Consistent with our prediction, contestants spend more resources on fighting in the experiment when $r = 4$. This yields a more competitive outcome with lower payoffs.

Public Goods game We now provide the tedious calculation of our index that we left out in the main text. In general for an N-player game, the index is given by:

$$\mathrm{CCI}(g) = \frac{\sum_{i=1}^N \left\|\pi_i - \overline{\pi}_i - \frac{1}{N}\left(\sum_{j=1}^N (\pi_j - \overline{\pi}_j)\right)\right\|^2}{\sum_{i=1}^N \|\pi_i - \overline{\pi}_i\|^2}.$$

where $\overline{\pi}_i$ is the mean payoff for player i. From the main text, recall that:

$$\pi_i - \overline{\pi}_i = x_i - \alpha \sum_{j=1}^N x_j + \frac{\alpha N - 1}{2}.$$

Then:

$$
\begin{aligned}
&\pi_i - \overline{\pi}_i - \frac{1}{N}\sum_{j=1}^{N}(\pi_j - \overline{\pi}_j), \\
&= \frac{N-1}{N}\left[(\pi_i - \overline{\pi}_i) - \sum_{j\neq i}^{N}\frac{\pi_j - \overline{\pi}_j}{N-1}\right], \\
&= \frac{N-1}{N}\left[x_i - \alpha\sum_{j=1}^{N}x_j + \frac{\alpha N-1}{2} - \sum_{j\neq i}^{N}\frac{x_j - \alpha\sum_{k=1}^{N}x_k + \frac{\alpha N-1}{2}}{N-1}\right], \\
&= \frac{N-1}{N}\left[x_i - \sum_{j\neq i}^{N}\frac{x_j}{N-1}\right].
\end{aligned}
$$

As such, for the numerator of our index, we obtain:

$$
\begin{aligned}
&\left\|\pi_i - \overline{\pi}_i - \frac{1}{N}\sum_{j=1}^{N}(\pi_j - \overline{\pi}_j)\right\|^2, \\
&= \frac{(N-1)^2}{N^2}\int_0^1 \cdots \int_0^1 \left(x_i^2 - 2x_i\sum_{j\neq i}\frac{x_j}{N-1} + \sum_{j\neq i}\frac{x_j^2}{(N-1)^2} + \sum_{j\neq i, k\neq i, j\neq k}\frac{x_j x_k}{(N-1)^2}\right) dx_N \ldots, dx_1, \\
&= \frac{(N-1)^2}{N^2}\left[\frac{1}{3} - 2\frac{1}{2}\frac{(N-1)}{(N-1)}\frac{1}{2} + (N-1)\frac{1}{3(N-1)^2} + \frac{(N-1)(N-2)}{4(N-1)^2}\right], \\
&= \frac{N-1}{12N}.
\end{aligned}
$$

This shows that the numerator is independent of α.

For the norms in the denominator, we obtain:

$$
\begin{aligned}
&\left\|\pi_i-\overline{\pi}_i\right\|^2, \\
&=\int_0^1 \ldots \int_0^1\left(x_i-\alpha \sum_{j=1}^N x_j+\frac{\alpha N-1}{2}\right)^2 \mathrm{d} x_N \ldots \mathrm{d} x_1, \\
&=\int_0^1 \ldots \int_0^1\left(x_i^2-2 \alpha x_i^2-2 \alpha x_i \sum_{j \neq i} x_j+2 x_i \frac{\alpha N-1}{2}\right) \mathrm{d} x_N \ldots \mathrm{d} x_1 \\
&+\int_0^1 \ldots \int_0^1\left(\alpha^2 \sum_{j=1}^N x_j^2+\alpha^2 \sum_{j \neq k} x_j x_k-2 \alpha \sum_{j=1}^N x_j \frac{\alpha N-1}{2}\right) \mathrm{d} x_N \ldots \mathrm{d} x_1 \\
&+\int_0^1 \ldots \int_0^1\left(\frac{\alpha N-1}{2}\right)^2 \mathrm{d} x_1 \ldots \mathrm{d} x_N, \\
&=\frac{1}{3}-\frac{2 \alpha}{3}-\frac{\alpha(N-1)}{2}+\frac{\alpha N-1}{2} \\
&+\frac{N \alpha^2}{3}+\frac{\alpha^2(N)(N-1)}{4}-\alpha N \frac{\alpha N-1}{2}+\left(\frac{\alpha N-1}{2}\right)^2, \\
&=\frac{1}{12}\left(1-2 \alpha+N \alpha^2\right).
\end{aligned}
$$

This is increasing in α for $\alpha > \frac{1}{N}$. Plugging in the two previous results, our index is given by:

$$
\begin{aligned}
\mathrm{CCI}(g) & =\frac{(N-1)^2}{N^2} \frac{N \frac{N}{12(N-1)}}{N \frac{1}{12}\left(1-2 \alpha+N \alpha^2\right)}, \\
& =\frac{(N-1)}{(\alpha N-1)^2+(N-1)},
\end{aligned}
$$

which is the result that we use in the main text.

References

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